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Abstract

If inspection shows that a collection of parts does not conform to the design specification, in traditional design a conforming subcollection can be culled by screening to the worst-case tolerance window. If a part is statistically specified, this approach need not be successful. However, if one assumes specification by process capability indices, assumes part values to be normally distributed, and obtains explicit formulas for means and variances of truncated normals, then a maximal balanced subinterval that is conforming can be found. This report documents this process.

Keywords: statistical part specification, process capability indices, truncated normals

1 Introduction

In traditional worst-case design, a dimension of a part is specified by giving as a target T some nominal value and by giving as a description of acceptable part variations of the dimension some tolerance window from a lower specification limit L to an upper specification limit U. If in inspection it is found that the values of the dimension of the parts in some collection of parts do not all fall within the tolerance window, then by screening each part against the window, it is obviously possible to find a maximal subcollection of parts that do fall within the tolerance window.

The most common form of design employing statistical specification uses process capability indices (PCIs). (See [1], [2], or [3] for details.) In this type of design the target T and the tolerance window $[L, U]$ recur, but acceptability is specified in terms of bounds on ratios of linear functions of L, T, U, the mean of values of the dimension of the parts, and the standard deviation of these values. In this situation, for a nonconforming collection of parts the simple process of screening to the tolerance window may not result in a conforming subcollection. If the screening window is made smaller and smaller around the target, then it can be seen that a conforming subcollection will eventually be found. But how small should we make it? This report addresses this question in a commonly occurring situation.

If the values of the population of parts are normally distributed, then, when the population is screened, a truncated normal results. In Section 2 we discuss truncated random variables. If we are to test for conformance of one of these truncated normals, then we will need to know its mean and standard deviation. In Section 3 we obtain explicit formulas for these means and standard deviations. If we restrict ourselves to windows that are symmetric about the target, we can think of the windows as being parameterized by the percentage of the tolerance window they represent. In Section 4 we show that we can use this parameterization to explicitly parameterize the PCIs of the random variables obtained by truncating the normally distributed variable to each of the windows. This allows us to rewrite the problem of finding a subwindow in terms finding a solution of an explicitly known function of one real variable. We close in Section 5 by discussing briefly certain issues that arise in the application of the results of Section 4.

2 Truncated Random Variables

If X is a random variable with distribution function F_X, then let X_L^U be the truncation of X to the interval $[L, U]$, that is, let X_L^U be a random variable with distribution function given by

$$F_{X_L^U}(x) = \begin{cases} 0 & \text{for } X < L \\ \frac{F_X(x) - F_X(L)}{F_X(U) - F_X(L)} & \text{for } L \leq x < U \\ 1 & \text{for } U \leq x \end{cases} \qquad (1)$$

Let dF_X be the measure associated with F_X, and $dF_{X_L^U}$ that associated with $F_{X_L^U}$. By construction subsets of $[L,U]$ are measurable with respect to dF_X, if and only if they are measurable with respect to $dF_{X_L^U}$. If A and B are any two such measurable subsets of $[L,U]$ with $dF_X(B) \neq 0$, then by (1)

$$\frac{dF_X(A)}{dF_X(B)} = \frac{dF_{X_L^U}(A)}{dF_{X_L^U}(B)} \ ,$$

that is, the relative probability of subsets of $[L,U]$ determined by dF_X coincides with that determined by $dF_{X_L^U}$.

If dF_X has a density f_X, then (1) also implies that

$$f_{X_L^U}(x) = \begin{cases} 0 & \text{for } x < L \\ \frac{f_X(x)}{F_X(U)-F_X(L)} & \text{for } L \leq x < U \\ 0 & \text{for } U \leq x \end{cases} \tag{2}$$

is the density of $dF_{X_L^U}$. Finally, recall that if we define $\Phi : x \to ax+b$ for reals a and b with $a > 0$, then $F_{\Phi(Y)}(y) = F_Y(\Phi^{-1}(y))$ for any random variable Y and for all y, so that

$$\begin{aligned} F_{\Phi(X_L^U)}(x) &= F_{X_L^U}(\Phi^{-1}(x)) \\ &= \begin{cases} 0 & \text{for } \Phi^{-1}(x) < L \\ \frac{F_X(\Phi^{-1}(x))-F_X(L)}{F_X(U)-F_X(L)} & \text{for } L \leq \Phi^{-1}(x) < U \\ 1 & \text{for } U \leq \Phi^{-1}(x) \end{cases} \\ &= \begin{cases} 0 & \text{for } x < \Phi(L) \\ \frac{F_{\Phi(X)}(x)-F_{\Phi(X)}(\Phi(L))}{F_{\Phi(X)}(\Phi(U))-F_{\Phi(X)}(\Phi(L))} & \text{for } \Phi(L) \leq x < \Phi(U) \\ 1 & \text{for } \Phi(U) \leq x \end{cases} \\ &= F_{\Phi(X)_{\Phi(L)}^{\Phi(U)}}(x) \ . \end{aligned}$$

Thus transforms like Φ "commute" with truncation, that is, the distribution function is the same, whether we first truncate and then transform, or we first transform and then truncate. More precisely, if we denote that two random variables X and Y have the same distribution function by $X \simeq Y$, then we have

$$aX_L^U + b \simeq \{aX+b\}_{aL+b}^{aU+b} \ \text{ for all } \ b > 0 \ . \tag{3}$$

3 Means and Variances of Truncated Normals

That a random variable X is normally distributed with mean μ and variance σ^2 is often denoted by $X \sim \mathcal{N}(\mu, \sigma^2)$. Let us extend this notation by writing $X \sim$

$\mathcal{N}(\mu, \sigma^2; L, U)$, when X is the truncation to $[L, U]$ of a normal variate with mean μ and variance σ^2. In this section we obtain the means and variances of any $X \sim \mathcal{N}(\mu, \sigma^2; L, U)$.

To begin, let $X \sim \mathcal{N}(0,1)$. For simplicity we will denote its distribution function by F and its density by $f(x)$, that is,

$$f(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \quad \text{and} \quad F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \quad . \tag{4}$$

Now if we truncate X to X_L^U, then by (2) it follows that $\mu(X_L^U)$, the mean of X_L^U, satisfies

$$\begin{aligned} \mu(X_L^U) &= \textstyle\int_{-\infty}^{\infty} t \, dF_{X_L^U} = \int_{-\infty}^{\infty} t f_{X_L^U}(t) dt = \frac{1}{F(U)-F(L)} \int_L^U t f(t) dt \\ &= \textstyle\frac{1}{\sqrt{2\pi}(F(U)-F(L))} \int_L^U t e^{-\frac{t^2}{2}} dt = \frac{e^{-\frac{L^2}{2}} - e^{-\frac{U^2}{2}}}{\sqrt{2\pi}(F(U)-F(L))} \quad . \end{aligned} \tag{5}$$

To evaluate $\mu_2(X_L^U)$, the second noncentral moment of X_L^U, we apply (2) and integration by parts to find that

$$\begin{aligned} \mu_2(X_L^U) &= \textstyle\int_{-\infty}^{\infty} t^2 \, dF_{X_L^U} = \int_{-\infty}^{\infty} t^2 f_{X_L^U}(t) dt = \frac{1}{F(U)-F(L)} \int_L^U t^2 f(t) dt \\ &= \textstyle\frac{1}{\sqrt{2\pi}(F(U)-F(L))} \int_L^U t^2 e^{-\frac{t^2}{2}} dt \\ &= \textstyle\frac{1}{\sqrt{2\pi}(F(U)-F(L))} \left\{ \int_L^U e^{-\frac{t^2}{2}} dt - t e^{-\frac{t^2}{2}} \Big|_L^U \right\} \\ &= 1 - \textstyle\frac{U e^{-\frac{U^2}{2}} - L e^{-\frac{L^2}{2}}}{\sqrt{2\pi}(F(U)-F(L))} \quad . \end{aligned} \tag{6}$$

In general, the mean μ, the second noncentral moment μ_2, and the variance σ^2 of a random variable are related by $\sigma^2 = \mu_2 - \mu^2$, so in particular

$$\sigma^2(X_L^U) = \mu_2(X_L^U) - \mu(X_L^U)^2 \quad ,$$

and of course $\sigma(X_L^U)$, the standard deviation of X_L^U, is just the square root of this value:

$$\sigma(X_L^U) = \sqrt{\mu_2(X_L^U) - \mu(X_L^U)^2} \quad . \tag{7}$$

It follows that if access to the appropriate values of the exponential function and the distribution function of the standard normal is available, then the mean and standard deviation of X_L^U can be easily determined.

Before turning to general truncated normal random variables recall that if Y' is an arbitrary random variable with mean μ' and standard deviation σ', then the mean and standard deviation of the transformed random variable $Y = a'Y' + b'$ for any real a' and b' satisfy

$$\mu(a'Y' + b') = a'\mu' + b' \tag{8}$$

and

$$\sigma(a'Y' + b') = |a'|\sigma' \ . \tag{9}$$

One special case of this type of transform

$$z_{Y'}(Y') = \frac{Y' - \mu'}{\sigma'}$$

is called the z-transform of Y'. Recall that

$$a'X' + b' \sim \mathcal{N}(a'\mu' + b', a'\sigma'^2) \text{ for any } a' > 0 \text{ and } X' \sim \mathcal{N}(\mu', \sigma'^2) \ , \tag{10}$$

so that in particular

$$z_{X'}(X') \sim \mathcal{N}(0,1) \text{ for } X' \sim \mathcal{N}(\mu', \sigma'^2) \ . \tag{11}$$

Now for $X' \sim \mathcal{N}(\mu', \sigma'^2)$ consider $X'^{U'}_{L'}$, its truncation to $[L', U']$. From (3) and (11) we have that

$$z_{X'}(X'^{U'}_{L'}) \sim \mathcal{N}(0, 1; z_{X'}(L'), z_{X'}(U')) \text{ for } X' \sim \mathcal{N}(\mu', \sigma'^2) \ . \tag{12}$$

It follows from (8) and (9) that

$$\mu(X'^{U'}_{L'}) = \sigma'\mu(z_{X'}(X'^{U'}_{L'})) + \mu' \text{ for } X' \sim \mathcal{N}(\mu', \sigma'^2) \tag{13}$$

and that

$$\sigma(X'^{U'}_{L'}) = \sigma'\sigma(z_{X'}(X'^{U'}_{L'})) \text{ for } X' \sim \mathcal{N}(\mu', \sigma'^2) \ . \tag{14}$$

We thus have that the mean and standard deviation of the truncation of any normal variate can be determined by (5) and (7), which give the mean and standard deviation of the associated truncation of a normal variate with mean 0 and variance 1.

We can obtain a converse to (12) by using (3) and (10) to get

$$z^{-1}_{X'}(X^U_L) \sim \mathcal{N}(\mu', \sigma'^2; z^{-1}_{X'}(L), z^{-1}_{X'}(U)) \text{ for } X \sim \mathcal{N}(0,1) \text{ and } X' \sim \mathcal{N}(\mu', \sigma'^2) \ . \tag{15}$$

4 PCIs and Truncated Normals

When a design has specified the values of some dimension of a part statistically, but the population of parts fails to meet the specification, it is natural to hope that by some method a subpopulation can be culled that will meet specification. In this section we consider one such method. More explicitly, assume that a tolerance interval $[L, U]$, from lower specification limit L to upper specification limit U with target $T = \frac{L+U}{2}$, has been specified. Assume that acceptability of the population is defined by bounds on two of the three PCIs of X, the random variable of values of the dimension of the parts in the population, that is, bounds on two of

$$C_c(X) = \frac{|\mu(X) - T|}{U - T}, \quad C_p(X) = \frac{U - L}{6\sigma(X)}, \text{ and } C_{pk}(X) = \min\left\{\frac{\mu(X) - L}{3\sigma(X)}, \frac{U - \mu(X)}{3\sigma(X)}\right\}.$$

Further assume that $X \sim \mathcal{N}(\mu, \sigma)$ is such that the bounds on the PCIs are not both met. If $0 \leq t \leq 1$, then say $I_t = [L_t, U_t]$ is a balanced $100t\%$-subinterval of $[L, U]$, if $\frac{U_t + L_t}{2} = T$ and $\frac{U_t - L_t}{U - L} = t$, and let X_t be the truncation of X to I_t. Clearly, as t approaches 0, the mean of X_t must approach T, and the variance of X_t approaches 0, so that PCIs of X_t must satisfy the prescribed bounds for some t. In this section we develop some formulas and make some observations that aid in determining such a t.

To begin, assume that L, T, and U have been given, that acceptability is specified by

$$C_c(X) \leq c \text{ and } C_{pk}(X) \geq k \ ,$$

and that $X \sim \mathcal{N}(0, 1)$ with

$$C_c(X) = \hat{c}_c < 1 \text{ and } C_{pk}(X) = \hat{c}_{pk} > 0 \ .$$

Finally, for concreteness, assume for now that $T \leq 0$. (See Figure 1.)

First, we show that L, T, and U are determined by $\hat{c}_c$ and $\hat{c}_{pk}$. Since $\hat{c}_{pk} = C_{pk}(X) = \frac{U-0}{3}$, solving for U yields

$$U = 3\hat{c}_{pk} \ . \tag{16}$$

Since $\hat{c}_c = C_c(X) = \frac{0-T}{U-T} = \frac{0-T}{3\hat{c}_{pk}-T}$, solving for T yields

$$T = \frac{-3\hat{c}_{pk}\hat{c}_c}{1 - \hat{c}_c} \ . \tag{17}$$

Since T is the midpoint of L and U,

$$L = T - (U - T) = 2T - U = \frac{-6\hat{c}_{pk}\hat{c}_c}{1 - \hat{c}_c} - 3\hat{c}_{pk} = -3\hat{c}_{pk}\frac{1 + \hat{c}_c}{1 - \hat{c}_c} \ , \tag{18}$$

and

$$T - L = U - T = 3\hat{c}_{pk} + \frac{3\hat{c}_{pk}\hat{c}_c}{1 - \hat{c}_c} = \frac{3\hat{c}_{pk}}{1 - \hat{c}_c} \ .$$

Similarly, L_t and U_t are determined by $\hat{c}_c$ and $\hat{c}_{pk}$. Since I_t is balanced, $\frac{T-L_t}{T-L} = t$ and $\frac{U_t-T}{U-T} = t$. Solving for L_t and U_t yields

$$L_t = T - t(T - L) = \frac{-3\hat{c}_{pk}\hat{c}_c - t3\hat{c}_{pk}}{1 - \hat{c}_c} = -3\hat{c}_{pk}\frac{t + \hat{c}_c}{1 - \hat{c}_c} \tag{19}$$

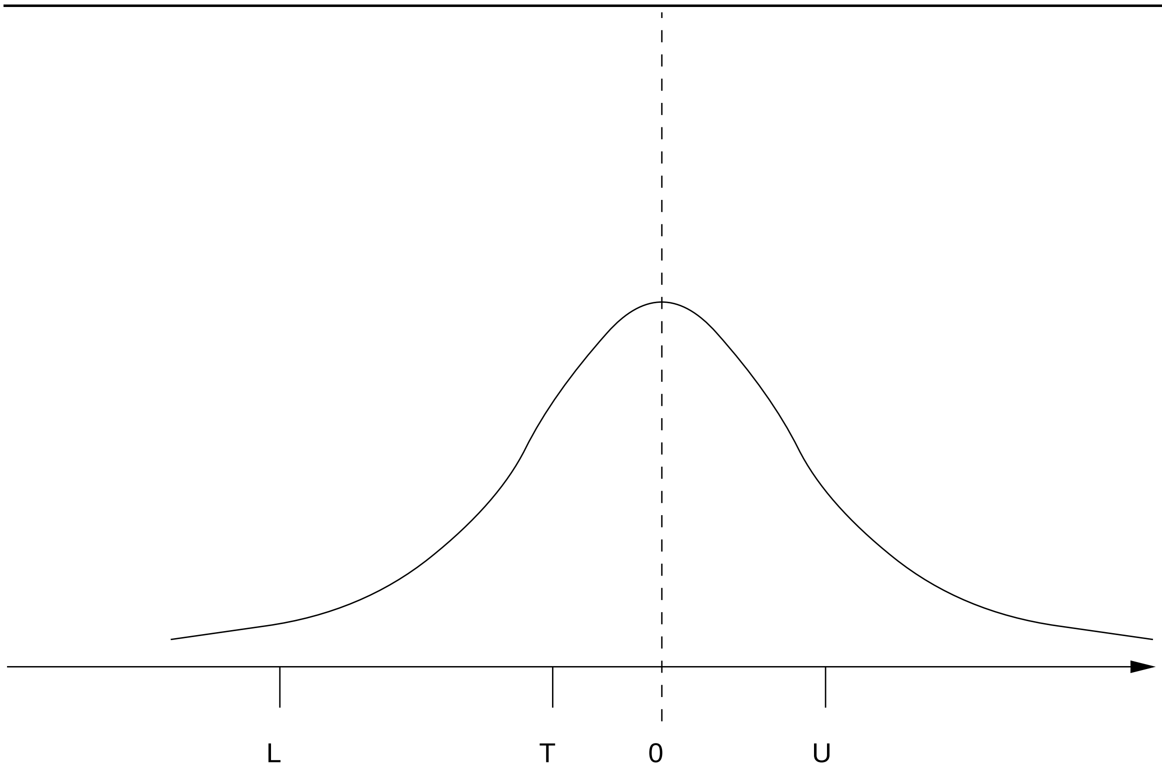

Figure 1: The figure depicts a standard configuration. The tolerance window is $[L, U]$. Target T satisfies $T = \frac{L+U}{2} \leq 0$. If X is the random variable of values of the dimension of the part, then $X \sim \mathcal{N}(0, 1)$ with $C_c(X) = \hat{c}_c$ and $C_{pk}(X) = \hat{c}_{pk}$. L, T, and U satisfy $L = -3\hat{c}_{pk}\frac{1+\hat{c}_c}{1-\hat{c}_c}$, $T = \frac{-3\hat{c}_{pk}\hat{c}_c}{1-\hat{c}_c}$, and $U = 3\hat{c}_{pk}$. The density of F_X is graphed.

and

$$U_t = T + t(U - T) = \frac{-3\hat{c}_{pk}\hat{c}_c + t3\hat{c}_{pk}}{1 - \hat{c}_c} = 3\hat{c}_{pk}\frac{t - \hat{c}_c}{1 - \hat{c}_c} \quad . \tag{20}$$

Let us denote the truncation of X to $I_t = [L_t, U_t]$ by X_t, that is, let $X_t = X_{L_t}^{U_t}$. Since $T \leq 0$ by assumption, $f_X(T - \delta) \leq f_X(T + \delta)$ for all $\delta \geq 0$. This implies that $T \leq \mu(X_t)$ for all t, because I_t is balanced. It follows that

$$C_c(X_t) = \frac{\mu(X_t) - T}{U - T} \text{ and } C_{pk}(X_t) = \frac{U - \mu(X_t)}{3\sigma(X_t)} \quad .$$

From (5) and (7) we see that $\mu(X_t)$ and $\sigma(X_t)$ are dependent only on L_t and U_t. From (16)-(20) we have that T, U, L_t, and U_t are dependent only on $\hat{c}_c$, $\hat{c}_{pk}$, and t. Thus $C_c(X_t)$ and $C_{pk}(X_t)$ are also dependent only on $\hat{c}_c$, $\hat{c}_{pk}$, and t. In fact, using (5), (7), and (16)-(20), we can explicitly write

$$C_c(t) = C_c(X_t) = \frac{\mu(X_t) - T}{U - T} \text{ and } C_{pk}(t) = C_{pk}(X_t) = \frac{U - \mu(X_t)}{3\sigma(X_t)} \quad , \tag{21}$$

where

$$\mu(X_t) = \frac{e^{-\frac{(3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c})^2}{2}} - e^{-\frac{(3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c})^2}{2}}}{\sqrt{2\pi}(F(3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c}) - F(3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c}))}$$

and

$\sigma(X_t) =$

$$\sqrt{1 - \frac{3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c}e^{-\frac{(3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c})^2}{2}} - 3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c}e^{-\frac{(3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c})^2}{2}}}{\sqrt{2\pi}(F(3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c}) - F(3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c}))} - \left(\frac{e^{-\frac{(3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c})^2}{2}} - e^{-\frac{(3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c})^2}{2}}}{\sqrt{2\pi}(F(3\hat{c}_{pk}\frac{t-\hat{c}_c}{1-\hat{c}_c}) - F(3\hat{c}_{pk}\frac{t+\hat{c}_c}{1-\hat{c}_c}))}\right)^2} \quad .$$

Thus we can repose the problem of finding a maximal conforming balanced subinterval of $[L, U]$ as finding maximal $\hat{t}$, such that

$$C_c(\hat{t}) \leq c \text{ and } C_{pk}(\hat{t}) \geq k \quad . \tag{22}$$

If we now change our assumption that $T \leq 0$ to the assumption that $T \geq 0$, then we can return to the case we just considered by treating the equivalent problem for $-X$. To be more precise we take $-U$ as lower specification limit, $-L$ as upper specification limit, $-T$ as target, and define acceptability by

$$C_c(Y) \leq c \text{ and } C_{pk}(Y) \geq k \quad .$$

We note that

$$C_c(-X) = C_c(X) = \hat{c}_c \text{ and } C_{pk}(-X) = C_{pk}(X) = \hat{c}_{pk} \quad ,$$

that

$$-X \sim \mathcal{N}(0,1) \ ,$$

and that

$$-T \leq 0 \ .$$

Thus the conditions previously assumed are met. If we find a maximal $\hat{t}$ satisfying (22), and $[-U_t, -L_t]$ is its associated balanced $100t\%$-subinterval of $[-U, -L]$, then $[L_t, U_t]$ is a balanced $100t\%$-subinterval of $[L, U]$. From (8) and (9) it follows that $\mu(X_{L_t}^{U_t}) = -\mu(-X_{-U_t}^{-L_t})$ and $\sigma(X_{L_t}^{U_t}) = \sigma(-X_{-U_t}^{-L_t})$, which implies that $C_c(X_{L_t}^{U_t}) = C_c(-X_{-U_t}^{-L_t})$ and $C_{pk}(X_{L_t}^{U_t}) = C_{pk}(-X_{-U_t}^{-L_t})$. The maximality of $[L_t, U_t]$ follows easily from that of $[-U_t, -L_t]$. In this way considering $-X$ leads to a solution for X. But now that we have seen that this is the case, we really need not consider $-X$ at all, and in fact can use $\hat{c}_c$, $\hat{c}_{pk}$, c, and k directly in (21) and (22).

In a similar manner, we can deal with the general class of problems, where $X \sim \mathcal{N}(\mu, \sigma)$. We use z_X to transform X to $z_X(X) \sim \mathcal{N}(0,1)$. $[L, U]$ is transformed to $[z_X(L), z_X(U)]$. Using (8) and (9) shows that $C_c(X) = C_c(z_X(X))$ and $C_{pk}(X) = C_{pk}(z_X(X))$. We find a maximal balanced $100t\%$-subinterval of $[z_X(L), z_X(U)]$ as before and use z_X^{-1} to transform it back to a maximal balanced $100t\%$-subinterval of $[L, U]$. And again we note that it is sufficient to use $\hat{c}_c$, $\hat{c}_{pk}$, c, and k directly in (21) and (22).

Finally, if acceptability is defined in terms of two PCIs, other than C_c and C_{pk}, since any two PCIs determine the value of the third, we can always obtain the values of C_c and C_{pk}. Thus we can rewrite the formulas for μ and σ in (21) in terms of the available data. This allows us to write the two PCIs of the truncated variables in terms of the available data, and hence we can make the obvious changes to (22) to complete the restatement of the problem.

5 Comments on Applications

Due to the explicitness and relative simplicity of $\mu(X_t)$ and $\sigma(X_t)$ of (21), of (21) itself, and of (22), developing computer code evaluating the PCIs of any balanced subinterval of the given tolerance window is virtually straightforward in almost any programming language, statistical package, or even spreadsheet environment. All the necessary functions and calculations are immediately available with the possible exception of $F(x)$, the distribution function of a normal variate $X \sim \mathcal{N}(0,1)$. Although, $F(x)$ has no closed form expression, the function can easily be well approximated by a Taylor series expansion, since any order of the derivative of $F(x)$ is easily obtained, and for any choice of points on which to base the expansion high precision values of $F(x)$ are readily available from many sources.

If we were to know that the PCIs of (21) are monotonic in t, then an application of Newton's method or even a bisection method would suffice. However, although it may be reasonable to assume that they are monotonic, and experiments indicate that this is probably the case, no proof of monotonicity has been attempted. On the other hand, we know that as t approaches 0, any conditions of the form of (22) are met, and so the continuity of (21) implies that the conditions of (22) are met in some interval. Thus, simply decrementing t and evaluating (21) yields a (possibly) suboptimal choice for t. This approach often suffices in applications.

We have developed an implementation of this last approach in the Lotus 123 spreadsheet program. It uses a fifth degree Taylor series expansion with 25 tabulated values spaced evenly between 0 and 5. For our applications this yields sufficiently accurate results in the covered range, and any values outside this range yield only insignificant probability.

A typical application of these results can be found in inspection. If the statistical design specification of a part is not met, a balanced subinterval of the tolerance window can be used for a 100% screening process, with the hope of obtaining a conforming subcollection of parts. Several words of caution are essential. First, the assumption of normality of the underlying population was critical in all that followed. Whatever divergence from normality is found in the real population will have an unknowable effect on the predicted PCIs of the subpopulation. Second, the functions of (21) that give the values of the PCIs of the subpopulation are strongly dependent on the PCIs of the population. In real applications these values can only be estimated. Whatever error exists in the estimate will again have an unknowable effect on the predicted PCIs of the subpopulation. Thus care should be exercised in applying these results. It might be advisable to require slightly stronger PCI bounds, unless the population is very close to normal, and the PCIs of it are known to great precision. It might be wise to consider confidence intervals rather than point estimates of the PCIs. In any case, the PCIs of the culled population should be calculated after the screening process, which is easy to do, since in the screening process the values of all members of the population will be obtained. Finally, we note that if the subpopulation fails this postscreening evaluation, the results should not be reapplied to the subpopulation, since it will surely not be normal. However, with complete knowledge of the values of this subpopulation more effective means of choosing a new subpopulation are easy to envision.

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