Nature of the Superconducting State in High-T_c Cuprates

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Recent evidence of three-dimensional xy-critical behavior in the Meissner phase of cuprate superconductors strongly suggests that, close to the transition temperature, the fluctuations of the order parameter are essentially those of a neutral superfluid or xy-model. Full confirmation of xy-behavior requires the sound waves of the superfluid to dominate the low-temperature properties. Our calculation of the sound wave contribution to the magnetic penetration depth, which yields $\lambda(T)/\lambda(0) = 1 + AT^4$ in the limit $T \to 0$, is in remarkable agreement with recent experiments. Thus, the nature of the superconducting state in the cuprates appears to be essentially that of superfluidity.

75.40.-S, 67.40.-Db, 74.30.Ek, 74.30.Ci

Considerable debate has arisen over the nature of the ground state and the symmetry of the order parameter in high-T_c superconductors [1–8]. Measurements of the temperature dependence of the magnetic penetration depth λ are one way to probe the nature of the low-energy excitations and the symmetry of the order parameter. In the macroscopic description of extreme type II superconductors the penetration depth is given by the London expression

$$\frac{1}{\lambda(T)^2} = \frac{16\pi\rho_{\rm s}(T)e^2}{M^2c^2} , \qquad (1)$$

where $\rho_{\rm s}(T)$ denotes the superfluid mass density and $2e$ the charge of the pairs with mass M. In the low-temperature regime, $T \to 0$, and close to T_c, where low-energy and long-wavelength fluctuations are expected to dominate, a macroscopic description is appropriate. It has recently been shown that the Meissner phase data on specific heat, magnetization and penetration depth, taken close to T_c, is well described by three-dimensional (3D) xy-behavior [9–12]. Thus, there is strong evidence of a complex scalar order parameter, $\Psi(\vec{r}) = |\Psi(\vec{r})| \exp(i\varphi(\vec{r}))$, which describes a superfluid where the fluctuations in Ψ are essentially those of a neutral one like ^{4}He or the xy-model [12]. Introducing the correlation length ξ, this behavior can be understood in terms of the dimensionless charge, $\tilde{e} = 1/\kappa = \xi/\lambda$, which determines the strength of the screening. In cuprate superconductors, this charge is small, as reflected by $\lambda \gg \xi$. In other words, fluctuations in the magnetic field can be ignored and the Landau-Ginzburg free energy functional reduces to that of ^{4}He rotated at constant frequency [13,14]. For this reason, one expects that the fluctuations in the order parameter Ψ, describing the macroscopic wave function of pairs of charge $2e$, are essentially those of an uncharged superfluid or xy-model [12]. In this context it is essential to recognize that the order parameter $\Psi(\vec{r})$, with modulus $|\Psi(\vec{r})|$ and phase $\varphi(\vec{r})$, represents the macroscopic wave function of the pair with $\vec{r}$ being the center of mass coordinate. Indeed, when long-wavelength fluctuations with low-energy dominate, the internal pair degrees of freedom can be integrated out because their energy scale, set by the binding energy of the pair, exceeds $k_{\rm B}T_c$. When applying a mean-field or BCS treatment, however, fluctuations arising from the center of mass motion of the pairs are neglected. In this case, the symmetry of the order parameter is fixed by the pairing interaction. When fluctuations dominate, a mean field treatment is not applicable and the symmetry of the order parameter is no longer simply related to that of the pairing interaction because the pairs interact with one another. For these reasons and in view of the overwhelming evidence of fluctuation effects in cuprate superconductors [9–12], it is essential to clarify whether fluctuations in Ψ are indeed essentially those of a neutral superfluid like ^{4}He or the xy-model. Confirmation of the uncharged superfluid scenario is tantamount to showing that the low-temperature behavior is dominated by the sound waves of the superfluid. As far as measurable low-temperature properties are concerned, the magnetic penetration depth appears to be particularly suitable. It is directly accessible and not, as in the case of the specific heat, subject to large background subtractions. Although the temperature dependence of λ has been studied rather extensively [5–8,11], only the study of Sun *et al.* [4] extends to sufficiently low temperatures ($T < 4.2$ K).

In this Letter we explore the compatibility of these measurements with the behavior of an uncharged superfluid like ^{4}He. The resulting dependence of maximum supercurrent through the Josephson junction (Pb/insulator/$Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$) upon the flux threading the junction and temperature dependence $\lambda(T)/\lambda(0) = 1 + AT^4$, for $T \to 0$, turn out to be in remarkable agreement with the available experimental data [4]. Thus, the evidence of 3D-xy-critical behavior around T_c and sound wave dominated low-temperature properties strongly suggest that the nature of the superconducting state in the cuprates is essentially that of superfluidity.

To calculate the macroscopic low-temperature properties, we adopt the standard effective Hamiltonian for an uncharged superfluid describing the hydrodynamic regime [15,16]

$$\mathcal{H} = \frac{1}{2}\int d^3r \left(\rho\vec{v}(r)^2 + s^2\frac{(\rho(\vec{r})-\rho)^2}{\rho}\right) , \qquad (2)$$

where ρ denotes the mass density, $\rho(\vec{r}) - \rho$ mass density fluctuations, $v(\vec{r})$ the local velocity and s the speed of sound in the superfluid. Introducing the velocity potential Φ in terms of $\vec{v} = \nabla\Phi$, Hamiltonian (2) reduces to

$$\mathcal{H} = \frac{1}{2}\int d^3r \left(\rho(\nabla\Phi)^2 + s^2\frac{(\rho(\vec{r})-\rho)^2}{\rho}\right) . \qquad (3)$$

Velocity and density operators satisfy the commutation relation [15]:

$$[\Phi(\vec{r}),\ \rho(\vec{r}\,') - \rho] = -i\hbar\delta(\vec{r} - \vec{r}\,') \qquad (4)$$

revealing that $\Phi(\vec{r})$ and $\rho(\vec{r}) - \rho$ are canonical conjugate "momentum" and "coordinate" operators. In the second quantization representation they can be written in terms of sound wave creation and annihilation operators, yielding for the energy

$$E = \sum_p \hbar\omega(p)\left(n(p) + \frac{1}{2}\right) ,$$
$$n(p) = \frac{1}{\exp\left(\frac{\hbar\omega(p)}{k_{\rm B}T}\right) - 1} , \qquad \omega = sp . \qquad (5)$$

To calculate the superfluid mass density ρ_s entering the London penetration depth (Eq. (1)), it is convenient to consider a superfluid moving with velocity $\vec{v}$ [15–17]. The distribution function of the superfluid moving as a whole is then obtained from the distribution function $n(E)$ for the gas at rest by replacing the excitation energy $E(p) = \hbar\omega(p)$ by $E = E(p) - \hbar\vec{p}\vec{v}$. Thus the total momentum of the quasiparticle gas is

$$\begin{aligned}\vec{P} &= \sum_{\vec{p}} \hbar\vec{p}n\left(E(\vec{p}) - \hbar\vec{p}\vec{v}\right) \\ &\approx \sum_{\vec{p}} \hbar\vec{p}(\hbar\vec{p}\vec{v})\left(-\frac{dn(p)}{dE(\vec{p})}\right) \\ &= \frac{\hbar^2}{3}\vec{v}\sum_{\vec{p}} p^2\left(-\frac{dn(p)}{dE(\vec{p})}\right) = \rho_{\rm N}\vec{v}\ . \end{aligned} \tag{6}$$

The proportionality constant $\rho_{\rm N}$ defines the normal fluid mass density, and the superfluid mass density is then obtained from

$$\rho_{\rm s} \equiv \rho - \rho_{\rm N}\ , \quad \rho_{\rm s}(T=0) = \rho\ . \tag{7}$$

Using Eqs. (1) and (5–7) the asymptotic temperature dependence of the London penetration depth is now readily calculated:

$$\begin{aligned}\frac{\lambda(T)}{\lambda(0)} &= 1 + \frac{\pi^2(k_{\rm B}T)^4}{45s^5\hbar^3\rho} \\ &= 1 + \frac{16\pi^3(k_{\rm B}T)^4\ e^2\lambda(0)^2}{45s^5\hbar^3M^2c^2} \\ &= 1 + AT^4\ . \end{aligned} \tag{8}$$

Clearly, this term dominates only at sufficiently low temperatures. At higher temperatures the interaction of the excitations and deviations from the linear dispersion law ($E(p) = \hbar\overline{s}p$) become important. The temperature range where the long-wavelength description applies can be estimated by introducing into Eq. (6) a cutoff at $p_c \approx \pi/a$, where a is the lattice constant. For the long-wavelength regime to be valid it is necessary that the upper limit of the integral in Eq. (6) becomes large. This restricts the temperature range where Eq. (8) applies to

$$T \ll \frac{\pi\hbar s}{k_{\rm B}a}\ . \tag{9}$$

Recently, remarkably accurate low-temperature penetration depth measurements have been performed [4] using planar Josephson junctions (Pb/insulator/$Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$), tunneling into the c-axis and using an applied magnetic field H parallel to the ab-plane. In the macroscopic description, where the order parameter is a complex scalar, and neglecting screening effects, the maximum supercurrent that can be sent through the junction is given by [18]

$$\begin{aligned} I_{\max}(\Phi) &= I_c\left|\frac{\sin\left(\frac{\pi\Phi}{\Phi_0}\right)}{\frac{\pi\Phi}{\Phi_0}}\right|\ , \\ \Phi &= HY(\lambda + \lambda_{\rm Pb} + d)\ , \\ \Phi_0 &= \frac{hc}{2e}\ , \end{aligned} \tag{10}$$

where d denotes the barrier thickness and λ and $\lambda_{\rm Pb}$ the penetration depths of $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ and Pb, respectively. The value of $I_c = YZJ_c$ is the maximum critical current and J_c the critical current density over the YZ area of the junction. The "diffraction pattern" and its temperature dependence have been verified experimentally [4]. The resulting estimates for the penetration depth are shown in Fig. 1 together with fits to Eq. (8), yielding estimates for $\lambda(0)$ and the prefactor A. Apparently, the experimental data is remarkably consistent with a T^4 dependence and yields the following estimates: $\lambda(0) = 1831$ Å, $A = 3.29 \times 10^{-5}$ K^{-4} for $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$, and $\lambda(0) = 1818$ Å, $A = 2.78 \times 10^{-5}$ K^{-4} for $YBa_2Cu_3O_{7-\delta}$. Nevertheless, to attribute this agreement to a mode with frequency $\omega = sp$ it remains to be shown that condition (9) is satisfied. For this reason we evaluated velocity s from the estimated fit parameter A as a function of the pair mass M. The resulting values are plotted in Fig. 2 in terms of s versus M/m, where m is the mass of the electron. The magnitude of s clearly reveals that condition (9) is satisfied ($\pi\hbar s/(k_{\rm B}a) \approx 150$ K, $a \approx 4 \times 10^{-8}$ cm, $s \approx 2.5 \times 10^5$ cm sec^{-1}). Thus, the experimental data extends to sufficiently low temperatures where the sound wave with frequency $\omega = sp$ dominates, while excitations corresponding to the binding energy Δ of the pairs do not contribute significantly, because $\Delta > k_{\rm B}T_c$. In this respect it is not surprising that the measured zero temperature product I_cR deviates markedly from that of BCS superconductors [4]. The macroscopic description, however, yields $I_c \propto \sqrt{\rho_s\rho_s^{\rm Pb}}$ [18] and, with Eq. (1), $I_c \propto 1/(\lambda(0)\lambda_{\rm Pb}(0))$. Thus, I_c should decrease with increasing Pr concentration x because of the variation of $\lambda(0)$ in $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ with x (see Fig. 1 and Refs. [10,19]).

In summary we have shown that the model of an uncharged three-dimensional superfluid accounts remarkably well for the measured temperature dependence of the magnetic penetration depth in the limit $T \to 0$. Thus, we provided strong evidence that cuprate superconductors and uncharged superfluids like ^{4}He share the universal 3D-xy properties in the fluctuation-dominated regimes $T \to 0$ and $T \to T_c$. These properties include the asymptotic low-temperature behavior and the critical phenomena close to T_c. As a consequence, the nature of superconductivity in the cuprates appears to be essentially that of superfluidity. In both cases T_c signals the appearance or disappearance of phase ordering.

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FIG. 1. Temperature dependence of the penetration depth λ_{ab} for $YBa_2Cu_3O_{7-\delta}$: $\diamond$ and $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ ($x = 0.1$): $\triangle$. Data taken from [4]. The solid line is a fit to $\lambda(T) = \lambda(0)(1 + AT^4)$ yielding $\lambda(0) = 1831$ Å, $A = 3.29 \times 10^{-5}$ K^{-4} for $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$, and $\lambda(0) = 1818$ Å, $A = 2.78 \times 10^{-5}$ K^{-4} for $YBa_2Cu_3O_{7-\delta}$.

FIG. 2. Estimated superfluid sound velocity s versus pair mass M in units of the electron mass m for $YBa_2Cu_3O_{7-\delta}$ (dashed line) and $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ (solid line).