Comment on "Evolution from BCS Superconductivity to Bose-Einstein Condensation: Role of the Parameter $k_{\mathrm{F}}\xi$ in Interpreting the Experimental Plot by Uemura et al."

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Abstract

The problem of crossover from BCS to Bose-Einstein condensation has recently attracted considerable interest owing to the discovery of high-T_c cuprates. Their short coherence length, ξ, places these materials in the interesting region between BCS and Bose-Einstein condensation. In the paper of F. Pistolesi and G.C. Strinati (Phys. Rev. B **49**, 6356 (1994)), the Nozières and Schmitt-Rink approach (NSR) is taken, which is valid for the weak coupling regime. They derive a relation between T_c and T_{F}, which they insist is valid for any coupling strength. We present arguments that their assumptions are incorrect by using our fully self-consistent T-matrix formalism in two dimensions, and show that the NSR approach produces unphysical results in this case.

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In a recent paper, Pistolesi and Strinati [1] attempt to justify a variable that is able experimentally to describe the transition from extended pairs (BCS approach) to local pairs (Bose condensation). They do so because they feel that there must be a better variable than the interaction parameter, $-V$. Such a variable should be independent of the superconducting pairing mechanism. In the end, they conclude that such a variable should be $k_{\rm F}\xi$, where $k_{\rm F}$ is the Fermi k-vector and ξ is the coherence length. With this variable they locate the high-$T_{\rm c}$ superconductors close to the instability $k_{\rm F}\xi \approx 2\pi$ in the plot of $T_{\rm c}$ vs. $T_{\rm F}$, which they call the Uemura plot [2].

Let us preface our discussion of the details of Pistolesi and Strinati's paper with the statement that the important quantities to plot are $T_{\rm c}$ vs. $n_{\rm s}/m$, where $n_{\rm s}$ is the superfluid density and m is the effective mass of the pairs. The value of $T_{\rm c}$ vs. $n_{\rm s}/m$ increases in the underdoped regime, saturates for maximum doping and then decreases in the overdoped regime. This is not discussed in the paper of Pistolesi and Strinati. For example, $\varepsilon_{\rm F}$ and $T_{\rm F}$ are quantities that increase with doping. Therefore, the variable $\varepsilon_{\rm F}$ is not the correct scaling variable because it cannot explain the variation of $T_{\rm c}$ vs. $\sigma(0)$, where $\sigma(0)$ is the zero temperature muon relaxation rate. To be more specific, plotting $T_{\rm c}$ vs. doping produces different curves for different materials. However, when normalizing $T_{\rm c}$ by the maximum transition temperature, all the curves collapse on a generic curve [3]. This cannot be explained with the standard BCS approach.

In addition, recent experiments with high-$T_{\rm c}$ superconductors also reveal features that cannot be explained by the standard BCS approximation. For example, upon doping, the materials go from an insulating to a superconducting phase, where $T_{\rm c}$ increases until a maximum doping of about 0.16 electrons/CuO [4] is reached. As doping continues, $T_{\rm c}$ decreases again and a transition to a metallic state occurs around 0.27 electrons/CuO [4]. Hence, the simple BCS description of the problem is not satisfactory, whereas a Bose scenario appears more appropriate, as has been found by Schneider and Pedersen [5], among others.

We shall now raise a few technical points regarding the arguments of Pistolesi and Strinati for using Eq. (9) in their paper.

1. They do not take the discreteness of the lattice into account, even when the high-$T_{\rm c}$ superconductors are strongly correlated electron systems and the band plays an important role. The continuum approximation is valid only in the dilute limit. The assumption of a parabolic band is clearly not valid for all dopings. The reason they retain the variable $k_{\rm F}$ is that they have not considered the full band structure.

2. They use the Nozières and Schmitt-Rink [6] approach (NSR) to omit the interaction potential in favor of $k_{\rm F}\xi$. However, $k_{\rm F}$ is clearly a variable that is only meaningful for a weakly interacting, many-particle Fermi system. For example, with the two-dimensional attractive Hubbard model [7], which adopts the T-matrix approach, we have found that the distribution function, $n(k)$, differs from the the Fermi distribution function by 20% when $U/t = -4.0$ and $n = 0.32$, where t is the hopping matrix element between nearest neighbors, U the on-site attraction between electrons with opposite spins and n the total number of carriers. In this case, therefore, $k_{\rm F}$ is not defined exactly, and the best we can do is to define it as the point where $n(k)$ reaches half its maximum value.

3. The BCS trial wave function is robust at zero temperature, where fluctuations are irrelevant. But at higher temperatures, it is better to use the Thouless criterion to evaluate T_c with the T-matrix approach. The coherence length, ξ, should be calculated from the correlation function at large distances. Hence, the use of Eq. (9) in Pistolesi and Strinati's paper is not correct. More appropriate would be two-particle correlation functions, which in the language of the T-matrix approach is the T-matrix function itself.

4. The NSR scheme fails for strong couplings for which the approach of Alvarez and Balseiro [8] has proved to be a good one. As a consequence, we conclude that there is no unique approach that allows us to go from the weak coupling regime to the strong coupling one. On the other hand, the NSR scheme has the questionable feature that for all densities, in 2D [9], the ground state is defined by bound pairs for any value of the interaction U.

   We strongly emphasize here that the T-matrix approach is not valid down to zero temperature, unless we generalize it to include the anomalous Green function. In fact, according to our calculations (Fig. 1) with the fully self-consistent T-matrix approximation, we obtain the critical temperature, T_c, for a particular density, and $T_c \neq 0$. It is determined using the divergence of $\mathrm{Re}(T(\vec{0},0))$ or the Thouless criterion. Re(...) denotes the real part of what follows. For $T < T_c$, we cannot use the T-matrix approach in its simple form, because the instability breaks the symmetry. Consequently, we should not use the T-matrix formalism in 2-D in the way proposed by Nozières and Schmitt-Rink, because the temperature cannot be lowered to approach the ground state. It produces unphysical results like those obtained by Schmitt-Rink et al. [9], see Fig. 3 in [9] in particular. For this reason, Nozières and Schmitt-Rink's approach must be used with care.

5. Pistolesi and Strinate plot μ vs. $k_F\varepsilon$ and normalize μ differently for $\mu > 0$ and $\mu < 0$. There is no reason *a priori* why μ should interpolate smoothly in the entire range of $k_F\xi$. In fact, it does not. They take the value $k_F\xi \approx 2\pi$ as the criterion for an instability towards Bose condensation. From this condition, they conclude that $\xi = \lambda_F$, where λ_F is the Fermi wavelength. However, the important question to ask is why this instability occurs at $\xi = \lambda_F$, which they do not explain.

6. High-T_c materials are highly anisotropic, which is reflected by such quantities as the coherence length parallel and perpendicular to the copper-oxide planes, i.e., $\xi_{||}$ and $\xi_{\perp}$, respectively. This is not discussed in Pistolesi and Strinati's paper either. For example, Schneider et al. [10] find that the behavior of T_c vs. ρ, where ρ is the band filling, depends on the parameter α, defined by $\alpha = t_{\perp}^2/t_{||}^2$, where $t_{\perp}$ and $t_{||}$ are the hopping integrals perpendicular to and on the planes.

In conclusion, the approach taken by Pistolesi and Strinati lacks of physical meaning and does not take recent measurements [11] of T_c vs. $\sigma(0)$ into account, where T_c is observed to increase and then to decrease with doping. The term $\sigma(0)$ is the muon spin relaxation rate at zero temperature. Pistolesi and Strinati have used the parabolic band approximation, but it is the very fact that the bands are narrow that enables the high-T_c superconductors to have universal properties [10]. In Ref. [10], T_c is found to increase with increasing carrier concentration, reach a maximum and then come down to zero. See, for example, Fig. 2 in [10]. The universal properties of high-T_c superconductors must therefore be studied using a narrow band description. Furthermore, above T_c, the description of the high-T_c materials must include pair fluctuations, and the critical temperature should be evaluated from the fully self-consistent T-matrix formalism by Thouless criterion. The value of T_c should not be calculated from the BCS gap equation, i.e., from a mean field approximation. We have used the fully self-consistent T-matrix formalism above T_c and found that the approach followed by Nozières and Schmitt-Rink produces unphysical results. In addition, we have calculated the distribution function [7] in the strongly correlated limit ($U/t = -4.0$) and found that the Fermi energy is not a well-defined quantity. As the approach taken by Pistolesi and Strinati does not have a real physical basis, their conclusion is not justified.

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FIGURES

FIG. 1. The real part of the T-matrix at zero momentum and zero frequency as a function of temperature. The number of points in the Brillouin zone is 22×22 and the number of the Matsubara frequency is 1024. We have chosen a damping factor of $\delta = 0.1$ to go to real frequencies.