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ISO Deliberates Statistical Tolerancing

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Abstract

ISO is currently in the first of its three-phase process for developing an international standard for statistical tolerancing of mechanical parts. In this phase the technical scope of the future standard is being defined by a group of international experts. This paper describes the technical deliberations that have been undertaken thus far and the standards development process used for this purpose.

Keywords: ISO, statistics, tolerance

1 Introduction

On June 14, 1996 the International Organization for Standardization (ISO) created a Technical Committee 213 on Dimensional and Geometrical Product Specifications and Verification. Under this new technical committee Advisory Group 4 on Statistical Tolerancing of Mechanical Parts was constituted, thus formally recognizing earlier technical work carried out by an *ad hoc* group. This advisory group is to conduct a preliminary study of statistical tolerancing and report the results to ISO/TC 213 before June 1998 for further action. If the results are encouraging, ISO/TC 213 will formally start the business process of drafting and issuing a chain of ISO standards in this area within a period of three years.

It is important for the international technical community to understand and influence the technical decisions on statistical tolerancing standardization now before it is too late. It is also important that the community understands the ISO standards development process so that it can participate in setting future standards. In this paper I will summarize the technical deliberations that ISO has undertaken thus far and the standards development process used for this purpose.

2 Why Statistical Tolerancing?

ISO is interested in statistical tolerancing, because industry is interested in it. There are at least three reasons why industry is interested in statistical tolerancing.

The first is related to cost. The alternative is to use worst–case tolerancing which may lead to prohibitively expensive tight part tolerances. Worst–case tolerancing is based on the philosophy of complete interchangeability of parts in an assembly. Here the design objective is to ensure that even if the parts are manufactured at their worst limits they can be assembled without interference. Probabilistically, the simultaneous occurrence of such worst–case part–level events is rather rare. On the other hand, in statistical tolerancing the designer takes advantage of the fact that part variations often cancel themselves out, particularly if they are independent. Thus he ends up with looser specifications of part variations, but explicitly gives up the lofty goal of 100% interchangeability of parts in an assembly. The cost advantage of having looser part specifications more than offsets the lack of 100% interchangeability.

Secondly, some industries find themselves with no choice but to adopt statistical tolerancing. Such is indeed the case in electronics industry, for example, where the rate of miniaturization far exceeds the rate of improvement in manufacturing process capabilities.[1] Hence we have the popular saying that "tolerances don't scale down as the nominals". Also, the high cost of capital investment demands that several generations of aggressive designs be produced with a given generation of manufacturing technology.

Finally, statistical ideas and methods that have found successful applications in production (in the form of SPC, that is, Statistical Process Control) and inspection (in the form of SQC, that is, Statistical Quality Control) are influencing product developers to design statistically. If part lot acceptance is based on SPC data, or SQC–based sampling plans, does it make sense to design the parts with the determinism of 100% interchangeability? One may view this as a natural outcome of closer relationship between design and manufacturing functions encouraged in successful companies.

3 What is Statistical Tolerancing?

We can now place statistical tolerancing in the right context. In the late 1930's Walter A. Shewhart, who pioneered statistical methods for industrial quality control, observed that [1]:

> "Broadly speaking, there are three steps in a quality control process: the *specification* of what is wanted, the *production* of things to satisfy the specification, and the *inspection* of the things produced to see if they satisfy the specification."

Shewhart went further and argued that these three steps were not independent but should be viewed as forming a cycle shown in Figure 1.

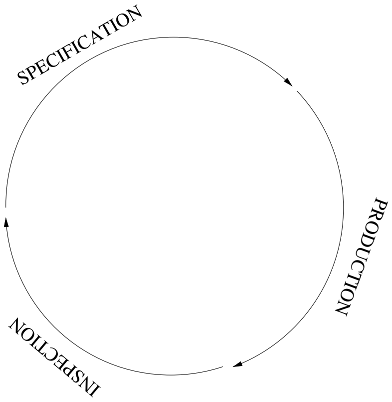


Figure 1: The Shewhart Cycle linking the three steps in a quality control process. Statistical tolerancing deals with statistical specification of what is wanted.

Since then, statistical methods have been developed and applied successfully in production (e.g., SPC) and in inspection (e.g., SQC), in part due to the Allied efforts during the Second World War. Some of the best SPC and SQC practices are now codified in several national and ISO standards. However, statistical notions have been notably absent in product specifications. Although engineers have been using statistical methods in their designs for several decades, they don't have standardized means to specify them in design documents. Statistical tolerancing (ST) is intended to fill this gap. When fully developed ST, SPC, and SQC methodologies will enable statistical implementation of the specification, production, and inspection steps, respectively, thus completing the "Shewhart's Statistical Cycle" envisioned nearly sixty years ago.

ST specifies what populations of parts are acceptable and what are not.[2] Under ST, given a single part we cannot judge whether it conforms to the specification; we need a population of parts before we can decide whether the whole population is acceptable or not. This is a subtle but important fact to bear in mind while practicing ST.

[1] For the moment, think of the range of variation of the output in a manufacturing process as a measure of the process capability. It will be defined more precisely later.

[2] Whereas worst–case tolerancing specifies what parts of the part populations are acceptable and what are not.

4 The ISO Process

ISO is a worldwide federation of national standard bodies from some 100 countries, one from each country.[3] In developing standards ISO follows three principles:

1. **Consensus**: The views of all interests are taken into account: manufacturers, vendors and users, consumer groups, testing laboratories, governments, engineering professions and research organizations.
2. **Industry–wide**: Global solutions to satisfy industries and customers worldwide.
3. **Voluntary**: International standardization is market-driven, and therefore is based on voluntary involvement of all interests in the market-place.

There are three main phases in the ISO standards development process.
The need for a standard is usually expressed by an industry group, which communicates this need to a national member body. The latter proposes the new work item to ISO as a whole. Once the need for an International Standard has been recognized and formally agreed, the first phase involves definition of the technical scope of the future standards. This phase is usually carried out in working groups of technical experts from countries interested in the subject matter. Once an agreement has been reached on which technical aspects are to be covered in the standard, a second phase is entered during which countries negotiate the detailed specifications within the standard. This is the consensus building phase. The final phase comprises the formal approval of the resulting Draft International Standard (the acceptance criteria stipulate approval by two-thirds of the ISO members that have participated actively in the standards development process, and approval by three-fourth of all members that vote), following which the agreed text is published as an ISO International Standard.
The responsibility for developing international standards for the specification of product geometry is now with the newly formed ISO Technical Committee 213 on Geometrical Product Specifications and Verification. Prior to the Summer of 1996 this technical committee was operating as an *ad hoc* entity called the Joint Harmonization Group (JHG). A need for statistical tolerancing standard was communicated to the JHG in their January 1994 meeting at Longboat Key, Florida, U.S.A. by the U.S. delegation based on input from the U.S. industry. This need was recognized by the JHG and a Technical Group 6 on Statistical Tolerancing was promptly appointed to do the work.
The TG 6 first met at Gothenburg, Sweden in May 1995. It was attended by fourteen experts from seven countries. They reviewed several documents on existing national and company standards. They discussed expositions on various industrial statistical tolerancing practices [2, 3].[4] It was resolved in that meeting to apply statistical tolerancing to all tolerances and not be restricted to those affecting assemblies. It was also agreed that ideas presented in the meeting will be taken back to the national committees for further discussion and comments to provide feedback to TG 6.
The second meeting of the TG 6 was held at Paris, France in June 1996. Fifteen experts from six countries participated in the meeting. Consensus was reached on many issues which are described in the rest of the paper. Soon after the meeting, TG 6 was reconstituted as an Advisory Group 4 on Statistical Tolerancing of Mechanical Parts in the new ISO/TC 213, and was instructed to continue the work without interruption.

5 Process Capability Indices

One of the two major approaches used in industry for statistical tolerancing is based on process capability indices (PCI). Let x be a random variable, and LSL and USL be the lower and upper specification limits. If μ and σ are the mean and standard deviation of x, then

$$C_p = \frac{USL - LSL}{6\sigma}$$

and

$$C_{pk} = \min\left\{\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right\} = \frac{\frac{USL-LSL}{2} - |\frac{LSL+USL}{2} - \mu|}{3\sigma}$$

[3] More information on ISO can be obtained via internet at the URL http://www.iso.ch/infoe/ intro.html.

[4] One of these expositions was a case for ISO standardization of statistical tolerancing presented in the last CIRP Seminar on Computer Aided Tolerancing at the University of Tokyo during April 1995.

define two of the main process capability indices [4]. In addition, it is useful to define

$$C_c = \frac{|\mu - \frac{LSL+USL}{2}|}{\frac{USL-LSL}{2}}$$

to quantify the mean shift from the target value of $(LSL + USL)/2$. It should be emphasized that the definitions above implicitly assume that the design target is the midpoint of the specification limits.
These definitions are applicable for any variable characteristic x that is of interest. However, staying within the scope of the ISO/TC 213, we focus only on geometric characteristics. Then the variable characteristic x can be derived from a collection of actual parts (or features) by Gaussian or Chebyschev fitting. In this case, x is called an *actual value*. One of the standing resolutions of the TC 213 is that these actual values will be defined for all tolerances covered by the TC 213 standards. A particular type of actual value is the *actual mating size*, which is found to be very useful for assembly analysis.
For some geometric tolerances, such as perpendicularity, parallelism, runout and so on, $LSL = 0$ and the design target is also 0. The PCIs for such cases are defined as [4]

$$C_p = \frac{USL}{6\sigma}, C_{pu} = \frac{USL - \mu}{3\sigma}, \quad \text{and} \quad C_c = \frac{\mu}{USL}.$$

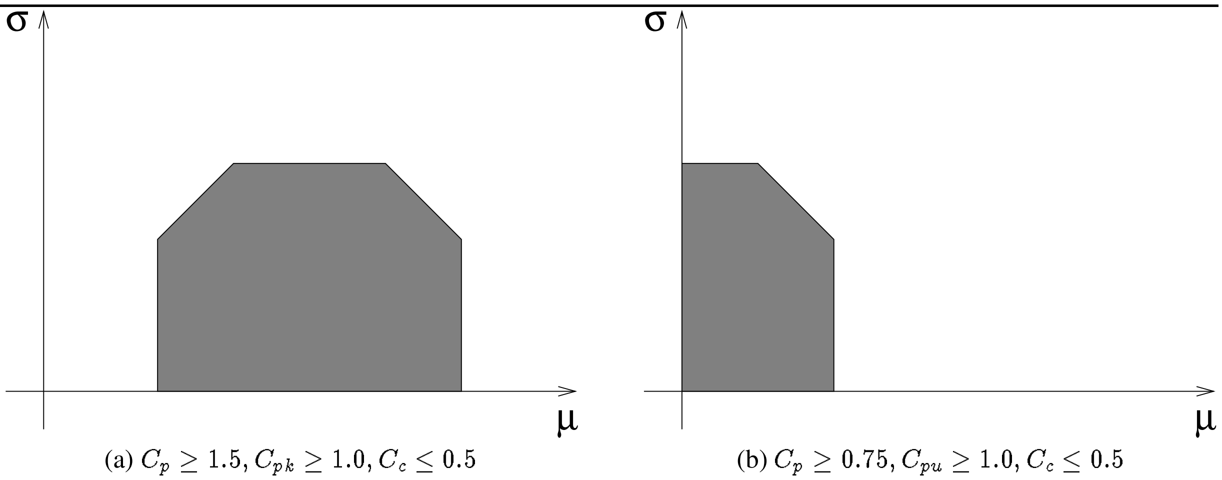
Here C_{pu} takes the place of C_{pk}.

6 A Survey of PCI–based Practices

In the PCI–based industrial practices a population of acceptable parts (or features) is statistically specified by

$$C_p \geq P, \quad C_{pk} \geq K, \quad \text{and} \quad C_c \leq F$$

for particular values P, K, and F, or some subset of these inequalities. This leads to a set of acceptable (μ, σ) pairs, that is, a statistical tolerance zone, hereafter referred to as an STZone, in the parametric space of the population. Figure 2 illustrates two STZones, one involving C_{pk} and the other C_{pu}. STZones in the parametric space of the



(a) $C_p \geq 1.5, C_{pk} \geq 1.0, C_c \leq 0.5$ (b) $C_p \geq 0.75, C_{pu} \geq 1.0, C_c \leq 0.5$

Figure 2: STZones in the μ-σ plane. (a) The target is the midpoint of the specification interval. (b) $LSL = 0$ and the target is also 0.

population of parts (or features) have proved to be very useful in understanding, comparing, and analyzing various PCI-based methods of statistical tolerancing. The zone can take different shapes depending on the indices used and the bounds applied. Table 1 summarizes the results of our survey of current part-level statistical tolerancing practices in some of the leading companies around the world.

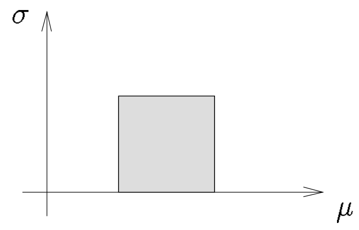
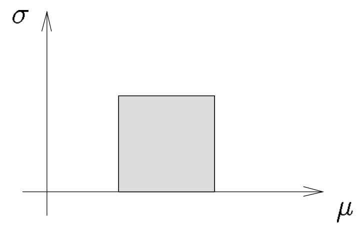
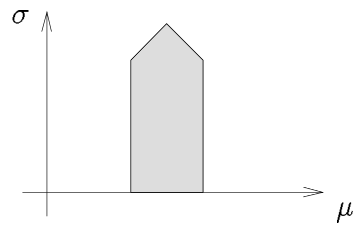
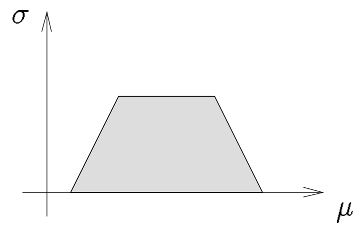
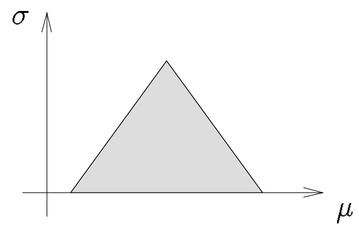
Company	Practice	Shape of STZone
A	$C_p \geq 2.0, C_{pk} \geq 1.5, C_c \leq 0.25$	
B	$C_p \geq 2.0, C_{pk} \geq 0.75C_p$	
C	$C_{pk} \geq 1.0, C_c \leq 0.20$	
D	$C_p \geq 2.0, C_{pk} \geq 1.5$	
E	$C_{pk} \geq 1.33$	

Table 1: Summary of current part-level statistical tolerancing practices in companies surveyed.

Qualitative comparisons of these practices can be made by considering the mean and standard deviation as reasonable measures of the location and spread, respectively, of the population of a desired geometric characteristic. In the practices of Companies A and B, allowable variations in the location and spread are specified independently. This is also true of Company D for a subset of the allowed variation in location. In the remaining cases variation in location from the target is penalized by reducing the allowable variation in spread. Viewed another way, in the latter examples by maintaining a smaller spread the manufacturer can gain a "bonus" in the allowable variation in location.
In addition it was found that the companies invoke an additional condition that these parts be produced with statistical process controls. This requires an examination of the motive behind coupling statistical tolerancing with statistical process control.

6.1 State of Statistical Control

ISO standards that use statistical notions are required to follow basic ISO standards that define statistical vocabulary and symbols [5, 6]. For our purpose, the relevant definitions are:

assignable cause A factor (usually systematic) that can be detected and identified as contribution to a change in a quality characteristic or process level. (Assignable causes are sometimes referred to as special causes of variation. Many small causes of change are assignable, but it may be uneconomical to consider or control them. In that case they should be treated as chance causes.)

chance causes Factors, generally many in number but each of relatively small importance, contributing to variation, which have not necessarily been identified. (Chance causes are sometimes referred to as common causes of variation).

state of statistical control A state in which variations among the observed sampling results can be attributed to a system of chance causes that does not appear to change with time. (Such a system of chance causes will generally behave as though the results are simple random samples from the same population.)

process in control; stable process A process in which each of the quality measures (e.g. the average and variability or fraction nonconforming or average number of nonconformities of the product or service) is in a state of statistical control.

It is widely believed in industry that applying statistical tolerancing without first getting the manufacturing process under a state of statistical control is a bad practice. When a process is not in control it makes little sense to talk about a population, since we don't know when we have finished defining it, that is, when the process has settled down. Also PCIs use only the first two moments of the distribution to characterize the process capabilities. Unless further assumptions can be made about the distribution, mean and standard deviation alone cannot characterize the whole process. Getting the process under a state of statistical control ensures, within economic limits, that only chance causes are at work and this enables us to make some reasonable assumptions about the population distribution.
In the Paris meeting, TG 6 resolved that features that are statistically toleranced shall be produced by a manufacturing process in which the quality measure of the feature is in a state of statistical control. This does not require that control charts (by which statistical control is usually assessed) be maintained after the manufacturer has established that the process is in a state of statistical control and stable for the desired features.

6.2 C_{pk} (or C_{pu}) vs Proportion of Nonconforming Items

It is a common practice to associate C_{pk} or C_{pu} with a proportion of nonconforming items in a population of parts. This stems from the fact that $\frac{x-\mu}{\sigma}$, known as the *standardized random variable* [5], closely resembles C_{pk} and C_{pu}, and that for normally distributed populations readily available tables can be consulted to determine the fraction of the population that lies outside the specification limits for this standardized random variable. For example, specifying that $C_{pu} \geq 1.33$ is often assumed to mean that the proportion of nonconforming items (that is, the fraction of the population for which the characteristic value x exceeds USL) will be less than thirty-two parts per million. What is not common is the realization that such association of C_{pk} or C_{pu} with proportion of nonconforming items is valid only if there is strong reason to believe that the population is normal.
It has been shown that the population of actual values of some geometric tolerances are not normally distributed and associating C_{pk} or C_{pu} limits with proportion of nonconforming items can lead to grossly erroneous results [7, 8].

This is true even if the underlying manufacturing process is in a state of statistical control. Realizing this, TG 6 at the Paris meeting resolved to caution users of PCIs for statistical tolerancing about uncritical association of C_{pk} or C_{pu} with proportion of nonconforming items. It is important to remember that specifications in the PCI-based approach deal primarily with the mean and the standard deviation of the population, and any secondary implication (such as the proportion of nonconforming items) the designer may assume should be backed by further knowledge.
It is more important to consider proportion of nonconforming items at the assembly-level than at the part-level. Then we can reap the benefit of the central limit tendency and can often assume that an assembly-level characteristic, which is affected by several part-level variations, is normally distributed even if we are unsure of the part-level distribution details. This seems to be the reason for the overall success of PCI-based methods of statistical tolerancing in industry.

6.3 C_{pm} and the Quadratic Loss Function

Let τ be the target for a quality characteristic x. Taguchi argues that the total "loss to the society" due to deviation from the target can be captured in the cost per part as $C_0 + C_2(x-\tau)^2$ which is shown graphically in Figure 3. If

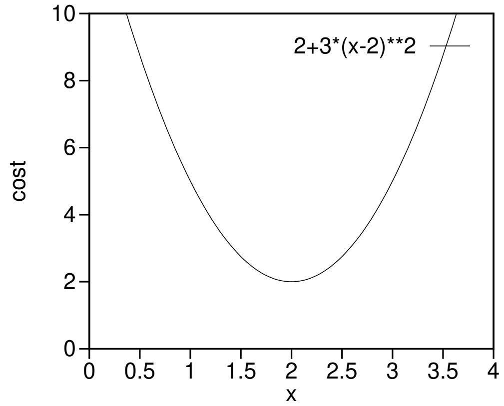


Figure 3: Cost per part using the quadratic loss function with $C_0 = 2$, $C_2 = 3$ and $\tau = 2$.

we now have a population of parts (with probability density function $p(x)$, mean μ, and standard deviation σ), the "average" cost per part is

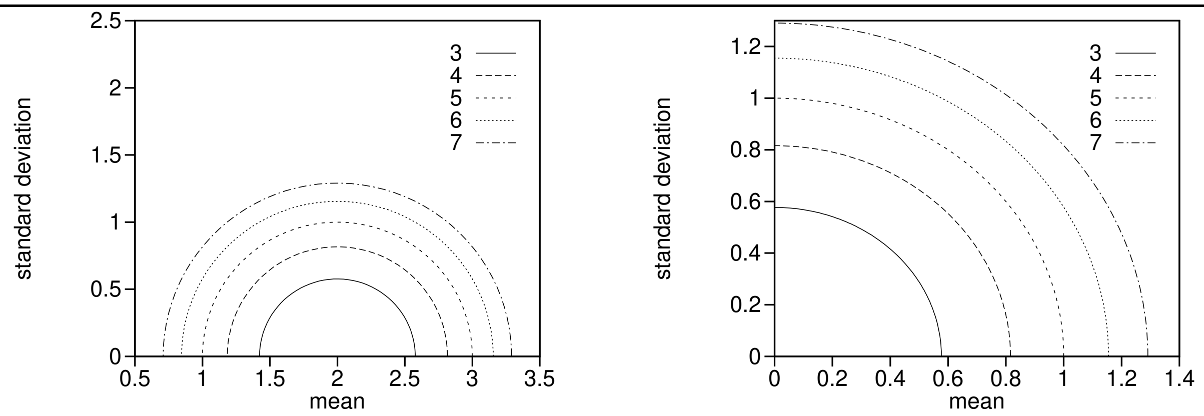
$$\begin{aligned}
&\int_{-\infty}^{+\infty}\{C_0 + C_2(x-\tau)^2\}p(x)dx \\
&= C_0 + C_2\{\int_{-\infty}^{+\infty} x^2p(x)dx - 2\tau\int_{-\infty}^{+\infty} xp(x)dx + \tau^2\} \\
&= C_0 + C_2\{\mu^2 + \sigma^2 - 2\tau\mu + \tau^2\} \\
&= C_0 + C_2\{(\mu-\tau)^2 + \sigma^2\}
\end{aligned}$$

Interestingly, the above result is independent of the type of distribution of x.
Iso-value contours for the average cost are shown in Figure 4 in the μ-σ plane. If such cost models are available, populations of parts for which the average cost per part is less than a preset limit can be specified by an STZone in the μ-σ plane that is bounded by simple circular arcs. The same goal is achieved by using an index defined as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\mu-\tau)^2 + \sigma^2}}$$

and setting an upper bound for it [9]. The above index is a modified form of C_p wherein σ is replaced by $\sigma_\tau = \sqrt{(\mu-\tau)^2+\sigma^2}$. It can be seen that σ_τ is the root-mean-square of the deviation of x from the target τ, because

$$\begin{aligned}
&\{\int_{-\infty}^{+\infty}(x-\tau)^2p(x)dx\}^{\frac{1}{2}} \\
&= \{\int_{-\infty}^{+\infty} x^2p(x)dx - 2\tau\int_{-\infty}^{+\infty} xp(x)dx + \tau^2\}^{\frac{1}{2}} \\
&= \{\mu^2 + \sigma^2 - 2\tau\mu + \tau^2\}^{\frac{1}{2}} \\
&= \sqrt{(\mu-\tau)^2 + \sigma^2}
\end{aligned}$$



$C_0 = 2, C_2 = 3, \tau = 2$ $C_0 = 2, C_2 = 3, \tau = 0$

Figure 4: Iso-cost contours and zones for different target values in the μ-σ plane.

and can be considered as a single measure that statistically captures total variation of x including its deviation from the target and spread. C_{pm} has the advantage that it does not involve the cost coefficients C_0, C_2 which are difficult to determine.

In spite of its simplicity, the use of C_{pm} is not wide-spread in industry. Some in industry have rationalized that the use of C_p, C_{pk} and C_c that leads to STZones as shown in Figure 2 can be considered as piece-wise linear approximation to the zones of Figure 4 that can be obtained by specifying limits for C_{pm}. In its Paris meeting TG 6 appreciated this fact, and decided to stay with the prevalent practice in industry of using C_p, C_{pk} and C_c.

7 Current Proposals

While the PCI-based approach is popular in many industries, some German industries follow a different approach based on distribution function zones (DFZ) [3]. At the Paris meeting of the TG 6 a consensus was reached that both the PCI– and DFZ–based approaches should be considered for ISO standardization of statistical tolerancing, with the stipulation that these parts shall be produced under a state of statistical control.

Figure 5 illustrates how one may statistically specify a diametral tolerance on a shaft using the PCI–based approach. In

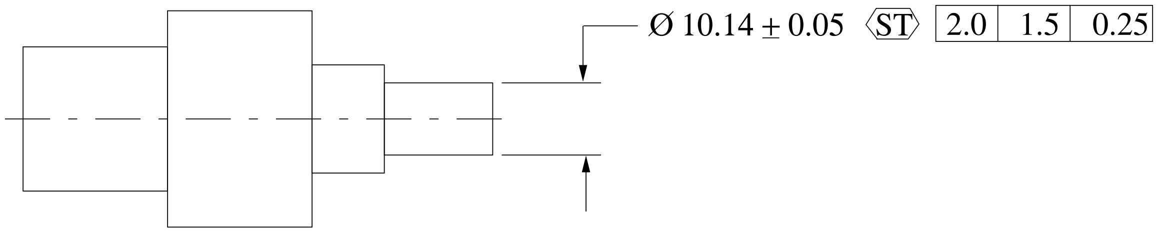


Figure 5: Proposed syntax for the PCI–based specification of statistical tolerancing. The semantics is that the acceptable populations of diameters shall have $C_p \geq 2.0$, $C_{pk} \geq 1.5$, and $C_c \leq 0.25$, with $LSL = 10.09$ and $USL = 10.19$.

this syntax a three-box frame follows the ST symbol and each box contains the appropriate limit for C_p, C_{pk}, and C_c in that order from left to right. If statistical tolerancing is applied to a geometric tolerancing, such as perpendicularity, lower limit for C_{pu} is entered in the second box from the left. Figure 6 illustrates cases where some of the PCIs are not used as in Companies C, D, and E in Table 1. Boxes that are not applicable are crossed out.

Alternatively, using the DFZ–based approach the diameter can be toleranced as shown in Figure 7. The semantics is graphically illustrated in Figure 8. DFZ-based practice is less common, but has greater generality as it deals with the

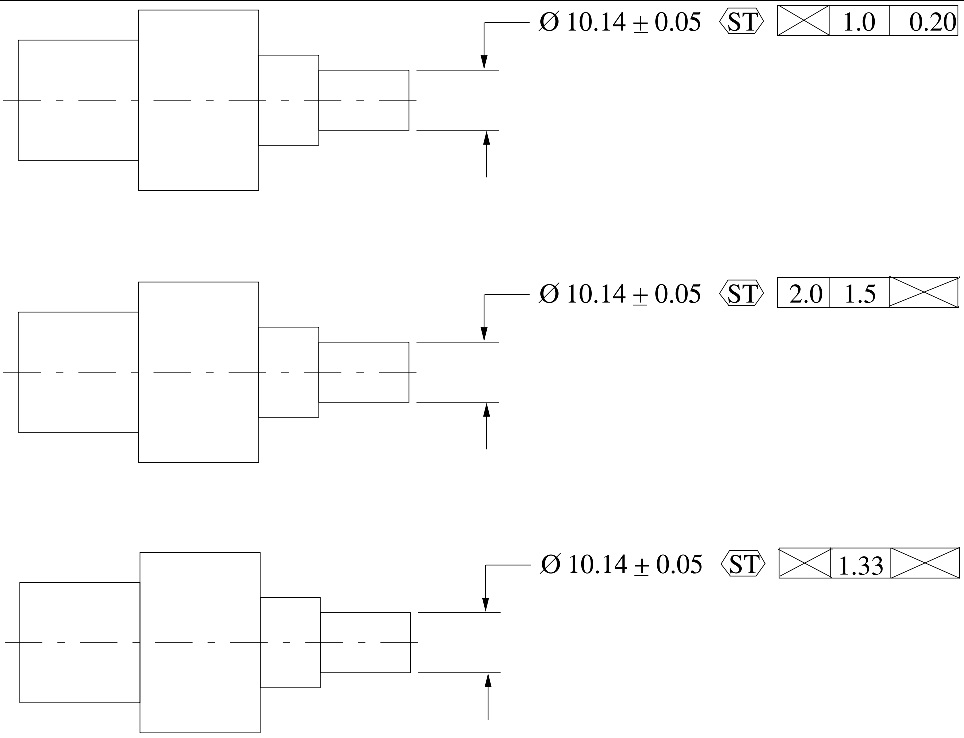


Figure 6: Proposed syntax for the PCI–based specification of statistical tolerancing when some of the PCIs are not used.

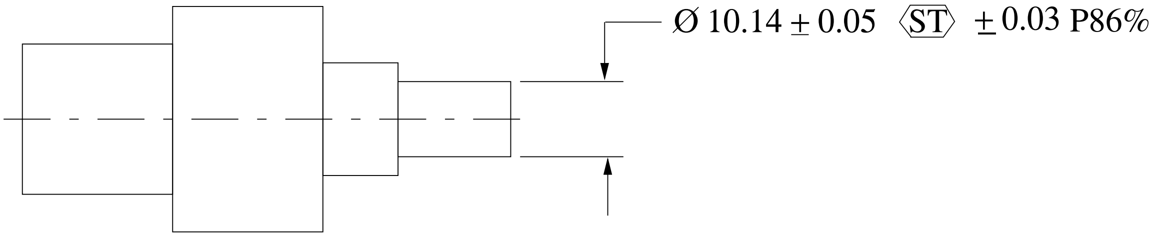


Figure 7: Proposed syntax for the DFZ–based specification of statistical tolerancing. The semantics is that in any acceptable population of parts, at least 86% of the diameters shall be within 10.14 ± 0.03, at most 7% shall be within $10.14^{-0.03}_{-0.05}$, and at most 7% shall be within $10.14^{+0.05}_{+0.03}$.

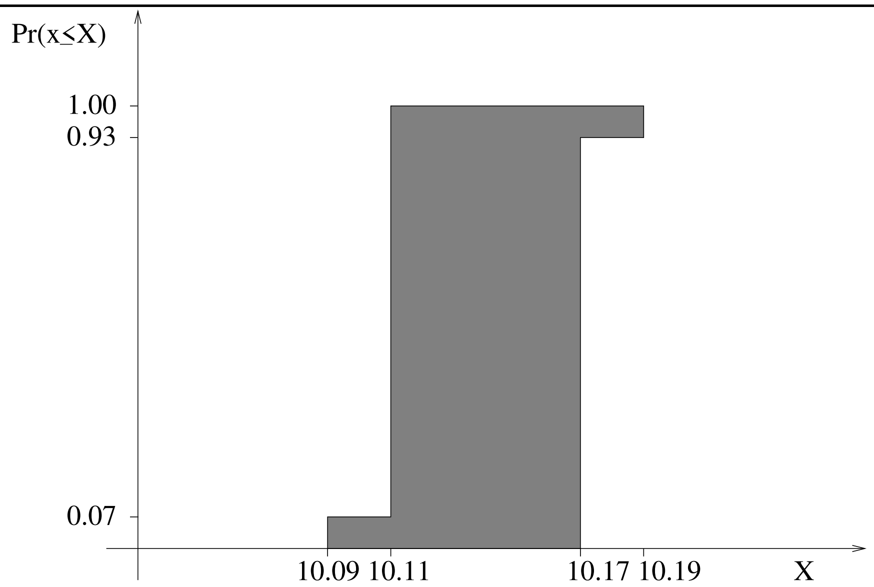


Figure 8: Semantics of the proposed DFZ–based specification. The actual value of a diameter is denoted as x. Any population of diameters whose distribution function falls within the shaded zone is acceptable.

entire distribution function and makes no assumption about sufficiency of μ and σ to describe the distribution.
These two specifications lead to different classes of acceptable populations of parts. The designer can choose the one that best meets his needs. It should be emphasized that these are merely proposals and may undergo changes before being adopted as a standard.

8 Concluding Remarks and Future Work

ISO standards may address only the statistical tolerancing of parts, but the designer has to consider the impact of part–level variations on assembly–level characteristics. If he chooses the PCI–based approach for part–level statistical tolerancing, under some mild assumptions the assembly–level characteristics can be obtained by composing equivalent tolerance zones in the parametric space of the part populations [10]. Similar techniques are being developed for the DFZ–based specifications [11].
There still remain several issues that need to be resolved. Thus far we have handled only those cases wherein the target value is the midpoint of the specification interval or equal to the lower limit. Should the designer be allowed to specify a different target value? Should the dimension (that is, the nominal value) be considered as the target? What is the actual value (which is one value per feature) for size, circularity, circular runout and so on wherein the current practice gives one or more values per cross-section of the feature? What will be the sampling plans for inspecting statistically toleranced parts? These questions are currently being investigated and will be considered in the future meetings of the new AG 4 of ISO/TC 213. Once these questions are answered satisfactorily, the first phase of the ISO standard development process for statistical tolerancing of mechanical parts will be completed.

9 Acknowledgment and a Disclaimer

I would like to thank my colleagues, particularly in the ASME and ISO standard committees, for their valuable input. However, opinions expressed in this paper are my own and not those of ASME, ISO, or any of its member bodies.

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