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Samer Takriti, Chonawee Supatgiat, and Lilian S.-Y. Wu

IBM Research Division
T.J. Watson Research Center
Yorktown Heights, New York

Research Division
Almaden · T.J. Watson · Tokyo · Zurich

Coordinating Fuel Inventory and Electric Power Generation Under Uncertainty

Samer Takriti* Chonawee Supatgiat Lilian Wu

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Abstract

We discuss the problem of hedging between the natural gas and electric power markets. Based on multiple forecasts for natural gas prices, natural gas demand, and electricity prices, a stochastic optimization model advises a decision maker on when to buy or sell natural gas and when to transform gas into electricity. For relatively small models, branch-and-bound solves the problem to optimality. Larger models are solved using Bender's decomposition and Lagrangian relaxation. We apply our approach to the system of an electric utility and succeed in solving problems with 50,000 binary variables in less than 4 minutes to within 1.16% of the optimal value.

Keywords: Bender's decomposition, Branch-and-bound, Lagrangian relaxation, Spark spread, Stochastic programming, Unit commitment.

1 Introduction

Both the natural gas and electricity markets are going through deregulation and consumers are no longer under the obligation of purchasing their energy from local utility companies. Consumers now have the option to choose between different suppliers and to select what type of energy contract they prefer. For example, one may choose to have an interruptible electric service in exchange for a cheaper rate, while others may prefer firm contracts that guarantee uninterrupted service. At the other end of the spectrum, energy suppliers are facing greater uncertainty in this new market as a result of customers' freedom of choice. We refer the reader to [7] and [8] for an in-depth discussion of this subject.

Due to deregulation, utilities—gas suppliers and electric power generating companies—are trying to position themselves in the newly emerging energy market. The utility industry has never seen so many acquisitions and mergers as it sees today. A common strategy for a natural gas supplier is to purchase electric generating assets at different locations. The goal of this approach is to hedge between the natural gas market and the electric power market. This is the case in the summer season when natural gas prices are low while those of electricity peak. It is beneficial for a natural gas supplier to turn some of its supply into electricity and profit from the soaring electricity

*Contacting author: IBM T. J. Watson Research Center, P. O. Box 218, Yorktown Heights, New York 10598, takriti@watson.ibm.com

prices. The difference between electricity price and fuel cost is called the spark spread [5]. It is an essential risk management tool in an open energy market.

In this paper, we discuss the problem of hedging between the gas and electricity markets. Our model assumes that gas prices, gas demand, and electricity prices are stochastic. Uncertainty is modeled using a set of scenarios that represent possible future outcomes of the random parameters in the model. We describe the problem in Section II and provide a mixed-integer formulation for it. If the number of generating units is relatively small, then the resulting model can be solved using branch-and-bound. Larger models are tackled by decomposing the problem as suggested in Section III. This approach creates two optimization problems: a mixed-integer program of smaller size than the original model and a linear program. If the resulting mixed-integer program is still large to be solved directly, we use Lagrangian relaxation to further decompose it into smaller programs. This is described in Section IV. Section V presents numerical results for test problems that are based on the system of a midwestern US utility.

2 Problem Description

Let us consider an energy marketer who purchases natural gas from the open market and sells it to contracted customers. Our marketer has a firm contract with his/her customers, meaning that the uncertain demand on any day must be met. In order to protect against uncertainty in demand as well as the changing spot-market price of natural gas, the marketer uses a storage tank to hold natural gas as needed. Whenever gas is stored, a holding cost, which is a linear function of the amount of gas in the tank, is incurred. The marketer can pay a fee and sell the stored gas back to the market in any period at the spot-market price. Another option is to use the gas to generate electricity and sell the produced electric power in the open electricity market. We assume the knowledge of all physical properties of the available generating units at his/her disposal. That is, the heat-rate curves, start-up requirements, minimum up and down times, and operating ranges of all units are known. Note that the minimum up-time and minimum-down time constraints complicate the decision of switching a unit on or off because they link some periods in the planning horizon with each other. To maximize profit, our marketer considers a planning horizon of T periods. At the beginning of a period t, he/she observes the spot-market price of natural gas, the spot-market price of electricity, and the gas demand of customers, then makes a decision on how much gas to purchase or sell in the open market and how much power to generate. The decision attempts to maximize future revenue given the current state—gas storage and status of the electric generators—of the system.

To fix notations, we assume that there are I generating units. The status of generator i at time t is denoted by $u_t^i \in \{0,1\}$. Due to the minimum up and down time constraints, $u^i = (u_1^i, \dots, u_T^i)$ can only take values from the set of feasible solutions U_i. The amount of power generated z_t^i is limited by the operating capacity of the generator as well as the status of the unit at time t. We write these constraints in the form $z^i \in Z_i(u^i)$. The energy, y_t^i, required to run the unit at time t—start-up and generation requirements—is a function of u_{t-1}^i, u_t^i and z_t^i which we denote by $g_i(u_{t-1}^i, u_t^i, z_t^i)$. To simplify notations, we use $u \in U$ and $z \in Z(U)$ to denote the constraints $u^i \in U_i$, $i = 1, \dots, I$, and $z^i \in Z_i(U_i)$, $i = 1, \dots, I$, respectively. Furthermore, we write the constraints $y_t^i = g_i(u_{t-1}^i, u_t^i, z_t^i)$, $t = 1, \dots, T$, $i = 1, \dots, I$, in the form $y = g(u, z)$.

To model uncertainty in prices and demand, we use a set of scenarios. Each scenario represents a future forecast that could occur with a certain probability. A scenario in our case is a vector of T tuples each of which contains three pieces of information: natural gas price, electricity price, and gas demand. We use the letter s to denote a scenario and S to denote the total number of scenarios in the model. For example, binary decisions associated with generator i under scenario s are referred to by $u^{s,i}$ while the amount of power generated on i is denoted by z^i. It is understood that z^i is the collection of vectors $z^{s,i}$, $s = 1, \ldots, S$. The probability of scenario s is denoted by p_s. One can think of scenarios and their associated probabilities as a discrete approximation of the joint distributions of prices and demand. This approximation yields (under mild assumptions) solutions that converge to the solution of the stochastic model as the number of scenarios increases.

The use of multiple scenarios to model uncertainty requires adding a new set of constraints to the model. If two scenarios are indistinguishable at time t, then the decisions adopted by these scenarios at t must be the same. This condition is known as non-anticipativity. It simply requires a one-to-one correspondence between the scenario tree and the decision tree. Two scenarios are indistinguishable at t if the information—gas prices, gas demand, and electricity prices—associated with them, prior and up to t is the same. Mathematically, if s_1 and s_2 are indistinguishable at t, then we must enforce $u_t^{s_1,i} = u_t^{s_2,i}$, $i = 1, \ldots, I$. We assume that these constraints are embedded in the sets of feasible decisions U and $Z(U)$. We refer the reader to [4] and [12] for more information.

Given a certain amount of fuel, $y_t^{s,i}$, to be used in electric generation, the goal of the marketer is to maximize profit from electric production. That is, $\{\max_{u,z} E(cz) : u \in U, z \in Z(U), g(u,z) = y\}$ where $E(.)$ denotes the mathematical expectations and $c = (c_t^{s,i})$ is a row vector that represents the electricity spot-market price at the location of generator i during period t under scenario s. On the other hand, if we assume that the amount of power that needs to be generated at any time period and under any scenario is known, the gas inventory problem is a linear program which we write in the form $\{\min_x E(fx) : Ax = b - By, x \geq 0\}$, where x represents the level of gas in the reservoir, amounts of gas bought and sold, and any other variables that may be needed to represent the activities related to natural gas. For those entries of x that represent purchasing natural gas, the row vector f contains the gas spot-market prices. Other entries in f are filled appropriately in correspondence with x. The vector By represents the contribution of electric power generation into the gas model.

By merging the natural gas and the electric generation models, the goal of the marketer is to $\{\min E(fx - cz) : Ax + By = b, x \geq 0, y = g(u,z), u \in U, z \in Z(u)\}$. This model is a mixed-integer program. Furthermore, its continuous relaxation is non-linear and non-convex. In the following section, we use Bender's decomposition to handle it effectively.

3 Bender's Decomposition

Assuming that the amount of power that needs to be generated at each time period and under each scenario is known, the gas inventory problem is a linear program $\psi(y) = \{\min_x fx : Ax = b - By, x \geq 0\}$. The function ψ is a convex polyhedron. As a result, available dual solutions can be used to construct a lower bound on the value of ψ. To do so, note that if π is a dual feasible solution, then $\psi(y) \geq \pi[b - By]$ for any y. Therefore, the gas inventory problem can be approximated using a set of K cuts—constraints—of the form $\Psi \geq \pi^k[b - By]$, $k = 1, \ldots, K$, where Ψ is the approximate value of ψ and π^k is the kth dual solution available. The approximate description of ψ is passed on to the electric-generation model to determine an optimal generation level. The results, y, of the electricity model are passed to the gas inventory problem and a new cut is created. The process is repeated until an optimal solution is found. This approach is known as Bender's decomposition. A comprehensive discussion of this subject can be found in [11].

Here is a formal description of the suggested algorithm.

- Initialization. Set the number of cuts K to zero and Ψ to $-\infty$.
- General Step.
  1. Solve the power generation problem, $\{\min -E(cz) + \Psi : u \in U, z \in Z(U), g(u,z) = y, \Psi + \pi^k By \geq \pi^k b,\ k = 1, \ldots, K\}$.
  2. Solve the gas inventory problem $\{\min E(fx) : Ax = b - By, x \geq 0\}$. Let π^{K+1} be the row vector of the resulting dual multipliers.
  3. If $\Psi = \pi^{K+1}[b - By]$, then the current solution is optimal. Terminate the process.
  4. Set $K \leftarrow K + 1$. Go to 1.

The electric-power generation program $\{\min -E(cz) + \Psi : u \in U, z \in Z(U), g(u,z) = y, \Psi + \pi^k By \geq \pi^k b,\ k = 1, \ldots, K\}$ has similar structure to that of the unit commitment problem [10]. If one relaxes the coupling constraints $\Psi + \pi^k By \geq \pi^k b,\ k = 1, \ldots, K$, then each generator has its own optimization problem that can be solved using dynamic programming. One may argue that the original program $\{\min E(fx - cz) : Ax + By = b, x \geq 0, y = g(u,z), u \in U, z \in Z(u)\}$ can be solved directly by relaxing $Ax + By = b$. However, due to the large size of the fuel model in applications, we expect the number of cuts to be smaller than the size of b.

4 Lagrangian Relaxation

Note that the constraint $\Psi + \pi^k By \geq \pi^K b$ is satisfied as an equality in an optimal solution to $\{\min -E(cz) + \Psi : u \in U, z \in Z(U), g(u,z) = y, \Psi + \pi^k By \geq \pi^k b,\ k = 1, \ldots, K\}$. Let us associate a multiplier $\lambda_k \geq 0$ with each of the constraints $\Psi + \pi^k By \geq \pi^k b,\ k = 1, \ldots, K-1$. By relaxing these constraints, the objective function becomes $\mathcal{L}(\lambda) = -E(cz) + \Psi - \sum_{k=1}^{K-1} \lambda_k[\Psi - \pi^k[b - By]]$ or $-E(cz) + \sum_{k=1}^{K-1} \lambda_k \pi^k[b - By] + [1 - \sum_{k=1}^{K-1} \lambda_k]\Psi = -E(cz) + \sum_{k=1}^{K} \lambda_k \pi^k[b - By]$ where λ_K is defined to be $1 - \sum_{k=1}^{K-1} \lambda_k$.

The Lagrangian function can be written as $\mathcal{L}(\lambda) = \sum_{i=1}^{I} \mathcal{L}_i(\lambda) + \sum_{k=1}^{K} \lambda_k \pi^k b$, where $\mathcal{L}_i(\lambda) = \{\min_{u,z} \sum_{s=1}^{S} \sum_{t=1}^{T} [-p_s c_t^{s,i} z_t^{s,i} + \phi_t^{s,i} g_i(u_{t-1}^{s,i}, u_t^{s,i}, z_t^{s,i})] : u^i \in U^i, z^i \in Z_i(u^i)\}$. The constant $\phi_t^{s,i}$ is defined to be $\sum_{k=1}^{K} \lambda_k \pi^k B_t^{s,i}$. The value of $\phi_t^{s,i}$ is the average price of supplying fuel to generator i at t under scenario s. It is worthwhile mentioning that this cost could be negative. An example is the case in which the holding cost of gas is high and electricity prices are low. Although the utility does not profit from the direct generation of electricity, it benefits from reducing the cost of its inventory.

The value of $\mathcal{L}_i(\lambda)$ is computed using stochastic dynamic programming [2]. This approach is the same as that adopted in solving a single-generator problem in stochastic unit commitment. Starting from the end of the planning horizon, backward recursion is used to find an optimal strategy, u, for the generating units. The only difference between our case and that of the unit commitment is that the cost function of a generator could be concave and negative. However, this does not cause any problem since this is a single-variable function and its minimum is attained by setting the variable to the maximum generating capacity.

In order to find an optimal set of multipliers λ, we maximize $\mathcal{L}$ over the feasible set $\lambda_k \geq 0$, $k = 1, \ldots, K-1$, $\lambda_K = 1 - \sum_{k=1}^{K-1} \lambda_k > 0$. The function $\mathcal{L}$ is piecewise linear and concave. To find its maximum, we use a line-search approach that is based on subgradient information [1]. A subgradient, $\partial\mathcal{L}/\partial\lambda$, for $\mathcal{L}$ at λ can be computed as $(\partial\mathcal{L}/\partial\lambda)_k = \pi^k[b - By]$, $k = 1, \ldots, K-1$ and $(\partial\mathcal{L}/\partial\lambda)_K = 0$. Here is the description of the suggested algorithm.

- Initialization. Set $\lambda_k \leftarrow 0$, $k = 1, \ldots, K-1$, and $\lambda_K \leftarrow 1$. Select an upper bound, $\theta^* > 0$, for the step size. Set the number of iterations $l \leftarrow 1$.
- General Step.
  1. Define $\Delta \in \Re^K$ to be the search direction, where
  $$\Delta_k = \begin{cases} 0 \text{ if } \lambda_k = 0 \text{ and } (\partial\mathcal{L}/\partial\lambda)_k < 0, \\ (\partial\mathcal{L}/\partial\lambda)_k \text{ otherwise.} \end{cases}$$
  2. If $\Delta = 0$, terminate. The current solution λ is optimal.
  3. Compute $\tilde{\theta} = \min\{-\lambda_k/\Delta_k : \Delta_k < 0,\ k = 1, \ldots, K-1\}$.
  4. Using a derivative-free line-search method [1], determine an optimal step size θ in the range $[0, \min\{\theta^*, \tilde{\theta}\}]$.
  5. Update the multipliers $\lambda \leftarrow \lambda + \theta\Delta$ and normalize the new λ.
  6. Set $l \leftarrow l + 1$. If l is less than the permitted number of iterations, go to 1.

In the following section, we apply branch-and-bound and the aforementioned decomposition approach to the problem of maximizing the marketer's return $\{\min E(fx - cz) : Ax + By = b, x \geq 0, y = g(u,z), u \in U, z \in Z(u)\}$.

5 Numerical Experience

The goal of our numerical tests was to determine the effectiveness of Bender's decomposition and Lagrangian relaxation in solving $\{\min E(fx - cz) : Ax + By = b, x \geq 0, y = g(u,z), u \in U, z \in Z(u)\}$. The test problems were based on the operations of a utility in the midwestern US. We adopted three methods for solving the underlying model. The first is a direct approach in which the mixed-integer model—fuel inventory and electric generation scheduling—was solved to optimality using branch-and-bound. To do so, we used the branch-and-bound implementation provided by OSL—the Optimization Subroutine Library of IBM [6].

The second was the decomposition approach of Section III. The fuel inventory $\{\min E(fx) : Ax = b - By, x \geq 0\}$ was handled using the dual simplex method routine `ekksslv` of OSL. The scheduling of the generating units $\{\min -E(cz) + \Psi : u \in U, z \in Z(U), g(u,z) = y, \Psi + \pi^k By \geq \pi^k b,\ k = 1, \ldots, K\}$ was performed to optimality using the branch-and-bound routine `ekkmslv` of OSL. Cuts were added to the model as they were generated by the linear programming solver.

The third used Lagrangian relaxation of Section IV to incorporate the cuts $\Psi + \pi^k By \geq \pi^k b$ into the objective function of the electric scheduling problem. This approach—Bender's decomposition coupled with Lagrangian relaxation—was implemented using the C language with considerable effort put into optimizing the code and tuning its performance.

Table 1: Exact Solutions Using Pure Branch-and-Bound or Bender's Decomposition and Branch-and-Bound

	Direct		Bender's	
Size	Optimum	CPU	K	CPU
1×30	-19,320	1	29	30
2×30	-40,020	3	27	268
5×30	-108,192	7	23	233
10×30	-206,120	283	29	4973
20×30	-371,040	228	19	16,303
1×100	-113,275	1	7	9
5×100	-500,363	296	38	29,383
$\geq 10 \times 100$	—	—	—	—
1×1000	-1,067,910	286	4	405
$\geq 2 \times 1000$	—	—	—	—

For the fuel inventory problem, we assumed that gas can be bought from or sold to the market at a charge. At each period t and for every scenario s, there is a balance constraint requiring that the inventory level at the end of t must be equal to the storage level at the beginning of t minus the flow out the utility's system—gas delivered to customers, burnt to generate power, or sold on the spot—plus the amount of fuel purchased during t. Another constraint is that of the maximum capacity of the storage tank. In this simple set up, the matrix B contains zeros and ones only.

The stochastic information—gas price, electricity price, and gas demand—was stored in a compact form similar to that used in storing stochastic linear programs [3]. Only distinct information was stored to reduce computer requirements. In this case, the size of the problem can be determined using the number of distinct nodes in the scenario tree and the number of generating units used. We do not discuss the details of this scheme in this document. We refer the reader to [13] for a comprehensive description of our data structure.

Table I presents the results for cases that we were able to solve to optimality. The first column contains the size of the problem. For example, a problem of size 20×30 refers to a model in which the number of generating units was 20 and the number of distinct nodes in the scenario tree was 30. Hence, the number of binary variables in the model is $20 \times 30 = 600$. The optimal objective function value is listed in the second column. Note that these values are negative indicating that we made profit from purchasing natural gas in certain periods and selling it when prices were higher, or from generating electricity and selling it in the open market. The execution times are reported in the column labeled "CPU" Execution times were measured in seconds on an IBM RS/6000 running AIX 4.1 with 125 MBytes of memory and a 133 MHz processor.

To study the effectiveness of Bender's decomposition, we decomposed the problem and passed cuts from the fuel inventory program to the electric generation problem. The electric generation problem was solved to optimality using branch-and-bound. As expected, the optimal objective values were the same as those of column 2. Although the number of cuts, K, decreased as the model became larger, the execution time of this approach was disappointing.

Entries containing "—" in them indicates that we could not solve the problem since the maximum CPU time of 8 hours was reached.

Table II presents the results obtained from using Lagrangian relaxation to handle the power generation problem as described in Section IV. In this case, our approach constructs an upper and lower bound on the optimal value of the objective function. The column labeled "Upper" contains the value of the best feasible solution obtained. The duality gap is given in "Gp%" as a percentage of the upper bound. In general, the gap seems to become smaller as the size of the problem increases. In "Er%", we display the error of the best feasible solution obtained. This error is computed using the upper bound of Table II and the optimal value from Table I. The Lagrangian relaxation seems to produce good results—error less than 4%.

The last column in Table II contains the CPU time in seconds. It is clear that this method outperforms the others when it comes to execution time. Furthermore, we were able to solve significantly larger models using this approach in a reasonable time. Since the mixed-integer program was not solved to optimality in this case, the number of cuts was large. We limited the maximum number of cuts that could be to $K = 40$ and $K = 100$. The size of K did not affect the quality of the solution obtained significantly. The largest model that we solved had 50,000 binary variables and took less than 15 minutes to obtain a solution within 1.03% of the optimal solution.

6 Conclusion

We discussed a stochastic formulation for the problem of controlling fuel inventory in a deregulated environment. Our model links the gas and electricity markets. Stochasticity in gas price, gas demand, and electricity price, is modeled using a set of scenarios. The resulting model is a mixed-integer program that we solve using branch-and-bound or through a decomposition approach that iterates between the fuel inventory and power generation problems. In the case of large models, the power generation problem can be approximated by solving its Lagrangian dual. We solved problems with 50,000 binary variables to within 1.2% of the optimal solution in less than 4 minutes.

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Table 2: Approximate Solutions Using Bender's Decomposition Combined with Lagrangian Relaxation

Size	K	Upper	Gp%	Er%	CPU
1×30	40	-19,243	15.69	0.40	1
	100	-19,302	15.37	0.09	2
2×30	40	-39,888	3.21	0.33	3
	100	-39,888	3.09	0.33	5
5×30	40	-107,989	2.17	0.19	1
	100	-107,989	2.15	0.19	2
10×30	40	-200,071	5.20	2.93	3
	100	-201,126	4.61	2.42	7
20×30	40	-360,418	3.21	2.86	4
	100	-360,616	3.13	2.81	12
1×100	40	-113,095	0.16	0.16	1
	100	-113,095	0.16	0.16	2
5×100	40	-479,514	4.45	4.17	3
	100	-481,329	4.05	3.80	13
10×100	40	-582,122	9.46	—	3
	100	-584,701	8.94	—	28
20×100	40	-1,017,138	6.99	—	10
	100	-1,019,303	6.75	—	27
50×100	40	-2,593,743	4.49	—	17
	100	-2,596,043	4.40	—	62
1×1000	40	-1,067,661	0.15	0.02	20
	100	-1,067,671	0.15	0.02	60
2×1000	40	-2,050,221	1.90	—	39
	100	-2,051,560	1.80	—	143
10×1000	40	-5,680,656	4.47	—	97
	100	-5,704,408	3.89	—	326
50×1000	40	-23,367,161	1.16	—	239
	100	-23,392,437	1.03	—	755

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