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Price Negotiations for Procurement of Direct Inputs

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PRICE NEGOTIATIONS FOR PROCUREMENT OF DIRECT INPUTS

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Abstract. Strategic sourcing relates to the procurement of direct inputs used in the manufacture of a firm's primary outputs. Such transactions are usually very large (in total quantity and the dollar value) and require the use of special price negotiation schemes that incorporate the appropriate business practices. In this paper, we present two auction mechanisms that have been developed for price negotiations in the context of strategic sourcing for a large food manufacturer. The first auction solicits supply curves as bids for procuring forecast demand for a direct input over a long planning horizon (such as a quarter). Typically suppliers provide *volume discounts* for such large orders and we call such auctions *volume discount auctions*. The second auction aggregates short-term (weekly) demand over direct inputs across multiple manufacturing plants in an effort to increase the total transaction size and allows suppliers to provide bundled all-or-nothing bids. Such auctions are called *combinatorial auctions*. A fundamental consideration that arises in the design of these schemes is the incorporation of business rules that appear as side constraints in the mathematical formulation for determining the winning bids (called the *winner determination problem*). We model the winner determination problem for these auctions as integer programs and provide some computational results using commercial integer programming solvers to illustrate the efficacy of these formulations.

1. Introduction. Strategic sourcing relates to the procurement of direct inputs used in the manufacture of a firms primary outputs. For example, the primary inputs for the manufacture of computers are processors, RAM (random access memory), hard drives, monitors, etc. Typically up to 90% of a firm expenditures are related to procuring direct inputs and as a result these transactions are large in volume as well as in dollar amount. As a result there is considerable room for price negotiations. However, a fundamental concern in such sourcing decisions is related to the reliability of suppliers, since defaulting suppliers might have considerable impact on the firm's ability to satisfy demand obligations. As a result these negotiations are generally confined to a restricted number of pre-certified suppliers having established relationships with the company.

The total quantity of direct inputs that needs to be procured is usually based on the forecasted demand for a planning horizon, typically a quarter. However, since there is considerable uncertainty in the forecast, a strategic decision is made regarding the fraction of the forecast demand that is procured up front[1]. The short-term (weekly) demand fluctuation over base level (which is procured using a long-term contract) is procured on a

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[1]This decision is made based on the uncertainty in the demand and price forecast and typically attempts to optimize some expected benefit. This decision problem is outside the scope of this paper.

weekly basis. However, the short-term demand is usually much smaller and there is less room for price negotiations. An approach adopted to address this is to aggregate demand over several commodities and over different locations and negotiate price for the entire bundle. In addition, in order to exploit the cost complementarities that suppliers might have for different commodities or locations it becomes necessary to allow all-or-nothing bids over bundles.

In this paper auction mechanisms are proposed for price negotiations related to the procurement of both the long-term and short-term demand. A *volume discount* auction is proposed for the procurement of large volume (long-term) demand. Volume discount auctions are a generalization of the multi-unit auctions [Ausubel, 1997] in that they allow bidders to specify different price levels for different quantities[2]. Typically for large volume price negotiations, suppliers provide volume discounts, i.e. as the total quantity bought increases the price drops monotonically. *Combinatorial auctions* are proposed for the procurement of short-term demand. Combinatorial auctions allow the buyer to procure a variety of commodities simultaneously, and allow the suppliers to bid on sets or bundles of items. Because of the complementarities (or substitution effects) between the production or transportation costs of different commodities, suppliers can provide lower prices on sets of commodities.

The focus of this paper is the *winner determination problem*: given a set of bids for some specified demand, which subset of the bids should the buyer accept so as to minimize the total procurement cost for this total demand. A fundamental consideration in formulating these price negotiation schemes is the business rules that firms use to constrain their selection of suppliers. Typical rules are motivated by risk hedging considerations. For example the number of winning suppliers chosen is constrained to have a minimum since dependence on too few suppliers might expose the firm to the misfortunes or supply fluctuations of the chosen suppliers. On the other hand, too many suppliers would lead to high administrative costs and hence the number of winning suppliers is also constrained from above. Other constraints emerge from limits that firms impose on the total volume allocated to transactions with each supplier. We provide mathematical formulations for both the volume discount auction and the combinatorial auction taking into account the business rules as side constraints. The side constraints fundamentally alter the structure of the problems for both cases and lead to novel formulations that have not been studied in the auctions literature [de Vries & Vohra, 2000].

The price negotiation mechanisms discussed in this paper have been developed and deployed at a large food manufacturer for their strategic sourcing operations. The auctions are implemented in the context of a pri-

[2]Notice that auctions in the context of procurement are reverse auctions that are buyer driven.

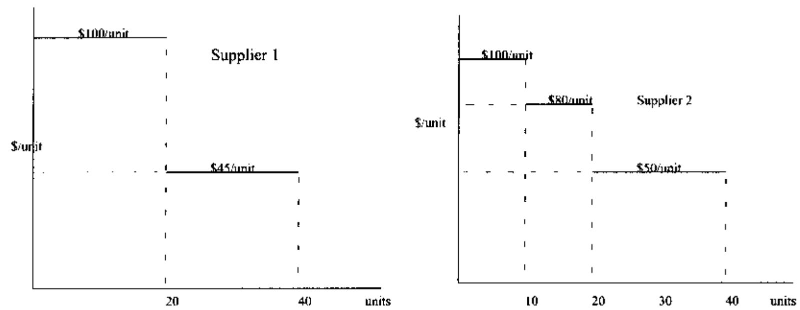
vate business-to-business electronic exchange (B2B exchange). Such private electronic markets increasingly seem to be the emergent model for most large firms.

Since the focus of this paper is on the decision problems that lie at the core of the price negotiation scheme we first provide a general description of the volume discount auction and the combinatorial auction problem and the optimization models that need to be solved to determine the winners for a given set of bids. Section 2 provides a description of the volume discount auction, the business rules and the associated mathematical formulation of the winner determination problem as a linear integer program. Section 3 provides a description of the reverse combinatorial auction, the business rules and the mathematical formulation of the winner determination problem. Computational results investigating the impact of the side constraints on solving the winner determination problem for randomly generated data sets are also presented in these two sections. Section 4 discusses some modeling issues that are required to capture some of the operational details of the auctions. Section 5 provides a discussion of some of the research issues that arise from this work.

2. Volume discount auctions. Volume discount auctions are used in a procurement context where there is a single buyer and multiple sellers. A buyer wishes to purchase some quantity of an item (which we also refer to as *lots*), and invites sellers to submit bids for these items. Bids in a volume discount auction allow the seller to specify the price they charge for an item as a function of quantity that is being purchased. For instance, a computer manufacturer may charge $1000 per computer for up to 100 computers, but for each additional computer over 100 computers would charge only $750 per computer. Bids take the form of *supply curves*, specifying the price that is to be charged per unit of item when the quantity of items being purchased lies within a particular quantity interval.

2.1. Example. We consider an example of a volume discount procurement auction where the buyer wishes to purchase 60 units of some commodity. Figures 1(a) and (b) illustrate two supply curves that could be provided by two different sellers bidding in this auction. Each supply curve consists of a list of pairs, composed of a price and a quantity interval. Each pair specifies the price that is being offered per unit of commodity, if the buyer purchases a number of units within this quantity interval. For instance, in supply curve 1 (Figure 1(a)) the buyer is offering a price of $100 per unit, for quantities between 1 and 20 units. If the buyer is willing to buy more units from this seller, say 25 units, then they can get a better price per unit, in this case $45 per unit for each additional unit over 20.

The winner determination problem for the volume discount auction is to select a set of winning bids, where for each bid we select a price and quantity of items to be bought, such that the total demand of the buyer is satisified at the lowest price.



(a) supply curve 1 (b) supply curve 2

FIG. 1. *Example supply curves.*

The winner determination problem for this type of auction is straightforward if there are no constraints on the maximum number of units that can be purchased from a single seller, i.e. the total quantity of the commodity requested by the buyer can be met by a single seller. In this case, it suffices to determine the best price that each seller is offering for the quantity requested by the buyer, and buy the entire quantity from this seller. For instance, if the buyer has a demand for 30 units, and we are given the supply curve bids as shown in Figure 1, then we can determine the best price is given by seller 2 at a total cost of $2300 per unit for the entire demand.

For the particular project we were involved with, there were constraints on both the minimum and maximum number of units that the buyer may purchase from each seller. Such a constraint may state that the maximum quantity that can be purchased from a single seller is 40 units. If the buyer wishes to buy more than 40 units, the winner determination problem becomes more complex. For instance in our example, if the total demand is 60 units, the best solution is to buy 30 units from seller 2 for a cost of $2300, and 30 units from seller 1 for a cost of $2450, resulting in a total cost of $4750. By buying 40 units from seller 1 we could get more units at a lower price of $50 per unit (cost $2800), however this would decrease the quantity, and thus increase the price per unit of the items we buy from seller 2 for a cost of $2000 resulting in a total cost of $4800.

2.2. Mathematical formulation. We represent a volume discount procurement auction in the following way:

1. The buyer has K lots that s/he needs to procure, and requires a quantity $Q^k, k = 1, \ldots, K$ for each lot;

2. The buyer also identifies a list of potential suppliers $i = 1, \ldots, N$ that can bid in the auction;
3. Each supplier responds with a bid composed of a supply curve (at most one for each lot). A supply curve from supplier i for lot k given by a bid B_i^k consists of a list of M_i^k price-quantity pairs, $\{(P_{i1}^k, [Q_{i1,low}^k, Q_{i1,high}^k]), \ldots, \quad (P_{i|M_i^k|}^k, [Q_{i|M_i^k|,low}^k, Q_{i|M_i^k|,high}^k])\}$. Each price-quantity pair $(P_{ij}^k, [Q_{ij,low}^k, Q_{ij,high}^k])$ specifies the price P_{ij}^k that the supplier i is willing to charge per unit of the lot k if the number of units bought from this supplier lies within the interval $[Q_{ij,low}^k, Q_{ij,high}^k]$. We assume that the quantity intervals within a single supply curve are all pairwise disjoint.

The winner determination problem for the volume discount auction can be formulated as a mixed integer programming problem (MIP) in the following way:

1. We associate a decision variable x_{ij}^k to each price-quantity pair $(P_{ij}^k, [Q_{ij,low}^k, Q_{ij,high}^k])$ for each bid B_i^k. This variable takes the value 1 if we buy some number of units of the lot through this bid within the quantity range $[Q_{ij,low}^k, Q_{ij,high}^k]$ at the price P_{ij}^k; it takes the value 0 otherwise.
2. We associate a continuous variable z_{ij}^k with each price-quantity pair, which specifies the exact number of units of the lot that is to be purchased from the bid B_i^k within this price-quantity pair.

The MIP formulation is then given as follows:

$$\textit{minimize} \quad \sum_{k \in K} \sum_{i \in N} \sum_{j \in M_i^k} (z_{ij}^k \, P_{ij}^k + x_{ij}^k \, C_{ij}^k)$$

$$\textit{subject to}$$

$$(1) \qquad \begin{array}{lll}
z_{ij}^k - (Q_{ij,high}^k - Q_{ij,low}^k)\, x_{ij}^k \leq 0 & \forall i \in N, \forall j \in M_i^k & (a) \\
\sum_{j \in M_i^k} x_{ij}^k \leq 1 & \forall i \in N, \forall k \in K & (b) \\
\sum_{i \in N} \sum_{j \in M_i^k} (z_{ij}^k + x_{ij}^k \, Q_{ij,low}^k) \geq Q^k & \forall k \in K & (c) \\
x_{ij}^k \in \{0, 1\} & \forall i \in N, \forall j \in M_i^k, \forall k \in K & \\
z_{ij}^k \geq 0 & \forall i \in N, \forall j \in M_i^k, \forall k \in K. &
\end{array}$$

The coefficient C_{ij}^k is a constant and computed apriori as:

$$(2) \qquad C_{ij}^k = \sum_{\hat{j}=1}^{j-1} P_{i\hat{j}}^k (Q_{i\hat{j},high}^k - Q_{i\hat{j},low}^k).$$

Constraints (a) specifies for each price-quantity pair $(P_{ij}^k, [Q_{ij,low}^k, Q_{ij,high}^k])$, that if we buy some quantity of the commodity from the bid at the price P_{ij}^k, this quantity must lie within the range $[Q_{ij,low}^k, Q_{ij,high}^k]$. Constraint (b) specifies that for each winning bid, we can only buy at a price and quantity that corresponds to a single price-quantity pair. Constraint (c) states that we must determine a winning set of bids such that the total demand of the buyer for each lot k is satisfied.

We have included all lots that the buyer wishes to procure in this formulation, even though, as it stands, we could solve the winner determination problem for each lot independently. This is done for convenience: in the next section we introduce side constraints which involve all the lots in the auction.

2.3. Side constraints. In a real world setting there are several considerations beside cost minimization. These considerations often arise from business practice and/or operational considerations and are specified as a set of constraints that need to be specified while picking a set of winning suppliers. We discuss four such business rules/constraints that we encountered in our application with the food industry.

2.3.1. Number of winning suppliers. An important consideration while choosing winning bids is to make sure that the entire supply is not sourced from too few suppliers, since this creates a high exposure if some of them are not able to deliver on their promise. On the other hand, having too many suppliers creates a high overhead cost in terms of managing a large number of supplier relationships. These considerations introduce constraints on the minimum, S_{min}, and maximum, S_{max}, number of winning suppliers in the solution to the winner determination problem.

These constraints are encoded in the MIP formulation in the following way. We introduce an indicator variable y_i for each supplier i, which takes the value 1 if the supplier has any winning bids and 0 otherwise. The first constraint sets y_i to 1 if supplier i has any winning bids. Note that the constant multiplier ensures that the right hand side is large enough when y_i is one[3].

$$
\begin{aligned}
&\sum_{k\in K}\sum_{j\in M_i} x_{ij}^k \le y_i\, K && \forall i\in N\\
&y_i \in \{0,1\} && \forall i \in N\\
&S_{min} \le \sum_{i\in N} y_i \le S_{max}.
\end{aligned} \tag{3}
$$

2.3.2. Local lot level constraints. Local lot level constraints state for each supplier i and lot k, the minimum $q_{i,min}^k$ and maximum $q_{i,max}^k$ quantity that can be allocated to supplier of this lot type. For instance, a constraint may state that some supplier must be allocated at least 500 tons and at most 12,000 tons of a particular lot. These constraints are once

[3]The constant multiplier K follows from constraint (c) in the formulation.

again motivated by concerns similar to the ones related to the number of winning suppliers. Rather than add these constraints to the formulation, they can be used to prune the supply curves of the bids from each supplier so that they lie within a feasible range with respect to this constraint.

2.3.3. Global lot level constraints. For each supplier i, the buyer provides bounds on the total allocation, across all lots, to lie between $W_{i,min}$ and $W_{i,max}$. In the winning allocation, if the supplier i has *any allocation at all* then it must lie in this range. These constraints can be expressed in the following way in the formulation:

$$
\begin{aligned}
&y_i W_{i,min} - \sum_{k \in K} \sum_{j \in M_i^k} (z_{ij}^k + x_{ij}^k Q_{ij,low}^k) \leq 0 \quad \forall i \in N \\
&\sum_{k \in K} \sum_{j \in M_i^k} (z_{ij}^k + x_{ij}^k Q_{ij,low}^k) - y_i W_{i,max} \leq 0 \quad \forall i \in N.
\end{aligned} \tag{4}
$$

Note that these constraints imply the first constraint in Equation 3.

2.3.4. Reservation prices. A maximum allowable price per unit can be provided by the buyer for each lot. This constraint is not encoded in the MIP formulation for the winner determination problem. Instead bids, or portions of the supply curve in each bid, that fail to satisfy this constraint are disregarded by the solver engine.

2.4. Relevant literature. The restriction of the volume discount auction winner determination problem to a single lot can be modeled as a variation of the *multiple choice knapsack* problem [Martello & Toth, 1989], and as such is NP-hard.

2.5. Auction mechanism and computational issues. The buyer in the auction specifies:

1. the lot to be purchased, required quantity and a reservation price;
2. a list of acceptable suppliers for each lot;
3. lot level constraints (discussed below);

Each supplier submits a supply curve bid for each lot. The supplier may submit up to 10 price-quantity pairs within each bid, ranging from their minimum to maximum supply limits. The supplier may choose to submit bids over a smaller quantity range than is required by the buyer (e.g., the minimum required volume is 200 tons, but the supplier chooses only to place bids for quantities greater than 300 tons.)

This is a multi-round sealed bid auction. Once the bids have been submitted, the system processes the bids, solves the winner determination problem and returns a minimum cost optimal solution to the buyer. Suppliers are informed of the winning bids in each round, and may submit new bids. Typically there are up to 30 lots and about 10 suppliers for each lot, and the optimization engine needs to find a solution within a couple of minutes.

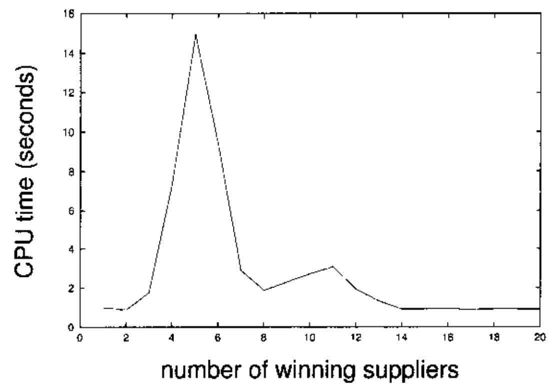
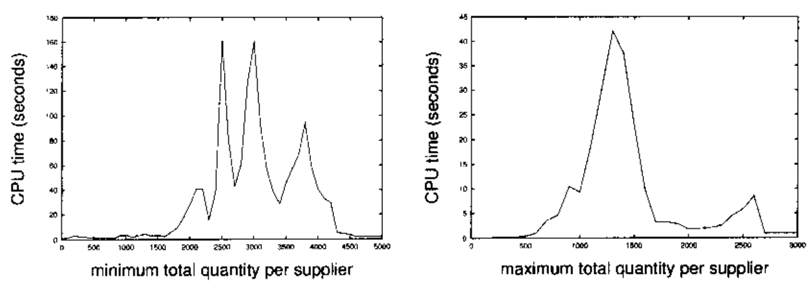


FIG. 2. *Experimental results for a volume discount auction, varying the number of allowed winning suppliers constraint.*



(a) Minimum total quantity per supplier constraint

(b) Maximum total quantity per supplier constraint

FIG. 3. *Experimental results for a volume discount auction, varying the constraints on total quantity that can be awarded to each supplier.*

A dynamic programming approach can be used to solve this problem. We found however that commercial integer programming software using a branch-and-bound approach was able to solve problems specified by the customer on the order of seconds or minutes.

Figures 2 and 3 present CPU time results (in seconds) for solving a randomly generated instance of the volume discount winner determination problem, where we vary side constraints. This problem had 10 lots available in the auction, with 100 suppliers submitting up to 5 bids each. Figure 2 illustrates how varying the number of winning suppliers affects problem difficulty. We set the minimum number of winning suppliers to be equal to the maximum number of winning suppliers, which we varied between 1 and 20. No other side constraints were set for this experiment. The optimal allocation for this problem, without any side constraints on the number of winning suppliers, has 13 winning suppliers. The problem becomes harder to solve as we decrease the number of permitted winning suppliers below this number. For very small numbers of winning suppliers, the problem becomes easier to solve, since with few suppliers it may not be possible to find an allocation which satisfies the total demand of the buyer. Note that the results presented here were generated using an instance that uses a slightly different objective function than the one presented in Equation 1. However, the qualitative behavior presented here persists across both models.

Figure 3 investigates the minimum and maximum total quantity assigned to each supplier constraint. Once again, no other side constraints were set for this experiment. Here the optimal allocation, without any side constraints, gives a minimum quantity of 100 and a maximum quantity of 2700 to some winning suppliers[4]. We see here that deviating from this optimal allocation, by decreasing the maximum or increasing the minimum quantity that can be assigned to each supplier, makes the problem harder to solve.

3. Combinatorial auctions. Combinatorial auctions provide a mechanism whereby bidders can submit bids on combinations of items [Rassenti et al., 1982, Fujishima et al., 1999, Sandholm, 1999, de Vries & Vohra, 2000]. Such auctions are used when, due to complementarities or substitution effects, bidders may have preferences not just for particular items but also for sets, or bundles, of items. In this paper, we describe a reverse combinatorial auction. The auction is run as a procurement auction, where the buyer wishes to purchase different items of varying quantities, for the cheapest overall price. The total quantity of each item is called a *lot* and is treated as indivisible unit of some weight. Suppliers can bid on combinations of items, however, a bid on any item has to be for the entire lot for that item. The winner determination problem for the single unit reverse combinatorial auction problem is to select a winning set of bids such that each item is included in at least one winning bid, and the total cost of procurement is minimized. This problem is a weighted set covering problem, which is known to be NP-hard.

3.1. Mathematical formulation. We are given a set of K items, where for each item $k \in K$ there is a demand for d^k units of the item (called a lot). Each supplier $i \in N$ is allowed up to M bids indexed by j. We associate with each bid B_{ij} a zero-one vector a_{ij}^k, $k = 1, \dots, K$ where

[4]Note that the allocation found for both the combinatorial and volume discount auction winner determination problems may not satisfy the total demand of the auction. This may not be possible with the side constraints, in which case the mechanism described in Section 3.5 is used to return a partial allocation.

$a^k_{ij} = 1$ if B_{ij} will supply the (entire) lot corresponding to item k, and zero otherwise. Each bid B_{ij} offers a price p_{ij} at which the bidder is willing to supply the combination of items in the bid. A mixed integer programming formulation for the reverse combinatorial auction can be written as follows:

$$
\text{(4)} \qquad \begin{array}{lll}
\textit{minimize} & \sum_{i \in N} \sum_{j \in M} p_{ij} x_{ij} & \\
\textit{subject to} & & \\
& \sum_{i \in N} \sum_{j \in M} a^k_{ij} x_{ij} \geq 1 \quad \forall k \in K & (a) \\
& x_{ij} \in \{0,1\} \qquad \forall i \in N, \forall j \in M. &
\end{array}
$$

The decision variable x_{ij} takes the value 1 if the bid B_{ij} is a winning bid in the auction, and 0 otherwise. Constraint (a) states that the total number of units of each item in all the winning bids must satisfy the demand the buyer has for this item. In this auction goods complement each other, so the valuations of sets of items are sub-additive. That is, the price offered by a particular supplier for two non-disjoint sets of items A and B, $p(A)$ and $p(B)$, is such that $p(A) + p(B) \geq p(A + B)$. Such sub-additive cost functions result from complementary costs such as the use of a common warehouse, or unused capacity in a carrier.

3.2. Side constraints. The auction mechanism and side constraints are similar to those of the volume discount auction. The side constraints are:

1. minimum and maximum number of winning suppliers;
2. minimum and maximum total quantity allocated to each supplier;
3. reservation prices on each lot.

These constraints can be added to the MIP formulation as follows:

$$
\text{(5)} \qquad \begin{array}{lll}
W_{i,min}\, y_i \leq \sum_{k \in K} \sum_{j \in M} a^k_{ij} d^k x_{ij} & \forall i \in N & (a) \\
\sum_{k \in K} \sum_{j \in M} a^k_{ij} d^k x_{ij} \leq W_{i,max}\, y_i & \forall i \in N & (b) \\
\sum_{j \in M} x_{ij} \geq y_i & \forall i \in N & (c) \\
S_{min} \leq \sum_{i \in N} y_i \leq S_{max} & & (d) \\
y_i \in \{0,1\} & \forall i \in N. &
\end{array}
$$

$W_{i,min}$ and $W_{i,max}$ relate to the minimum and maximum quantity that can be allocated to any supplier i. Constraints (a) and (b) restrict the total allocation to any supplier to lie within ($W_{i,min}$, $W_{i,max}$). Note that y_i is an indicator variable that takes the value 1 if supplier i is allocated any lot. Notice that if $W_{i,min} = 0$ then y_i becomes a free variable. In order to fix this we introduce constraint (c) which ensures that $y_i = 0$ if no bids from supplier i are chosen. S_{min} and S_{max} relate to the minimum and maximum number of winners required for the allocation. Constraint (d) restricts the total number of winners to be within the range (S_{min}, S_{max}).

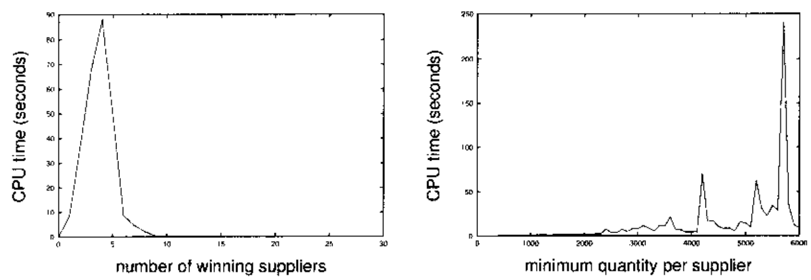
A typical problem is specified to have 6–10 suppliers and 250 lots for auction. The maximum problem size is given as 30 suppliers and 400 lots. There is no limit on the number of lots in each bid, although this could be limited if required to improve solution time. Solutions for the winner determination problem are needed on the order of 5–10 minutes.

3.3. Relevant literature. The formulation (4) provided for the reverse combinatorial procurement auction is a set covering problem, which is NP-hard [Garey & Johnson, 1979]. Since this remains the core even after adding the side constraints, the overall formulation with side constraints remains NP-hard. There is a greedy approach for set covering which yields an $O(\log_2 n)$ approximation, and it has been shown that this cannot be significantly improved [Shmoys, 1995].

The addition of the side constraints makes a fundamental impact on the feasibility of the problem. Without these we could always choose all bids and get a feasible (albeit) expensive solution. However, with limits on the quantity allocated to each supplier and on the total number of winners, a feasible solution might not exist or if one exists it might be difficult to find.

3.4. Auction mechanism and computations. We ran some experiments to investigate the difficulty of solving the winner determination problem with respect to the side constraints in the auction. Figure 4 presents results for one example of a single round reverse combinatorial auction winner determination problem. This particular instance was generated randomly. For each supplier we generated a set of lots the supplier was interested in, and a set of bids for different subsets of this set. A single bundled bid for a set of lots $\mathcal{S}$ would be for a lower price than that of the sum of the prices of any set of bids by the same supplier which also, in total, covers all the lots in $\mathcal{S}$[5]. This particular problem had 30 suppliers and 15 lots available for auction. Each supplier made multiple bids: in total there were 352 bids. With no side constraints, there were 12 winning bids and 12 winning suppliers. Figure 4(a) illustrates how varying the number of permitted winning suppliers affects problem solving time. Once again, we set the minimum number of winning suppliers to be equal to the maximum number of winning suppliers, which was varied between 1 and 30. Decreasing the number of permitted winning suppliers below 14 increased problem solving time significantly, reaching a peak at 4 winning suppliers. The total demand of the auction could not be satisfied with less than 5

[5]This property, which should hold for bids in a procurement auction, would not hold for the combinatorial auction problems discussed in [Sandholm, 1999, de Vries & Vohra, 2000].



(a) Varying the permitted number of winning suppliers constraint (minimum = maximum)

(b) Varying the total minimum quantity per supplier constraint

FIG. 4. *Experimental results for a combinatorial auction with varying side constraints.*

suppliers. Increasing the number of permitted suppliers did not make the problem harder to solve. Figure 4(b) shows what happens when we vary the minimum quantity assigned to each supplier constraint. For these experiments, each supplier has the same minimum total quantity constraint. With no side constraints, the solution to the winner determination problem assigns a maximum quantity of 1800 to any supplier. We see here that as we increase this minimum quantity, the problem becomes increasingly harder to solve. Conversely, we found that problem difficulty was not very much affected by varying the maximum quantity allocated to each supplier constraint.

3.5. Feasibility. As discussed earlier, finding feasible solutions for some instances of this problem is difficult. This also has a big impact on the run time. A common trick for dealing with this situation is to introduce a dummy bid for each lot with a price set to K times the largest bid price in the auction. Dummy bids are added to the integer programming formulation for the demand cover constraints, but do not appear in the side constraints for min/max quantity or min/max number of winning suppliers. A dummy bid for a particular lot type will only appear in an optimal solution to the winner determination problem for these auctions if there is no way to satisfy the demand for the lot using an existing real bid. Thus any solution to the winner determination problem will still satisfy the entire demand for lots in the auction. Should there be dummy bids in the set of winning bids for the winner determination problem, these are removed from the set before the real winning bids are returned to the buyer and infeasibility is indicated.

4. Operational issues. In this section we discuss two operational issues that emerged in this application. Both of these issues arise as requirements in a practical setting and have interesting implications in terms of refining the formulations provided above.

4.1. Feasibility. In a multi-round auction mechanism, there may not be enough bids placed in earlier rounds in the auction to satisfy the total demand requested by the procurer. In such cases, the formulations we have presented earlier for the volume discount and combinatorial auction winner determination problem (formulations (1) and (4)) will be infeasible. In practice, we found that the procurer preferred to receive a partial allocation if the entire demand could not be met[6]. We can model this requirement using the existing formulations by adding *dummy bids* to the formulation as described earlier.

4.2. Timestamps. One issue which arises in the context of multi-round auctions is the treatment of bids made in different rounds of the auction, for the same bundle of items at the same price or supply curve. Consider the following example: A combinatorial procurement auction is created to purchase some quantities of items $\{1, 2, 3\}$. In the first round of the auction Supplier 1 makes a bid B_1 for items $\{1, 2, 3\}$ at a price of \$100, and Supplier 2 bids B_2 of \$30 for item $\{1\}$. The solution to the winner determination problem for this round is that Supplier 1 wins with bid B_1. During the second round a new supplier, Supplier 3, enters the auction with a bid B_3 for items $\{2, 3\}$ at \$70. Using the integer prgramming formulation presented earlier (Equation 4), there are two potential solutions to this combinatorial auction winner determination problem: either $\{B_1\}$ or $\{B_2, B_3\}$. In both cases the total cost to the procurer is \$100.

From a business point of view these two solutions are not both equally desirable. In a multi-round procurement auction, new bids should only supercede existing winning bids if they result in a lower price being paid for the items in the auction. The usual way to handle such situations is to have a rule stating that, given two identical bids, the bid that was made earlier in time is to be preferred. This rule is straightforward to enforce in a simple, single or multi-item forward or reverse auction. In the context of combinatorial and volume discount auctions this rule becomes harder to enforce, since the number of possible solutions to the winner determination problem may be exponential in the number of bids placed in the auction.

In practice, each bid B_i is associated with a timestamp TS_i, representing the time it was accepted into the auction[7]. The new objective for the winner determination problem then becomes to select the set from the set

[6]The customer specified that the side constraints be satisifed at all times, and only the demand constraints could be relaxed.

[7]For the purposes of this paper, we will regard the timestamp as being an integer. In the customer implementation of these auctions, the timestamp was of the SQL type 'timestamp'.

of all sets of bids which minimizes the total cost, such that this set minimizes the sum of the timestamps of all the winning bids[8]. That is, we have a further timestamp objective, which is secondary to the cost objective, which is:

$$\text{minimize} \sum_{i \in N} TS_i x_{ij}. \tag{6}$$

We have considered and implemented two ways of dealing with this issue. We describe these techniques below.

4.2.1. Multiple formulations. The first proposal to deal with the timestamp issue works by solving two integer programming problems. The first problem we solve is the winner determination problem (for either the combinatorial or volume discount auction) without considering timestamps at all, i.e., by using the formulations (1) and (4).

We then formulate a new integer programming problem, which differs from the first formulation in two ways:

1. We take the value of the objective v in the solution to the first integer programming problem, which represents the minimum cost price to the procurer to purchase their demand in the auction. We add as a constraint that the demand should be bought at this price. For the combinatorial auction formulation (4), this constraint is written as:

$$\sum_{i \in N} p_i x_{ij} = v. \tag{7}$$

   For the volume discount auction formulation 1 this constraint is:

$$\sum_{i \in N} \sum_{j \in M_i^k} z_{ij}^k P_{ij}^k = v. \tag{8}$$

2. The new objective becomes to minimize the sum of the timestamps of winning bids (Equation 6).

4.2.2. Price modification. The second technique encodes information concerning the time a bid was made into the price of the bid that is seen by the winner determination solver. Typically, prices are expressed to a fixed number of decimal places, usually two. Thus within the representation of the bid price as a floating point number, we can use digits beyond two decimal places to encode timestamp information. For instance, in our previous combinatorial auction example we had three bids: B_1 of \$100 for items $\{1,2,3\}$, B_2 of \$30 for items $\{1\}$ and B_3 of \$70 for items $\{2,3\}$. If

[8]In practice we found it to be more efficient and scalable to sort the bids by their timestamps into a list, and use the order that bids appear in this list, rather than their timestamps, in the new objective.

these bids B_1, B_2 and B_3 were made at times 1, 2 and 4 respectively, we might encode the timestamps into the bid prices such at p_1 = \$100.001, p_2 =\$30.002 and p_3 = \$70.004. We then solve the integer programming problem with the same objective to minimize the total cost to the procurer. The set of winning bids that minimizes the total cost will also be the one whose bids were made the earliest. In this example, bid B_1 has a lower cost (100.001) than bids B_2 and B_3 combined (100.006), so B_1 will be the winning bid.

Some care must be taken when encoding timestamp information into bid prices. Firstly, using the scheme outlined above, the following situation may occur. Suppose that bid B_3 was accepted at time 2, B_2 accepted at time 4 and bid B_1 accepted at time 5. The corresponding prices on these bids with timestamps become p_3 = \$70.002, p_2 = \$30.004 and p_1 = \$100.005. Bid B_1 would be the winning bid in this example, even though bids B_2 and B_3 were made earlier and for the same total (real) cost. The problem here is that the sum of the timestamp portions of the bid prices for bids B_2 and B_3 is greater than that for bid B_1, even though their individual timestamp parts of the bid price are less. To resolve this problem we must take into account the number of items within each bid when determining timestamp bid price modifications. To do this, we sort all the bids by their timestamps into a list. We set a counter variable *ts = 0*. We then take each bid B_i from the list in order, increment the variable *ts* by the number of items in bid B_i, and set the timestamp portion of the bid B_i to the current value of the variable *ts*. For our running example, the bid prices become p_3 = \$70.002, p_2 = \$30.003 and p_1 = \$100.006.

The advantage of this price modification technique for dealing with timestamps is that we only have to solve one integer programming problem in order to determine the winners of the auction. The main disadvantage is the precision of the floating point arithmetic limits how many bids we can deal with in this way, while guaranteeing that the timestamp part of the modified bid price does not interfere with the real part of the modified bid price[9].

5. Discussion. The sell side forward combinatorial auction has received much attention in recent years [Rassenti et al., 1982, Fujishima et al., 1999, Sandholm, 1999, de Vries & Vohra, 2000], especially with respect to its economic properties and the computational difficulty of solving its winner determination problem. However, little has been said about the reverse combinatorial auction. We have presented details of an application of the reverse combinatorial auction to procurement of directs inputs, where short term demand is aggregated across multiple manufacturing plants, allowing suppliers to provide bundled all-or-nothing bids. In

[9]In practice, given n bids each for m items, we need ceil($\log_2(\frac{nm \times (nm+1)}{2})$) exclusive digits within the bid price in which to store the timestamp information, in order to guarantee that the timestamp part of the bid price does not interfere with the real (price) part of the bid price.

such a context, much of the work in combinatorial auctions may not be applicable to real world situations such as direct procurement, since they ignore side constraints arising as a result of business rules on how demand can be allocated across different suppliers. Such constraints not only impact the computational difficulty of finding an optimal solution to such problems (as illustrated in our empirical studies), but also make determining whether a feasible solution exists an NP-complete problem. In contrast, finding a feasible solution for the forward and reverse combinatorial auctions without side constraints is trivial.

The volume discount auction is common practice in industry for large transactions that are made over a long term time horizon. They provide a way for suppliers to give an indicator of their production costs without revealing details of their manufacturing and capacity constraints. Once again, this type of auction has received little attention in the research literature, and little is known about its computational and economic properties.

There are three functions a decision support system can provide in an auction setting: winner determination, price signalling and bid reformulation. This paper has concentrated on winner determination. In a multi-round auction, price signalling and bid reformulation refer to what information can be given to a losing bidder to enable them to make a winning bid in the next round of an auction. This is a significant research issue. The mechanism used for the project described in this paper informed all bidders of the winning bids in each round of the auction. This mechanism is weak, in the sense that it does not tell bidders how they should reformulate their bids for the next round. For the combinatorial and volume discount auctions, designing price signalling mechanisms to give bidders such information is complex, given the combinatorial nature of the problem.

Recent work [Bickchandani & Ostroy, 1998] has provided an extended formulation for the set packing problem which is integral. The dual of this formulation provides nonlinear prices (with price discrimination) for various bundles. Based on these properties it has been shown that efficient and incentive compatible iterative auctions can be designed [Parkes & Ungar, 2000, Bickchandani & Ostroy, 1998]. However, little attention has been paid to the set covering formulation with regard to designing efficient and incentive compatible mechanisms. In practice, the most likely use of combinatorial auctions seems to be in the context of procurement where the set covering formulation is central. The volume discount auction provides another interesting problem where identifying extended formulations that provide dual prices would be very useful. An additional direction for investigation is the impact of side constraints (the business rules used in practice) on the extended formulation even for the set packing case. The robustness of an auction design would greatly depend on whether the economic properties can be shown to hold under various business rules that appear as side constraints in practice.

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