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Procurement Auctions for Differentiated Goods

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Abstract: We consider two mechanisms to procure differentiated goods: the request for quote and an English auction with bidding credits. In the request for quote, each seller submits a price and the inherent quality of his good. Then the buyer selects the seller who offers the greatest difference in quality and price. In the English auction with bidding credits, the buyer assigns a bidding credit to each seller conditional upon the quality of the seller's good. Then the sellers compete in an English auction with the winner receiving the auction price and his bidding credit. Game theoretic models predict the request for quote is socially efficient but the English auction with bidding credits is not. The optimal bidding credit assignment under compensates for quality advantages, creating a market distortion in which the buyer captures surplus at the expense of the seller's profit and social efficiency. In experiments, the request for quote is less efficient than the English auctions with bidding credits. Moreover, both the buyer and seller receive more surplus in the English auction with bidding credits.

I. Introduction

Large enterprises often procure goods for which the alternatives differ in quality. In many situations, it is neither feasible nor profitable for sellers to modify the non-price attributes of their offerings. Examples of such goods are office furniture, hotel stays, and contract software programming. Traditionally, such goods are bought through a Request For Quote (RFQ).

As part of their e-procurement agendas, many enterprises search for mechanisms that out perform the RFQ. Most of these searches lead to an English auction.[1] The promise to deliver the lowest cost seller through cutthroat price competition entices the enterprise. Unfortunately, an English auction can undermine the enterprise when goods vary in quality: often the lowest cost seller doesn't provide the best combination of price and quality. To accommodate differentiated goods, we introduce a pre-auction stage in which the buyer can assign quality based bidding credits to sellers. Our primary concern is whether an enterprise's interest is better served by the English auction with bidding credits (EBC) or the RFQ.

Our research is part of the IBM procurement organization's evaluation of online auctions to procure differentiated goods. In particular we focus on the procurement of contract programming. The numerical examples we provide and experimental parameter values we use are based upon this market.[2] Again the RFQ is the baseline for current practice while the EBC is an alternative we speculate is easy to implement, has good theoretical performance, and has a dominant strategy sellers will adopt. With an eye towards implementation, we provide both experimental and game theoretic evaluations of both mechanisms.

A RFQ starts with the buyer providing her evaluation criteria for the non-price attributes, i.e. how she measures quality, to potential sellers. The buyer provides the evaluation criteria, even though the non-price attributes of good are fixed, so a seller knows the quality level assigned to his good.[3] Next, each seller sets a price and provides the fixed description of his product. The buyer, then, selects the seller who offers the largest margin between quality and price. The winning seller receives his submitted price.

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[1] Within the e-procurement community the English auction is often called the Reverse auction.

[2] There are several website with auctions for contract programming services. Some examples are www.freelance.com, www.guru.com, www.bizbuyer.com, and www.freeagent.com.

[3] Hence, we are not considering the buyer making a strategic choice of evaluation functions for RFQ's as in studies such as Dasgupta and Spulber (1990) and Che (1993). Rather, we wish to choose a benchmark RFQ formulation that accurately reflects common practice in the procurement community.

In our game theoretic analysis of the RFQ, we perform a change of variables and define a seller's type as his realized surplus – quality minus cost – and a seller's bid as surplus offered -- quality minus price. After this change of variables, the strategic formulation of the RFQ is the same as a first price sealed bid auction for selling an object to potential buyers with private values.[4] A realized surplus is equivalent to a buyer's private value; both are the potential gains from exchange that the auction participant provides. A surplus offer is equivalent to a buyer's bid; both are the utility gain offered by the action participant. We exploit this equivalence to derive a symmetric Nash equilibrium for the RFQ. The symmetric equilibrium strategy is an increasing function of realized surplus, ensuring the seller with the highest realized surplus wins and the auction is socially efficient.

In an EBC, as in the RFQ, the buyer provides the evaluation criteria to the sellers, who in turn provide their product descriptions. Observing the quality of each good, the buyer assigns a bidding credit to the seller of each good. Then the sellers, each with his bidding credit in hand, compete in an English auction. In the auction, sellers offer successively lower prices until the winning seller submits a price that no other seller is willing to improve upon. Again the winning seller receives the auction price and his bidding credit.

We derive the equilibrium strategies of the EBC through a backward induction approach. For any bidding credit assignment, a seller's optimal strategy is to exit the auction when the price falls below his cost less his bidding credit. Anticipating the sellers' following this strategy, the buyer's optimal bidding credit rule is discriminatory. Although the buyer assigns the seller of the higher quality good the largest credit, the credit is smaller than his quality advantage. The buyer's optimal rule is reminiscent of the discriminatory policies of optimal auctions when sellers have asymmetric cost distributions.[5] In these optimal auctions and the EBC, the optimal discriminatory policies promote competitive pressure by subsidizing the disadvantaged seller and enrich the buyer at the expense of sellers' profits and social efficiency.

Our experimental results differ from the game theoretic predictions. In our experiments, the EBC is more socially efficient and provides a better outcome to the seller and the buyer than the RFQ provides. In the RFQ experiments, many subjects' choices don't correspond to the Nash equilibrium strategy. The Nash equilibrium strategy of our RFQ is non-linear and other studies,

[4] First presented in Vickrey's (1961) seminal paper.
[5] Myerson (1981) and MacAfee and McMillan (1989) derive the optimal auctions for procurement when sellers have asymmetric costs.

such as Chen and Plott (1998) and Georee and Offerman (2002), demonstrate that subjects tend not to play non-linear Nash equilibrium strategies. Our EBC experiments are similar to the experiments of Cornes and Schotter (1999) that consider sealed bid procurement auctions that use price preferences to promote minority representation. Cornes and Schotter use fixed levels of the price preference as their treatment variable. They find the level of minority representation increases with the level of the price preference, but procurement costs are minimized at some interior level of the price preference. In our experiments we allow the buyer to choose the bidding credits. Buyers, left to their own devices, select discriminatory bidding credits, but not as discriminatory as advocated by the optimal bidding credit. The combination of the sellers' non-equilibrium behavior in the RFQ and the buyer's overly generous bidding credits in the EBC gives rise to the outcome of the EBC Pareto dominating the outcome of the RFQ.

II. Game Theoretic Models and Predictions

In our analysis and experiments, we consider the case of two sellers, (s_1, s_2), and a buyer. Levels of cost and quality characterize a seller. Seller i's cost to produce a unit is a random variable, denoted c_i, which is uniformly distributed on the interval $[c_L, c_H]$. A seller incurs this cost only when he makes a sale. The quality of seller i's good is a random variable, denoted v_i, which is uniformly distributed on the interval $[v_L, v_H]$. You can think of this quality as the buyer's maximum willingness-to-pay for the good. The cost and the quality of each seller's good are independent random variables. We ensure that the quality of a seller's good always exceeds its cost by assuming $v_L \geq c_H$. At the time of the auction, each seller knows his quality and cost, but only the distributions of the quality and cost of the other seller's good. Also, the buyer knows the quality of each seller's good but only the distributions of his costs. This information structure is common knowledge.

II.1 Request For Quote Auction

In this mechanism, potential sellers simultaneously submit prices. Seller i's sumitted price is denoted p_i. Seller i wins the auction if $v_i - p_i$ is the maximum of $\{v_1 - p_1, v_2 - p_2\}$ and receives the price p_i. The winning seller i's profit is $p_i - c_i$, the other seller's profit is zero, and the buyer's payoff is $v_i - p_i$.

Although a seller's type is the pair (v_i, c_i), the relevant economic information is simply the difference of the two variables. The potential gains from exchange seller *i* provides is $s_i = v_i - c_i$. We call s_i seller *i*'s "realized surplus." The random variable s_i has a distribution function, denoted $F(\,)$, which is the convolution v_i and $-c_i$. Instead of explicitly considering the submitted bid, we consider seller *i*'s "surplus offer," $o_i = v_i - p_i$. Under this formulation, the seller who submits the largest offered surplus wins the RFQ auction.

With this change of variables, the RFQ has the same formulation as a first price sealed bid auction used to sell a single object to buyers with private values. In such a setting, each buyer's value is her realized surplus, or the potential gains from exchange he provides (assuming the seller has a cost of zero). When a participant makes a bid he is making a surplus offer to the seller. And the highest bid is simply the greatest surplus offer. The equivalence of the RFQ, under the change of variables, and the first price sealed bid auction allows us to apply standard arguments to derive a symmetric Bayes-Nash equilibrium.

The expected profit for seller *i* for surplus offer o_i when he has realized surplus s_i and all other sellers make surplus offers according to the strictly increasing function $O(s)$ is

$$\pi_i(o_i) = (s_i - o_i)\Pr\left[o_i \geq O(s_j)\,\right].$$

Apply the inverse of $O(s)$ and the calculate the probability to get

$$\pi_i(o_i) = (s_i - o_i)F\left[O^{-1}(o_i)\right].$$

Seller *i*'s optimal response, $o_i{*}$, satisfies

$$\frac{\partial \pi_i(o_i{*})}{\partial o_i} = 0. \quad (1)$$

Seller *i*'s maximal profit adjusts to changes in s_i according to

$$\frac{d\pi_i(o_i{*})}{ds_i} = \frac{\partial \pi_i(o_i{*})}{\partial s_i} + \frac{\partial \pi_i(o_i{*})}{\partial o_i} * \frac{\partial o_i}{\partial s_i}.$$

Simplify and substitute (1) – this is the envelope theorem— and get

$$\frac{d\pi_i(o_i{*})}{ds_i} = F\left[O^{-1}(o_i{*})\right]$$

In a symmetric Bayes-Nash equilibrium the each seller must adopt the same strategy and the strategy must be an optimal response with the characteristics we've derived. This implies the Nash equilibrium surplus offer function must satisfy $O^{-1}(o_i) = s_i$. Substitution yields the following differential equation

$$\frac{d\pi_i(o_i*)}{ds_i} = F(s_i).$$

Solve the differential equation by integrating both sides and setting the expected profit to zero for the lowest realized surplus type. Then obtain the symmetric Bayes-Nash equilibrium strategy by substituting for the definition of expected profit:

$$o_i* = O(s_i) = s_i - \frac{\int_{V_L - C_H}^{s_i} F(z)dz}{F(s_i)}. \quad (2)$$

This expression calculates how much of seller *i*'s realized surplus he offers in equilibrium. From the equilibrium surplus offer function and the definition realized surplus we get the equilibrium bid function

$$p_i*(v_i, c_i) = c_i + \frac{\int_{V_L - C_H}^{s_i} F(z)dz}{F(s_i)}.$$

This expression provides the margin demanded by the seller as a function a cost and quality. The same analysis applied to the n-seller case and yields the equilibrium strategies,

$$o_i* = O(s_i) = s_i - \frac{\int_{V_L - C_H}^{s_i} [F(z)]^{n-1} dz}{[F(s_i)]^{n-1}} \text{ and } p_i*(v_i, c_i) = c_i + \frac{\int_{V_L - C_H}^{s_i} [F(z)]^{n-1} dz}{[F(s_i)]^{n-1}}.$$

Consider the following example, which is based upon the parameters of our experiment. Let $[c_L, c_H] = [40, 80]$ and $[v_L, v_H] = [100, 130]$. Then the distribution of realized surplus, s_i, is

$$F(s) = \begin{cases} \dfrac{(\text{s}-20)^2}{2400} & \text{for } 20 \le \text{s} < 50 \\ \dfrac{\text{s}-35}{40} & \text{for } 50 \le \text{s} < 60 \\ 1 - \dfrac{(90-\text{s})^2}{2400} & \text{for } 60 \le \text{s} \le 90 \end{cases}.$$

The equilibrium surplus offer and bid functions resulting from this distribution are

$$o*(s_i) = \begin{cases} \dfrac{2s_i + 20}{3} & \text{for } 20 \le \text{s} < 50 \\ s_i - \dfrac{150 + (s_i - 20)(s_i - 50)}{2(s_i - 35)} & \text{for } 50 \le s < 60 \quad \text{and} \\ s_i - \dfrac{2400(s_i - 55) + \frac{1}{3}(90 - s_i)^3}{2400 - (90 - s_i)^2} & \text{for } 60 \le s \le 90 \end{cases}$$

$$p^*(v_i, c_i) = \begin{cases} \dfrac{2c_i + v_i + 20}{3} & \text{for } 20 \le s < 50 \\ c_i + \dfrac{150 + (v_i - c_i - 20)(v_i - c_i - 50)}{2(v_i - c_i - 35)} & \text{for } 50 \le s < 60 \\ c_i + \dfrac{2400(v_i - c_i - 55) + \frac{1}{3}(90 - v_i + c_i)^3}{2400 - (90 - v_i + c_i)^2} & \text{for } 60 \le s \le 90 \end{cases}.$$

The equilibrium surplus offers and bids are depicted in Figure 1.

II.2 English Auction with Bidding Credits

There are two stages in this mechanism. In the first stage, the buyer assigns to each seller a bidding credit, conditioning it upon the sellers' quality number. Seller *i*'s bidding credit is denoted b_i. In the second stage of the auction, each seller is told their respective bidding credit and the sellers participate in an English auction. The winning seller receives a monetary amount equal to the auction price and his assigned bidding credit.

This auction has a unique sequential Nash equilibrium. Each of a seller's information sets in stage two is defined by his bidding credit and the cost and quality of his good. A seller *i*'s behavioral strategy is to set an exit price for the English auction, i.e. the continuation game, at each of his information sets. Also we require that the seller update his belief about the other seller's type at each of his information sets via Baye's Rule. Anyway, this is only a formality because each seller has a weakly dominant strategy.

Proposition 1: Seller *i* has a weakly dominant strategy: $p_i^*(b_i, c_i, v_i) = c_i - b_i$.

In other words seller *i* remains in the auction as long as the standing price is greater than or equal to the seller's cost less his assigned bidding credit.

Proof: Apply one of the standard arguments, such as Krishna (2002) p.15, that establish the weakly dominant strategy in second price sealed bid or English auctions. Just recalibrate the seller's zero payoff price to cost less the bidding credit.

Now we derive the buyer's optimal bidding credit assignment in stage one. A buyer's payoff is the difference between the quality and the price paid (auction price plus bidding credit) of the procured object. The buyer's expected payoff for a pair of bidding credits -- when sellers adopt their dominant strategies -- is

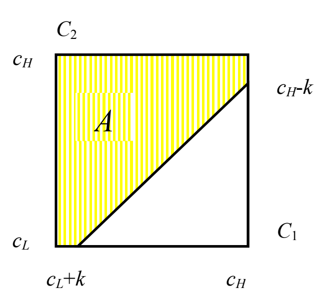
$$
\begin{aligned}
E[\Pi(b_1,b_2)] = &\Pr[c_1 - b_1 \le c_2 - b_2](v_1 - E(c_2 - b_2 \mid c_2 > c_1 - b_1 + b_2) - b_1) \\
&+ \Pr[c_1 - b_1 > c_2 - b_2](v_2 - E(c_1 - b_1 \mid c_1 > c_2 + b_1 - b_2) - b_2)
\end{aligned}
\quad \text{or}
$$

$$
\begin{aligned}
E[\Pi(b_1,b_2)] = &\Pr[c_1 - b_1 \le c_2 - b_2](v_1 - E(c_2 \mid c_2 > c_1 - b_1 + b_2) + b_2 - b_1) \\
&+ \Pr[c_1 - b_1 > c_2 - b_2](v_2 - E(c_1 \mid c_1 > c_2 + b_1 - b_2) + b_1 - b_2)
\end{aligned}
.
$$

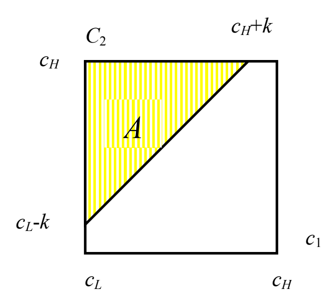
Inspection of this payoff function reveals that there are payoff equivalent strategy classes for the buyer: two pairs of bidding credits (b_1, b_2) and (b'_1, b'_2) yield the same expected payoff if $b_1 - b_2 = b'_1 - b'_2$. Let K be the set of payoff equivalent strategies with the typical element k, where $k \in K = \{(b_1, b_2) : b_1 - b_2 = k\}$. With this notation, the buyer's expected payoff function is

$$
\begin{aligned}
E[\Pi(k)] = &\Pr[c_1 - k \le c_2](v_1 - E(c_2 \mid c_2 > c_1 - k) - k) \\
&+ \Pr[c_1 - k > c_2](v_2 - E(c_1 \mid c_1 > c_2 + k) + k).
\end{aligned}
\quad (4)
$$

The term $\Pr[c_1 - k \le c_2]$ is the probability that Seller one wins the auction. This corresponds to the probability of the event $A = \{c_1 - k \le c_2\}$. The figure below show the two shapes this event can take in the support of (c_1, c_2).



Case 1: $k > 0$



Case 2: $k < 0$

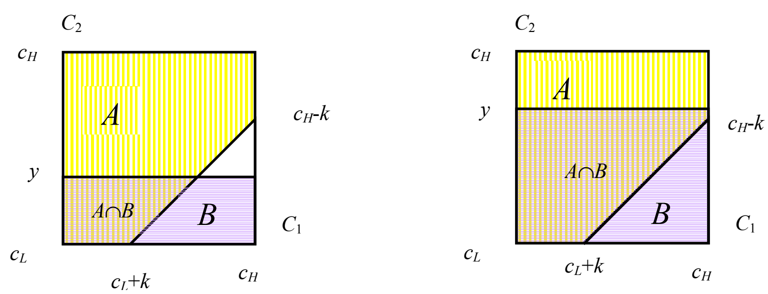
With a rectangular distribution on the support, the probability that Seller one wins the auction is

$$\Pr[c_1 - k \le c_2] = \begin{cases} \dfrac{2(c_H - c_L)^2 - (c_H - c_L - k)^2}{2(c_H - c_L)^2}, \text{ if } k \ge 0 \\ \dfrac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}, \text{ otherwise} \end{cases}. \quad (5)$$

Start with the case $k \ge 0$. When seller one wins the auction the expected auction price is $E(c_2 \mid c_2 \ge c_1 - k)$. To calculate this expectation we need the probability density function for the auction price. The cumulative distribution function for this random variable is

$$F(y) \equiv \Pr(\{c_2 \le y \mid c_2 \ge c_1 - k\}) = \frac{\Pr(\{c_2 \le y\} \cap \{c_2 \ge c_1 - k\})}{\Pr(\{c_2 \ge c_1 - k\})}. \quad (6)$$

Recall $A = \{c_1 - k \le c_2\}$ and let $B = \{c_2 \le y\}$. The figure below shows the relevant events in the support of (c_1, c_2) for the case where $k > 0$.



When $c_L \le y \le c_H - k$ When $c_H - k \le y \le c_H$

Direct calculation yields

$$\Pr(A \cap B) = \begin{cases} \dfrac{(y - c_L)(2k + y - c_L)}{2(c_H - c_L)^2}, \text{if } c_L \le y \le c_H - k \\ \dfrac{2(y - c_L)(c_H - c_L) - (c_H - c_L - k)^2}{2(c_H - c_L)^2}, \quad \text{if } c_H - k \le y \le c_H \end{cases}. \quad (7)$$

Substitution of (4) and (6) into (5) gives us

$$F(y) = \begin{cases} \dfrac{(y - c_L)(2k + y - c_L)}{2(c_H - c_L)^2 - (c_H - c_L - k)^2}, \text{if } c_L \le y \le c_H - k \\ \dfrac{2(y - c_L)(c_H - c_L) - (c_H - c_L - k)^2}{2(c_H - c_L)^2 - (c_H - c_L - k)^2}, \quad \text{if } c_H - k \le y \le c_H \end{cases}.$$

Differentiate this expression to obtain the density function of y:

$$f(y)=\begin{cases}\dfrac{2(k+y-c_L)}{2(c_H-c_L)^2-(c_H-c_L-k)^2}, \text{if } c_L\le y\le c_H-k\\ \dfrac{2(c_H-c_L)}{2(c_H-c_L)^2-(c_H-c_L-k)^2}, \quad \text{if } c_H-k\le y\le c_H\end{cases}.$$

The expected value of c_2 conditional upon k and seller one winning the auction is

$$E(y)=\int_{c_L}^{c_H} yf(y)dy=\frac{1}{2(c_H-c_L)^2-(c_H-c_L-k)^2}\left[\int_{c_L}^{c_H-k}\left(2y^2+2ky-2c_Ly\right)dy+\int_{c_H-k}^{c_H}2(c_H-c_L)y\,dy\right]$$

$$=\frac{2}{3}c_H+\frac{1}{3}c_L-\frac{k(c_H-c_L)^2+k^2(c_H-c_L-k)}{6(c_H-c_L)^2-3(c_H-c_L-k)^2}. \quad (8)$$

The first term in this expression is the expectation of the first order statistic of the draw of two costs and the second term is the deviation from this expectation as k changes.

For the parameters of our experiment,

$$E(y)\equiv E(c_2\mid c_2\ge c_1-k)=66\tfrac{2}{3}-\frac{k}{3}\left(1-\frac{40k}{1600+80k-k^2}\right).$$

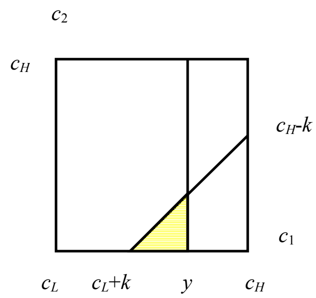
One consistency check on this expression is to set k=0, i.e. the special case of a simple English auction. When k=0, the expected value of the auction price conditional upon seller one wining is the expected value of the maximum cost statistic, $66\tfrac{2}{3}$. A second consistency check is to set k=40 and guaranteeing seller one wins the auction. Here the expected value of y is 60, the unconditional expectation of seller two's cost.

Now we calculate the expected price when seller two winning the auction, i.e. $E(c_1\mid c_1-k\ge c_2)$. In this instance,

$$\Pr(\{c_1-k\ge c_2\})=\frac{(c_H-c_L+k)^2}{2(c_H-c_L)^2} \text{ and}$$

$$\Pr(\{c_1\le y\}\cap\{c_1-k\ge c_2\})=\begin{cases}0, & \text{if } y\le c_L+k\\ \dfrac{(y-c_L-k)^2}{2(c_H-c_L)^2}, & \text{if } c_l+k\le y\le c_H\end{cases}.$$

One can verify these probabilities from the following diagram.



$c_1 \leq y$ and $c_1 - k \geq c_2$

The cumulative distribution function of the auction price when seller two wins is

$$F(y) = \begin{cases} 0, & \text{if } y \leq c_L + k \\ \dfrac{(y - c_L - k)^2}{(c_H - c_L - k)^2}, & \text{if } c_l + k \leq y \leq c_H \end{cases}.$$

This is the CDF of the maximum statistic for two independent draws from a uniform distribution on the interval $[c_L + k, c_H]$, permitting us to state:

$$E(c_1 \mid c_1 - k \geq c_2) = \frac{2}{3}c_H + \frac{1}{3}c_L + \frac{1}{3}k \,. \quad (9)$$

After substituting (5), (8), and (9) into (4) and simplifying, the buyer's expected payoff function for $k \geq 0$ is

$$E[\Pi(k)] = \left[\frac{2(c_H - c_L)^2 - (c_H - c_L - k)^2}{2(c_H - c_L)^2}\right]\left(v_1 - \frac{2}{3}c_H - \frac{1}{3}c_L + \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2} - k\right)$$
$$+ \left[\frac{(c_H - c_L - k)^2}{2(c_H - c_L)^2}\right]\left(v_2 - \frac{2}{3}c_H - \frac{1}{3}c_L + \frac{2}{3}k\right).$$

Let's consider the case where $k<0$. The symmetry of the probability and conditional expectation calculations allows us to immediately state the expected payoff in this case:

$$E[\Pi(k)] = \left[\frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right]\left(v_1 - \frac{2}{3}c_H - \frac{1}{3}c_L - \frac{2}{3}k\right)$$
$$+ \left[\frac{2(c_H - c_L)^2 - (c_H - c_L + k)^2}{2(c_H - c_L)^2}\right]\left(v_2 - \frac{2}{3}c_H - \frac{1}{3}c_L + \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2} + k\right).$$

Without loss of generality, assume seller one is the seller with the higher quality good. The following lemma indicates that is never in the buyer's interest to give a larger bidding credit to the lower quality seller, i.e. choose $k<0$.

Proposition 2: $E[\Pi(0)] > E[\Pi(k \mid k < 0)]$.

Proof:

$$E[\Pi(0)] - E[\Pi(k \mid k<0)] = \frac{1}{2}(v_1 - v_2) - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2} v_1 - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] v_2$$

$$+ \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\frac{2k}{3} - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] k - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2}.$$

The term $\frac{1}{2}(v_1 - v_2) - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2} v_1 - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] v_2$ is strictly positive because a negative *k* reduces the probability of seller one winning below one-half. Also the term $\frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\frac{2k}{3} - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] k$ is strictly positive. Finally the term $-\left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2}$. is also strictly positive. Therefore, $E[\Pi(0)] - E[\Pi(k \mid k > 0)] > 0$. Q.E.D.

With this proposition, we examine the case of *k*>0 for the optimal bidding credit assignment *k**. The first order condition for the maximization of the buyer's payoff is

$$\frac{dE[\Pi(k^*)]}{dk} = \frac{2k^{*2} - [3(c_H - c_L) + (v_1 - v_2)]k^* + (c_H - c_L)(v_1 - v_2)}{4} = 0.$$

The second order condition is

$$\frac{d^2 E[\Pi(k^*)]}{dk^2} = \frac{4k^* - 3(c_H - c_L) - (v_1 - v_2)}{4} \le 0.$$

The first order condition is a quadratic. Of the two roots, only the negative one satisfies the second order conditions for the maximum. The negative root and optimal bidding credit rule is

$$k^* = \frac{3(c_H - c_L) + (v_1 - v_2) - \sqrt{[3(c_H - c_L) + (v_1 - v_2)]^2 - 8(c_H - c_L)(v_1 - v_2)}}{4}.$$

Proposition 3: The optimal bidding credit assignment is less than the differences in quality, i.e. $k^* < v_1 - v_2$.

Proof: $k^* < (v_1 - v_2) \quad \Rightarrow$

$$3(c_H - c_L) - 3(v_1 - v_2) - \sqrt{[3(c_H - c_L) + (v_1 - v_2)]^2 - 8(c_H - c_L)(v_1 - v_2)} < 0 \quad \Rightarrow$$

$$9(c_H - c_L)^2 - 18(c_H - c_L)(v_1 - v_2) - 9(v_1 - v_2)^2 < 9(c_H - c_L)^2 - 2(c_H - c_L)(v_1 - v_2) + (v_1 - v_2)^2 \Rightarrow$$

$$-16(c_H - c_L)(v_1 - v_2) - 10(v_1 - v_2)^2 < 0.$$ Q.E.D

According to proposition three, the buyer's best strategy in the EBC is to use a discriminatory rule that assigns a bidding credit to the high quality seller that is less than his quality advantage. The impact of the rule bolsters the low quality seller's competitiveness and leads the high quality seller to receive lower expected surplus than in the RFQ. Similar discriminatory policies appear in the literature on optimal auctions when the auction participants are asymmetric, for example see Myerson (1981) and McAfee and McMillan (1989).

Again consider the environment of our experiment; two sellers who independently and uniformly draw costs from the interval [40, 80] and qualities from the interval [100, 130]. After the buyer observes each seller's quality, has assigns to the higher quality seller the bidding credit

$$k^* = \frac{(120 + v_1 - v_2) - \sqrt{(40 + v_1 - v_2)^2 + 12800}}{4}.$$

An inefficient outcome occurs when the high quality seller's costs is in the interval $[c_2 + k^*, c_2 + (v_1 - v_2)]$. For example, if (v_1, c_1)=(120,60) and (v_1, c_1)=(110, 55) seller one has the greatest realized surplus but seller two wins the EBC. Specifically, the buyer assigns a 3.24 bidding credit to seller one, and seller two wins the auction at a price of 56.76.

II.3 Economic Performance

Using the Nash equilibrium strategies for the RFQ and EBC we can generate theoretical predictions of economic variable such as efficiency, market price, the average quality and cost of the procured good, and buyer's welfare. Table I presents the expected values of various economic variables for the economic environment of our experiment. We obtained all of the expected values, except '% of Efficient Outcomes', by simulating each auction ten million times.

The predicted outcomes of the two mechanisms differ for many variables. The RFQ always generates a socially efficient outcome because the symmetric Nash equilibrium strategy is strictly increasing in realized surplus. In contrast, the EBC picks the inefficient seller over 16% of the time. Also, the buyer's discriminatory bidding credit assignments reduce the average winning seller quality. Of course, the EBC more than makes up for this lower quality with a reduced price and a bias towards the lower cost seller. Also, the EBC leads to a gain in buyer welfare and a reduction in seller profit.

From a theoretical perspective, the EBC better serves the buyer's interest than the standard RFQ. But does human behavior conform to the models used to derive our predictions? Past experimental studies show that human choice often differs from game theoretic predictions and we will see this occur again in our experiments.

III. Experimental Design

All of our experimental sessions, except one, were executed via a computer software application at the UCSD Department of Economics EEXCL facility. We conducted the other session at the IBM's TJ Watson Research facility. Every session was a RFQ or EBC session.

Each subject received a show up fee of five US dollars prior to participating in a session (except for the IBM session at which a twenty dollar payment was given.) Before the decision-making portion of a session, each subject read a paper copy of the instructions and then had to successfully complete a simple written test of how the auction worked and how earnings were calculated. After the experiment each subject was privately paid his or her earning in US currency.

In a RFQ session, subjects participated in five practice periods with no payments and then fifty additional periods with cash payments proportional to their experimental earnings. Prior to each period, subjects were randomly paired to participate in different auctions. At the start of each period, each subject was informed of his or her quality and cost. Then, each seller privately submitted a price, and a winner was determined. Subjects were informed of whether they won the auction, the winning auction price, and their period earnings. A complete history of this information was always available to a subject.

In an EBC session, two-thirds of the subjects were randomly designated sellers and one-third of the subjects were randomly assigned to the buyer roll. After two practice periods, subjects participated in twelve to sixteen periods in which cash earnings accumulated.[6] Before each period, a collection of independent auctions was formed by randomly assigning two sellers and one buyer to each auction. At the beginning of an auction, the buyer was informed of the quality of each seller's good, and each seller was informed of the quality and unit cost of his or her good. Then the buyer had the opportunity to assign a credit to each of the sellers. Once these credits were assigned, they were revealed to the respective sellers.

Finally, and iterative English auction took place starting with sellers making opening offers. In subsequent iterations, the seller who did not have the lowest current offer could either make an offer lower than the current lowest offer, or they could exit the auction. The seller with the current best offer could either maintain their current offer or improve it. When one of the sellers exited, the auction concluded. The auction price was the current lowest price when they exited.

The winning seller received the auction price and their assigned credit less his or her unit cost. The buyer received the difference between the quality of winning seller's good and the total payment to the winning seller. The losing seller received zero earnings. Over the course of the session, subjects could see a complete history of the information that had been revealed to them. Instructions for both RFQ and EBC sessions are in the appendix.

We conducted four RFQ sessions with forty-four total subjects; two with 12 subjects each and two ten subjects each. Each RFQ session was completed in less than ninety minutes. We conducted four RBC sessions: The number of participants and periods in each session is given in the following table. All RFQ sessions were completed in 105 minutes.

RFQ Session	Total Periods	Practice Periods	Sellers	Buyers
1	14	2	10	5
2	18	2	6	3
3	16	2	10	5
4	16	2	8	4

[6] An EBC auction lasted significantly longer than a RFQ auction, and consequently we used and 20 experimental dollar to one US dollar exchange rate in RBC sessions and a four to one exchange rate in the RFQ sessions.

IV. Results on Empirical Economic Performance

Under Nash equilibrium play, the RFQ is socially efficient, while the EBC better serves the buyer's interest at the expense of efficiency and seller profit. Contrary to these predictions, in experiments, the EBC is more efficient, and nominally makes both buyer and seller better off than the RFQ. The difference between the comparative theoretical and empirical economic performances mostly results from play that diverges from the theory in the RFQ experiments.

In Table II, we provide statistics for various performance variables in the RFQ and EBC auctions. Each of the following variables is associated with one the columns in Table II: "% of Efficient Outcomes" or the percentage of auctions that select the seller with the greatest difference between quality and cost, "Avg. Realized Surplus" or the sum of the buyer's surplus and the winning seller's profit, "Avg. Winning Seller's Quality," Avg. Auction Price (for the EBC this is the auction price and bidding credit paid), "Avg. Winning Seller's Cost," "Avg. Buyer's Surplus" or the winning seller's quality less total price paid, and "Avg. Wining Seller's Profit." For both the RFQ and EBC sessions, we provide the sample mean and standard deviation for each variable. Also, for each variable, we provide the *z*-statistic and its p-value for the hypothesis test that the means are the same both auction types. A bold-faced p-value indicates the hypothesis is rejected at a five percent level-of-significance.

The EBC, not the RFQ, provides a more socially optimal outcome in the experiments. In over 84% of the EBC auctions, the higher surplus seller wins and the average total realized social surplus is 63.25, while the higher surplus seller wins only 79% of RFQ auctions and the average total realized social surplus is 61.53. Dividing realized social surplus into its two components, we see that the average buyer surplus is about 1.7% greater and the average seller profit is about 5.2% greater in the EBC than in the RFQ.[7]

How buyers and sellers in the EBC benefit from the advantage in average social surplus is found by examining the average realized qualities, prices, and costs. The RFQ auction generates a higher quality level than the EBC, 117.69 versus 116.26, but also an increase in costs, 56.15 versus 53.01. The net effect of these two differences is the 1.71 advantage in total social surplus enjoyed by the EBC. Also, the average EBC price is 2.52 lower than the RFQ price. From the seller's perspective, the net effect of the price and cost reductions is a .62 increase in profit in the

[7] However, the improvement in surplus for the buyer and seller in the EBC is not significant according to the *z*-test.

EBC. From the buyer's perspective, the reduction in quality is more than offset by the reduction in cost, and result in a 1.09 increase in buyer surplus in the EBC.

The differences between the relative empirical performances and the game theoretic predictions must mean at least one of the auctions is performing differently than its Nash equilibrium predictions. Table III presents the observed and theoretical values of the reported performance variables, and hypothesis tests that the observed and theoretical values are the same. The theoretical predictions of the RFQ are rejected at the 5% level of significance for all variables except Avg. Winning Seller's Quality. On the other hand, for the EBC, the theoretical prediction is only rejected for a single variable. The observed buyer surplus is significantly less than predicted. Clearly the Nash Equilibrium predictions fare worse for the RFQ than the EBC. Subjects not using Nash equilibrium strategies must be the source of the theoretical prediction's failures.

V. Analysis of Individual Behavior

To understand how the equilibrium predictions fail we must identify how subjects' behavior is deviating from the equilibrium strategies. First, we consider subject behavior in the RFQ experiments. Here we show that sellers offer too much surplus when the have high realized surplus types. Also subjects offer different surpluses to the buyer for distinct quality-costs pairs that provide the same realized surplus. Regression analysis shows that there are two distinct types of bidders: those who make nonlinear bids that are correlated with the Nash bids, and those who submit bids linear in cost and quality. This mixture model explains the why sellers are "too generous" when they receive a high realize surplus type. In the EBC, we see that buyers are too generous with their bidding credit assignments and that sellers follow their dominant strategy with one caveat; the losing seller on average exits the auction about three dollars before their zero profit price. This bias takes away some of the buyer's surplus in the EBC.

V.1 Behavior in the RFQ

To what extent do subjects' surplus offers correspond to the Nash Equilibrium surplus offer function? When we plot surplus offers versus realized surplus as in Figure II we don't find evidence that subject follow the Nash surplus offer function. We do see at low levels of realized surplus subjects will offer less than Nash levels of surplus, while at middle levels of realized surplus the surplus offers exceed the Nash Levels, and at high levels of realized surplus there is tremendous variation in the level of surplus offers. The scatter plot of Surplus Offers also has several linear bands. Each of the bands represents a focal amount of profit demanded by a seller such as ten, twenty, or thirty dollars. These bands could be indicative of subjects who only ask for a fixed absolute margin independent of their quality. The presence of these bands and the large variation in the surplus offers raises a question; Is a subject's surplus offer determined by the difference in quality and cost or more generally by the absolute values of quality and costs?

Defining realized surplus as a seller's type is key to solving for the symmetric Nash equilibrium strategy, but assuming a subjects' behavior is solely characterized by his realized surplus, or the difference in quality and cost, may be erroneous. To understand how subjects condition their choices on the absolute levels of cost and quality we consider the difference between submitted and Nash bids for different quality-cost pairs. In Figure III, we present the average difference between submitted and Nash bids for different ranges of cost-quality pairs. We start by defining 100 equal sized bins that cover the range of the cost and quality variables. For each bin we select all the instances some subject drew a cost-quality pair in the range of the bin. Then we calculate the average deviation of the submitted bids from the Nash bids.

The graphs of these averages reveal systematic patterns. First, for each level of cost the difference between the submitted and Nash bid falls as quality increases. Evidently, subjects do not appreciate the competitive advantage associated with higher quality levels. Second, when costs are high and quality is low – i.e., a low level of realized surplus – submitted bids are greater than Nash bids. This is counter to the Nash equilibrium feature that the lowest type demands zero profit. Third, for low cost-high quality bins the bids are below the Nash levels. Finally, if the subjects condition their behavior only on the level of realized surplus, then we would expect the average bid deviation to be the same for a constant level of realized surplus. Bins corresponding to the same level realized surplus lie on off-diagonal lines of the cost-quality range. Inspection of the bar graphs does not suggest equal bid biases for bins lying on these off-diagonals. Hence, subjects' behavior is not invariant to the absolute levels of cost and quality.

Given the significant variation observed when we pool subject behavior, we now ask whether subjects decisions are noisy or whether their is systematic heterogeneity in the subjects' bidding rules. We proceed by allowing for two possibilities; a subject's bids could either be a linear function of cost and quality or correlated with the non-linear Nash bid function. A linear bid for subject i function has the following form:

$$p_i(c_i, v_i) = \beta_0 + \beta_1\, c_i + \beta_2\, v_i,$$

where the betas are unknown coefficients. We formulate the nonlinear Nash bid model as

$$p_i(c_i, v_i) = \gamma_0 + \gamma_1\, p^*(c_i, v_i)\,,$$

where $p^*(\,)$ is the Nash bid function. If a bidder exactly follows the Nash bidding rule, then $\gamma_0 = 0$ and $\gamma_1 = 1$. The two models allow us to characterize linear bidders and bidders who follow non-linear rules that are close to the Nash equilibrium strategy. We want to ascertain, for each subject, whether either of these models is appropriate.

We use the J-test of Davidson and MacKinnon (1981) to determine the selection from the two non-nested models. First we imbed the two models into one specification

$$p_i(c_i, v_i) = (1-\alpha)\,[\beta_0 + \beta_1\, c_i + \beta_2\, v_i\, p_i(c_i, v_i)] + \alpha\,[\gamma_0 + \gamma_1\, p^*(c_i, v_i)].$$

We use OLS to estimate this model. Then we run two hypothesis tests: $\alpha = 0$ and $\alpha = 1$. There are four possible outcomes to this exercise. First, we could reject both hypotheses and we select the larger nesting model. Second, we could not reject both hypotheses. This would indicate that both models are adequate and that the models highly co-linear. Third, we could reject $\alpha = 0$ but not $\alpha = 1$. In this case we select Nash model. Finally we could reject $\alpha = 1$, not $\alpha = 0$, and consequently select the linear bid model. Recall the Nash bid function is linear over part of the cost-quality range and in this range the two models can correspond. This can confound the identification of which bidding function a subject follows. Nevertheless, our results allow us to make a definitive model assignment for half of the subjects.

We apply the *J*-test to each subject's data in the RFQ and find substantial heterogeneity in the bidding strategies. In Table III, we report for each subject the model selected, the estimated parameters of the selected model, and the r-square statistic. For twenty-two of the forty-four subjects we are able to select a single model. Six subjects follow the Nash bidding model and sixteen follow the linear bidding model. The *J*-test selects both models for ten subjects. For these subjects we report the coefficient estimates for the model with the cost, quality and Nash bid parameters. Inspection of the regression results reveals some classic signs of multicollinearity: a high r-square, insignificant coefficient, and coefficients with the wrong sign. For these ten subjects the two models are too similar to differentiate. For the remaining twelve subjects, we reject both the linear and Nash model in favor of the nested model. Here we report the regression results for the nested regression and again observe the signs of multicollinearity.[8]

The *J*-test exercise demonstrates there are large contingencies of both non-linear Nash bidders and linear bidders. The presence of linear bidders leads to inefficient auction outcomes; linear bidding rules leads to low prices for high quality goods. Consequently, the Buyer's surplus is significantly higher and the Seller's profit is significantly lower than under the Nash equilibrium.

V.1 Behavior in the EBC

Subject behavior in the EBC is adheres closer to the game theoretic predictions than it does in the RFQ. Again, a seller has a weakly dominant strategy in the auction: exit only when the price falls below cost minus bidding credit. Subjects do follow this prescription with a caveat. The losing seller exits the auction, on average, three dollars above his threshold price. The buyer's Nash strategy is not as apparent as the seller's. Most of the time the Buyers do assign non-zero bidding credits, but their assignments are on average too generous. The combination of sellers exiting the auction slightly early and the buyer's assigning overly generous bidding credits leads to greater efficiency and seller profit than predicted.

How closely do sellers adhere to the dominant strategy? In Figure IV, we provide a histogram of the difference between the losing seller's exit price and his dominant strategy exit price. Most losing sellers exit slightly above their zero profit prices. Specifically, over 81% of the deviations are between zero and four. In contrast, only 2.4% of the losing sellers exit after the zero profit price and only three out of 248 auction winners lose money. Sellers clearly understand the dominant strategy and exit close to, but not below, their zero profit prices. We conjecture that the tediousness of the English auction is responsible for the early exit behavior.

The Buyers' bidding credit assignments greatly vary and on average are more generous than the optimal bidding credits. First, approximately twenty five percent of the time the buyer does not utilize the bidding credits to give an advantage to the high quality seller. Specifically, in over eighteen percent of the auctions the buyer assigns the same bidding credit to both sellers, and in almost seven percent of the auctions the buyer assigns a larger bidding credit to the lower

[8] Of course finding multicollinearity is not surprising in these regressions because for almost half of its domain the Nash bidding function is linear in cost and quality.

quality. In these cases the buyer is certainly not using the bidding credits to manage quality differences. At the other end of the spectrum, in almost six percent of the auctions the difference in the assigned bidding credits is equal to the difference in quality. Here, while buyer is ensuring the best seller is selected, he is not capturing any additional surplus over the Nash equilibrium outcome of the RFQ. In Figure V, we provide a scatter plot of the difference in bidding credit versus the difference in quality, a graph of the optimal bidding credit rule, and a graph of the OLS trend line. The trend line is above the optimal assignment line, but the OLS trend also has a low r-square that reflects highly variable Buyer behavior. Although the EBC has a transparent dominant strategy for sellers, the optimal bidding assignment rule proves elusive to the buyers. However, as seen in Figure V, the majority of the time the buyers do use discriminatory assignments.

The experimental results provide an assessment of the impact the choice of auction has on the effectiveness of procurement activities. The empirical performance of the RFQ leaves opportunities for other mechanisms to improve upon status quo. As our experiments show, the EBC produces greater total surplus and/or better serves the buyer's interest depending upon the buyer's objective and rule for assigning bidding credits.

VI. Concluding Remarks

In this paper, we assess two alternative ways enterprises can procure differentiated goods. The first alternative is the traditional method of request for quote. We show that under a symmetric Nash equilibrium the RFQ is an efficient mechanism. The second alternative is an English auction in which the buyer can assign bidding credits to the sellers based upon the qualities of the goods offered. This mechanism has a Nash equilibrium in which a seller has a transparent weakly dominant strategy, and the buyer has an optimal bidding credit assignment, which under compensates the high quality supplier for his quality advantage. This discriminatory policy improves the buyer's welfare over what she receives in the RFQ at the expense of social efficiency and seller profit. However, in our experiments, we find the EBC outperforms the RFQ for both buyers and sellers because (1) sellers don't follow the symmetric Nash strategy in the RFQ and because (2) buyers assign overly generous bidding credits in the EBC.

The transformation of how enterprises procure goods and services is one promise the emergence of e-commerce has actually fulfilled. Part of this transformation is an increase in the use of English auction variations. In practice, these English auctions for procurement are not equivalent to the English auctions typically studied by economists. For example, at FreeMarkets, Inc. (the world's largest third party provider of procurement auctions) an English auction does not determine the supplier; the auction only sets each participating seller's price. After the auction, the buyer selects the winning seller and pays that seller his exit price from the auction. This is how implementations of English procurement auctions manage product differentiation — Kinney (2000).

The EBC is a potential attractive alternative to current procurement English auction practices. In the current business use of English auctions, a seller no longer has a transparent dominant strategy and, more importantly, a buyer can't credibly commit to a discriminatory policy when they select a seller after the auction. Evaluating the non-price attributes of goods after the auction, the buyer is less likely to use a discriminatory policy because that would involve sometimes selecting a seller who does not provide the best combination of price and quality. In the EBC, the evaluation of quality prior to auction is an opportunity to pre-commit to a discriminatory policy, which doesn't suffer from the credibility problem of exercising the policy after the auction. Of course, to answer whether the EBC does outperform the current practice of English auctions in procurement we need to perform the appropriate game theoretic analysis and experimental evaluation.

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Table I: Nash Equilibrium Predictions for the RFQ and EBC

Auction	% of Efficient Outcomes	Avg. Winning Seller Quality	Avg. Auction Price	Avg. Winning Seller Cost	Avg. Buyer Surplus	Avg. Winning Seller Profit	Avg. Realized Social Surplus
RFQ	100%	117.90	71.17	54.65	46.74	16.52	63.26
EBC	83.41%	115.55	66.79	53.37	48.76	13.41	62.17

Table II. Empirical Auction Performance: RFQ versus EBC

Auction	% of Efficient Outcomes	Avg. Realized Social Surplus	Avg. Winning Seller Quality	Avg. Auction Price	Avg. Winning Seller Cost	Avg. Buyer Surplus	Avg. Winning Seller Profit
RFQ	79%	61.53	117.69	68.37	56.16	49.32	12.21
Stand. Dev.	0.407	12.67	8.18	9.39	10.64	9.86	8.33
EBC	84.27%	63.25	116.26	65.85	53.01	50.41	12.83
Stand. Dev.	0.64	3.56	2.86	3.06	3.26	3.14	2.89
$m_{RFQ} - m_{EBC}$	-5.27%	-1.71	1.43	2.52	3.14	-1.09	-0.62
z-stat	-2.01	-2.00	2.37	3.45	4.74	-1.25	-0.94
$F(z)$	**0.022**	**0.023**	**0.991**	**1.000**	**1.000**	0.106	0.173

Table III: Auction Performance: Theoretic Predictions and Empirical Measurements

Auction		% of Efficient Outcomes	Avg. Realized Social Surplus	Avg. Winning Seller Quality	Avg. Auction Price	Avg. Winning Seller Cost	Avg. Buyer Surplus	Avg. Winning Seller Profit
RFQ n =1100	Theoretical	100%	63.26	117.90	71.17	54.65	46.74	16.52
	Observed	79%	61.53	117.69	68.37	56.16	49.32	12.21
	Stand. Dev.	0.407	12.67	8.18	9.39	10.64	9.86	8.33
	Z-stat	-17.092	-4.51	-0.85	-9.91	4.69	8.69	-17.16
	$F(z)$	**0.000**	**0.000**	0.198	**0.000**	**1.000**	**1.000**	**0.000**
EBC n= 248	Theoretical	83.41%	62.17	115.55	66.79	53.37	48.76	13.41
	Observed	84.27%	63.25	116.26	65.85	53.01	50.41	12.83
	Stand. Dev.	0.638	3.56	2.86	3.06	3.26	3.14	2.89
	Z-stat	0.374	1.40	1.29	-1.40	-0.61	2.01	-0.94
	$F(z)$	0.646	0.919	0.902	0.081	0.270	**0.978**	0.173

Tabel IV: Estimated Bidding Models with *J*-test Selection Criteria For each Subject

(**Bold face** indicates *t*-test does reject coefficient is zero; 50 observations for each subject)

Subject	Intercept	Cost	Quality	NE Price	R-Square	Model Selected
1	**-4.10**	-	-	1.04	0.85	Nash Price
2	**-8.11**	-	-	1.08	0.64	Nash Price
3	14.10	-	-	0.78	0.62	Nash Price
4	**9.10**	-	-	0.91	0.54	Nash Price
5	**13.80**	-	-	0.79	0.54	Nash Price
6	24.19	-	-	0.66	0.53	Nash Price
7	**5.29**	0.74	0.15	-	0.94	Linear Cost and Bid
8	**-1.69**	0.92	0.12	-	0.93	Linear Cost and Bid
9	**-2.25**	0.68	0.27	-	0.88	Linear Cost and Bid
10	**0.05**	0.89	**0.10**	-	0.85	Linear Cost and Bid
11	**10.20**	0.66	0.19	-	0.82	Linear Cost and Bid
12	**8.28**	0.68	0.21	-	0.81	Linear Cost and Bid
13	**9.54**	0.76	**0.11**	-	0.81	Linear Cost and Bid
14	**12.44**	0.81	**0.08**	-	0.77	Linear Cost and Bid
15	**2.20**	0.65	0.26	-	0.75	Linear Cost and Bid
16	**12.88**	0.86	**0.02**	-	0.67	Linear Cost and Bid
17	**13.82**	0.61	0.22	-	0.65	Linear Cost and Bid
18	**10.80**	0.63	**0.18**	-	0.64	Linear Cost and Bid
19	**6.93**	0.61	0.23	-	0.62	Linear Cost and Bid
20	**18.35**	0.91	**0.05**	-	0.55	Linear Cost and Bid
21	**18.45**	0.67	**0.14**	-	0.47	Linear Cost and Bid
22	**38.65**	0.59	**0.05**	-	0.33	Linear Cost and Bid
23	**-4.05**	0.40	**-0.06**	0.75	0.93	Both Models Selected
24	**4.43**	**-0.01**	**-0.15**	1.12	0.87	Both Models Selected
25	**12.30**	-1.20	-0.71	2.89	0.86	Both Models Selected
26	**-5.12**	-0.63	**-0.26**	2.03	0.85	Both Models Selected
27	**1.68**	**0.20**	**-0.07**	0.88	0.84	Both Models Selected
28	**3.19**	-1.48	**-0.60**	3.15	0.83	Both Models Selected
29	35.84	**-0.45**	-0.66	1.86	0.83	Both Models Selected
30	**12.41**	-0.89	**-0.36**	2.12	0.70	Both Models Selected
31	40.41	**-0.31**	**-0.45**	1.39	0.60	Both Models Selected
32	57.49	-0.75	-0.60	1.76	0.53	Both Models Selected
33	**-11.02**	**0.08**	**0.01**	1.00	0.82	Neither Model Selected
34	**15.39**	**0.33**	**0.01**	**0.54**	0.79	Neither Model Selected
35	**-3.13**	**0.41**	**0.49**	**0.06**	0.71	Neither Model Selected
36	**-2.15**	**0.30**	**-0.02**	**0.77**	0.70	Neither Model Selected
37	**-7.04**	**0.19**	**0.30**	**0.53**	0.67	Neither Model Selected
38	**24.90**	**-0.78**	**-0.49**	2.14	0.58	Neither Model Selected
39	**8.10**	**-0.17**	**-0.02**	**1.04**	0.57	Neither Model Selected
40	36.39	**0.23**	**0.06**	**0.25**	0.41	Neither Model Selected
41	**19.83**	**-0.71**	**-0.36**	**1.95**	0.33	Neither Model Selected
42	43.29	**0.16**	**0.03**	**0.33**	0.26	Neither Model Selected
43	**37.00**	**0.43**	**-0.05**	**0.31**	0.21	Neither Model Selected
44	**27.52**	**0.64**	**0.48**	**-0.63**	0.12	Neither Model Selected

Figure 1: Contour Plots of Equilibrium Bid Function and Graph of $O^*(s_i)$

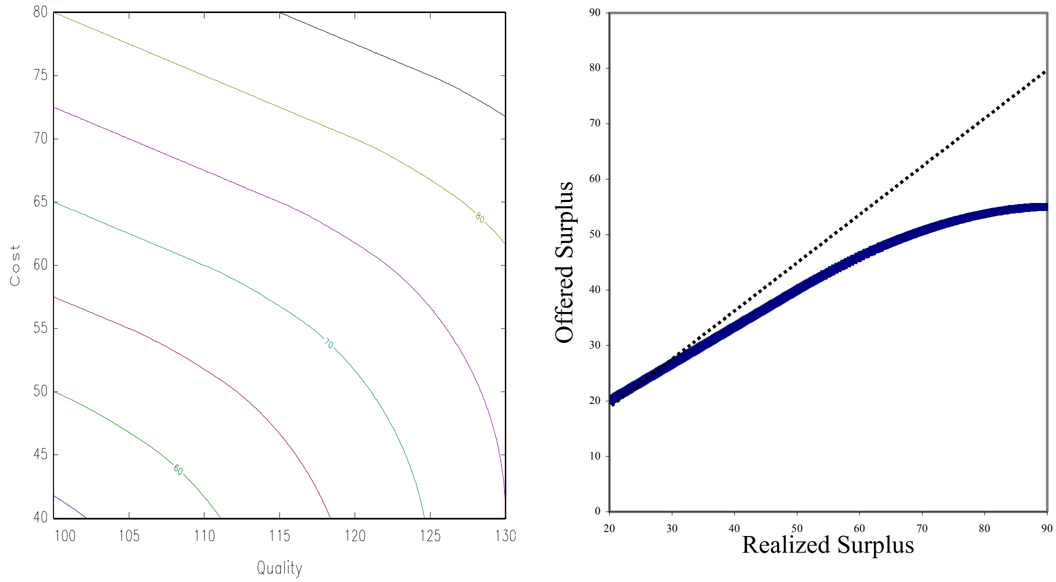

Figure II: Surplus Offered

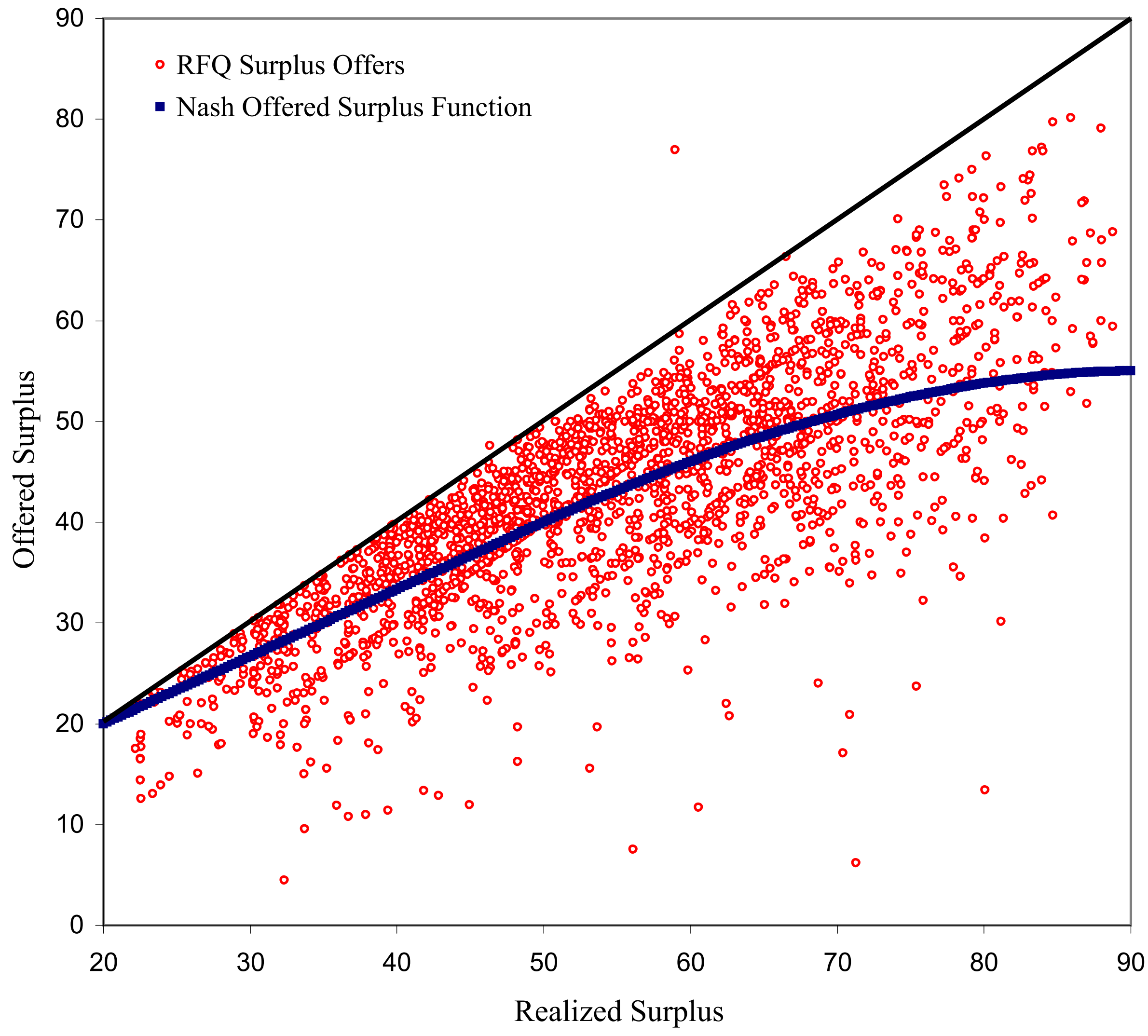

Figure III: The Average Difference Between the Actual Bid and the Predicted
Bid for Different Cost-Quality Types

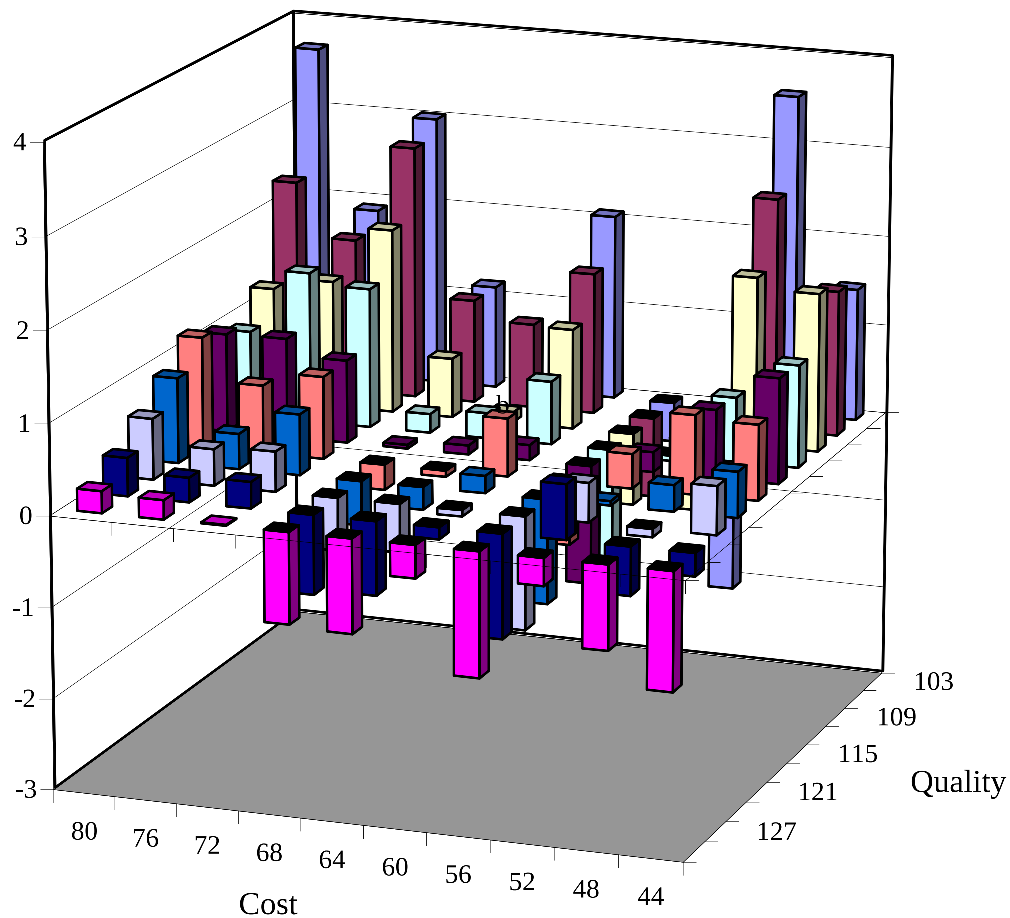

Figure IV: Devation of Losing Seller's Exit Price From Dominant Strategy Exit Price

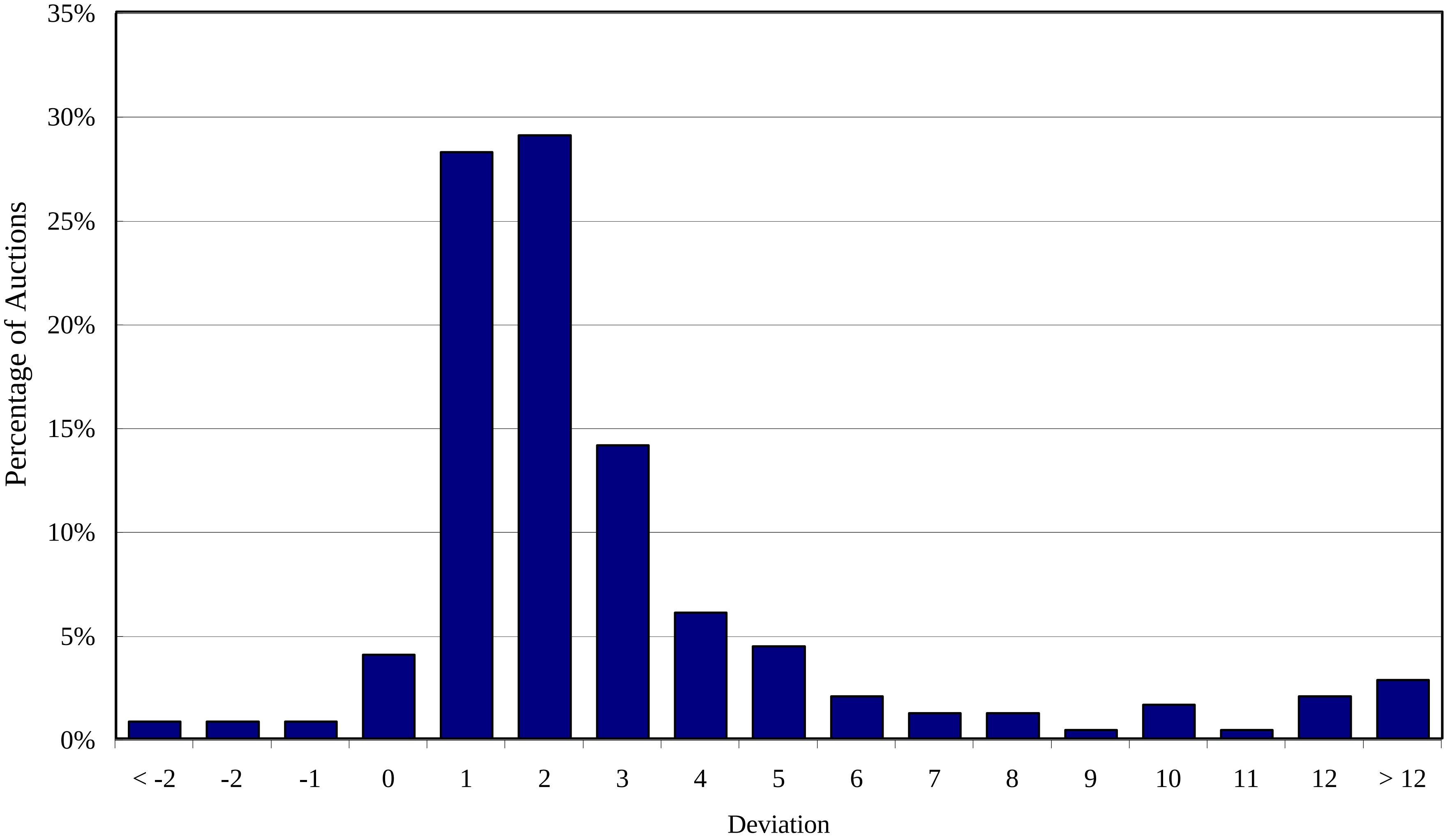

Figure V: Differences in Assigned Bidding Credits versus Differences in Quality