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Lossy Transmission Line Macromodeling based on Optimal Matrix Rational Approximations - Three Case studies

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Lossy Transmission Line Passive Macromodeling Algorithm - Three Case Studies

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Abstract: *The rapid increase in operating speeds, density and complexity of modern integrated circuits has made interconnect analysis a requirement for all state-of-the-art circuit simulators. Recently passive closed-form optimal matrix-rational approximation based algorithm is developed for macromodeling of lossy multiconductor transmission lines. This paper presents three case studies for optimal matrix rational approximation and provides the performance comparison with the conventional lumped discretization.*

I. Introduction

Recent trend in the VLSI industry towards higher operating speeds, sharper rise times and smaller devices has made the signal integrity analysis a challenging task. The major difficulty usually encountered while linking the distributed transmission line models and nonlinear simulators is the problem of mixed frequency/time. This is because distributed elements are usually characterized in the frequency-domain whereas nonlinear components such as drivers and receivers are represented only in time-domain. Several publications can be found in the literature, which address this issue [1] - [9]. Approaches based on conventional lumped segmentation of transmission lines provide a brute force solution to the problem of mixed frequency/time simulation. However, these methods lead to large circuit matrices, rendering the simulation inefficient.

In order to address the above problem, a passive transmission line macromodeling algorithm was recently suggested in [4]. The algorithm is based on closed-form optimal matrix-rational approximation (OMRA) of exponential functions describing Telegrapher's equations [3]-[5]. The method uses pre-determined (stored) coefficients given by the closed-form matrix-rational approximation and the per-unit-length parameters to obtain a time-domain macromodel. The proposed model can be easily incorporated with conventional circuit simulators such as SPICE and also with the recent passive model-reduction techniques [6]. In this paper, we provide results of three case studies [8] and performance comparison of OMRA with conventional lumped discretization.

II. Development of the Proposed Algorithm

Distributed interconnects can be described by a set of Telegrapher’s equations in the Laplace-domain as

$$\begin{bmatrix} \boldsymbol{V}(d,s) \\ -\boldsymbol{I}(d,s) \end{bmatrix} = e^{\boldsymbol{Z}(s)} \begin{bmatrix} \boldsymbol{V}(0,s) \\ \boldsymbol{I}(0,s) \end{bmatrix}; \quad \boldsymbol{Z}(s) = \begin{bmatrix} \mathbf{0} & -(\boldsymbol{R}+s\boldsymbol{L})d \\ -(\boldsymbol{G}+s\boldsymbol{C})d & \mathbf{0} \end{bmatrix} \qquad \textbf{(1)}$$

where $\boldsymbol{V}(s)$, $\boldsymbol{I}(s)$ are the terminal voltage and current vectors and d is the length of the line. $\boldsymbol{R}, \boldsymbol{L}, \boldsymbol{G}, \boldsymbol{C}$ are the per-unit-length (PUL) parameters, and are symmetric nonnegative definite matrices. Equation (1) does not have a direct representation in the time-domain, which makes it difficult to interface with nonlinear simulators. In order to address this, several techniques based on passive macromodeling have been suggested recently. It is to be noted that most of these algorithms employ some kind of approximation in the frequency-domain to match the impulse response up to a maximum frequency of interest (f_{max}). However, the behavior after f_{max} is generally not considered, which can lead to significant errors in the impulse transient response (especially in the early-time period) [7]. This can affect the accuracy of the transient response at all other time-points when the macromodel is included during the simulation of a large network. Also, the above problem can be aggravated in the presence of sharp rise times. To remove these ripples, the order of the approximation required would be very high, making the macromodel inefficient.

In order to address the above problem, a passive macromodeling algorithm based on OMRA was recently suggested [4]. It provides a mechanism to control the asymptotic behavior of the high-frequency impulse response while matching the response up to f_{max} accurately. This leads to significant reduction in errors of transient responses. Also, it guarantees the passivity of the macromodel, while ensuring that the macromodel orders are comparable to the ones published in the literature. The macromodel is obtained analytically, in terms of predetermined (stored) constants and the given PUL line parameters. In this paper, we provide results of three case studies [8] and performance comparison of OMRA with conventional lumped discretization. A brief review of the concept and the steps involved in the OMRA algorithm is given below.

The objective of the proposed algorithm is to provide a mechanism to control the macromodel impulse response beyond f_{max} so as to minimize early-time ripples while preserving the accuracy and passivity of the macromodel. The early-time impulse response is mainly influenced by the following relationship:

$$h(0^+) = \lim_{s \to \infty} sH_{MN}(s) \quad \textbf{(2)}$$

where 's' is the Laplace operator, $H_{MN}(s)$ represents the frequency-domain rational function, M and N are the numerator and denominator polynomial orders, respectively, and $h(0+)$ represents the early-time response $(t=0)$. Assuming that the k^{th} derivative of the impulse response, $h^{(k)}(0^+) = 0$, then

$$h^{(k+1)}(0^+) = \lim_{s \to \infty} s^{k+1} H_{MN}(s) \quad \textbf{(3)}$$

Observing (2) and (3) one can note that to obtain flat response around $t=0$, the transfer-admittances represented by $H_{MN}(s)$ must be a *strictly proper rational-function* [4] such that $k = N - M$ is as large as possible, while preserving the accuracy of the macromodel. Also, it is desired that the order of the denominator (N) is kept as small as possible for achieving efficient simulation. We will use the above principle to reduce the error in transient responses of distributed transmission line macromodels. To achieve the above objective, in the new algorithm, we use predetermined coefficients from two different orders of approximation of the scalar exponential matrix, (N) and $(N+1)$ that satisfy the passivity preserving theorem [3], to approximate the admittance matrix. Expressing the N^{th} and $(N+1)^{th}$ order approximations of the exponential function, in terms of even and odd polynomials as:

$$e^s \approx \frac{Q_1^{EV} + Q_1^{ODD}}{Q_1^{EV} - Q_1^{ODD}}; \quad Q_1^{EV} = \sum_{i=0}^{N} q_{1,i}\left[\frac{1}{2}(1+(-1)^i)s^i\right] \quad Q_1^{ODD} = \sum_{i=0}^{N} q_{1,i}\left[\frac{1}{2}(1-(-1)^i)s^i\right]$$

$$e^s \approx \frac{Q_2^{EV} + Q_2^{ODD}}{Q_2^{EV} - Q_2^{ODD}}; \quad Q_2^{EV} = \sum_{i=0}^{N+1} q_{2,i}\left[\frac{1}{2}(1+(-1)^i)s^i\right] \quad Q_2^{ODD} = \sum_{i=0}^{N+1} q_{2,i}\left[\frac{1}{2}(1-(-1)^i)s^i\right] \quad \textbf{(4)}$$

Next, the Y-parameters can be written as

$$\left.\begin{matrix} Y_{11} \\ Y_{22} \end{matrix}\right\} = \frac{\sum_{i=0}^{N} (\mu_i + \rho_i)(ab)^i}{a\left(\sum_{i=0}^{N-1} \varphi_i (ab)^i\right)}; \quad \left.\begin{matrix} Y_{12} \\ Y_{21} \end{matrix}\right\} = \frac{\sum_{i=0}^{N} (\mu_i - \rho_i)(ab)^i}{a\left(\sum_{i=0}^{N-1} \varphi_i (ab)^i\right)}; \quad \begin{matrix} a = R + sL \\ b = G + sC \end{matrix} \quad \textbf{(5)}$$

The predetermined coefficients μ_i, ρ_i and φ_i in (5) and can be obtained using (4) as follows.

$$\frac{Q_1^{EV} Q_2^{EV}}{2Q_1^{ODD} Q_2^{EV}} = \frac{\sum_{i=0}^{N} \mu_i s^{2i}}{\sum_{i=0}^{N-1} \varphi_i s^{(2i+1)}}; \quad \frac{Q_1^{ODD} Q_2^{ODD}}{2Q_1^{ODD} Q_2^{EV}} = \frac{\sum_{i=0}^{N} \rho_i s^{2i}}{\sum_{i=0}^{N-1} \varphi_i s^{(2i+1)}} \quad \textbf{(6)}$$

Due to the Hurwitz characteristics of the approximation, the coefficients μ_i, ρ_i and φ_i are all positive values. By appropriately choosing the values of ρ_i such that $\mu_i = \rho_i$, (for example $\mu_N = \rho_N$), *the final rational-form of the transfer-admittances* can be obtained with numerator-order less than the denominator order $(k > 1)$. The rate of decay can be speeded-up by setting higher values for k (removing more number of zeros from transfer admittances). Details about formulation of minimax objective function for obtaining predetermined coefficients can be found in [4]-[5]. *It should be emphasized that the minimax optimization is performed on the SCALAR function e^s and is independent of the number of coupled lines and the per-unit length parameters*. The results obtained are then stored and the macromodel can be obtained analytically in terms of the predetermined coefficients and per-unit-length parameters.

III. Computational Results

We present here three case studies [8], to validate the proposed algorithm. The first example corresponds to Line-2 transmission line network (Fig. 1 in [8]). Fig. 1 shows the comparison of transient responses using the proposed OMRA algorithm, with conventional discretization, at all four terminals of the transmission line subnetwork, and they match accurately. The second example corresponds to Line-4 transmission line network (Fig. 2 in [8]). Fig. 2 shows the comparison of transient responses using the proposed OMRA algorithm, with the conventional discretization, at both near and far end of the transmission line. The third example corresponds to the Line-6 transmission line network (Fig. 3 in [8]). This network is analyzed for different lengths of the transmission line (5cm, 20cm and 40cm). Fig. 3 shows the transient responses at nodes B1 and C2 when the line lengths are 5cm, Fig. 4a shows the response at node B2 (length = 20cm) and Fig. 4b shows the response at node C2 (length=40cm). As seen, they match accurately with the responses from conventional lumped discretization. Table-1 gives the comparison of the MNA sizes involved (for comparable accuracy) and the CPU expense. It can be noticed that, as the line length increases, OMRA yields significant speed-up (22, for line length = 40cm).

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Table I: Matrix Rational Approximation vs Conventional Lumped Model

Simulations	OMRA (MNA size)	Lumped (MNA size)	MNA savings	OMRA CPU time (seconds)	Lumped CPU time (seconds)	Speedup
line 2	355	2482	86%	1.65	13.46	8.2
line 4	8 281	48 001	83%	712.25	4036.1	5.7
line 6 (5cm)	914	6 002	85%	14	109	7.8
line 6 (20cm)	3 650	24 002	85%	86	707	8.2
line 6 (40cm)	7 298	80 002	91%	249	5662	22.7

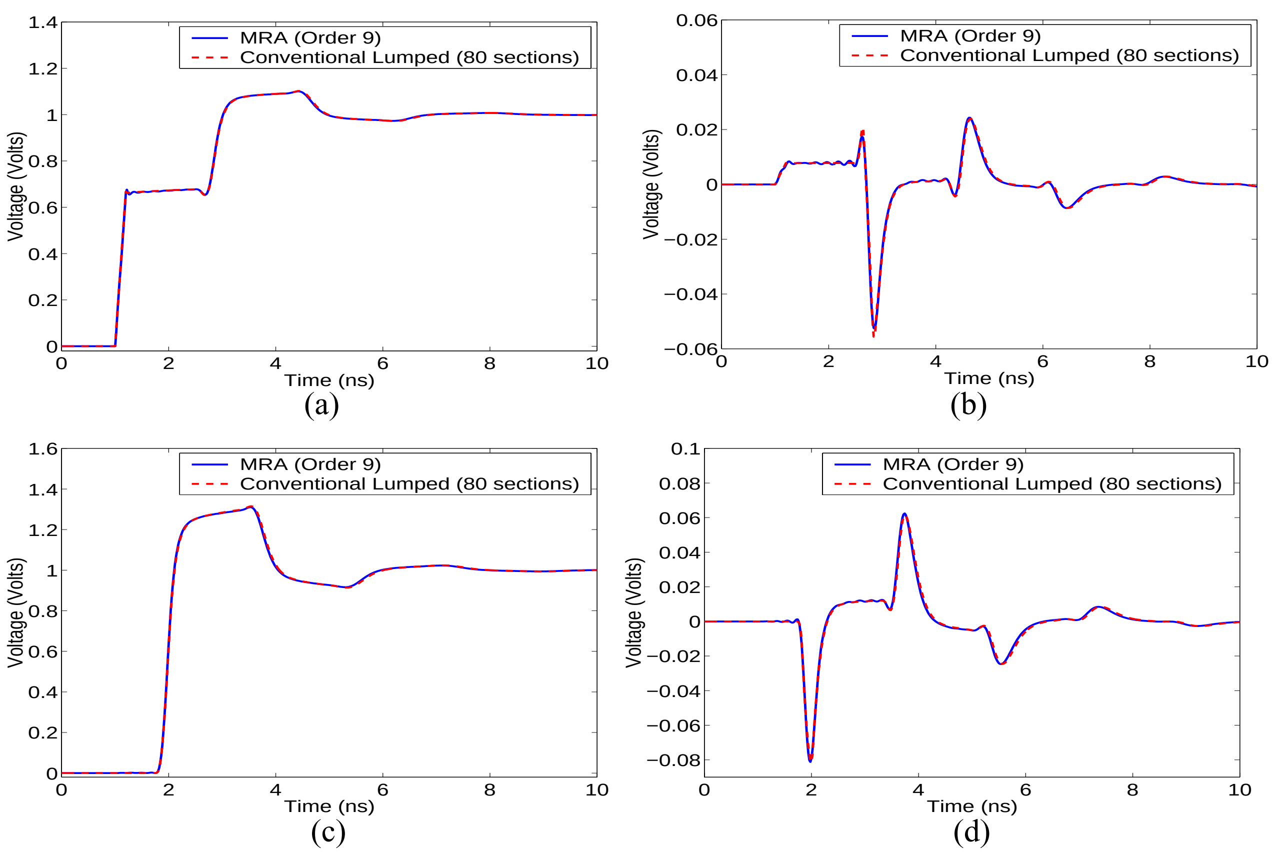
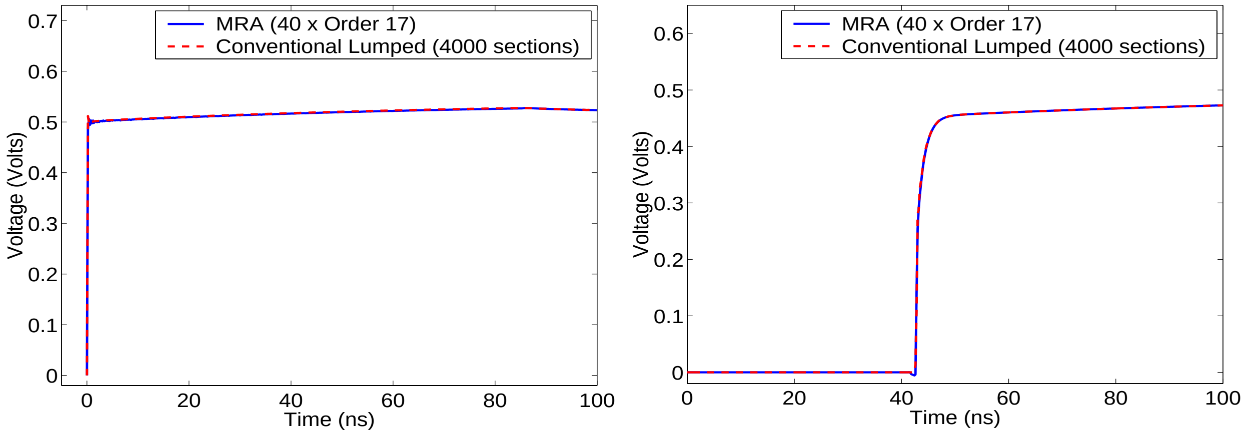
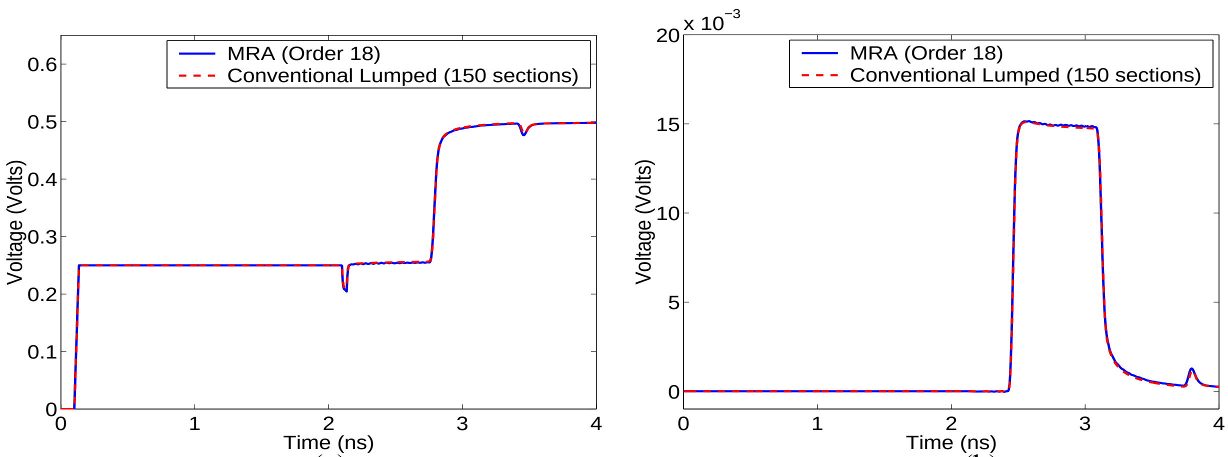


Fig. 1 : Time Domain Response (Example 1): a) Active line near end b) Quiet line near end c) Active line far end d) Quiet line far end



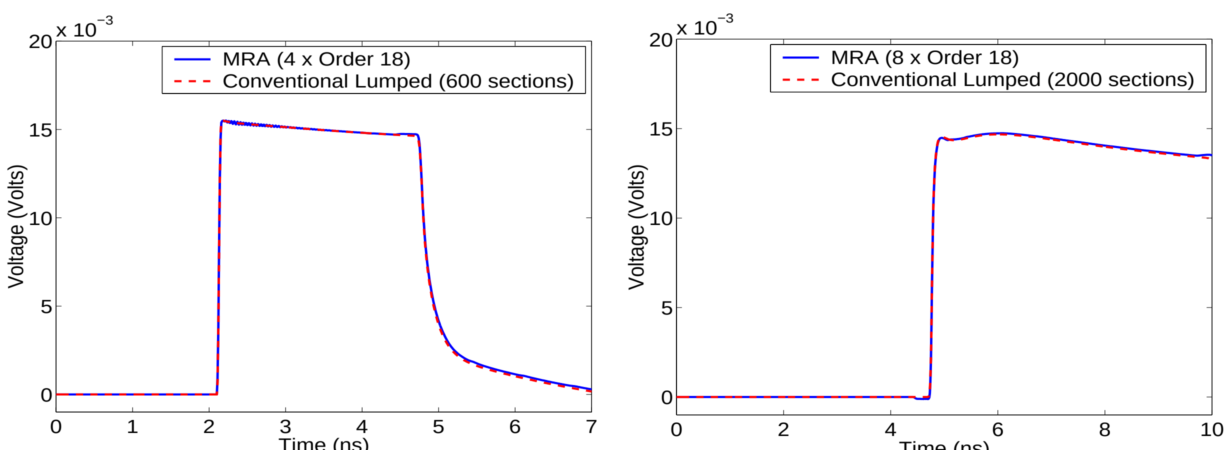
(a) (b)

Fig. 2 : Time Domain Response (Example 2) : a) Near end b) Far end



(a) (b)

Fig. 3 : Time Domain Response for 5cm line (Example 3) : a) At node B1 b) At node C2



(a) (b)

Fig. 4 : Time Domain Response (Example 3) : a) 20cm line - node B2, b) 40cm line - node C2