RC 21660(97613) February 2, 2000
Economics/Mathematics

IBM Research Report

Clearing Markets with Indivisible Supply and Demand

Jayant R. Kalagnanam

IBM Research Division
T.J. Watson Research Center
Yorktown Heights, New York 10598

Clearing Markets with Indivisible Supply and Demand

Jayant R. Kalagnanam
International Business Machines
T. J. Watson Research Center
Yorktown Heights, NY 10598

Abstract

We show that the problem of clearing markets using a continuous call auction with indivisible demand and supply requires solving hard computational problems such as the generalized assignment problem and the bin packing problem. This is in stark contrast to the case where the demand and supply is continuous and market clearing is computationally straightforward.

We examine the structure of the market clearing problem for the two institutions - the continuous call auction and the continuous double auction. The indivisible nature of the supply and demand introduces waste in allocating supply to demand and hence moves the equilibrium away from the point where the supply and demand curves intersect. An alternate formulation for assigning demand to supply that maximizes net revealed surplus is introduced. Several research issues pertaining to the allocation of the net revealed surplus to buyers and sellers are identified.

Additionally, we show that the institution of a continuous double auction is in fact an approximation of the continuous call auction where a greedy approach is used to allocate supply to demand in real time as bids and asks arrive. The approach used in a continuous double auction can be viewed as an online algorithm for solving the generalized assignment problem. Several theoretical and simulation based research issues are identified to characterize the efficiency of the continuous double auction.

Key Words: Electronic exchanges, supply, demand, market clearing, continuous double auction, continuous call auction, bin packing, multiple knapsack, generalized assignment, online algorithms, NP-hard.

1 Introduction

Electronic exchanges are becoming increasingly widespread in various industry sectors as hubs for vertical integration. As companies begin to move their procurement to the web these exchanges will need to maintain markets for various commodities and provide (among other things) one or more market clearing mechanisms. In this paper we examine the computational requirements of clearing mechanisms in markets where the supply and demand is indivisible and the volumes are not large enough to justify ignoring the indivisible nature of the demand and supply. With indivisible supply and demand the market clearing problem becomes computationally hard.

Process industries such as paper and steel are good examples of markets where the supply and demand is indivisible. In the paper industry, paper is manufactured in standard sized rolls with a few standard widths [KDTL97], [KWGMAKD98]. Similarly steel is also produced in standard sized slabs, rolls or sheets. Once again standarized geometries such as width, length and weight are used. As a result, the supply for paper and steel is an aggregated over standard sizes and this leads to an indivisible supply curve. We illustrate this with an example using the paper industry - let us assume that paper is produced using a standard widths of 2000 mm each corresponding to a weight of 5 tons. As a result of standard sizes, rolls that are partially assigned incur waste. For example, if 1700 mm from a standard roll is assigned to bids, the remaining 300 mm is often considered waste. The primary reason is that non-standard sizes are difficult to match and usually end up on secondary markets and have to be sold at much lower prices. For purposes of illustration, we assume that there are 9 suppliers in the market for paper with the following bids shown in Table 1 below.

Buyers typically bid for rolls of paper of desired width. For example, a newpaper printer might need an integer number (say 5) of 800 mm rolls. Notice however that the buyer requires that each roll be of 800 mm, and this cannot be substitued with two rolls of 400 mm. This leads to an interesting constraint on how the bids can be satisfied - each

Seller	Price $/ton	Units	Weight (tons)	Cumulative
S_1	100	1	5	5
S_2	100	1	5	10
S_3	110	1	5	15
S_4	110	1	5	20
S_5	120	2	10	30
S_6	140	2	10	40
S_7	160	2	10	50
S_8	160	2	10	60
S_9	175	2	10	70

Table 1: Seller's Asks

roll demanded by the buyer is indivisible in the sense that it has to be cut in a single piece from the one roll. This leads to an indivisible aggregate demand and the task of matching demand (presented as a list of bids) to the available supply requires finding an optimal packing of the bids into the available supply. Let us assume that the following bids for our example:

Buyer	Price $/ton	Width (mm)	Units	Weight (tons)	Cumulative
B_1	175	1200	1	3	3
B_2	165	1200	1	3	6
B_3	150	1600	1	4	10
B_4	145	800	1	2	12
B_5	145	1600	1	4	16
B_6	140	2000	1	5	21
B_7	125	800	1	3	24
B_8	125	1200	1	3	27
B_9	120	1200	2	6	33
B_{10}	120	1600	1	4	37
B_{11}	115	1600	1	4	41
B_{12}	110	2000	1	5	46
B_{13}	110	1200	1	3	49
B_{14}	110	800	1	2	51
B_{15}	105	2000	2	10	61
B_{16}	100	1200	1	3	64
B_{17}	100	800	1	2	66

Table 2: Buyer Bids

The aggregate demand and supply is plotted in figure 1. Notice that in this figure the supply and demand are discrete and indicated on the supply and demand curve with bold circles an tagged with (price, quantity). The indivisible nature of the demand and supply leads to a staircase structure for the demand and supply curve. Notice that the demand and supply curve intersect at ($120/ton, 27 tons) - however, this cannot be intepreted as the equilibrium point since supply is discrete at 25 or 30 tons for this price. An additional interesting issue arises in market clearing. In general, for any level of demand (say 27 tons at $120/ton) the total supply required is more than 27 tons since packing the individual bids (the first 7 bids in table 1) requires more than 30 tons of the available supply at this price. Since the bids are indivisible packing these 7 bids in standard size rolls of 5 tons requires 7 such rolls - however only 6 rolls are available at this price. If we are resigned to clearing less than 27 tons of demand at this price of $120/ton using the available supply of 30 tons we find that we can can atmost clear 24 tons of demand. This implies that an additional 6 tons are wasted and the sellers will not realize a value of 120x6= $ 720 for their supply. This gap arises due to the indivisible nature of the supply and demand and needs to be accounted for by the market maker.

A primary effect of indivisible supply and demand is that determining the equilibrium price using the intersection of the supply and demand curve is usually invalid since the intersection point (P^*, Q^*) might not correspond to the actual discrete demand and supply[1]. This general problem of how to determine the clearing price for markets with indivisible supply and demand is the focus of this paper.

The paper is organized as follows. Section 2 introduces three allocation problems from the operations research literature that are relevant for market clearing. These prob-

[1]This is the market clearing mechanism used in a continuous call auction.

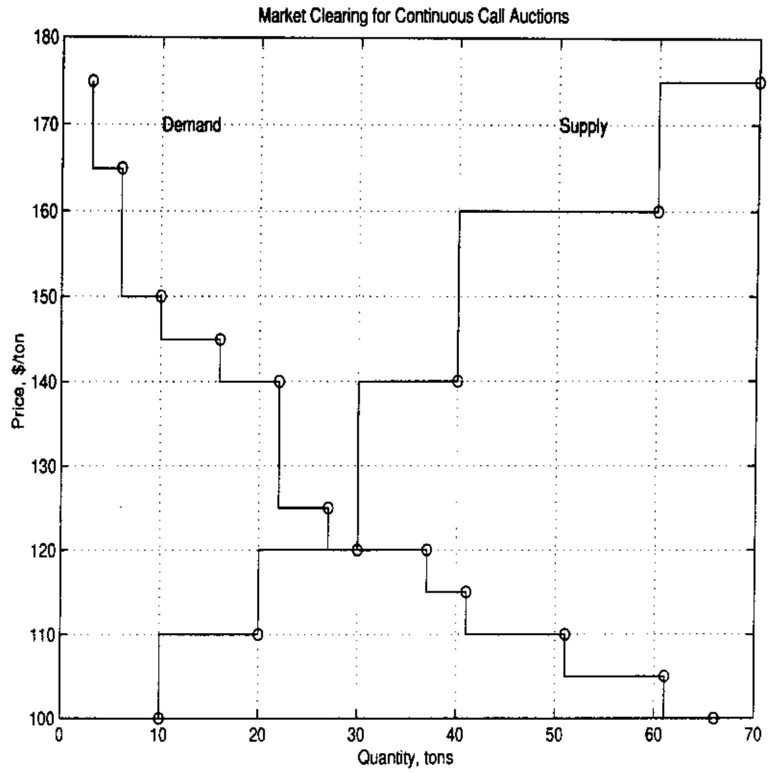

Figure 1: Market Clearing for Continuous Call Auctions

lems are introduced for different market clearing scenarios. Section 3 examines market clearing with indivisible demand and supply for the continuous call auction and the continuous double auction. The relevance of the allocation problems introduced in Section 2 is illustrated under different market clearing scenarios. The efficiency of the continuous double auction is compared to the continuous call auction. Section 4 outlines several directions for further research.

2 Computational Aspects of Solving Allocation Problems

This section introduces four allocation problems that are relevant for market clearing. A mathematical formulation for each of the problems is provided. There are no known efficient algorithms to solve these problems despite decades of efforts [MT90]. An efficient algorithm provides an optimal solution to the problem in time that is a polynomial function of the size of the problem [GJ79]. The four problems are the single knapsack problem, multiple knapsack problem, the generalized assignment problem and the bin packing problems.

2.1 Preliminaries

Let us first introduce the nomenclature that we will use to describe the problems. We will formulate these problems in the context of market clearing and hence use descriptions that refer to asks and bids.

We associate with each buyer one or more bids. Associated with each bid B_i is a bid price, p_{B_i} and a demand of one unit of weight q_{B_i} at this price. Note that for the purpose of this paper we will assume that the quantity is indivisible, i.e. the buyer wants the entire quantity q_{B_i} or nothing. In addition, we also assume that these units is non-decomposable. For example a buyer demanding a paper roll of width 1200 mm cannot be supplied 2 rolls of 600 mm width. Let there be m such bids. Notice that a buyer who needs more than one unit of the commodity would have multiple bids with the same price and quanitity.

We associate with seller one or more asks. Associated with each ask is a price p_{S_j} and the willing to supply/manufacture one unit at this price, q_{S_j}. Note that this quantity can be used to satisfy demand from multiple buyers, i.e. it is posible to cut a roll of 2000 mm into 2 rolls of 1200 and 800. Notice that a seller who is willing to sell multiple units would have multiple asks. Let there be n such asks.

Now we set up the four allocation problems. All these allocation problem are computationally intractable.

2.2 The Multiple Knapsack Problem

Consider the case where we identify an equilibrium clearing price p^* based on some consideration. The first issue relates to identifying the quantity of demand that can be cleared with the supply available at this level. Consider all the bids with $p_{B_i} \geq p^*$, and say that there are M such bids. Similarly, consider all asks with $p_{A_j} \leq p^*$ and say that there are N such asks. Notice that the bids and asks are such that all bids have $p_{B_i} \geq p_{A_j}$ for $i \in M$ and $j \in N$.

The goal is to find an assignment of bids to the asks such that the total assigned quantity of demand is maximized. That is, for each ask $j \in N$, we need to choose a subset S_j of bids in $i \in M$ to be assigned to the ask A_j, such that:

(1) All S_j's are disjoint. (Each bid is assigned to at most one ask.)

(2) $\sum_{i \in S_j} q_{B_i} \leq q_{A_j}$, for $i = 1, \ldots, M$. (Total weight of bids assigned to an ask does not exceed the quantity q_{A_j} of bid A_j.)

(3) $\sum_{i \in M} \sum_{i \in S_i} q_{B_i}$ is maximized. (Total weight of bids assigned is maximized.)

An integer programming formulation of Multiple Knapsack Problem is as follows.

$$\max \sum_{j\in N}\sum_{i\in S_j} q_{B_i} x_{ij}$$

st

$$\sum_{i\in S_j} q_{B_i} x_{ij} \leq q_{A_j},\ \ j\in N$$
$$\sum_{j\in N} x_{ij} \leq 1,\ \ i\in M$$
$$x_{ij}\in\{0,1\},\ i\in M,\ j\in N$$

where the 0-1 variable x_{ij} denotes whether a bid B_i is assigned to the ask A_j.

2.3 The Bin Packing problem

Consider the same case as before where we identify an equilibrium clearing price p^* based on some consideration. Another issue relates to identifying the quantity of supply required to clear the demand at this level. Consider all the bids with $p_{B_i} \geq p^*$, and say that there are M such bids. In order to calculate the supply required to satisfy this demand we assume that there are as many asks available as bids and then we solve the bin packing problem formulated below to identify the number of asks required to satisfy the demand.

Using the same notation as before we now introduce another integer variable z_j, $j \in M$ which counts the number of asks required. The goal is to find an assignment of all bids to the asks (which we have assumed are as many as there are bids) such that the total quantity of supply used is minimized. That is, for each ask $j \in M$, we need to choose a subset S_j of bids in $i \in M$ to be assigned to the ask A_j, such that:

$$
\begin{aligned}
\min \quad & \sum_{j \in M} z_j \\
\text{st} \quad & \\
& \sum_{i \in S_j} q_{B_i} x_{ij} \leq q_{A_j} \times z_j, \;\; j \in M \\
& \sum_{j \in M} x_{ij} = 1, \;\; i \in M \\
& x_{ij} \in \{0,1\}, \; i \in M, \; j \in N \\
& z_j \in \{0,1\}, \; j \in N
\end{aligned}
$$

where the 0-1 variable x_{ij} denotes whether a bid B_i is assigned to the ask A_j. The variable z_j is 1 if the corresponding ask is used and zero otherwise.

2.4 The Generalized Assignment Problem

The previous two formulations dealt with the case where the clearing price is assumed to be known. In this subsection we provide a formulation that does not assume an equilibrium price, instead we try to find an allocation that maximizes the net revealed surplus. The main implication of using such a formulation is that the equilibrium price would have to be derived from the allocation provided by this formulation.

Since we do not have an equilibrium price all bids and asks are considered in formulating the allocation problem. Therefore we use all the m bids and all n asks. Now we can formulate a maximization problem to identify an allocation that maximizes net revealed surplus. Note that if bid B_i is assigned to ask A_j then the net surplus is $(p_{B_i} - p_{A_j}) \times x_{ij}$.

The maximization problem can be written as follows:

$$
\begin{aligned}
\max \quad & \sum_{j \in n} \sum_{i \in m} (p_{B_i} - p_{A_j}) q_{B_i} x_{ij} \\
\text{st} \quad & \\
& \sum_{i \in m} q_{B_i} x_{ij} \leq q_{A_j}, \;\; j \in n \\
& \sum_{j \in n} x_{ij} \leq 1, \;\; i \in m \\
& x_{ij} \in \{0,1\}, \; i \in m, \; j \in n
\end{aligned}
$$

where the 0-1 variable x_{ij} denotes whether a bid B_i is assigned to the ask A_j.

Notice that the formulation does not account for waste in packing bids to asks. For example, we can get a large assigned quantity, however the waste could also be large. In order to model the minimization of waste we will modify the above formulation slightly by adding another term to the objective function which subtracts the surplus lost by the seller due to the waste quantity. The waste in seller surplus on any ask is represented by $p_{A_j} \times \sum_{j \in N} z_j (q_{A_j} - \sum_{i \in m} q_{B_i} x_{ij})$. As before, the variable z_j is 1 if the corresponding ask is used and zero otherwise. If we subtract this from the net revealed surplus in the objective function and noting that $z_j x_{ij} = x_{ij}$ we get a new objective function as follows:

$$\max \quad \sum_{j \in Y} \sum_{i \in X} (p_{B_i} q_{B_i} x_{ij} - p_{A_j} z_j q_{A_j})$$

This new objective essentially accounts for the fact that even if a portion of an ask is allocated the entire ask is used and the unallocated part is wasted.

2.5 The Single Knapsack Problem

Consider the case where a seller j arrives in a continuous double auction with a unit of quantity q_{A_j} and a ask price of p_{A_j}. He has access to the bid queue which is a list of bids sorted in non-increasing order by price, $\{(p_{B_1}, q_{B_1}), (p_{B_2}, q_{B_2}), \cdots, (p_{B_k}, q_{B_k})\}$ where $p_{B_1} \geq p_{A_j} \geq p_{B_k}$. The seller wishes to identify a subset of bids from the list K that maximizes his revealed surplus. This is formulated as follows:

$$\begin{aligned} \max \quad & \sum_{i \in K} (p_{B_i} q_{B_i} x_{ij} - p_{A_j} q_{A_j}) \\ \text{st} \quad & \\ & \sum_{i \in K} q_{B_i} x_{ij} \leq q_{A_j}, \ \ j \in n \\ & x_{ij} \in \{0, 1\}, \ \ i \in m, \ \ j \in n \end{aligned}$$

Notice that the term $p_{A_j} q_{A_j}$ is a constant and hence can be ignored for this formulation.

3 Market Clearing with Indivisible Supply and Demand

Economic markets are exchanges where buyers and sellers can find each other. Typically buyers come to the market with a demand for some good (q_D) alongwith a price (p_D) they are willing to pay for it. Similarly sellers arrive with a quantity they are willing to supply (or produce, q_S) and the asking price (p_S). The question then arises as to whether there is a price p^* that there is no excess positive demand in the market. The *Walsrasian equilibrium* identifies such a price p^* by finding the intersection of supply and demand [Var84]. The price discovery process depends on the market mechanism used and differs substantially across different market mechanisms. We discuss two market mechanisms - the continuous call auction (CCA) and the continuous double auction (CDA) [GD98].

3.1 The Continuous Call Auction

The continuous call auction clears the market at periodic intervals. During an interval, offers and bids are submitted continuously. At the end of the period, an aggregate supply curve and an aggregate demand curve is composed from the offers and bids. The crossover point between the demand and supply provides the equilibrium price p^* and equilibrium quantity q^*. All bids above p^* and all offers below p^* are cleared. This is based on the assumption that all offers and bids are divisible, i.e. a bid for 5 units at $2/unit will accept an allocation of only 3 units at the same price.

Consider the example provided in the introduction in tables 1 and 1. The aggregate supply and demand is listed in the cumulative column at each price level. The supply and demand is plotted in figure 1. Let us assume that the bids and asks quantities are divisible. Under this assumption the equilibrium price is $120/ton and the quantity cleared is 30 tons. To satisfy a demand of 30 tons using exactly 30 tons of supply requires that we are able to allocate any fraction of any bid to any fraction of any ask. For example, bid B_2 and bid B_5 are divided and allocated across two asks as shown in figure 2. Therefore assuming that the bids and ask quantities are divisible, we identify the equilibrium price by sorting the bids and asks which is only log-linear in the size of the bids and asks. An allocation corresponding to this case is shown in figure 2.

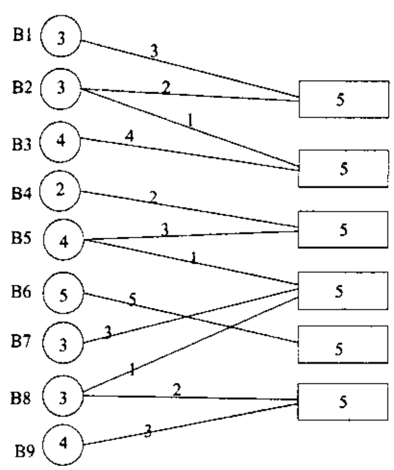

Figure 2: Allocation for Divisible Demand

Now let us return to the more realistic case when the bids are *not* divisible. Now the question of whether 30 tons of demand at $120/ton can be satisfied using 30 tons of supply becomes a NP-hard problem. There are two approaches to evaluate this question:

1. Let us identify an equilibrium price p* based on the cross-over of supply and demand in figure 1. For an equilibrium price of $125/ton, the available supply is

30 tons (made up of 6 asks of 5 tons each). The interesting question that arises is how much demand can be satisifed using these 30 tons. In order to answer this question it is necessary to solve a *multiple knapsack problem* where we try to maximize the allocation of the bids to available supply. It turns out that if the bids are indivisible then it is possible to only fulfill 24 tons of demand. An optimal allocation that leads to this solution is shown in figure 4. We are faced with a situation that 6 tons of supply is wasted. In addition it is important to remember that the multiple knapsack problem is an NP-hard.

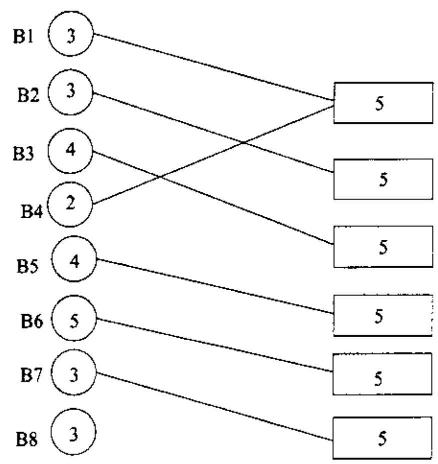

Figure 3: Allocation for Indivisible Demand

2. An alternate approach is to identify a level of demand for a given price and then determine the supply required to clear this demand. We need to solve the *bin packing problem* to answer this question. For instance, consider the p=$125/ton and a demand of 27 tons. The number of rolls of 5 tons required to satsify this demand is 7, i.e. we need 35 tons of supply out of which 7 tons are wasted. However, at this price only 30 tons are available. This might prompt us to examine the demand and supply at various prices to solve this problem. Once again the *bin packing* problem is NP-hard.

3. An alternate approach to clearing markets where demand is indivisible is to formulate an allocation problem where the net revealed surplus is maximized. This differs fundmentally in that the equilibrium price is not determined apriori, rather the allocation problem is solved first so as to maximize the net surplus. How to distribute the net surplus between the buyers and sellers is a question that has to be solved independently. Such a formulation leads to a generalized assignment problem which is also NP-hard. Solving for the net surplus without explicitly accounting for waste we get an allocation of 44 tons with a waste of 6 tons (as shown in 4) with a surplus of 1045 and a loss of seller surplus of 740 (due to 6 tons of waste) leading to a net surplus of 305. However, if we use the alternate formulation whereby the waste is accounted for as lost surplus for the seller, only 19 tons are allocated with total waste of 1 ton with a net surplus of 730 (as shown in figure 5).

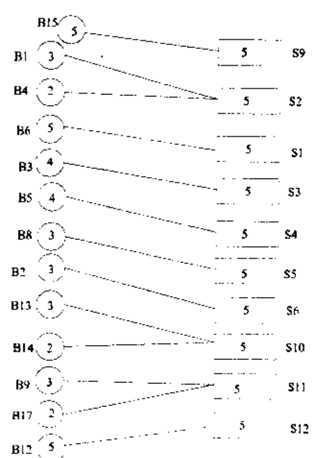

Figure 4: Allocation Maximizing Total Surplus

Essentially, in order to clear the markets with indivisible demand and supply, we need to address two fundamental issues:

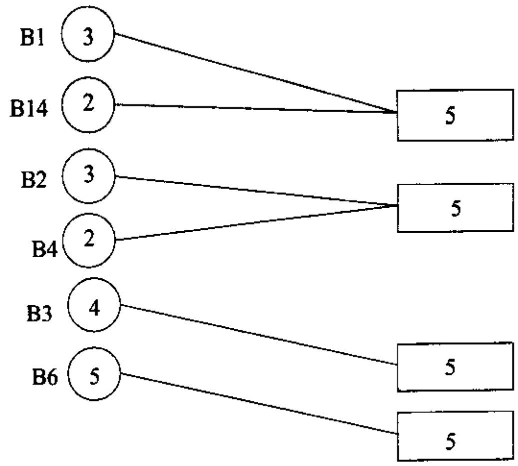

Figure 5: Allocation Maximizing Net Surplus

1. We need to solve NP-hard problems such as the multiple knapsack problem or the bin packing problem to identify the price at which there is sufficient supply to clear the demand or the generalized assignement problem to maximize net surplus.

2. In addition, the indivisible nature of demand and supply leads to a packing problem where even optimal allocations lead to a waste as illustrated in figure 4. We need to identify a policy whereby this waste (or loss of surplus) can be allocated between buyers and sellers. Such a reallocation would lead to a different clearing price which might perturb the equilibrium. Alternately, maximizing net surplus leads to an allocation where it may not be possible to find a single equilibrium price and a new policy for distributing the net surplus between the buyer and the seller has to be designed.

3.2 The Continuous Double Auction

In a continuous double auction bids and asks are continuously submitted during an interval of time. Exchange occurs when a buyers bid meets or exceeds the lowest standing ask or a sellers ask is less than or equal to the highest standing bid. As a result, the exchanges happen on a continuous basis and the asks and bids are matched in a greedy fashion that typically leads to some inefficiencies as compared to the continuous call auctions. Much experimental work has been done in the case of divisible demand and supply that indicates that such markets converge to equilibria quickly and efficiencies are within a few percent of the continuous call auctions [FR91][GD98].

However, when the demand and supply is indivisible then the exchange mechanism for a continuous double auction becomes a little more involved. A buyer's bid that meets or exceeds a sellers ask price is not sufficient for an exchange since such a greedy allocation might incur large waste. A more general mechanism needs to be used for an exchange whereby the net revealed surplus for the seller is non-negative.

A seller examines the bid list to identify a set of bids such that the net revealed seller surplus is positive. Using notations introduced in section 2, this means that for any seller S_j the net revealed surplus for each unit $NRS_j = p_{A_j} \times \sum_{j \in N} (z_j \, q_{A_j} - \sum_{i \in m} q_{B_i} \, x_{ij}) \geq 0$. Here the variable x_{ij} indicates the bids from the bid queue that is exchanged by seller j. If the seller is able to find a set of bids that provide non-negative seller revealed surplus then an exchange will occur. Note however, for a given bid queue there might be a large number of bid sets that provide non-negative seller surplus. Clearly, a rational seller would attempt to maximize his revealed surplus provided he can do this in a reasonable amount of time. In general, identifying the optimal set is the *knapsack problem* and is NP-hard.

We identify two mechanisms for exchange to occur when demand and supply is indivisible.

Greedy Seller: A greedy seller is one who looks at the bid list and greedily picks the first k bids that he can pack into his standard sized item and that provides a positive surplus. If he can find such a set, an exchange occurs. If the seller has any remaining amount he enters the ask queue. The advantage of a greed is that the computation is straightforward and such decision can be made very fast. However, if he cannot find such a set of bids he waits in the ask queue.

Rational Seller: A rational seller looks at the bid list and identifies the optimal bid set which maximizes his net revealed surplus. If the maximium surplus that he can obtain is positive then an exchange occurs. If the seller has any remaining amount or the maximum surplus is non-positive he enters in the ask queue.

Notice that both these mechanism lead to allocations that are suboptimal when compared against the continuous call auction that maximizes the net revealed surplus because the exchange is done with only the bids and asks available at any point. The interesting question that arises is whether we can characterize the inefficiency quatitatively. There are two approaches available to attempt to answer this question:

Average Case Analysis: We can identify average case difference between the continuous double auction (using one of the two mechanisms identified above) against the continuous call auction by computationally simulating the continuous double auction for randomized data. For each of the two mechanisms above we can compute the net revealed surplus for over a statistically significant number of runs and compare that against a continuous call auction by solving each instance optimally solving a generalized assignment problem. Such an analysis will provide an estimate of efficiency of the CDA for markets with indivisible supply and demand.

Worst Case Analysis: An alternate approach is a theoretical one that attempts to bound the worst case performance of the continuous double auction (for each mechanism) as compared to the optimal allocation of the continuous call auction. Notice that since the bids and asks are arriving continuously, the theoretical analysis needs to adopt an online algorithm approach [Hoch97].

4 Conclusions and Future Research

We have presented in this paper the implications of indivisibility of demand and supply on the computations required to clear markets using the continuous call auction or the continuous double auctions. In both cases, market clearing calls for solving problems which are NP-hard. In addition, we have identified the class of allocation problems that lie at the heart of market clearing. These are set packing problems that have been studied extensively in the context of operations research. The computational complexity arising due to indivisibility is in stark contrast to the more conventional market clearing with divisible demand and supply which requires only a sorting operation. In the context of marketplaces for commodities such as paper and steel such indivisible demand and supply is encountered and this calls for the development of optimization libraries that can be used for market clearing.

The conventional approach of clearing markets involves identifying an equilibrium price by identifying the crossover between aggregate demand and supply. For indivisible demand and supply such an approach leads to a fundamental difficultly - typically for a demand D^* a supply of $\alpha \times D^*$, where $\alpha > 1$ is required. This waste of $(1 - \alpha)$ D^* arises due to the indivisibility of demand and supply and result is a loss of net revealed surplus for the seller of $(1 - \alpha)$ D^* P^*. A policy for reallocation of this waste between buyers and sellers without perturbing the equilibrium price requires further research.

An alternative to the conventional market clearing approach was presented where the optimal assignment of demand to supply that maximizes net revealed surplus was presented. However, this approach does not provide a single clearing price p^* and hence the allocation of the net surplus between buyers and seller still remains unresolved. Designing good policies for distributing the surplus needs further research.

Exchanges in continuous double auctions with indivisible demand and supply is more complicated than otherwise. Two general mechanisms for conducting exchanges were presented. One of the mechanisms assumes that the seller identifies a bid set that maximizes his surplus. This optimization exercise is NP-hard. A set of optimization libraries to assist sellers in such marketplaces would be useful.

Finally we discussed the efficiency of the continuous double auctions as compared to the continuous call auctions. A quantitative comparison calls for research along two directions: (i) A computer simulation for a continuous double auction that provides an estimate of the net surplus created in the average case, and (ii) a theoretical analysis that characterizes the worst case performance of the mechanisms presented.

References

[FR91] Daniel Friedman and John Rust (Editors). *The Double Auction Market: Institutions, Theories and Markets.* Proceedings Volume XIV, Sante Fe Institute Studies in the Sciences of Complexity. Addison-Wesley Publishing Company, New York, 1991.

[GJ79] M.R. Garey and D.S. Johnson. *Computers and Intractibility:A Guide to the Theory of NP-Completeness.* W.H. Freeman and Co., San Francisco, 1979.

[GD98] S. Gjerstad and J. Dickhaut. Price Formation in Double Auctions. *Games and Economic Behavior*, Vol. 22(1):1-29, 1998.

[Hoch97] Dorit S. Hochbaum (Editor). *Approximation Algorithms for NP-hard Problems.* PWS Publishing Company, MA 1997.

[KDTL97] J. Kalagnanam, M. Dawande, M. Trumbo, and H.S. Lee. The surplus inventory matching problem in the process industry. Technical Report RC21071, IBM T. J. Watson Research Center, Yorktown Heights, NY, 1997.

[KWGMAKD98] P. Keskinocak, F. Wu, R. Goodwin, S. Murthy, R. Akkiraju, S. Kumaran, and A. Derebail. Scheduling solutions for the paper industry. Technical Report RC21297, IBM T. J. Watson Research Center, Yorktown Heights, NY, 1998.

[MT90] S. Martello and P. Toth. *Knapsack Problems: Algorithms and Computer Implementaions.* John Wiley and Sons, New York, 1990.

[Var84] Hal R. Varian. *Microeconomic Analysis.* W. W. Norton and Company, New York, 1984.