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Similarity Incertainty , and Time-Tversky (1969) Revisited

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Similarity, Uncertainty, and Time

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Tversky (1969) Revisited [1]

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Abstract

A model of choice building on the approach outlined in Tversky (1969) is presented. In this model decisions are based on comparisons regarding the similarity or dissimilarity of attributes across alternatives. This model explains a number of anomalies observed when risky alternatives are to be played once. The model also explains anomalies observed under repeat play conditions as well as those that occur when the options are intertemporal, as opposed to risky, prospects.

[1] Hal Arkes, Robert Baseman, Robyn Dawes, Catherine Eckel, Miguel Costa-Gomes, Jeryl Mumpower , Jason Shachat, Dailun Shi, and Patrick Sileo have all provided insightful comments on earlier drafts of this paper. Remain errors are the responsibility of the author.

behaviors observed in situations involving single-shot gambles, though still in ways inconsistent with expected utility.

Finally, it has become clear that irrationalities in choice are not unique to situations involving uncertainty -- choices over intertemporal prospects sometimes violate the requirements of the Discounted Utility model and do so in ways seemingly related to irrationalities observed under uncertainty.

In this paper I explore the conditions under which these behaviors also result from reliance on a simple decision heuristic involving comparisons of the similarity or dissimilarity of attributes across alternatives along the lines suggested by Tversky (1969), Rubinstein (1988) and Leland (1994, 1998).[2]

The paper is organized as follows. Section II presents the model of decision making. We assume that for the purposes of choosing between alternatives, agents begin by comparing prizes and their corresponding probabilities of occurrence (or time periods in the case of intertemporal choice) across alternatives with respect to their equality or inequality. Based on these comparisons, agents attempt to resolve the decision by appeal to stochastic dominance. If this procedure proves uninformative for the purposes of making a decision, the comparisons are repeated in terms of the similarity or dissimilarity of prizes and their probabilities (or time periods). Alternatives that are favored in more of these paired comparisons are chosen.

Section III discusses implications of the model for single-shot choices under uncertainty, focusing on *common ratio, common consequence, reflection,* and *event*

[2] Also see work by Medin, Goldstone and Markman (1995), Markman and Medin (1995), Mellers and Biagini (1994), Hsee (1996) Buchena and Zelberman (1995), and Azipurtha et al (1993).

I. Introduction

Over thirty years ago, Amos Tversky (1969) presented subjects with choices between simple lotteries {$$X_1$, p_1} versus {$$X_2$, p_2}, then {$$X_2$, p_2} versus {$$X_3$, p_3}, ...then {$$X_5$, p_5} versus {$$X_1$, p_1} (for $$X_1 > \$X_2 > \ldots > \$X_5$ and $p_1 < p_2 < \ldots < p_5$). In the experiment, lotteries were presented so as to make the payoff differences easier to discern than probability differences. Tversky speculated that between adjacent gambles (e.g., {$$X_1$, p_1} versus {$$X_2$, p_2}), this presentation of the alternatives might lead some subjects to ignore probability differences since the probabilities appeared similar, and choose on the basis of the apparently dissimilar payoffs. In the choice between the lotteries {$$X_5$, p_5} versus {$$X_1$, p_1}, where both prizes and probabilities appeared dissimilar, subjects were hypothesized to choose based on some other criterion (e.g., expected value or the probability of winning). If subjects behaved in these ways, their choices would exhibit systematic intransitivities – and they did.

Since 1969, the list of circumstances in which peoples' choices between simple gambles depart from requirements of rationality has grown much longer. For example, Starmer and Sugden (1993) found that the attractiveness of a lottery can be enhanced simply by splitting one of its prize-probability pairs (e.g., a 50% chance of winning $100) into two parts (e.g., two 25% chances of winning $100). This phenomenon is termed *event splitting.*

Evidence accumulated over the last thirty years also reveals that when choices under uncertainty involve repeat play, they differ from what we would expect based on *splitting* effects. Section IV presents an explanation of systematic risk seeking observed when risky alternatives are played repeatedly. According to this explanation, there is a direct connection between such behaviors and *event splitting.* In Section V we discuss *common difference, magnitude, and reflection* effects occurring in intertemporal choice -- all of which violate the Discounted Utility model of intertemporal choice.

The paper concludes with a summary of the implications of the model and their relationship to other explanations for anomalies in decision making.

II. A Model of Similarity Judgments Under Uncertainty

Consider a choice between lotteries L_1 and L_2 represented or perceived by agents as shown below, where for i =1,2 and j = 1,2,..,n, $p_{ij} \in [0,1]$, $\Sigma_j p_{ij} = 1$ and $x_{ij} \in$ **R**.

$$L_1 = \{ x_{11}, \quad p_{11} \quad ; \quad x_{12}, \quad p_{12} \quad ; \quad ...; \quad x_{1n}, \quad p_{1n}\}$$

$$L_2 = \{ x_{21}, \quad p_{21} \quad ; \quad x_{22}, \quad p_{22} \quad ; \quad ...; \quad x_{2n}, \quad p_{2n}\}$$

Given these choices agents begin by comparing the value of x_{11} with x_{21} in terms of their equality/inequality and p_{11} with p_{21} in terms of their equality/inequality, then x_{12} with x_{22} and p_{12} with p_{22}, and so forth, for a total of n pairs of comparisons. For each pair of comparisons they conclude whether the pair favors one alternative over the other (e.g., if $x_{11} > x_{21}$ and $p_{11} \geq p_{21}$ then the pair of comparisons "favors" L_1 when x_{11} and x_{21} are "good" outcomes, is "inconclusive" (e.g., $x_{11} > x_{21}$ and $p_{11} < p_{21}$ are "good" outcomes) or is "inconsequential" (i.e., prizes and probabilities are identical).[3] Once all n comparisons are made, agents choose L_1 (L_2) if L_1 (L_2) is "favored" in some comparisons with the remainder being "inconsequential."

For certain representations of the alternatives this procedure will detect stochastically dominated prospects, although other representations can obscure the dominance relationship. If the latter occurs, or if there is no dominance relationship to detect, agents attempt to base their choice on the similarity or dissimilarity of attributes across alternatives. For the purposes of modeling this procedure, assume that the binary relations $>^x$ and $>^p$ (reading "greater than and dissimilar") are strict partial orders[4] on consequences and probabilities, respectively. In this case, the similarity relations, $\sim^x$ and $\sim^p$, defined by the symmetric complements of $>^x$ and $>^p$ are not necessarily transitive in that for some prizes $x_{h(igh)} > x_{m(edium)} > x_{l(ow)}$, $x_h \sim^x x_m$, $x_m \sim^x x_l$ but $x_h >^x x_l$, with an analogous result holding for probabilities.

We will assume that three additional properties characterize similarity relations, two of which are general and one of which is unique to probabilities. To motivate the first of these, consider two numbers, say 17 and 12 (or -17 and -12), and suppose an individual perceives these as either dissimilar or similar. Intuition, in most contexts, suggests that numbers like 2017 and 2012 (or -2017 and -2012), obtained by increasing the absolute values of the original ones by a sufficiently large constant, will surely appear

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3 Leland (1994) contains an extended discussion of what determines whether prizes are perceived as good or bad. Summarizing, a prize is "good" if it is the best possible outcome in any of the alternatives or if it is greater than zero and not the worst possible outcome in any of the alternatives. Conversely, a prize is bad if it is the worst possible outcome in any alternative or less than zero and not the best possible outcome in any alternative.

4 That is, asymmetric and transitive.

similar. As such, assume that the similarity relation for prizes, probabilities (and for a later point in the paper, time periods) all obey the following property where $>^*$ and $\sim^*$ are generic relations defined on the real numbers:

Increasing Absolute Similarity (IAS) - For any two numbers a and b such that a $>^*$ b, there exists some sufficiently large c such that a +(-) c $\sim^*$ b +(-) c.[5]

Now suppose we take 17 and 12 (or –17 and –12) and scale them up proportionately to, say, 1700 and 1200 (or –1700 and –1200). Irrespective of whether the initial values were perceived as similar or dissimilar, it seems that this type of manipulation will eventually make the numbers appear dissimilar. As such, assume henceforth that similarity and dissimilarity relations over numbers (e.g., prizes, probabilities, time periods) obey the following:

Increasing Proportional Dissimilarity (IPD) - For any two numbers a > b such that a $\sim^*$ b there exists some sufficiently large $\alpha > 1$ such that $\alpha a >^* \alpha b$.[6]

Finally, we will assume the similarity and dissimilarity relationships regarding probabilities are symmetric in the sense that if p $>^p$ q then (1-p) $>^p$ (1-q) and if p $\sim^p$ q then (1-p) $\sim^p$ (1-q).

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[5] In the context of prizes, this property corresponds to diminishing marginal utility for gains and diminishing marginal disutility for losses. It is also closely related to the property of Absolute Decreasing Sensitivity discussed in Prelec and Lowenstein (1991).

[6] This property is closely related to the property of Increasing Proportional Sensitivity in Prelec and Lowenstein (1991) and the subproportionality property of the π function in Kahneman and Tversky's (1979) Prospect Theory.

For the purposes of trying to decide between L_1 and L_2 based on similarities and dissimilarities across attributes, agents repeat the comparisons of prizes (e.g., x_{11} with x_{21}) and their corresponding probabilities (e.g., p_{11} with p_{21}) made earlier. However, now the comparisons are made in terms of the similarity or dissimilarity of the prizes and probabilities rather than their equality / inequality.

For each of the n pairs of comparisons made, agents note whether the comparisons:

1a) "favor L_1" (e.g., if $x_{11} >^x x_{21}$ and $p_{11} >^p p_{21}$ or $x_{11} >^x x_{21}$ and $p_{11} \sim^p p_{21}$ where x_{11} is a good outcome.)

1b) "favor L_2"

1c) are "inconclusive" (e.g., if $x_{11} >^x x_{21}$ but $p_{21} >^p p_{11}$ in cases where x_{21} is a good outcome)

1d) are "inconsequential" (e.g., if $x_{11} \sim^x x_{21}$ and $p_{11} \sim^p p_{21}$).[7]

Once all n conclusions have been reached, agents choose according to the rule:

Choose the lottery "favored" in a plurality of comparisons and at random (denoted (E)ither) otherwise.[8, 9]

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[7] For the types of lotteries considered in this paper, the conclusions regarding when one lottery will be "favored" over another and when comparisons will be judged either "inconclusive" or "inconsequential" are obvious. Leland (1994) contains a general discussion as to when similarities and dissimilarities between prizes and probabilities contained in lotteries will "favor" one lottery over another, be "inconclusive", or be "inconsequential".

[8] The idea that decisions might be resolved by tallying reasons for and against the different alternatives is quite old (Benjamin Franklin ostensibly recommended such a procedure) and has been the focus of recurring interest [e.g., Montgomery (1983), Simonson (1989), Shafir, Simonson, and Tversky (1993)].

[9] Leland (1994 and 1998) assumed the weaker decision rule: choose L_1 (L_2) if it is favored in some comparisons and not disfavored in any, and at random otherwise. This pareto rule and the plurality rule assumed here are equivalent for prospects involving two

If a choice is not recommended by this process, we follow Rubinstein (1988) and assume it is reconciled in some other manner.

III. Similarity Judgments and Anomalies Under Uncertainty: Single-Shot Gambles

As was noted in the Introduction, there are a host of circumstances in which people violate the axioms of expected utility. Most of the violations identified to date involve simple one-shot choices between lotteries like S(afe) and R(isky) and between S' and R' shown below:

S:{ $3000, .90 ; $0, .10 }   S':{ $3000, .02 ; $0, .98 }

R:{ $6000, .45 ; $0, .55 }   R':{ $6000, .01 ; $0, .99 }

An implication of the independence axiom is that preferences between simple lotteries {$$x_1$, p_1 ; $0, 1-$p_1$ } and { $$x_2$, p_2 ; $0, 1-$p_2$ } must be invariant to changes in the values of p_1 and p_2 that leave their ratio undisturbed. In choices between S and R and between S' and R' above, the ratio of probabilities in both instances is 2 (i.e, .90/.45 and .02/.01). As such, independence requires either the choice of S and S' or the choice of R and R'. In fact, within and between subjects, the predominant response pattern is S over R but R' over S'; a result referred to as the *common ratio* effect.

Should the choices involve losses instead of gains, subjects tend to choose the risky option, R, over S but the safer option S' over R' -- the mirror image of the response

outcome comparisons but not necessarily for prospects involving three or more outcomes. All results and implications of similarity judgments reported in Leland (1994 and 1998) follow from either the plurality or pareto rule save those concerning violations of pattern exhibited for gains. This finding is an example of what is referred to as the *reflection effect.* Taken together, *common ratio* effects and *reflection* imply a four-fold pattern of risk attitudes - risk averse / risk seeking preference for gains / losses at high and moderate probabilities and risk seeking / risk averse preference for gains /losses at low probabilities.[10]

Now consider how this pattern of choices might arise if agents choose between alternatives as described in the previous section. To choose between prospects like S and R or S' and R', agents will first compare prizes and probabilities across alternatives in terms of their equality/inequality. Because neither alternative stochastically dominates the other, this procedure will prove uninformative, in which case agents will proceed to compare lottery components in terms of their similarity/dissimilarity. Specifically, they will compare the non-zero prizes in the lotteries (i.e., the \$3000 and \$6000) and their corresponding probabilities (e.g., .90 and .45 in SR) and then the common \$0 prizes and their corresponding probabilities (e.g., .10 and .55 in SR), both in terms of their similarity/dissimilarity. Given these pairs of comparisons the following four configurations of similarity and dissimilarity perceptions are possible:

2a) \$6000 $\sim^X$ \$3000 .90 $\sim^P$ .45 ; \$0 $\sim^X$ \$0 .55 $\sim^P$ .10

inconsequential inconsequential => Either

transitivity predicted by Regret theory which only follow from the pareto rule. Some of the behaviors discussed in this paper follow only from the plurality rule.

[10] This pattern of choices has been explained in a variety of ways: By relaxing the requirements of the independence axiom (e.g., Machina (1982)), by assuming that the weights attached to outcomes are not simply their corresponding probabilities (e.g., Quiggin (1982)), or by assuming decision weights different from probabilities and modifying the shape and domain of the utility function (e.g. Kahneman and Tversky (1979)).

2b) $\$6000 >^{x} \$3000$ $.90 \sim^{p} .45$; $\$0 \sim^{x} \$0$ $.55 \sim^{p} .10$

favors R inconsequential => Choose R

2c) $\$6000 \sim^{x} \$3000$ $.90 >^{p} .45$; $\$0 \sim^{x} \$0$ $.55 >^{p} .10$

favors S favors S => Choose S

2d) $\$6000 >^{x} \$3000$ $.90 >^{p} .45$; $\$0 \sim^{x} \$0$ $.55 >^{p} .10$

inconclusive favors S => Choose S

Agents with perceptions as in 2a will choose at random to the extent that both prizes and probabilities are perceived as similar. In 2b, agents perceiving \$6000 as greater than and dissimilar to \$3000 but .90 as similar to .45 will choose the risky option as it offers a noticeably better prize at similar probability (i.e., the first paired comparison favors R) and a similar probability of the same worst outcome (i.e., the second paired comparison is inconsequential). In 2c, agents choose the safer alternative to the extent that it offers a similar good prize at noticeably higher probability, and the same worse prize at noticeably lower probability (S wins here 2 comparisons to none). Finally, agents with perceptions as in 2d choose S since the first paired comparison is inconclusive (R offers a noticeably better prize but S offers a good prize at noticeably higher probability) but the second favors S (it offers a noticeably lower probability of the worst possible outcome).

For individuals choosing between S and R as described above to adhere to the independence axiom in the choice between S' and R', their similarity perceptions on probabilities must remain unaltered as values of the probabilities are reduced, their ratio held constant. However, Increasing Proportional Dissimilarity (IPD) implies that reductions in the values of dissimilar probabilities, their ratio held constant, will eventually produce values that appear similar at which point their complements also become similar. When this occurs, those initially choosing at random (case 2a) and those choosing the riskier option (case 2b) will continue to do so. Those choosing S over R because prizes appear similar but their probabilities dissimilar (case 2c) will choose at random once probabilities are reduced to the point where they too appear similar. Those choosing the safer option as in 2d, on the other hand, will eventually switch to the riskier option (their similarity/dissimilarity perceptions will change to those in 2b). Their response pattern corresponds to the *common ratio effect.* Given IPD, there are no circumstances under which the alternative response pattern RS' can occur.

For choices between lotteries SR and S'R' involving losses, identical reasoning implies that agents' choices here will be the reflection of those between gains.[11] For example, given a choice between S:{-\$3000,.90; \$0,.10} and R:{-\$6000, .45; \$0,.55}, R would be selected to the extent that while the first paired comparison is inconclusive (-\$3000 $>^x$ -\$6000 but .90 $>^p$ .45), the second favors R as it offers a noticeably greater probability (.55 $>^p$ .10) of the best possible outcome, \$0. In the choice between S' and R' involving losses, on the other hand, S' will be selected to the extent that it offers a similar

[11]Note that simple reflection of preferences when the gains in a pair of lotteries are replaced with losses occurs solely as a consequence of the decision rule.

probability (.02 ~P .01) of a noticeably better, albeit unfortunate, outcome (-$3000>X -$6000) and a similar probability (.99 ~P .98) of the best possible outcome, $0.

Common ratio effects constitute but one of the types of violations of the independence axiom originally pointed out by Allais (1953). The second is called the *common consequence* effect for reasons that will become apparent momentarily. To illustrate this anomaly, imagine being given choices between the options S and R and between S' and R' shown below, where each option involves drawing a colored marble from a corresponding urn. The description of each choice shows the percentage of marbles of different colors in each urn and the prizes to be won depending on the option chosen and the marble drawn.

Option S

100% red You win $2400			(62%))

Option R

33% red You win $2500	66% white You win $2400	1% blue You win $0

Option S'

34% white You win $2400	66% blue You win $0	(32%))

Option R'

33% red You win $2500	67% white You win $0

Options S' and R' are obtained from S and R by replacing the consequence $2400, .66 common to both S and R with the consequence $0, .66. According to expected utility, subjects should choose either SS' or RR' as the consequence common in each choice pair contributes equally to the expected utilities of the alternatives regardless of what that contribution is. If fact, however, replacing the consequence $2400, .66 with the consequence $0, .66 makes a big difference. When thirty-seven undergraduates were given these choices, the results are as shown below.[12]

	S'	R'	
S	8	15	23 (62%)
R	4	10	14 (38%)
	12	25	37
	(32%)	(68%)	

As indicated, a majority of subjects are risk averse in SR (often attributed to the existence of a "certainty effect") but risk seeking in S'R'. Within subjects, the modal response pattern is SR'.[13] Moreover, it occurs significantly more often than the other irrational pattern RS' (Conlisk's z = 2.74).[14]. As with common ratio effects, this pattern reflects when the alternatives involve losses.

To see how common consequence effects arise as a result of similarity judgments, note first that S and R are not described in the L_1L_2 format presented in the previous section in that there are not an equal number of prize-probability pairs across alternatives. On the other hand, given their description, it is clear that there are circumstances in which S yields $2400 and R yields $2500, others where both yield $2400, and still another where S yields $2400 and R nothing. This interpretation of the alternatives corresponds to the following L_1L_2 representation of the choice between S and R:

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[12] Subjects were Carnegie Mellon University undergraduates. Questions SR and S'R' were always the first two questions in a questionnaire consisting of three questions. Seventeen subjects were presented with SR and then S'R' while the rest of the subjects answered the questions in reverse order. The S or S' option was always presented first.

[13] Kahneman and Tversky (1979) report similar results (82% risk averse in SR and 17% risk averse in S'R') given a slightly different presentation of the options.

S:{ \$2400, .33; \$2400, .66; \$2400, .01}

R: { \$2500, 33; \$2400, .66;   \$0, .01}

Here, the choice depends on the similarity relationships between \$2500 and \$2400 and between \$2400 and \$0 (the comparison of \$2400 with itself is always inconsequential.) If \$2500 appears similar to \$2400 but \$2400 dissimilar and greater than \$0, the safer alternative will be recommended. Conversely, if \$2500 appears dissimilar to \$2400 but \$2400 similar to \$0, R will be selected. Finally, if all three outcomes are perceived as dissimilar, each alternative will be favored in one paired comparison and, therefore, the choice made by some other means.

The choice between S' and R' is in the L_1L_2 format described earlier. Here, agents will compare \$2500 with \$2400 and .34 with .33 and then \$0 with \$0 and .66 with .67. Given these comparisons and assuming that probabilities differing by 1% are perceived as similar, the riskier alternative will be recommended by similarity if \$2500 $>^x$ \$2400, with the choice resolved at random otherwise. Taken together with the results regarding the choice between S and R, the implication is that similarity judgments cannot produce systematic responses of the RS' variety. The observed pattern SR' is, however, consistent with choice based on similarity judgments. As with common ratio effects, this same reasoning allows the opposite pattern RS', but rules out systematic responses of the form SR', when the outcomes involved are losses.

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[14] Conlisk's z tests whether violations of the form SR' occur more frequently that RS' or vice versa. For additional discussion, see Conlisk (1989).

We have known for decades that choices may depart from the requirements of the independence axiom. Much more recently, Starmer and Sugden (1993) identified another class of behaviors departing from the requirements of expected utility as well as virtually all other models of choice under uncertainty. To demonstrate, consider choices RS, R'S', and R"S" below:

	R:{ $11, .45 ;	$0, .55 }	(76%)	
	S:{ $ 0, .45 :	$7, .55 }		
3a) $11 >^{x} 0, 7 >^{x} 0$	R	S	=>	E
3b) $11 >^{x} 0, 7 \sim^{x} 0$	R	I	=>	R

	R': {$11, .45 ;	$0, .10 ;	$0, .45}		(34% - 44%)
	S': { $ 7, .45 ;	$7, .10 ;	$0, .45}		
3a') $11 >^{x} 7, 7 >^{x} 0$	R'	S'	I	=>	E
3b') $11 >^{x} 7, 7 \sim^{x} 0$	R'	I	I	=>	R'
3c') $11 \sim^{x} 7, 7 >^{x} 0$	I	S'	I	=>	S'

	R": { $ 0, .45 ;	$11, .45;	$0, .10}		(50%)
	S": { $ 7, .45 ;	$0, .45;	$7, .10}		
3a") $11 >^{x} 0, 7 >^{x} 0$	S"	R"	S"	=>	S"
3b") $11 >^{x} 0, 7 \sim^{x} 0$	I	R"	I	=>	R"

Note that in all three choice pairs, the risky options correspond to the prospect {$11,.45;$0,.55} while the safe ones correspond to {$7,.55; 0,.45}. These equivalencies notwithstanding, people tend to be much more risk seeking in RS than either R'S' or R"S" (as indicated by the percentages of respondents selecting the risky alternative shown in parentheses). The tendency to be more risk seeking in RS than in R'S' has been termed a "juxtaposition effect" and attributed by Loomes and Sugden (1982) to "regret aversion." In the choice between R and S, regret aversion implies that agents will be particularly concerned about the possibility of choosing S and then experiencing regret upon finding out that if they had chosen R, they would have received an $11 prize instead of the $0 they got. However, by this reasoning, subjects should be risk seeking in R"S", yet here choices are roughly split between the safe and risky options. Starmer and Sugden (1993) attribute the tendency to be risk seeking in RS but not in R'S' nor R"S" to what they call "event splitting" -- the tendency for an event with given probability (here the 55% chance of $7) to be more heavily weighted if it is considered as two sub-events than if it is considered as a single event.

The explanation for these responses implied by Similarity is indicated below each choice problem. Specifically, note that in RS there is no pattern of similarities that could recommend the choice of S.[15] In R'S' and R"S", splitting the event $7,.55 in two produces circumstances in which the safe option can be recommended.

Note that by this explanation for "event splitting," common consequence and certainty effects are subsumed. In the common consequence problem SR, the fact that the safe lottery S pays off $2400 when the risky one pays off $0 cannot be overlooked. Stated differently, when S pays $2400 with certainty, this leads agents to split the event, juxtaposing the outcome $2400, .01 in S with $0,.01 in R. So long as $2400 and $0 are

[15] The only way S can get a favorable vote is if $\$7 >^x \$0$ but, if so, then $\$11 >^x \$0$, favoring R.

dissimilar, this splitting produces a vote for the safer alternative. To the extent this reasoning is correct, we should be able to induce risk aversion in the common consequence choice S':{$2400, .34; $0, .66} and R':{$2500,.33;$0,.67}discussed earlier, and eliminate independence violations between problems SR and S'R' in the process, simply by splitting the $2400, .34 component in S'R' into two components. To examine this possibility, thirty-seven other subjects were given choices S*R* and S*'R*' shown below.

Option S*			
33% red	66% white	1% blue	(73%)
You win $2400	You win $2400	You win $2400	
Option R*			
33% red	66% white	1% blue	
You win $2500	You win $2400	You win $0	

Option S*'			
33% red	1% white	66% blue	(73%)
You win $2400	You win $2400	You win $0	
Option R*'			
33% red	1% white	66% blue	
You win $2500	You win $0	You win $0	

Results for these questions are summarized below.

	S*'	R*'	
S*	22	5	27 (73%)
R*	5	5	10 (27%)
	27	10	37
	(73%)	(27%)	

Making the assumed comparisons in SR explicit, as in S*R*, increases the proportion of risk averse responses only modestly from 63% to 73% (z=.1.02). In contrast, reframing S'R' by splitting the $2400, .34 outcome into two outcomes as in S*'R*' increases the proportion of risk averse responses from 32% to 73% (z=3.87). In the process, common consequence violations within subjects vanish.

Common ratio, common consequence, event splitting and reflection effects represent only a small portion of the circumstances in which observed behavior departs from the requirements of expected utility in single-shot choice situations.[16] However, they do serve to illustrate how and why choices based on similarity judgments will produce robust departures from the requirements of expected utility. Transposing prize values around zero reverses the conclusions drawn regarding whether paired comparisons favor one lottery or the other, thus changing the recommendation of the decision rule in the process. Theoretically inconsequential arithmetic manipulations of lottery components, like dividing probabilities in lottery pairs by a constant, may matter to the extent that they alter the perceived similarity of those lottery components across alternatives. Finally, splitting events may influence choice to the extent that it alters the number of comparisons that can favor one lottery over another or, stated differently, the number of reasons for choosing one lottery over the other. The following sections examine how such manipulations produce anomalous choices in other decision contexts.

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16 Leland (1994, 1998) provide extensive discussions of the types of violations implied by similarity judgments in single-shot gambling situations. To summarize, for risky alternatives described as prospects, similarity judgments produce the types of choice anomalies upon which Kahneman and Tversky's (1979) Prospect Theory and Machina's (1982) Generalized Expected Utility were based (e.g., violations of the *independence axiom* and *reflection effects*). In situations where the alternatives are represented as actions involving state-contingent consequences, the model predicts the types of behaviors implied by Loomes and Sugden's (1982) Regret Theory (e.g., violations of the *independence axiom* under certain circumstances but adherence to its requirements in others), although under more general conditions. In addition, the model implies observed violations of stochastic dominance (Tversky and Kahneman (1986), Leland (1998), Hurley and Robinson (1995)), transitivity (Tversky (1969), Leland (1994)), and invariance (Leland (1998)) not accounted for by available alternatives to expected utility. All these predictions save those involving intransitivities predicted by Regret Theory also follow from the model presented here.

IV. Similarity Judgments and Anomalies Under Uncertainty - The Problem with Repeat Play

In an article written many years ago, Paul Samuelson (1963) tells a story about a colleague, a "...distinguished scholar -- who lays no claim to advanced mathematical skills..." The story goes that when Samuelson offered him a bet involving a 50:50 chance of either winning $200 or losing $100, the scholar responded:

> "I won't bet because I would feel the $100 loss more than the $200 gain. But I'll take you on if you promise to let me play 100 such bets...*because*... I am, so to speak, virtually sure to come out ahead on such a sequence..."

Samuelson proceeded to show that such behavior is inconsistent with expected utility maximization -- an expected utility maximizer offered the opportunity to play the gamble repeatedly must refuse if he or she would be unwilling to play it once at every wealth level achievable by playing it in the repeated context. He then went on to attribute his colleague's response to a failure to appreciate that while the probability of coming out ahead increases (and the probability of incurring a loss declines) with repetition, the magnitude of the losses, should they occur, also increases.

A number of recent studies [e.g., Keren and Wagenaar (1987), Wedell and Bockenholt (1990), Keren (1991) Redelmeier and Tversky (1992)] suggest that Samuelson's colleague was not alone in his confusion. Specifically, they show that in choices between a gamble or abstention and between a low-probability, high payoff (riskier) gamble and a high-probability low payoff (safer) gamble, people tend to prefer the safe option under single play conditions but the riskier one under repeat play. The latter tendency is encouraged the greater the number of repetitions involved. Moreover, Wedell and Bockenholt (1990) present evidence suggesting that subjects do this for the same reason Samuelson's colleague did -- because repetition increases the chances of winning.

Both the tendency to respond in a consistently risk seeking fashion in repeat versus single-shot decision contexts and the apparent role of the chance of "coming out ahead" follow if individuals choose based on similarity judgments. To see how, consider the following choices between a gamble and abstention presented to subjects by Redelmeier and Tversky (1992).

> Imagine you have the opportunity to play a gamble that offers a 50% chance of winning $2,000 and a 50% chance of losing $500. Would you play the gamble?

> Now suppose you have the opportunity to play the gamble five times, not just once. Would you play it five times?

For the first question, 80 of 185 (43%) of Redelmeier and Tversky's subjects accepted the gamble. When the same subjects were asked whether they would play the gamble five times, however, the percentage of subjects accepting the bet rose to 63%, a significant increase ($p<.005$).[17]

These choices, like one of the two choices involved in producing the common consequence effect discussed earlier, are not in L_1L_2 format. However, it is clear in both single-shot and repeat decision problems that there are circumstances where the gamble yields an inferior outcome. As such, then following the explanation for common consequence violations of independence and the existence of a certainty effect, the unique choice problem above would be interpreted as follows where the choice is either the $R_{(isky)}$ gamble or $A_{(bstention)}$.

R: { \$2000, .5; -\$500, 5}

A: { \$0, .5; \$0, .5}

Note here that since probabilities across alternatives are identical, predictions depend only on similarities or dissimilarities between prizes across alternatives. The four configurations of similarity perceptions between prizes are:

4a)	$2000 >^X 0$,	$0 >^X -\$500$		
	(favors R)	(favors A)	=>	choose Either
4b)	$2000 >^X 0$,	$0 \sim^X -\$500$		
	(favors R)	(inconsequential)	=>	choose R
4c)	$2000 \sim^X 0$,	$0 >^X -\$500$		
	(inconsequential)	(favors A)	=>	choose A

[17] Keren (1991) reports very similar results.

4d) $2000 \sim^X 0$, $0 \sim^X -\$500$

(inconsequential) (inconsequential) => choose Either

As indicated above, either the gamble or abstention can be recommended by similarity here.

But now consider what happens when the gamble is to be repeated, say, five times. Once again it is clear that there are circumstances where the gamble results in a loss relative to abstention. However, with repetition there are also a greater number of instances in which the gamble yields an outcome greater than abstaining does. For subjects up on their binomial expansion, the appropriate L_1L_2 representation of the repeated play decision problem is as follows:

R':{ \$10k, .03 ; \$7.5k, .16; \$5k, .31; \$2.5k, .31; \$0, .16; -\$2.5k, .03}

A':{ \$0, .03 ; \$0k, .16; \$0k, .31; \$0k, .31; \$0, .16; \$0k, .03}

Subjects perceiving the repeat choice in this fashion will make six comparisons of outcomes. Those individuals who chose R in the single-shot situation because $2000 >^X 0$ and $0 \sim^X -\$500$ must choose R' here as, even if we assume $\$0 >^X -\$2,500$ (favoring A'), there will be four votes favoring R' since if \$2000 is dissimilar and greater than \$0 then anything larger than \$2000 is also dissimilar and greater than zero. The same holds true for those who chose randomly in RA because if $\$2000 >^X 0$ and $0 >^X -\$500$, then R' again wins 4 to 1. Those individuals who choose A because \$2000 appears similar to \$0 but $0 dissimilar to -$500 (pattern 4c) and those who chose at random because all the outcomes appear similar (4d) can, if fact, systematically choose the safe alternative A' but only if $10,000 appears similar to $0 and $0 dissimilar to -$2,500. For the remaining possible configurations of similarities among the possible outcomes that individuals choosing according to 4c or 4d might possess, there are two ($10,000 $>^x$ 0, 0 $>^x$ -$2,500 and $10,000 $\sim^x$ 0, 0 $\sim^x$ -$2,500) that produce random choices with the remaining seven similarity configurations all recommending the riskier alternative, R'.

At this point, one might object that it is implausible to assume that subjects in experiments are actually producing binomial distributions for the purposes of making decisions, and particularly so given that these subjects (in contrast to the rational agents of normative theory) are assumed to have substantial difficulty making decisions even in single-shot situations. On the other hand, what is essential to produce the results above is that agents recognize that repetition produces more instances in which the riskier alternative offers a better payoff. If so, we would expect subjects to behave in a similar manner if presented with the choice between repeated alternatives described as a choice between a single-shot prospect and abstention. Tversky and Redelmeier (1992) did exactly this.[18] Qualitatively, their results are as predicted -- of 47 individuals given a choice between R described as a single-shot gamble and abstention, 39 (83%) selected the risky alternative.

As noted earlier, the repeat/single-shot disparity arises in situations involving two gambles as well as ones where one of the alternatives is abstention. A consequence of this

[18] Keren (1991) also presented repeat gambles in a unique frame with similar effect.

is that common ratio effects observed in single-shot contexts vanish with repetition. To illustrate consider choice-pairs RS and R'S' shown below.

R: {$25, .50; $0, .50 } R': {$25, .10; $0, .90 }

S: {$10, .99; $0, .01 } S': {$10, . 20; $0, .80 }

These are common ratio problems of the type described in the prior section -- the ratio of probabilities across problems being (approximately) 1/2.[19] As was shown earlier, under reasonable assumptions regarding the similarities and dissimilarity of prizes and probabilities across these lotteries, agents will choose SR' (and RS' for losses). Consistent with these predictions, Keren and Wagenaar (1987) report that for these problems and ones where the outcomes were multiplied by 10, 67% of subjects given SR choose S while for subjects given R'S' the majority, 55%, choose the riskier alternative R'. For losses, they find the opposite pattern -- 66% of respondents choose the risky alternative in RS and 60% of respondents choose the safe outcome in R'S'.

Keren and Wagenaar asked other subjects to choose between these lotteries assuming that the chosen alternative would be played 10 times. In this treatment, of subjects given RS, 65% selected the risky option R with a like proportion, 72%, selecting R' over S'. For losses, the opposite pattern obtained -- 58% choose S over R and 70% choose S' or R'. As these results demonstrate, introducing repetition drives out common

[19] These gambles were used in a study by Keren and Wagenaar (1987). They chose to use probabilities just less than 1 (and in RS) in their study to see whether certainty effect like behavior still occurred.

ratio effects although reflection effects remain -- Keren and Wagenaar found that the modal response pattern when these choices involved losses was SS'.

To see how such behavior follows from similarity judgments, consider the binomial expansions of choices R3S3 and R3'S3' shown below where the lotteries are to be played three times:

R3: {$75, .125;	$50, .375;	$25, .375;	$0, .125 }	
S3: {$30, .970299;	$20, .029403;	$10, .000297;	$0, .000001 }	
R3': {$75, .001;	$50, .027;	$25, .243;	$0, .729 }	
S3': {$30, .008;	$20, .096;	$10, .384;	$0, .512 }	

Given these alternatives, an agent with similarity perceptions producing the choice pattern SR' in the single shot case (i.e., $\$25 >^{x} \$10$, $.9 >^{p} .5$, $.2 \sim^{p} .1$) would deem the last comparison in either choice pair as "favoring S" if the probabilities of the common $0 appear dissimilar or as "inconsequential" if the probabilities were perceived as similar. In the remaining comparisons in either choice pair, we know that $\$25 >^{x} \$10$ and by Increasing Proportional Dissimilarity that $n(\$25) >^{x} n(\$10)$. As a result, these paired comparisons will either be judged "inconclusive" if the probabilities across alternatives are dissimilar or as "favoring R" if they are perceived as similar. As the number of repetitions increases, the differences in each pair of compared probabilities across alternatives approaches zero in which case probabilities become similar.[20] This fact,

[20] Given n repetitions of the risky and safe gambles, the difference in the probabilities associated with winning n times and losing n-m times equals $p^{(n-m)} (1-p)^{m} - (2p)^{(n-m)} (1-2p)^{m}$. The limit value for this expression (or, equivalently, the limit of $\{1/(2p)^{(n-m)}\} * \{(1-p)/(1-2p)\}^{m}\}$ as the number of repetitions, n, goes to infinity is 0.

coupled with the IPD property, assure that the riskier alternatives will eventually be selected.

In summary, according to this explanation for disparities between behavior revealed in repeated versus single-shot decisions, there is nothing fundamentally different about the two choice tasks nor is there an important distinction between short and long run. Instead, repetition plays the same role here as event splitting does in single-shot choice situations -- it increases the number of comparisons that can favor one alternative over the other.

V. Similarity Judgments and Anomalies in Intertemporal Choice

The independence axiom of the expected utility hypothesis places restrictions on how agents can choose between pairs of risky prospects. In the Discounted Utility model of intertemporal choice, the *stationarity* assumption plays a similar role. To demonstrate, consider simple intertemporal prospects T_j and T_k shown below where T_j offers an increment to consumption x_j in time period t_j and T_k offers an increment to consumption x_k in time period t_k.

$$T_j : \{ \quad x_j \quad , \quad t_j \quad \}$$

$$T_k : \{ \quad x_k \quad , \quad t_k \quad \}$$

Assuming, for simplicity, a common baseline level of consumption per period, c, agents deciding between these options according to the Discounted Utility model will choose as follows where U(.) is a concave, ratio-scaled, utility function, and δ is the one period discount factor:[21, 22]

5) $T_j \succsim T_k$ iff

$$U(c+x_j)\delta^j + U(c)\delta^k \gtreqless U(c)\delta^j + U(c+x_k)\delta^k$$

Dividing through by δ^j and rearranging terms, yields the following expression:

6) $T_j \succsim T_k$ iff

$$U(c+x_j) - U(c) \gtreqless [U(c+x_k) - U(c)]\delta^{k-j}$$

Two things are worth emphasizing regarding this model. First, note that preferences should be invariant to proportionate rescaling of the payoffs in intertemporal prospects. As such, individuals given choices between $T_1 T_2$ and $T_1' T_2'$ below (where the outcomes in the latter are obtained by scaling the receipts in the former by a factor of 10) should be consistent (i.e., agents should choose $T_1 T_1'$ or $T_2 T_2'$.)

T_1 :{ $20 , 1 month } T_1': { $200 , 1 months }

T_2:{ $25 , 2 month } T_2' :{ $250 , 2 months }

21 In the following expressions $\succsim$ signifies is "preferred to", "indifferent", "not preferred to" and $\gtreqless$ signifies is "greater than, equal to, less than."

22 This discussion follows Prelec and Lowenstein (1991).

Second, note that in expression 6, the only way discounting enters into the decision is through the absolute difference in the time periods. As such, agents given choices between T_1 and T_2 and between T_{11} and T_{12} shown below must either select T_1 and T_{11} or T_2 and T_{12} as the absolute time interval in both choices is identically 1 period.

T_1 :{	\$20 ,	1 month }	T_{11} :{	\$20 ,	11 months }
T_2:{	\$25 ,	2 month }	T_{12}:{	\$25 ,	12 months }

Neither of these requirements of the Discounted Utility model appear to hold. Prelec and Lowenstein (1991) review evidence suggesting that people indifferent between two intertemporal sequences, say T_1 and T_2, tend to prefer the one offering the larger-later payoff as the payoffs contained in the alternatives are scaled up proportionately (e.g., T_2' over T_1'). They refer to this response pattern as the *magnitude* effect.

They also review evidence suggesting that people indifferent between two intertemporal sequences, say T_1 and T_2, tend to prefer the one offering the larger-later payoff (e.g., T_{12} over T_{11}) as the sequences are deferred equally into the future. This response pattern is referred to as the *common difference* effect. Finally, Prelec and Lowenstein note that in cases where the intertemporal prospects involve future decrements to consumption rather than future increments, both magnitude and common difference response pattern *reflect* in much the same way as for losses versus gains in choices under uncertainty.

All three of these anomalies in intertemporal choice follow if agents base their choices on similarity judgments. To demonstrate, consider the choice between T_1 and T_2. Assume, as per the procedure assumed under uncertainty, that agents given these options compare consumption increments \$20 and \$25, and their dates of receipt, 1 month and 2 months, in terms of their similarity or dissimilarity. They then conclude whether the pair of comparisons favors one alternative over the other, is inconclusive, or is inconsequential. How these conclusions are drawn, in turn, depends upon how agents feel about more immediate versus delayed consumption. Assume, consistent with the assumption of impatience, that sooner receipts are preferred to later receipts, ceteris paribus. If so, then given the choice between T_1 and T_2 any of the four configurations of similarity/dissimilarity perceptions shown on the left below are possible where $>^t$ and $\sim^t$ are the time analogs to the dissimilarity and similarity relations on prizes and probabilities:

7a) $25 \sim^x 20,\quad 2 \sim^t 1$; $25 \sim^x 20,\quad 250 >^x 200$

inconsequential (Choose Either) favors T_2'(Choose T_2') => E T_2'

7b) $25 >^x 20,\quad 2 \sim^t 1$; $25 >^x 20,\quad 250 >^x 200$

favors T_2 (Choose T_2) favors T_2' (Choose T_2') => T_2 T_2'

7c) $25 \sim^x 20\quad ,\quad 2 >^t 1$; $25 \sim^x 20,\quad 250 >^x 200$

favors T_1 (Choose T_1) inconsequential (Choose Either) => T_1 E

7d) 25 $>^x$ 20, 2 $>^t$ 1 ; 25 $>^x$ 20, 250 $>^x$ 200

inconclusive (Choose Either) inconclusive (Choose Either) => E E

Agents with perceptions as in 7a will choose at random (Either) to the extent that consumption increments and their dates of receipt are perceived as similar (i.e., the pair of comparisons is inconsequential). Agents perceiving 25 as greater than and dissimilar to 20 but 2 months as similar to 1 month (as in 7b) will choose the larger-later option T_2 as it offers a noticeably better prize at a similar date in the future (i.e., the pair of comparisons favors T_2) . In 7c, agents choose the smaller-sooner alternative T_1 to the extent that it offers a similar consumption increment noticeably sooner (i.e., the pair of comparisons favor T_1). Finally, agents with perceptions as in 7d will choose at random since T_2 offers a noticeably better consumption increment but T_1 offers a desirable increment at a noticeably earlier date (i.e., the pair of comparisons is inconclusive.)

Now consider the choice between T_1'T_2'. By the *Increasing Proportional Dissimilarity* property, we know that similar values will eventually appear dissimilar if they are scaled up sufficiently. Assuming that scaling the prizes in T_1T_2 up by a factor of 10 (to produce T_1'T_2') achieves this effect, the consequences are shown of the right side in the table above. Such scaling up drives out impatience (case 7c) and encourages patience (case 7a). Together these results produce the magnitude effect.

Now suppose that rather than increasing receipts in T_1T_2 proportionately, we instead change the timing of these receipts by some absolute amount as in the choice, T_{11}

T_{12}. By the assumption of *Increasing Absolute Similarity*, we know that this manipulation will eventually make time periods appear similar in value. The consequences of this are illustrated below with all possible similarity perceptions between components of T_1T_2 shown on the left and those for T_{11} T_{12} assuming time periods 12 and 11 are perceived as similar, on the right.

8a) $25 \sim^x 20$, $2 \sim^t 1$; $25 \sim^x 20$, $12 \sim^t 11$

inconsequential (Choose Either) inconsequential (Choose Either) => EE

8b) $25 >^x 20$, $2 \sim^t 1$; $25 >^x 20$, $12 \sim^t 11$

favors T_2 (Choose T_2) favors T_{12}(Choose T_{12}) => T_2 T_{12}

8c) $25 \sim^x 20$, $2 >^t 1$; $25 \sim^x 20$, $12 \sim^t 11$

favors T_1 (Choose T_1) inconsequential (Choose Either) => T_1 E

8d) $25 >^x 20$, $2 >^t 1$; $25 >^x 20$, $12 \sim^t 11$

inconclusive (Choose Either) favors T_{12} (Choose T_{12}) => E T_{12}

As the table indicates, when the dates associated with consumption increments are deferred into the future, individuals choosing at random between T_1 and T_2 (case 8a) and those choosing the later-larger option T_2 (case 8b) continue to do. Those choosing the smaller-sooner option T_1 because it appears to offer a similar payoff noticeably sooner

(8c) switch to choosing at random for sufficient delays into the future. Finally, those choosing either alternative due to the configuration of similarity perceptions in 8d will eventually switch to the choice of later-larger option, T_{12}. This result constitutes the common-difference effect within subjects while the combination of patterns in 8c and 8d imply that the response pattern T_1 T_{12} should occur more frequently than the response pattern T_2 T_{11} between subjects. If we make the outcomes in choices T_1T_2 and T_1'T_2' or T_1T_2 and $T_{11}T_{12}$ consumption decrements, reflection effects also follow.

The analysis presented above suggests that the same decision process responsible for violations of the independence axiom and reflection effects under uncertainty also produces violations of the stationarity condition and reflection effects in intertemporal settings. If so, then we should also be able to sway intertemporal decisions by simply re-describing the alternatives in such a way as to influence what is compared to what (as with event splitting) and, indeed, there is evidence consistent with this prediction. To demonstrate, consider the following choices involving dinners A or B and between dinners C and D discussed in Lowenstein and Prelec (1993).

A.	French dinner (86%)	C.	French in 1 month (80%)
versus		versus	
B.	Greek Dinner	D	French in 2 month

As indicated by the percentages selecting A and C, people clearly tend to prefer French to Greek cuisine and are impatient. From these responses, we would infer that if given a choice between E and F below, alternative E, offering the preferred cuisine sooner, should be selected but it is not -- roughly half of respondents choose each alternative.

E	French in 1 month	;	Greek in 2 months (43%)
F	Greek in 1 month	;	French in 2 months

If subjects evaluate this choice based on similarity, the first paired comparison (French with Greek and 1 month with 1 month) will favor E since French is dissimilar and better than Greek (from AB) and 1 month is similar to 1 month. However, the second paired comparison yields the opposite conclusion in which case the choice is made at random. [23]

VII. Discussion

Most attempts to explain decision making anomalies start with the normative model and ask "what properties of this model must be relaxed or abandoned to accommodate observed behavior?" Choices are treated as synonymous with preferences. Tversky (1969) proposed instead that faced with complex alternatives, people would employ "various approximation methods that enable them to process the relevant information." In this case, choices might depart from properties preferences obey. He then postulated a seemingly reasonable approximation method, but one that, if employed, would produce systematically intransitive choices – and it did. In this paper, I have

[23] Lowenstein and Prelec (1993) present other data that are not consistent with the model presented here, at least without additional assumptions. For example, it appears that people like to spread out consumption of pleasurable things in ways the model presented here suggests should be irrelevant.

investigated the extent to which this type of approximation method produces other choice anomalies. Given certain assumptions regarding the specifics of the decision rule people employ and the nature of judgments regarding when magnitudes appear similar or dissimilar, choice based on similarity judgments explains a broad range of decision making anomalies. In the process, the model provides a unified account for behaviors heretofore attributed to many different causes. As summarized in Table 1, the decision rule assumed in the model implies that reflection effects in risky and intertemporal choice will be ubiquitous phenomena – by changing the signs of outcomes, votes favoring one outcome become votes favoring the other. Reflection effects are usually attributed

Table 1

Anomaly	**Similarity Model Explanation**	**Predominant Alternative Explanations**
Reflection Effects	Decision rule	S-shaped value function[24]
Juxtaposition effects / uncertainty	Decision rule	Regret [25]
Juxtaposition effects / intertemporal	Decision rule	
Event splitting	Decision rule	Overweighting small probabilities everywhere[26]
Common difference	Increasing absolute similarity (IAS)	Absolute Decreasing Sensitivity[27]
Common ratio	Increasing proportional dissimilarity (IPD)	Sub-proportionality of probability weighting function[28]
Magnitude effects	IPD	Increasing Proportional Sensitivity[29]
Single shot / repeat gamble disparity	Decision rule, IPD	Maximization of probability of coming out ahead[30]

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24 As in Prospect Theory (1979).

25 As in Loomes and Sugden (1982).

26 As in Starmer and Sugden (1993)

27 Assumed in Prelec and Lowenstein (1991).

28 As in Prospect Theory(1979).

29 Assumed in Prelec and Lowenstein (1991).

to an S-shaped value function as in Kahneman and Tversky (1979).

The decision rule, in some cases coupled with the assumption that similarity judgments may be intransitive, also implies that choices will be sensitive to the alignment of attributes across alternatives (as manifest in juxtaposition effects) and to the number of comparisons that need to be made (as manifest in event splitting effects). Juxtaposition effects and event splitting are elsewhere attributed to concern over post-decision regret [Loomes and Sugden (1982)], and systematic and universal overweighting of small probabilities [Starmer and Sugden (1993)], respectively.

Other behaviors follow from assumptions made regarding the nature of similarity and dissimilarity relations. Specifically, the common difference effect in intertemporal choice follows from the assumption of increasing absolute similarity. Prelec and Lowenstein (1991) explain common difference effects in a Prospect theory framework by the closely related property of absolute decreasing sensitivity.

Common ratio effects under uncertainty and magnitude effects in intertemporal choice both follow from the assumption that perceptions regarding similarity exhibit the property of Increasing Proportional Dissimilarity. Elsewhere, these behaviors are explained *by sub-proportionality* of the probability weighting function [Kahneman and Tversky (1979) and the property of *Increasing Proportional Sensitivity* [Prelec and Lowenstein (1991)], respectively.

Finally, Increasing Proportional Dissimilarity coupled with the voting characteristic of the decision rule imply behaviors observed in repeat gamble situations --

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[30] As in Lopes (1981).

behaviors elsewhere attributed to agents maximizing the probability of coming out ahead [Lopes (1969)].

In light of the range of anomalies addressed here, further investigation of the role similarity judgments in decision making would seem fruitful.

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