An Investigation of Application Level Performance in ATM Networks

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ABSTRACT

The goal of this paper is to investigate the relation that exists between the performance guarantees provided by a network, and what users of the network ultimately get. There are many different performance measures that may be of interest, but our focus is on the relation that exists between packet losses in the network and message losses experienced by the application. This investigation is carried out in the context of an ATM network where network packets are of fixed size (cells), and we concentrate mostly on the impact of one key parameter, the application peak transmission rate. As we shall see, varying this parameter for a given source while keeping network cell losses constant can translate into significant variations in the loss probability of application messages. In the paper, we develop and solve an analytical model that permits us to identify and evaluate this behavior. Based on the results available from this model, we then develop a set of *guidelines* that applications can use to determine which performance guarantees to require from the network, in order to ensure the appropriate level of service. In general, the ability to express performance in terms that are meaningful to applications is likely to play a major role in the acceptance and use of high speed networks. This paper attempts to take one step in this direction, and hopes to motivate further work in this important area.

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1 Introduction

ATM [1] has been identified as the approach of choice for next generation networks, and one of its main benefits is the flexibility it affords. In particular, it can support connections with very different characteristics, and is capable of providing a wide range of performances. However, these capabilities are currently specified in terms of *network performance,* i.e., cell loss, cell delay, cell jitter, etc., and it is unclear how they can/should be mapped into application level performance measures which are the only ones of interest to users (see [2, 3, 4, 5, 6, 7] for examples). A goal of this paper is, therefore, to investigate the kind of relations that exist between network and application level performance measures. We believe such an understanding is key to developing the guidelines that applications need in order to determine what to request from the network.

A complete investigation of this issue is obviously a topic which is beyond the scope of a single paper and we, therefore, limit our focus to one key aspect. Specifically, we are interested in the impact of source peak rate on the message/frame loss experienced by applications for a given level of network cell loss. Peak rate is one of the key traffic parameters that connections need to specify [1], and it certainly plays an important role in determining the amount of resources that need to be allocated to a connection, e.g., [8]. However, while such an allocation typically enables the network to guarantee a certain cell loss probability, the peak rate can still influence the relation that exists between cell and message loss probabilities[1] of individual connections. This is because the peak rate negotiated by a connection determines the minimum time interval that separates consecutive cells [1] which often belong to the same message. The length of this interval determines in turn the correlation between the queue lengths seen by consecutive cells. If this interval is small (high peak rate), the correlation will be high which can translate in the bunching of lost cells. The impact of such a bunching on the *mlp* experienced by a connection can, however, be either positive or negative.

Bunching of lost cells for a given connections can negatively affect its *mlp*, if it results in a higher *clp* for that connection than for others. In particular, lost cells occur in periods of overload, and a high peak rate source that generates many "back-to-back" cells may be more likely to trigger such an overload period. Hence, many of the cells lost by the network may belong to this connection which will then see a higher loss probability than lower peak rate connections. Such uneven distribution of lost cells among connections is likely to occur and may be difficult to control, unless the network closely monitors the performance of individual connections and adjusts its behavior accordingly. While the monitoring of individual connections and the scheduling of network resources on a per-

[1]Denoted by *clp* and *mlp*, respectively, in the rest of this paper.

connection basis may become feasible (cost effective) as technology progresses, it is unlikely to be the rule initially.

Bunching of lost cells can, however, be also beneficial to a connection if it does not translate into a higher *clp* for that connection. As mentioned before, this is because the peak rate can influence the distribution of lost cells from the same connection across multiple messages, i.e., bunched vs spread. In general, assuming the *clp* remains constant, it is obviously better for the *mlp* to have the lost cells affect the minimum possible number of messages (ideally, all lost cells are in the same message which then yield $clp = mlp$), and a higher peak rate will contribute in that direction. Note that here we assume that a message is lost if any of its cells is lost. This is in line with the use of ATM adaptation layer AAL5 which does not allow for cell-level retransmissions. In addition, some recent work [9] seems to indicate that message-level retransmissions often provide better performance than cell-level retransmissions.

In this paper, we investigate why and when either one of the two above phenomena dominates, and attempt to use this understanding to help applications determine which *clp* they need to request from the network to ensure a desired *mlp*. In addition, this information can also be useful in assisting users determine how to select the parameters of their Generic Cell Rate Algorithm (GCRA), so that they achieve an adequate trade-off between connection characteristics and requested network performance. For example, assuming a network whose tariffs are heavily dependent on the peak rate, a connection may opt to lower its peak rate and request a lower *clp*, so as to get a similar *mlp* but a lower connection cost. Understanding such relations between network-level and application-level performances has been recognized as an important issue, with implications in the design of forward error correction schemes, timeout mechanisms, playback algorithms, etc. There has, therefore, been a number of previous works which have tackled some aspects of this problem, and we now briefly review some of them.

Message related measures, such as loss, delay and jitter were analyzed in [10, 11, 12] under a "dispersed" message generation model, while simulations were used in [13, 2, 3] and [5] to investigate message loss and jitter, respectively. The relation between *clp* and *mlp* was also studied in [7] and [6]. [7] focused on providing a simple model, independent of the actual queueing behavior, to relate *clp* and *mlp* in the context of the SMDS protocol. [6] investigated the impact of message length under the assumption of constant length messages and back-to-back cell transmissions, i.e., source peak rate is constant and equal to link speed. Finally, [4] provided a general, albeit complex, model to investigate *mlp* as a function of source peak rate. However, it did not consider the impact of changing peak rates on *clp*, and did not study its influence on queue correlation and distribution of lost cells among different sessions.

This paper differs from these previous works in that it provides an accurate and yet analytically tractable model to study the relation that exists between the *clp* and *mlp* of a connection as some key parameters are varied. In particular, the method developed in the paper permits an in depth study of the influence of the peak rate on the *clp* and *mlp* a connection experiences. This enables us to develop simple guidelines to help applications estimate the level of performance they can expect from the network as a function of their traffic characteristics and the network guarantees.

The paper is structured as follows. In Section 2, we introduce more precisely the traffic and queueing models we rely on. Section 3 describes the analytical techniques and steps used to compute both the *clp* and *mlp* for our model (some of the proofs are relegated to an appendix). A wide range of numerical investigations based on the method of Section 3 are then conducted in Section 4, where we study the impact of varying the peak rate, the message size, and the utilization of the data source associated with the tagged connection. The results obtained from these investigations are discussed and used to provide guidelines, that should help sources negotiate traffic parameters with the network. A short conclusion summarizes the findings of the paper and points to possible extensions and further works.

2 The Queueing Model and The Problem Formulation

The model analyzed in the paper consists of two superposed traffic sources transmitting over a common ATM link (network). The first corresponds to a tagged source whose *clp* and *mlp* we want to evaluate as a function of its peak rate. The other one represents the aggregate traffic from all other connections sharing the link and, therefore, affecting the performance experienced by the tagged connection. We study the interactions of these two sources at a network link and associated buffer, through the use of a discrete-time, infinite-buffer, single-server queueing model. In this model, time is slotted with each slot equal to the transmission time of one cell.

The ATM connection associated with the tagged source is assumed controlled by a Leaky Bucket (GCRA) which is characterized by three parameters, the Peak Cell Rate (PCR), the Sustained Cell Rate (SCR) and the Maximum Burst Length (MBL). Messages or Application Data Units (ADUs) generated by the source are segmented by the adaptation layer into a number of fixed size cells before transmission. Cell transmissions are regulated by the GCRA to ensure compliance with the "contract" negotiated with the network. In particular, consecutive cell transmissions are always separated by an amount of time at least equal to the inverse of the PCR. We will assume that this is the only impact of the GCRA on cell transmissions from the source, i.e., the source is sufficiently well-behaved that the GCRA is essentially transparent [14] to the source.

The data source for the tagged connection consists of an ON-OFF source with geometrically distributed ON and OFF periods. Note that the assumption of geometric message size does not fully agree with the maximum message size (MBL) imposed by the GCRA, but the resulting difference is minimal. We further assume that all cells generated during an ON period belong to the same message, i.e., each transmission burst corresponds to a single message. All cells from the same message are, therefore, transmitted back-to-back, but as mentioned above, separated by a time interval equal to the inverse of the connection's peak rate. By changing the peak rate of the source, the duration of its ON period, and its frequency, this model allows us to vary the time between consecutive cells, the average message size, and the load induced by this connection on the network link. This flexibility enables us to investigate the contribution of these different parameters to the relation that exists between *clp* and *mlp*.

Note that the above model can be used to represent what happens at either the access point to the network (UNI) or at a network node along the path of the source (virtual connection). In the second case, the assumption of fixed peak rate, i.e., fixed cell interarrival times for cells from the same message, is less accurate due to the cell delay jitter introduced by intermediate network nodes.

Coming back to the description of the data source for the tagged connection, we assume that transitions between ON (state 1) and OFF (state 0) states take place only at slot boundaries, and that the transition probability from state i, $i = 0, 1$, to state $\bar{i}$ is fixed in time and given by $P_{\bar{i}}$. The transition probabilities are independent of any other quantity, and we assume that $P_0 + P_1 \neq 1$, otherwise the source degenerates to the uncorrelated case. As mentioned earlier, an ON period corresponds to one message and the size (in cells) of each message is, therefore, geometrically distributed with mean $1/P_0$. While in the ON state the source generates one cell every N slots ($1/N$ corresponds to the PCR of the data source). The arrival of the first cell of a message corresponds to the start of a new ON period which ends with the transmission of the last cell of the message. The arrival process to the system from this ON-OFF source can then be described as a modulated Markov process with an homogenous discrete-time Markov chain as shown in Fig. 1. The ON-OFF source generates a single cell at the beginning of a slot, if and only if the modulating Markov chain is in state 1. We further assume that a cell can be served in the same slot it arrives.

The second source to our queueing system corresponds to the background traffic aggregating contributions from all other sources sharing the link. This traffic is assumed to be arbitrary and with i.i.d. cell arrivals from slot to slot. We denote by A_t the number of cells that arrive from the background source in time slot $[t, t+1)$, and let the probability distribution and the probability generating function (PGF) of the random variable A_t be $Pr(A_t = i) \triangleq a_i,\ i = 0, 1, ...,$ and $A(z)$, respectively.

The arrival process to the buffer preceding the link consists, therefore, of the superposition of the two above cell sources, and we are interested in obtaining the message loss probability of the tagged ON-OFF source which we derive in the next section. As mentioned before, a message is lost if any of its cells is lost. Our assumption is that a cell is lost if its delay exceeds some pre-specified threshold $T+1$, i.e., there are already T or more cells in the buffer upon its arrival. This obviously represents an approximation of the finite buffer system, but it is reasonably accurate for low *clp* values.

3 The Analysis

In order to derive the *mlp* of the tagged ON-OFF source, we proceed in two steps. In the first step, we obtain the probability distribution of the delay seen by the *first* cell of a message. This first step relies mainly on the analytic properties of the PGFs of the number of cells in the system when the modulating Markov chain is in some state. Only the key steps of this derivation are given, and details of the proofs are relegated to appendices or references when available. This first step provides us with the *clp* of the first cell in a message. In our second step, we extend this result using a recursive procedure to the other cells in a message, which enables us to finally obtain the message loss probability.

3.1 The Delay of The First Cell of A Message

Define the state of the system at time slot $t,\ t = 0, 1, ...$ to be the couple (q_t, s_t), where q_t is the number of cells in the system at time t^+, and s_t is the state of the Markov chain modulating the ON-OFF source at time t. We assume that departures take place at the end of slots so that q_t includes all background cells that arrive at slot $t-1$, the cell generated by the ON-OFF source at the beginning of slot t, but not the cell that leaves the system at the end of slot $t-1$. Let $I^i(t),\ i = 0, 1, ..., N$, denote the indicator function which takes the value 1 if $s_t = i$, and zero otherwise. Clearly, $(q_t, s_t),\ t = 0, 1, ...,$ is a vector Markov chain. The evolution equation of q_t is given by,

$$q_{t+1} = (q_t - 1)^+ + A_t + I^1(t+1) \tag{1}$$

Assuming that this Markov chain is ergodic, we define the following steady-state PGF for the number of cells in the system when the modulating Markov chain is in state i,

$$G^i(z) \triangleq \lim_{t\to\infty} E\left(z^{q_t} I^i(t)\right) \qquad i = 0, 1, ..., N$$

Following standard $z-$transform arguments, we have

$$\left(z\underline{\underline{I}} \;-\; A(z)\underline{\underline{P}}(z)\right)\underline{G}(z) \;=\; (z-1)A(z)\underline{\underline{P}}(z)\underline{G}(0) \tag{2}$$

where, $\underline{\underline{I}}$ is the identity matrix,

$$\underline{\underline{P}}(z) = \begin{pmatrix} \overline{P}_1 & 0 & 0 & \ldots & 0 & P_0 \\ P_1 z & 0 & 0 & \ldots & 0 & \overline{P}_0 z \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 & 0 \end{pmatrix}, \tag{3}$$

$\underline{G}(z) \triangleq \left(G^0(z), G^1(z), ..., G^N(z)\right)$ and $\overline{P}_i \triangleq 1 - P_i,\ i = 1, 2.$

In order to uniquely determine the vector $\underline{G}(z)$ of PGFs we still need to obtain the vector $\underline{G}(0)$ of boundary functions. First, note that $G^1(0)$ is always zero because the ON-OFF source always generate a cell at the beginning of a slot in which the modulating Markov chain is in state 1. Denote by ρ the probability that the system is not empty (i.e., the fraction of time the server is busy) and let $\overline{A}$ denote the average number of cells generated by the background source in a slot, that is $\overline{A} \triangleq E[A_t]$. The condition for steady-state can be obtained from the normalization condition ($\sum_{i=0}^{N} G^i(1) = 1$) and is given by $\rho = \overline{A} + \frac{P_1}{NP_1+P_0} < 1$. By successive substitutions we have from (2) that,

$$\begin{aligned} G^N(z) \;&=\; \Bigg\{(z-1)(A(z))^N\left[P_1 G^0(0)z + (\overline{P}_0 z - (1-P_0-P_1)A(z))G^N(0)\right] \\ &+\; (z-1)(z-\overline{P}_1 A(z))\sum_{j=1}^{N-1} z^{N-j-1}(A(z))^j G^{N-j}(0)\Bigg\} \times K^{-1}(z) \end{aligned} \tag{4}$$

where, $K(z) \triangleq z^N - \overline{P}_1 A(z) z^{N-1} - \overline{P}_0 (A(z))^N z + (1-P_0-P_1)(A(z))^{N+1}$.

Note that $K(z)$ is the determinant of the characteristic matrix $\left(z\underline{\underline{I}} \;-\; A(z)\underline{\underline{P}}(z)\right)$. Following [15] we assume that $A(z)$ is analytic on the unit circle $|z| = 1$ (Assumption (A1) of [15]. It is clear that

$A(z)$ is also analytic in the unit circle $|z| < 1$) and $K(z)$ doesn't vanish on the unit circle except at $z = 1$ (Assumption (A2) of [15]). We have (Theorem 1 of [15]) that $K(z)$ has exactly $N-1$ zeros in the open unit disk and a simple zero at $z = 1$ if $\frac{d}{dz}K(z)\Big|_{z=1} > 0$ which holds whenever $\rho < 1$. Let $z_i,\ i = 1, 2, ..., N-1$, denote the $N-1$ zeroes of $K(z)$ in the open unit disk. Assume that these zeros are simple (Assumption (A3) of [15]). Since $G^N(z)$ is analytic in the open unit disk, then within this disk the numerator must vanish wherever the denominator of $G^N(z)$ vanishes. Since the denominator of $G^N(z)$ vanishes at $z_i,\ i = 1, 2, ..., N-1$, we have from (4) that,

$$\begin{aligned}&\left(P_1 z_i(z_i-1)(A(z_i))^N\right)\ G^0(0) + \sum_{j=1}^{N-1}\left((z_i-1)(z_i-\overline{P}_1 A(z_i))z_i^{j-1}(A(z_i))^{N-j}\right)\ G^j(0)\\ + &\left((z_i-1)(A(z_i))^N(\overline{P}_0 z_i-(1-P_0-P_1)A(z_i))\right)\ G^N(0)\ = 0 \quad i = 1, 2, ..., N-1 \qquad (5)\end{aligned}$$

Another equation for the computation of the vector $\underline{G}(0)$ is given by $\sum_{j=0}^{N} G^j(0) = 1-\rho$. Hence, we have obtained N equations from which the vector $\underline{G}(0)$ can now be obtained (remember that $G^1(0) = 0$). Under assumptions (A1)-(A3), Theorem 4 of [15] implies that these equations are linearly independent. In Appendix A we give a simple alternative proof that $K(z)$ has exactly $N-1$ zeros in the unit disk $|z| \leq 1$ and a simple zero at $z = 1$ whenever $\rho < 1$. This relaxes assumption (A2) of [15]. However, assumptions (A1)-(A3) of [15] were relaxed in [16]. This completes the determination of the vector $\underline{G}(0)$. Then, the generating function $G^N(z)$ is uniquely determined from (4) and the generating functions $G^i(z),\ i = 0, 1, ..., N-1$, are obtained recursively (in this order) from (2).

According to our arrival model (Section 2), a cell generated by the ON-OFF source arrives at the beginning of a slot. Hence it joins the queue after background cells which arrive in the preceding slot, but before background cells which arrive in the same slot. Let $P(j,i) \triangleq \lim_{t\to\infty} Pr(q_t = j, s_t = i),\ j = 0, 1, ...,\ i = 0, 1, ..., N$, be the limiting probability of the vector Markov chain $(q_t, s_t),\ t = 0, 1, ...$, under the steady-state assumption ($\rho < 1$). Denote by $P^a(i),\ i = 0, 1, ...$, the probability that the first cell of a message arriving from the ON-OFF source sees i cells in the system in steady-state. For the First-Come-First-Serve service discipline this equals to the probability distribution of the waiting time of the first cell of a message (the delay is given by the waiting time plus one). This probability distribution is easily obtained and is given by,

$$P^a(i) \ = \ \begin{cases} a_0(P(0,0)+P(1,0))(nP_1+P_0)/P_0 & i = 0 \\ \left(a_i P(0,0) + \sum_{j=1}^{i+1} a_{i+1-j} P(j,0)\right)(nP_1+P_0)/P_0 & i \geq 1 \end{cases} \qquad (6)$$

The probabilities $P(j,0)$, $j = 0,1,...$, are obtained by taking the inverse z-transform of the PGF $G^0(z)$, which from equations (2) and (4) is found equal to

$$\begin{aligned} G^0(z) &= \Big\{(\overline{P}_1 G^0(0) + P_0 G^N(0))(z-1)A(z) + G^0(0)P_0 P_1 z(z-1)(A(z))^{N+1} \\ &+ \sum_{j=2}^{N-1} G^j(0)P_0(z-1)(z-\overline{P}_1 A(z))z^{j-1}(A(z))^{N-j+1} \\ &+ G^N(0)P_0(z-1)(A(z))^{N+1}(\overline{P}_0 z - (1-P_0-P_1)A(z))\Big\} \\ &\times \Big((z-\overline{P}_1 A(z))K(z)\Big)^{-1} \end{aligned} \tag{7}$$

In Appendix B, we describe an alternative procedure which enables us to recursively compute all the probabilities $P(j,i)$, $0 \le i \le N$, $j \ge 1$, using the probabilities $P(0,i) = G^i(0)$, $0 \le i \le N$, as initial conditions. The computational complexity of this method for obtaining the probabilities $P(j,i)$, $0 \le i \le N$, $1 \le j \le J$, is of the order of $O(NJ^2)$.

Note that the overall cell loss probability for the two sources (both tagged and background) is equal to the probability that the cell delay exceeds $T+1$, and is given by $1 - \sum_{i=0}^{N}\sum_{j=0}^{T+1} P(j,i)$. Similarly, the cell loss probability of the ON-OFF source is also the probability that one of its cells sees a delay greater than or equal to $T+1$. As a cell from the ON-OFF source always arrives at the beginning of a slot in which the modulating Markov chain is in state 1, its loss probability is then given by $1 - \frac{P_0+NP_1}{P_1}\sum_{j=1}^{T+1} P(j,1)$.

3.2 The Message Loss Probability

Let $Q(j)$, $j \ge 0$, be the steady-state probability of no cell losses in a message, given that the first cell of the message sees the system state $(j,1)$, $j \le T$. Conditioning on the waiting time of the first cell of the message, we have that the message loss probability is given by

$$1 - \sum_{j=0}^{T} P^a(j)Q(j) \tag{8}$$

In what follows we describe a method to compute of the probabilities $Q(j)$, $0 \le j \le T$. We first define the probabilities $b_{i,j}^N$, $i,j \ge 0$, to be the N-step transition probabilities of the queueing system without the ON-OFF source, i.e., b_{ij}^N corresponds to the probability in a system consisting of only the background traffic, that given i cells in the buffer at slot 0 there are j cells in the system at the end of slot N. The motivation for introducing these probabilities is that they characterize the evolution of the system between two successive cell arrivals from the tagged source. The initial state i is, however, dependent on both the tagged and background traffic streams and taken from equation (6). Given our assumption on when a cell is lost, we are only interested in the probabilities $b_{i,j}^N$ for $1 \le i \le T+1,\ 0 \le j \le T$. These probabilities can be obtained recursively as follows. The recursion is initiated for $n = 0$ with the relation,

$$b_{ij}^0 \;=\; 1\{i = j\} \tag{9}$$

For $1 \le n \le N$, $1 \le i \le n$ and $0 \le j \le T + N - n$, we have

$$\begin{aligned} b_{i0}^n &= a_0 \left(b_{i0}^{n-1} + b_{i1}^{n-1} \right) \\ b_{ij}^n &= a_0 b_{i(j+1)}^{n-1} + \sum_{k=0}^{j-1} a_{k+1} b_{i(j-k)}^{n-1} + a_j b_{i0}^{n-1} \qquad j \ge 1 \end{aligned} \tag{10}$$

For $i > n$ we have $b_{ij}^n = b_{(i-1)(j-1)}^{n-1}$, $j \ge i - n$ and $b_{ij}^n = 0$, $j < i - n$. The recursion of (10) is obtained by conditioning on the number of background cells that arrive in the n-th transition slot, and using the fact that we are interested only in cases where the number of cells at the end of the n-th transition slot doesn't exceed T. The computation complexity of this recursive procedure is of the order of $O([N(N+T)]^2)$.

Now, conditioning on the next cell that arrives from the ON-OFF source (and using the fact that the distribution of the message size is geometric) we obtain the following set of $T+1$ linear equations from which the probabilities $Q(i)$, $0 \le i \le T$, are obtained,

$$Q(i) \;=\; P_0 \;+\; \overline{P}_0 \sum_{j=0}^{T} b_{(i+1)j}^N Q(j) \qquad 0 \le i \le T \tag{11}$$

The above set of equations can be expressed in matrix form, and we have $\underline{Q} \triangleq P_0(\underline{\underline{I}} - \overline{P}_0 \underline{\underline{B}}^N)^{-1}$, where $\underline{Q} \triangleq (Q(0), Q(1), \ldots, Q(T))$, $\underline{\underline{I}}$ is a $(T+1) \times (T+1)$ identity matrix, and $\underline{\underline{B}}^N$ is a $(T+1) \times (T+1)$ matrix with entries (i,j), $1 \le i \le T+1,\ 0 \le j \le T$, equal to b_{ij}^N. The matrix $\underline{\underline{B}}^N$ is substochastic and hence the matrix $(\underline{\underline{I}} - \overline{P}_0 \underline{\underline{B}}^N)$ is strictly diagonally dominant. Therefore, by the Levy-Desplanques Theorem (see [17]) it is non-singular, so that all probabilities $Q(i)$, $0 \le i \le T$, can indeed be obtained. This completes the computation of the message loss probability. In the next section, we use this result to carry out numerical investigations on the effect of various parameters on the relation between *clp* and *mlp* of the tagged source.

4 Numerical Investigation

This section relies on the model and solution techniques developed in the previous sections to investigate how peak rate and other parameters such as source utilization and message length determine the relation between the *clp* and *mlp* experienced by a connection. For the purpose of these investigations, we assume that the background traffic corresponding to cells from other connections has a geometric distribution, i.e., the probability of i cells arrivals in a slot is given by $a_i = q^i p, \ i \geq 0$, where $q \stackrel{\Delta}{=} 1 - p$, and $0 < p < 1$. The PGF of the probability distribution a_i is then given by $A(z) = \frac{p}{1-qz}$. Under these assumptions, $\overline{A} = q/p$ is the average load contributed by the background traffic, and the total load is $\rho = \frac{q}{p} + \frac{P_1}{NP_1+P_0}$. We further assume that $\rho < 1$, so that the system is stable. The tagged source generates cells according to the ON-OFF model described earlier, where the average message size is $1/P_0$ and N corresponds to the number of slots separating consecutive cells from the same message ($1/N$ is the ratio of the source peak rate and the link rate).

In order to capture the two possible environments mentioned in Section 1, our investigations distinguish between two types of networks. Networks that only control the *overall clp* of their connections, and networks which can guarantee the *clp* of *individual* connections. The latter is obviously preferable, but its cost and complexity make it unlikely to be the general rule, at least initially. It is, therefore, likely that users will need to understand how to deal with both environments. This makes it important that we explore the relation between *clp* and *mlp* as the source peak rate changes for both cases. Especially, since as alluded to earlier, the two are likely to yield very different (as a matter of fact, opposite) behaviors. As a result, and in order to better understand the impact of various parameter changes in each environment, we track both global and connection specific performance measures in all the scenarios we consider. Specifically, the quantities we compute are as follows:

- *clp*: The overall cell loss probability.
- clp^s: The cell loss probability of the tagged connection.
- mlp^s: The message loss probability of the tagged connection.

- *mlpind*^{s}: The message loss probability of the tagged connection under the assumption that each cell of the message is lost independently of the other cells of the message, and with the same loss probability clp^s.

- *mlpind*: The message loss probability of the tagged connection under the assumption that each cell of the message is lost independently of the other cells of the message, but with the same loss probability clp.

Note that under the assumption of independent cell losses, we have

$$\begin{aligned} mlpind^s &= 1 - \frac{P_0(1-clp^s)}{1-\overline{P}_0(1-clp^s)} \\ mlpind &= 1 - \frac{P_0(1-clp)}{1-\overline{P}_0(1-clp)} \end{aligned}$$

In all our numerical investigations, we assume that the threshold used to determine when a cell is considered lost is always $T = 20$. The results from these investigations are summarized in Tables 1 to 6, with odd-numbered tables reporting on scenarios where the network guarantees individual connections' performance, and even-numbered tables providing results for scenarios where the network only gives global performance guarantees. In other words, in Tables 1, 3, and 5, we adjust the average background cell rate so that clp^s is kept approximately constant, and in Tables 2, 4, and 6, we adjust the average background cell rate so that clp is kept approximately constant. In all cases the average load of the ON-OFF source is kept constant.

The first set of experiments we conduct focuses directly on the impact of changing the peak rate of the tagged source as we vary N from 2 to 25. The first scenario (Tables 1 and 2) focuses on a very small connection which only contributes 0.1% to the average link load and has an average message size of about 3.3 cells (recall that the link buffer can hold 20 cells as $T = 20$). Table 1 reports on the case where clp^s is kept constant, while Table 2 provides corresponding results for a network which only guarantees that clp is kept constant.

The first conclusions from the results of Tables 1 and 2 are that peak rate does have a significant influence on mlp^s, and that the two network environments considered do indeed correspond to opposite behaviors. Specifically, Table 1 shows that as N increases (peak rate decreases) mlp^s increases as well for a network where clp^s is kept constant. As expected [3], as N gets large mlp^s eventually approaches $mlpind^s$ which is always an upper bound for mlp^s (see [12]). Users could, therefore, use

$mlpind^s$ as a conservative estimate for the mlp they will experience. A "better" approximation is, however, available using a simple interpolation between clp^s and $mlpind^s$. Specifically, a user can approximate *mlps* using from the following heuristic expression:

$$\begin{aligned} mlp^s &\approx \alpha(N)\, clp^s + (1-\alpha(N))\, mlpdind^s \\ \text{with} \quad & \alpha(1) \approx 1 \;\; \text{and} \;\; \lim_{N\to\infty} \alpha(N) = 0 \end{aligned} \tag{12}$$

Based on a number of numerical experiments, a possible function for $\alpha(N)$ appears to be $\alpha(N) = 1/N^{0.5}$. While using this expression provides reasonable and typically conservative estimates of mlp^s, additional work is needed to both refine it and ensure its consistency across scenarios.

In the alternative scenario of a network where only *clp* is kept constant (Table 2), the opposite behavior was observed with mlp^s decreasing as the peak rate of the connection decreases. This is essentially because clp^s itself decreases as the peak rate goes down. It should also be noted, that in this scenario *mlpind* is not an upper bound for mlp^s but remains consistently close to it. It could, therefore, serve as the base for a possible estimate of mlp^s. From our numerical experiments (Tables 2 and 4), we found that the ratio between mlp^s and *mlpind* varied only between 2 and 1.1. Using a multiplicative factor of 2 appears, therefore, to provide users with a conservative estimate of the message loss probability they will experience. A possible estimate of mlp^s in networks which only guarantee *clp* could then be of the form:

$$mlp^s \approx 2 \times mlpind \tag{13}$$

Tables 3 and 4, essentially repeated the experiments of Tables 1 and 2, but for a slightly larger connection (it contributed about 5% to the overall link load, instead of only 0.1%). The conclusions from this other set of experiments were largely identical to the previous ones, and confirmed both the impact of the connection peak rate and the opposite behaviors of the two types of networks we consider.

The last set of experiments we conducted, investigates the impact of message length, i.e., by changing P_0 we vary the average message length from 2 to $1,000$ cells. As we assume that the link buffer is still only capable of holding 20 cells ($T = 20$), we clearly expect a significant increase in mlp^s as the average message length increases. In addition, we assume a source peak rate of one tenth the link speed ($N = 10$). The results are reported in Tables 5 and 6 for the cases where clp^s and *clp* are kept approximately constant, respectively. As expected, in both cases mlp^s increases significantly as P_0 decreases, but the heuristic guidelines (equations (12) and (13)) provided earlier are found to still hold.

Table 5 also shows a maybe less intuitive phenomenon. Specifically, we see that increasing the average message size of the tagged connection while keeping clp^s approximately constant, also lowers the value of clp, i.e., improves the level of service of other connections. This must obviously be qualified as the load induced by the background traffic is lowered as P_0 decreases in order to keep clp^s constant. The conclusion one can draw from this behavior is, that in order to guarantee the same level of service to a tagged connection of increasing burstiness, the network is forced to lower the overall utilization of the link. Hence, benefiting the level of service experienced by other connections. This is actually consistent with a number of recently proposed resource allocation algorithms, e.g., [8], which compute how much bandwidth to allocate a connection by looking at it in isolation. Such resource allocation rules should actually help minimize the differences that exist between networks which guarantee performances to individual connections, and those which only provide aggregate guarantees. A detailed investigation of this issue is, however, beyond the scope of this paper.

5 Conclusion

This paper has investigated one aspect of the relation that exists between network level performance guarantees and how they translate in application level performances. In particular, we have focused on the relation between cell and message losses in ATM networks as a function of a connection's peak rate. For that purpose, we developed and solved an analytical model that allows us to accurately account for the impact of a connection's peak rate on the pattern of cell losses it experiences. We used this model to study the relation between cell and message losses as source parameters such as peak rate, message length, and utilization change. We also considered two different network environments which we believe will coexist in the foreseeable future, namely networks which guarantee the level of service of individual connections, and networks which only provide global guarantees.

From our numerical investigations, we found that source peak rate does indeed play a major role in determining the message loss of an application, which confirms the initial simulation based findings of [3]. Furthermore, we also established that the two types of networks we were considering exhibited very different (opposite) behaviors. We then attempted to provide some heuristic guidelines to help applications determine the level of service they should request from the network, in order to ensure a desired application level performance. While these heuristic estimates were found to be generally accurate across the many experiments we conducted, much remains to be done to further refine them and this is the subject of ongoing work. However, one of the goals of this paper was not only to provide such initial guidelines to users, but also to foster recognition of the need for a better understanding of how to translate networks performances into quantities which are meaningful to applications. It is our hope that this paper will trigger additional activities in this area.

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A Number of zeros of $K(z)$

We rewrite equation $K(z) = 0$ as follows

$$(z - \overline{P}_1 A(z))(z^{N-1} - \overline{P}_0 (A(z))^N) \; - \; P_0 P_1 (A(z))^{N+1} = 0 \tag{14}$$

One of the solutions of equation (14) is clearly $z = 1$. The other solutions are different from 1, otherwise it can be shown that the steady-state condition is violated. To prove that equation (14) has exactly N solutions within the unit circle $|z| \leq 1$, we apply Rouche's theorem [18].

Rouche's Theorem: Given two functions $f(z)$ and $g(z)$ analytic in a region R, consider a closed contour C in R; if on C we have $f(z) \neq 0$ and $|f(z)| > |g(z)|$, then $f(z)$ and $f(z) + g(z)$ have the same number of zeros within C.

To apply this theorem we identify

$$\begin{aligned} f(z) &= (z - \overline{P}_1 A(z))(z^{N-1} - \overline{P}_0 (A(z))^N) \\ g(z) &= -P_0 P_1 (A(z))^{N+1} \end{aligned}$$

The region R is the disk of radius $1+\epsilon$ (i.e. $|z| < 1+\epsilon$) for some $\epsilon > 0$. If ϵ is small enough, $f(z)$ and $g(z)$ are both analytic in R since they are analytic in $z \leq 1$ (we assume $A(z)$ is analytic on $|z| = 1$). Also, because ϵ is strictly positive we can find some δ such that $\epsilon > \delta > 0$ so that $|z| = 1 + \delta$ is an appropriate contour for Rouche's theorem. If ϵ is small enough, then on this contour, $|z| = 1 + \delta$, we have that,

$$\begin{aligned} |A(z)| &\leq 1 + \overline{A}\delta + 0(\delta) \\ |A(z)|^N &\leq 1 + N\overline{A}\delta + 0(\delta) \end{aligned} \tag{15}$$

where $\lim\limits_{s \to 0^+} \dfrac{0(\delta)}{\delta} = 0$. Using the triangle inequality we have

$$\begin{aligned} |f(z)| &= |z - \overline{P}_1 A(z)||z^{N-1} - \overline{P}_0 (A(z))^N| \\ &\geq (|z| - \overline{P}_1 |A(z)|)(|z|^{N-1} - \overline{P}_0 |A(z)|^N) \end{aligned} \tag{16}$$

For $P_j > 0, \; j = 0, 1$, we can choose $\epsilon > \delta > 0$ such that

$$\begin{aligned} P_0 &> (1 + (\overline{P}_0 \overline{A} - 1)N)\delta + 0(\delta) \\ P_1 &> (\overline{P}_1 \overline{A} - 1)\delta + 0(\delta) \end{aligned}$$

which implies that for $|z| = 1 + \delta$,

$$\overline{P}_1|A(z)| < |z| \quad ; \quad \overline{P}_0|A(z)|^N < |z|^{N-1} \tag{17}$$

Using (15)-(17) we have,

$$\begin{aligned} |f(z)| &\geq (1 + \delta - \overline{P}_1(1 + \overline{A}\delta + 0(\delta)))((1+\delta)^{N-1} - \overline{P}_0(1 + N\overline{A}\delta + 0(\delta))) = \\ &= P_0P_1 + (P_1(N - 1 - \overline{P}_0N\overline{A}) + P_0(1 - \overline{P}_1\overline{A}))\delta + 0(\delta) \\ |g(z)| &= P_0P_1|A(z)|^{N+1} \leq P_0P_1 + P_0P_1\overline{A}(N+1)\delta + 0(\delta) \end{aligned} \tag{18}$$

Using the steady-state condition ($\rho < 1$) we have from (18) that, $|f(z)| > |g(z)|$ implying that $f(z)$ and $f(z) + g(z)$ have the same number of roots within $|z| \leq 1 + \delta$. Now applying Rouche's theorem twice, to $z - \overline{P}_1A(z) = 0$ and $z^{N-1} - \overline{P}_0(A(z))^{N+1} = 0$, on the contour $|z| = 1 + \delta$, it follows from (17) that $f(z)$ has exactly N roots within $|z| \leq 1 + \delta$ and the result follows.

B Computation of the probabilities $P(j, i),\ 0 \leq i \leq N,\ j \geq 1$

In steady-state, Fig. 1 and the evolution equation (1) give the following recursive equations for the computation of these probabilities from the probabilities $P(0, i),\ 0 \leq i \leq N$.

$$P(1, i) = \frac{1}{a_0}P(0, i+1) - P(0, i) \qquad 1 \leq i \leq N - 1 \tag{19}$$

$$\begin{aligned} P(1, 0) &= \frac{(\overline{P}_0 - (1 - P_0 - P_1)a_0)P(0,0) - P_0P(1,1)}{(1 - P_0 - P_1)a_0} \\ P(1, N) &= \frac{-P_1P(0,0) - (1 - P_0 - P_1)a_0P(0,N) + \overline{P}_1P(1,1)}{(1 - P_0 - P_1)a_0} \end{aligned} \tag{20}$$

For $j \geq 2$,

$$P(j, i) = \frac{1}{a_0}\left[P(j-1, i+1) - a_{j-1}P(0, i) - \sum_{k=1}^{j-1} a_{j-k}P(k, i)\right] \qquad 1 \leq i \leq N - 1 \tag{21}$$

$$P(j, 0) = \Big\{ \overline{P}_0P(j-1, 0) - (1 - P_0 - P_1)a_{j-1}P(0,0) - P_0P(j,1)$$

$$- \quad (1 - P_0 - P_1) \sum_{k=1}^{j-1} a_{j-k} P(k,0) \Bigg\} \ / \ [(1 - P_0 - P_1) a_0]$$

$$P(j,N) = \Bigg\{ \overline{P}_1 P(j,1) - P_1 P(j-1,0) - (1 - P_0 - P_1) a_{j-1} P(0,N)$$

$$- \quad (1 - P_0 - P_1) \sum_{k=1}^{j-1} a_{j-k} P(k,N) \Bigg\} \ / \ [(1 - P_0 - P_1) a_0] \qquad (22)$$

For $j = 1$, the probabilities $P(1,i)$, $1 \leq i \leq N-1$, are computed from (19). Then, the probabilities $P(1,0)$, $P(1,N)$ are computed from (20). For $j \geq 2$, the probabilities $P(j,i)$, $1 \leq i \leq N-1$, are computed from (21). Then, the probabilities $P(j,0)$, $P(j,N)$ are computed from (22). The computation complexity of this procedure for the computation of the probabilities $P(j,i)$, $0 \leq i \leq N$, $1 \leq j \leq J$, is of the order of $O(NJ^2)$.

C Figures and Tables

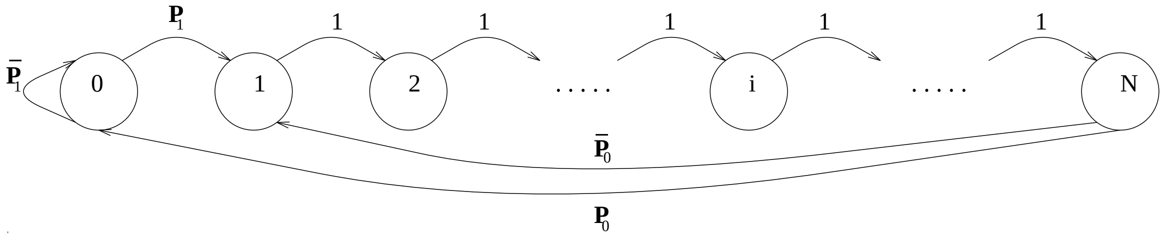

Figure 1: Modulating Markov Chain.

N	ρ	clp	clp^s	mlp^s	$mlpind^s$	$mlpind$
2	0.64738	0.00007395	0.00077072	0.00089803	0.00256445	0.00024648
3	0.65922	0.00010786	0.00077007	0.00101479	0.00256229	0.00035944
4	0.6667	0.00013680	0.00077002	0.00111821	0.00256214	0.00045588
5	0.67208	0.00016224	0.00077108	0.00121123	0.00256567	0.00054061
6	0.67612	0.00018433	0.00077041	0.00129198	0.00256343	0.00061417
7	0.6794	0.00020441	0.00077094	0.00136656	0.00256520	0.00068106
8	0.68208	0.00022239	0.00077082	0.00143336	0.00256480	0.00074094
9	0.68435	0.00023883	0.00077088	0.00149472	0.00256498	0.00079566
10	0.68627	0.00025364	0.00077022	0.00154985	0.00256282	0.00084498
11	0.688	0.00026777	0.00077091	0.00160315	0.00256510	0.00089200
12	0.6895	0.00028062	0.00077094	0.00165143	0.00256520	0.00093481
13	0.6908	0.00029225	0.00077016	0.00169467	0.00256259	0.00097350
14	0.692	0.00030340	0.00077019	0.00173653	0.00256271	0.00101062
15	0.6931	0.00031398	0.00077057	0.00177639	0.00256397	0.00104586
16	0.69408	0.00032372	0.00077045	0.00181272	0.00256355	0.00107825
17	0.695	0.00033312	0.00077089	0.00184801	0.00256502	0.00110955
18	0.6958	0.00034151	0.00077017	0.00187885	0.00256265	0.00113746
19	0.6966	0.00035011	0.00077093	0.00191113	0.00256517	0.00116609
20	0.6973	0.00035780	0.00077104	0.00193938	0.00256554	0.00119170
21	0.698	0.00036567	0.00077067	0.00196877	0.00256432	0.00121786
22	0.69856	0.00037207	0.00077092	0.00199136	0.00256513	0.00123917
23	0.6991	0.00037835	0.00077083	0.00201362	0.00256482	0.00126006
24	0.6998	0.00038666	0.00077049	0.00204588	0.00256369	0.00128773
25	0.701	0.00040139	0.00077095	0.00210884	0.00256524	0.00133673

Table 1: The probabilities clp, clp^s, mlp^s, $mlpind^s$, $mlpind$ as a function of the ON-OFF source peak rate, for $T = 20$, $P_0 = 0.3$ and average cell rate of the ON-OFF source equal 0.001. (clp^s is approximately constant)

N	ρ	clp	clp^s	mlp^s	$mlpind^s$	$mlpind$
2	0.69985	0.00040166	0.00242674	0.00274053	0.00804361	0.00133761
3	0.7001	0.00040150	0.00206007	0.00259683	0.00683405	0.00133709
4	0.7003	0.00040177	0.00181300	0.00250545	0.00601789	0.00133798
5	0.7004	0.00040139	0.00163101	0.00243796	0.00541610	0.00133673
6	0.70053	0.00040179	0.00149489	0.00239034	0.00496565	0.00133805
7	0.7006	0.00040170	0.00138655	0.00235112	0.00460695	0.00133774
8	0.70065	0.00040154	0.00129891	0.00231901	0.00431664	0.00133723
9	0.7007	0.00040153	0.00122703	0.00229289	0.00407842	0.00133718
10	0.70075	0.00040162	0.00116702	0.00227123	0.00387952	0.00133748
11	0.7008	0.00040179	0.00111621	0.00225299	0.00371103	0.00133805
12	0.7008	0.00040139	0.00107110	0.00223446	0.00356143	0.00133674
13	0.7008	0.00040105	0.00103189	0.00221804	0.00343139	0.00133560
14	0.70085	0.00040138	0.00099896	0.00220627	0.00332213	0.00133670
15	0.7009	0.00040175	0.00096993	0.00219588	0.00322580	0.00133792
16	0.7009	0.00040151	0.00094276	0.00218366	0.00313565	0.00133714
17	0.7009	0.00040130	0.00091841	0.00217238	0.00305484	0.00133644
18	0.7009	0.00040111	0.00089637	0.00216188	0.00298166	0.00133581
19	0.70092	0.00040119	0.00087688	0.00215323	0.00291697	0.00133608
20	0.70095	0.00040142	0.00085907	0.00214574	0.00285786	0.00133681
21	0.701	0.00040191	0.00084463	0.00213988	0.00280990	0.00133845
22	0.701	0.00040177	0.00082733	0.00213154	0.00275247	0.00133800
23	0.701	0.00040166	0.00081693	0.00212343	0.00271792	0.00133764
24	0.701	0.00040152	0.00079379	0.00211597	0.00264108	0.00133717
25	0.701	0.00040139	0.00077095	0.00210884	0.00256523	0.00133672

Table 2: The probabilities clp, clp^s, mlp^s, $mlpind^s$, $mlpind$ as a function of the ON-OFF source peak rate, for $T = 20$, $P_0 = 0.3$ and average cell rate of the ON-OFF source equal 0.001. (clp is approximately constant)

N	ρ	clp	clp^s	mlp^s	$mlpind^s$	$mlpind$
2	0.5176	0.00000640	0.00004319	0.00006294	0.00021590	0.00003199
3	0.5662	0.00000929	0.00004315	0.00008070	0.00021574	0.00004648
4	0.5903	0.00001214	0.00004311	0.00009550	0.00021553	0.00006073
5	0.605	0.00001487	0.00004318	0.00010819	0.00021588	0.00007438
6	0.61495	0.00001737	0.00004316	0.00011886	0.00021580	0.00008686
7	0.6222	0.00001965	0.00004314	0.00012804	0.00021567	0.00009824
8	0.62775	0.00002172	0.00004312	0.00013606	0.00021558	0.00010861
9	0.6322	0.00002366	0.00004319	0.00014337	0.00021592	0.00011829
10	0.63575	0.00002535	0.00004314	0.00014950	0.00021551	0.00014168
11	0.6387	0.00002690	0.00004310	0.00015500	0.00021550	0.00013452
12	0.6412	0.00002834	0.00004311	0.00016003	0.00021551	0.00014168
13	0.64335	0.00002967	0.00004315	0.00016469	0.00021572	0.00014833
14	0.64515	0.00003082	0.00004312	0.00016868	0.00021559	0.00015412
15	0.6467	0.00003188	0.00004311	0.00017235	0.00021555	0.00015938
16	0.64805	0.00003285	0.00004316	0.00017587	0.00021576	0.00016423
17	0.64915	0.00003365	0.00004315	0.00017893	0.00021574	0.00016824
18	0.65	0.00003427	0.00004312	0.00018161	0.00021556	0.00017134

Table 3: The probabilities clp, clp^s, mlp^s, $mlpind^s$, $mlpind$ as a function of the ON-OFF source peak rate, for $T = 20$, $P_0 = 0.2$ and average cell rate of the ON-OFF source equal 0.05. (clp^s is approximately constant)

N	ρ	clp	clp^s	mlp^s	$mlpind^s$	$mlpind$
2	0.56585	0.00003423	0.00020276	0.00027420	0.00101301	0.00017115
3	0.60117	0.00003426	0.00014523	0.00025034	0.00072573	0.00017128
4	0.6176	0.00003428	0.00011424	0.00023545	0.00057096	0.00017141
5	0.62685	0.00003424	0.00009507	0.00022436	0.00047520	0.00017120
6	0.63273	0.00003421	0.00008233	0.00021593	0.00041151	0.00017105
7	0.6368	0.00003423	0.00007340	0.00020947	0.00036688	0.00017113
8	0.63975	0.00003423	0.00006677	0.00020418	0.00033375	0.00017115
9	0.642	0.00003426	0.00006172	0.00019991	0.00030855	0.00017129
10	0.64375	0.00003427	0.00005773	0.00019625	0.00028860	0.00017136
11	0.6451	0.00003422	0.00005442	0.00019280	0.00027204	0.00017110
12	0.64623	0.00003422	0.00005177	0.00019005	0.00025880	0.00017109
13	0.6472	0.00003426	0.00004964	0.00018792	0.00024815	0.00017131
14	0.648	0.00003429	0.00004783	0.00018603	0.00023914	0.00017143
15	0.6486	0.00003422	0.00004620	0.00018400	0.00023097	0.00017109
16	0.6492	0.00003428	0.00004500	0.00018300	0.00022498	0.00017142
17	0.64959	0.00003420	0.00004385	0.00018168	0.00021922	0.00017102
18	0.65	0.00003427	0.00004312	0.00018161	0.00021556	0.00017134

Table 4: The probabilities clp, clp^s, mlp^s, $mlpind^s$, $mlpind$ as a function of the ON-OFF source peak rate, for $T = 20$, $P_0 = 0.2$ and average cell rate of the ON-OFF source equal 0.05. (clp is approximately constant)

P_0	ρ	clp	clp^s	mlp^s	$mlpind^s$	$mlpind$
0.5	0.754	0.00200129	0.00358545	0.00479971	0.00714529	0.00399460
0.4	0.749	0.00173254	0.00358755	0.00538175	0.00892087	0.00432012
0.3	0.7418	0.00140554	0.00357221	0.00630689	0.01180894	0.00466982
0.2	0.732	0.00105479	0.00357715	0.00818441	0.01763346	0.00525181
0.1	0.718	0.00069641	0.00351864	0.01354765	0.03410637	0.00692078
0.08	0.715	0.00063687	0.00353219	0.01631901	0.04242898	0.00790308
0.05	0.71	0.00054839	0.00353032	0.02441210	0.06616824	0.01085486
0.03	0.707	0.00050151	0.00358470	0.03895213	0.10707896	0.01645050
0.01	0.703	0.00044454	0.00354517	0.10358336	0.26241651	0.04258053
0.008	0.7025	0.00043783	0.00353074	0.12538890	0.30695470	0.05191095
0.005	0.7017	0.00042728	0.00350363	0.18470517	0.41286696	0.07875905
0.003	0.7013	0.00042214	0.00350024	0.27310109	0.53934998	0.12340338
0.001	0.701	0.00041840	0.00350986	0.52926094	0.77886990	0.29506788

Table 5: The probabilities clp, clp^s, mlp^s, $mlpind^s$, $mlpind$ as a function of the message size parameter of the ON-OFF source, P_0, for $T = 20$, $N = 10$ and average cell rate of the ON-OFF source equal 0.001. (clp^s is approximately constant)

P_0	ρ	clp	clp^s	mlp^s	$mlpind^s$	$mlpind$
0.5	0.70225	0.00041834	0.00086288	0.00123645	0.00172428	0.00083632
0.4	0.7022	0.00041876	0.00100051	0.00163033	0.00249753	0.00104625
0.3	0.70205	0.00041834	0.00121071	0.00234914	0.00402435	0.00139311
0.2	0.70185	0.00041807	0.00155797	0.00393416	0.00774164	0.00208690
0.1	0.7016	0.00041873	0.00219235	0.00913389	0.02149936	0.00417166
0.08	0.7015	0.00041854	0.00237896	0.01180220	0.02894513	0.00520671
0.05	0.70133	0.00041830	0.00271958	0.01984133	0.05171922	0.00830006
0.03	0.7012	0.00041819	0.00299907	0.03396008	0.09113217	0.01375374
0.01	0.70105	0.00041809	0.00333351	0.09919660	0.25063668	0.04014800
0.008	0.70105	0.00041830	0.00337231	0.12151123	0.29724260	0.04970964
0.005	0.70102	0.00041822	0.00342860	0.18218419	0.40761105	0.07721845
0.003	0.701	0.00041817	0.00346742	0.27162851	0.53700075	0.12238452
0.001	0.701	0.00041840	0.00350986	0.52926094	0.77886990	0.29506788

Table 6: The probabilities clp, clp^s, mlp^s, $mlpind^s$, $mlpind$ as a function of the message size parameter of the ON-OFF source, P_0, for $T = 20$, $N = 10$ and average cell rate of the ON-OFF source equal 0.001. (clp is approximately constant)