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Research Report

Role of Statistics in Achieving Global Consistency of Tolerances

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Abstract: This paper proposes and defends a thesis that global consistency of tolerances is achievable only by consistent use of statistics across specification, production, and inspection. It starts with a global view of variation in a product of an engineering enterprise, embodied in two major axioms: imprecision in manufacturing and uncertainty in measurements. It traces the successful evolution of statistical practices in production and inspection to deal with these variations. Finally, designer specified tolerances are brought into the global picture. Challenges that still lie ahead are summarized at the end.
Keywords: tolerances, statistics, consistency.

> "The conception of statistics as the study of variation is the natural outcome of viewing the subject as the study of populations; for a population of individuals in all respects identical is completely described by a description of any one individual, together with the number in the group. The populations which are the object of statistical study always display variation in one or more aspects."
>
> R. A. Fisher, *Statistical Methods for Research Workers*

1. INTRODUCTION

The main theme of this 6th CIRP International Seminar on Computer Aided Tolerancing is *global consistency of tolerances*. Let's start with the hypothesis that global consistency warrants a uniformity of principles and practices applied across an enterprise - a set of engineering firms in our case - and that it doesn't exist today. Observation indicates that industry struggles with this problem, and has developed islands of practices that achieve local consistency by *ad hoc* means. Theory and principles to unify these practices are still evolving.

This paper advances a thesis that global consistency of tolerances is achievable only by consistent use of statistics across the engineering enterprise. To put it simply, this is because the only thing that is global is *variation*, and statistics is the appropriate discipline for studying variation as observed by Ronald Fisher [Fisher, 1950]. The rest of the paper provides supporting evidence for this thesis.

We start with a global view of variation in Section 2, and argue that inevitable variations in production and inspection influence product specification. Section 3 describes how industry uses statistical tools to deal with variations in production. Section 4 addresses the same issue for variations in inspection. Statistical metrics to characterize variations in design are discussed in Section 5. The paper concludes with a summary of challenges ahead.

2. A GLOBAL VIEW OF VARIATION

We begin with a basic question: Why do engineers tolerance industrial artifacts? The answer lies in the fact that we can never manufacture exact parts, and even if we do we can never tell by measurement that they are exactly the same This inevitability of variability is embodied in the following two axioms:

1. *Axiom of manufacturing imprecision*: All manufacturing and assembly processes are inherently imprecise and produce parts and products that vary.
2. *Axiom of measurement uncertainty*: All measurement systems and measuring instruments contain inherent sources of error, and the result of a measurement is always subjected to some uncertainty.

The axiom of manufacturing imprecision implies that no two manufactured objects are exactly the same, and they are definitely not the same as the ideal shape the designer had envisioned. The axiom of measurement uncertainty implies that measurements can never guarantee with certainty that the manufactured object has conformed to the design specification – at best, we can only assure conformance with a high degree of confidence. It may be argued that these two axioms are provable facts from basic laws of physics; we will not succumb to this temptation here and leave them as axioms without proof. There is ample industrial evidence to suggest that these axioms have never been violated.

The designer now has the challenge of dealing with the inevitable variations in the parts of the product he develops. Fortunately, the final functionality of his product is not adversely affected by the variations in its constituent parts as long as they can be kept within some bounds. So he consciously accommodates these variations in his design and documents them by specifying tolerances. In fact, a product specification is incomplete – indeed, the product is not manufacturable – without explicit information about allowable variations in the form of tolerances. The connection between product specification, production, and inspection is emphasized in Shewhart's quality control cycle [Shewhart, 1986] shown in Figure 1.

It is instructive to note that Shewhart viewed the specification-production-inspection process as a cycle, highlighting the iterative nature of the problem. Tolerance specification is not an isolated activity; it strongly affects the production and inspection phases and, in turn, is affected by them. Because of this we will first examine how variations in production and inspection are handled in Sections 3 and 4, respectively.

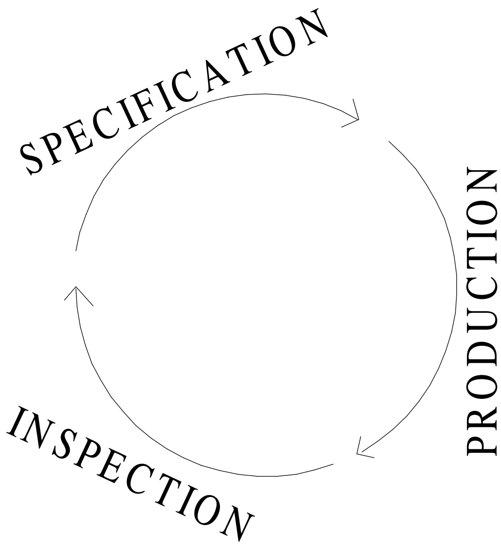

Figure 1. *Shewhart's quality control cycle of specification, production, and inspection.*

3. VARATION IN PRODUCTION

Controlling variation in the *output* of manufacturing processes is the chief concern of any quality conscious production operation. The output is the effect of several causes – namely the input, production system parameters, and the environment – and the goal in manufacturing process control is to reduce variations in these causes to the minimum while operating within economic constraints. The only successful practical solution known to date to this problem is provided by *statistical process control* (SPC).

3.1 Process control

A basic premise in SPC is that the causes of variation can be grouped under two categories: assignable causes and chance causes. Variation due to assignable causes can be reduced to vanishingly small level, but this is not possible for chance causes owing to economic reasons. A manufacturing process is in a "state of statistical control" if it is only under the influence of chance causes. In practice, control charts as shown in Figure 2 provide simple graphical methods for evaluating whether or not the process has attained, or continues in, a "state of statistical control" [ISO7870, 1993]. So current SPC practices, as codified by ISO, are based on an operational definition of a state of statistical control and are dominated by control chart methodology.

Is there a more precise definition of being in a "state of statistical control"? It can be argued that being in a state of statistical control is the same as a random process being stationary [Voelcker, 1996]. Intuitively a stationary random process is one whose

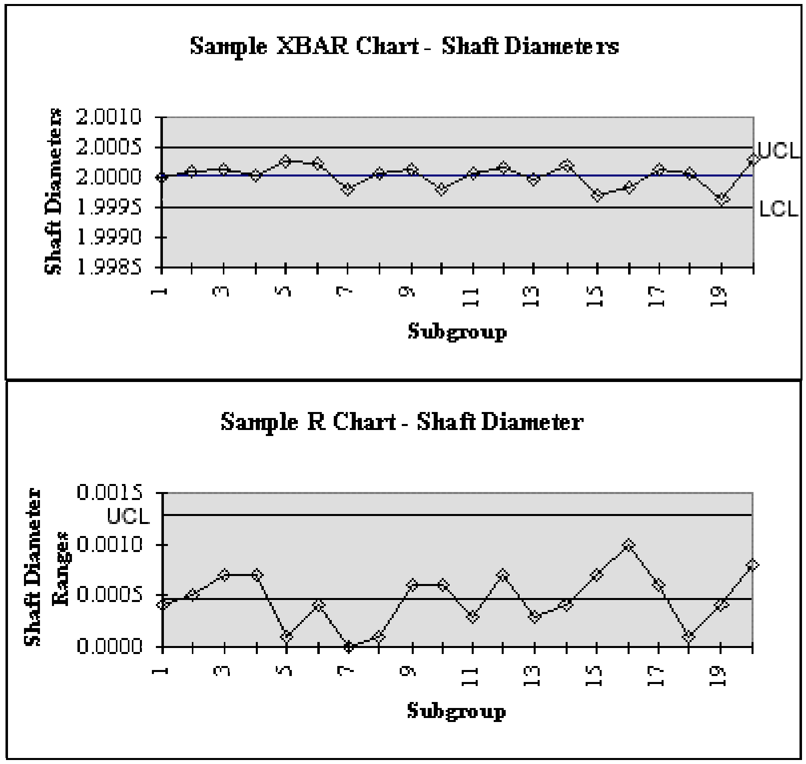

Figure 2. *Examples of X-bar and R control charts used widely in statistical process control. UCL = upper control limit, LCL = lower control limit.*

distribution remains the same as time progresses, because the underlying mechanism does not change with time [Parzen, 1962]. One can then view control charts as practical means of ensuring that a given process is indeed stationary.

An important characteristic of SPC is that the concept of statistical control is *intrinsic* to the process. The upper and lower control limits established for control charts are derived from the output of the process itself. Therefore, it is perfectly reasonable to conclude that an old milling machine is operating in a state of statistical control even though parts machined in it have large variations compared to, say, a modern CNC milling machine. This brings us to the second question on variability in production: Is the process *capable* of producing good parts? This leads us to process capability.

3.2 Process capability

The notion of process capability is relatively old. Table I is an extract from a product design handbook listing of variations that can be expected from typical manufacturing processes to produce rounded holes [Bralla, 1986]. This shows that even if a process is in control, it may or may not be capable of achieving the accuracy we want.

Table I characterizes capability by the range of variation. This is a simple statistical metric that is easy to measure and understand. It is also *intrinsic* to the process. Modern approach to process capability relies on other statistical metrics, such as the mean and the standard deviation, to characterize the variation. These are captured as non-dimensional ratios in various process capability indices (PCI) [Kotz et al., 1993], such as

Process	Usual material	Normal dimensional accuracy of 25mm hole, in mm	Surface finish, in µm	Maximum depth-to-diameter ratio
Drilling	All machinable	+0.2,-0.05	1.6-6.6	8:1
Reaming	All machinable	±0.025	0.8-3.2	8:1
Boring	All machinable	±0.04	0.4-6.3	5:1
Gun drilling	All machinable	±0.05	0.8-2.5	300:1
Sand casting	Gray iron, aluminum	±1.50	6.3-25	1.5:1
Die casting	Aluminum, zinc	±0.10	0.8-1.6	4:1
Injection molding	Thermoplastics	±0.10	0.2-3.2	2:1 (blind) 4:1 (through)
Punching	Sheet metal	±0.15	0.8-1.6	1:1
Power metal	Iron, steel, bronze	±0.06	0.2-1.6	1.35:1 (blind) 4:1 (through)

***Table I**. Capabilities of manufacturing processes that produce rounded holes.*

$$C_p = \frac{USL - LSL}{6\sigma},$$

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right), \text{ and}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\mu - \tau)^2 + \sigma^2}}$$

where *USL* is the upper specification limit, *LSL* is the lower specification limit, τ is the target value, μ is the mean, and σ is the standard deviation.

Modern approach to process capability differs from the old approach in two major aspects. First is the use of statistical metrics other than just the range to characterize variation. Second, in defining PCIs as non-dimensional ratios it introduces the specification limits *LSL* and *USL* that are *extraneous* to the process itself. Thus the PCIs do not provide metrics intrinsic to the capability of a process itself; they indicate how capable a particular process is in staying within some externally specified limits.

We can now examine the relationship between process capability and process control. A popular view is that a process capability study is used to choose or qualify a particular process before mass production begins, and then process control is imposed to make sure that it stays capable. This may suggest that there is no role for process capability studies once mass production begins, but, in practice, the PCIs are computed in parallel with plotting control charts to keep a dynamic account of the capability. In fact, many researchers view PCIs computed in the absence of a state of statistical control as meaningless. Modern SPC practice includes both control charts and PCIs, thereby monitoring statistical control as well as statistical capability of the manufacturing process.

4. VARIATION IN INSPECTION

Parts are inspected to ensure that they conform to design specifications or to ascertain the quality of their manufacturing processes. As noted in Section 2, measurements made during the inspection process are always subject to some uncertainty. Moreover, it is not always possible to measure and inspect every part that is produced. Shortcuts, such as inspecting only a sample, are often used which then introduce an additional level of uncertainty about the final result. Let's look at these in some detail.

4.1 Estimation by sampling

The goal of acceptance sampling is to come up with an inspection plan requiring only a small number of parts from a population to be inspected to draw a conclusion about the acceptance of the whole lot [ISO8550, 1994; Duncan, 1986]. First popularized by military in its procurement operations, such sampling plans are now routine in industry. Literature on this topic is quite vast and the reader should consult the references quoted above for details. A scientific basis for acceptance sampling is provided by the statistical estimation theory.

The quantity that is estimated can differ from application to application. In some sampling plans, called inspection by attributes, the quantity that is estimated is the fraction of the population that does not conform to specification limits. Other sampling plans that deal with inspection by variables estimate the population mean, standard deviation, or some ratios of these two.

In all these plans a probabilistic statement of the degree of confidence that they are indeed good estimates of the correct values for the population accompanies the estimated values. In fact, one can design a sampling plan based on an acceptable degree of confidence (e.g., acceptable quality level (AQL)) for the consumer.

4.2 Measurement uncertainty

In spite of our earlier assertion in Section 2 about the inevitability of measurement uncertainties, only recently this problem has received much attention. One plausible reason is the increasing demand for high precision in mass produced goods. When uncertainties in measurements were an order of magnitude smaller than the variations they were expected to detect (the infamous 10:1 rule), it was possible to ignore them in practice. As we move to high precision world, however, the margin has narrowed to as low as 4:1 or worse, forcing us to pay greater attention to measurements.

The international authority on measurement uncertainty is the ISO's "Guide to the expression of uncertainty in measurement" [GUM, 1993]. It defines an expanded uncertainty U as a "quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed" to the particular quantity subject to measurement. Figure 3 illustrates how the expanded uncertainty U shrinks the specification zone for the purpose of conformance assessment. Only those parts whose measurements fall within the conformance zone can be accepted as satisfying the specification with high degree of confidence.

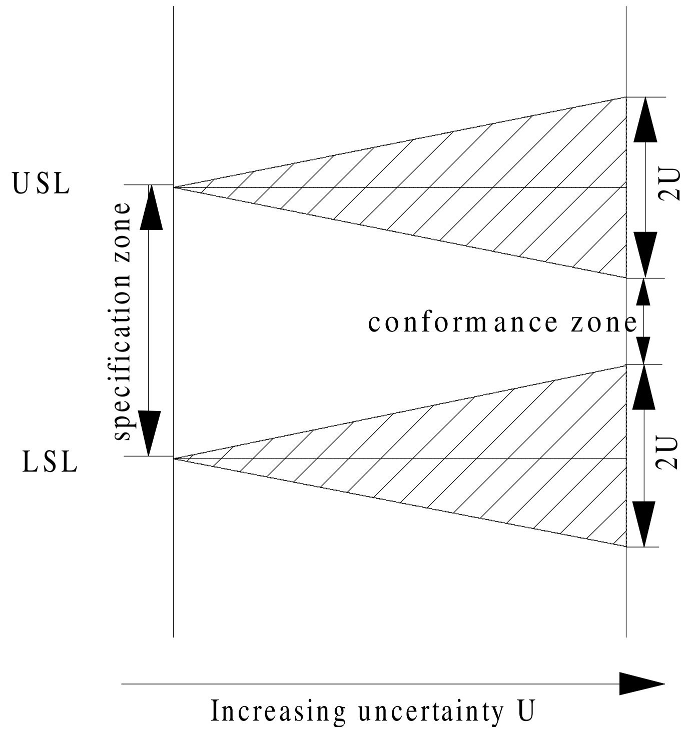

Figure 3. *Influence of uncertainty U on the assessment of measurement results. USL = upper specification limit, LSL = lower specification limit. [Weckenmann et al., 1998]*

The next natural question is how measurement uncertainty affects the assessment of a collection of parts. It clearly has an impact on sampling plans for inspection. It also affects process capability analysis [Weckenmann et al., 1998] because PCIs are calculated using measured values that are sampled from a population. These important issues are still under investigation.

5. BACK TO TOLERANCING

Based on the evidence presented in Sections 3 and 4, it is clear that statistics plays a central role in dealing with variations in production and inspection. We will now see what is the role of statistics in tolerancing. First a few words about tolerancing.

Tolerancing is a specification. Formally, it is a representation that has a syntax and associated semantics. Therefore, it is treated like a language. ISO/TC 213 is the ISO technical committee charged with standardizing the language, and it calls this the "geometrical product specification" (GPS) language. GPS replaces the older acronym

GD&T (geometrical dimensioning and tolerancing) due to the recognition that we are indeed dealing with product specification. ISO/TC 213's charter includes specification as well as verification (particularly, metrology of inspection) and is expected to harmonize these practices. Although the goal of ISO/TC 213 is to define a product specification language, it deals mainly with defining a language for specifying features. Why is this the case?

5.1 From product to features

Figure 4 shows a simple hierarchy of taxonomy that will be useful in understanding GPS. A product is recursively defined as an assembly of subassemblies and parts. Parts, in turn, are composed of features, which are then subjected to various conditions (or, synonymously, constraints). A couple of observations are worth making here.

First, as noted above, the recursive definition of a product ends in parts. It has proven to be remarkably difficult to come up with an acceptable definition for parts. A scientific approach is to define a part as that which can be modeled as a solid, say, as a regular set. Industry, however, is used to calling anything that has a "part number" as a part. This has some practical appeal because it is more in line with the way an engineering organization, as opposed to an individual engineer, views a product. A second observation is that the conditions, or constraints, on features come in two types: first is intrinsic, such as size and form, and these are on the features themselves; second is relational, such as position, and these are defined for two or more features.

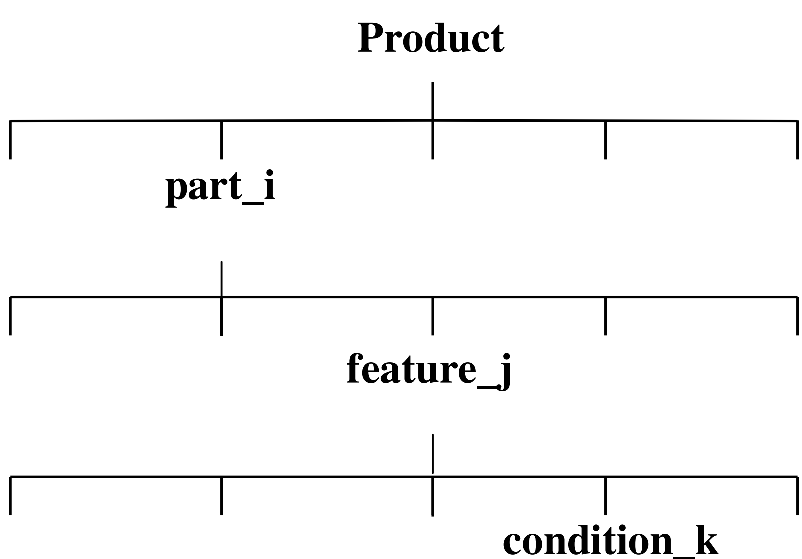

Figure 4. *A hierarchy of taxonomy in ISO's GPS language.*

It turns out that the difficulty in arriving at a good definition of parts to end the recursion on the product definition is not a showstopper. A key to overcoming this difficulty is to focus on features, for the relationships among features can be defined irrespective of whether the features belong to the same part or to different parts. This is an important observation because it allows standards to focus exclusively on features and constraints imposed on these features; these can then be combined upwards to specify parts or products as needed. This is the conclusion of a recent study conducted by an advisory group ISO/TC 213/AG 10 of international experts that looked into the needs of assembly-level tolerancing.

5.2 Risk at the product level

In tolerance analysis we start with variations at the feature-level and propagate it up to the product-level (see Figure 4). In tolerance synthesis (also called tolerance allocation) we start with variation at the product-level and flow this down the hierarchy to the feature-level. In either case, variation at the product-level is associated with some risk and this risk is minimized subject to some economic constraints. Risk, whether in tolerancing or in any other endeavor, is best characterized using the language and methods of statistics.

To be specific, let's assume a functional relationship between product-level characteristic vector $\boldsymbol{y}$ and feature-level characteristic vector $\boldsymbol{x}$ as $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$. At this stage, these characteristics are not restricted to geometry. For example, a component of $\boldsymbol{y}$ can very well be the stress at a critical point in the product. There are several statistical metrics that can be used to characterize the variation in $\boldsymbol{y}$ at the product-level, such as:

1. *Range*. This is the simplest statistical metric. It is closely related to the following metric, which is more popular.
2. *Maximum deviation from the target*. This can be either one-sided deviation (e.g., $y \leq 0.05$, target for y is 0.00) or two-sided deviation (e.g., $9.95 \leq y \leq 10.05$, target for y is 10.00). Sometimes the two-sided variation is split asymmetrically (e.g., $9.92 \leq y \leq 10.04$, target for y is 10.00).

   This is the familiar Chebyschev metric for variation and follows the common practice of worst-case tolerancing. Its power lies in its simplicity, but it brings with it a false sense of determinism (e.g., *100%* interchangeability, zero defect). As we saw in Sections 3 and 4, we can never assure in down-stream activities in production and inspection that the characteristic in question is within specified bound(s). We can only make statistical statements about the confidence we have in making such conformance assessment.
3. *Root-Mean-Square (RMS) deviation from the target*. This is the familiar Gaussian metric for variation. It is also the metric used to control variation in Taguchi's quadratic loss function approach [Srinivasan, 1997].
4. *Fraction of the population of products falling outside a limit*. This is a better statistical metric than the maximum deviation from the target. It is used in the six-sigma approach to design for quality.

If these are some of the popular metrics for the product-level characteristics $\boldsymbol{y}$, what are then the appropriate metrics for feature-level characteristics $\boldsymbol{x}$? A little reflection on our discussion in Section 5.1 tells us that the issues are the same, for what is a part to a customer is a product to a supplier due to the recursive nature of the problem. So, to maintain consistency, relationship between features, whether they belong to the same part or to different parts, will be specified using the same types of statistical metrics described above. In fact, there are compelling economic pressures that favor the use of statistical metrics to specify feature-level variations. Following this reasoning, ISO/TC 213/WG 13 on Statistical Tolerancing of Mechanical Parts is working towards defining a statistical extension to the GPS language using several statistical metrics similar to those mentioned above [Srinivasan, 1997].

6. CHALLENGE AHEAD

In spite of past and recent developments in the application of statistics to specification, production, and inspection, several challenges lie ahead. As noted in the body of the paper, research and standards organizations are working on issues in each of these three areas. A sharper definition of a "state of statistical control", better understanding and definition of PCIs, quantifying measurement uncertainty and its influence on sampling plans and PCIs, and statistical tolerancing are some of the issues that are receiving attention. To a lesser extent, efforts are underway to integrate these developments to maintain consistency across them.

REFERENCES

[Bralla, 1986] J. G. Bralla, *Handbook of Product Design for Manufacturing*, McGraw-Hill Book Co., New York, 1986.

[Duncan, 1986] A. J. Duncan, *Quality Control and Industrial Statistics*, 5th Edition, Irwin, Illinois, 1986.

[Fisher, 1950] R. A. Fisher, *Statistical Methods for Research Workers*, 11th Edition, Hafner Publishing Company, New York, 1950.

[GUM, 1993] *Guide to the Expression of Uncertainty in Measurement*, International Organization for Standardization, Geneva, 1993.

[ISO7870, 1993] ISO 7870:1993 *Control charts – General guide and introduction*, International Organization for Standardization, Geneva.

[ISO8550, 1994] ISO/TR 8550:1994 *Guide for the selection of an acceptance sampling system, scheme or plan for inspection of discrete items in lots*, International Organization for Standardization, Geneva.

[Kotz et al., 1993] S. Kotz and N. L. Johnson, *Process Capability Indices*, Chapman & Hall, London, 1993.

[Parzen, 1962] E. Parzen, *Stochastic Processes*, Holden-Day, San Francisco, 1962.

[Shewhart, 1986] W. A. Shewhart, *Statistical method from the viewpoint of quality control*, Dover, New York, 1986.

[Srinivasan, 1997] V. Srinivasan, ISO Deliberates Statistical Tolerancing, In: *Proceedings of the 5th CIRP Seminar on Computer Aided Tolerancing*, Toronto, Canada, 1997.

[Voelcker, 1996] H. B.Voelcker, private communication, 1996.

[Weckenmann et al., 1998] A. Weckenmann and M. Knauer, Causes and Consequences of Measurement Uncertainty in Production Metrology, In: *Proceedings of the 6th IMEKO Symposium on Metrology for Quality Control in Production*, Vienna, Austria, Sept. 8-10, 1998.