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The Impact of Options Trading on Supply Chain Management

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Abstract

This paper explores the impact of options on supply chain performance when consumer demand is uncertain. The analysis is based on an environment involving a single supplier and multiple retailers. Each retailer can either buy product directly from the supplier, or purchase options on product. An option gives the retailer the right, but not the obligation, to buy an additional unit of product. As such, the retailer will exercise an option if and only if that unit of product is needed to meet demand. The retailer must balance the reduced uncertainty that options afford against the price premium that must be paid to procure product with options.

Unlike other options and option-like contract arrangements that have been studied in the supply chain management literature, the options considered in this paper are independent of any contractual arrangement made between a particular buyer and seller. As such, these instruments can be traded among retailers as more information about the actual demand each faces becomes available, allowing contingency claims to flow to those retailers whose supply needs are greatest.

We first derive optimal replenishment policies for the retailers, the optimal production policy for the supplier, and closed-form solutions for optimal expected profits. We then show how options enhance information flows, encourage risk sharing, and improve supply chain efficiency. The paper includes a discussion of how options can be used to align the incentives of supply chain partners, and to improve supply chain responsiveness to changes in the business environment. We then consider an open market for trading supply chain derivatives, and characterize how an individual retailer should react to the opportunity to buy and sell options on product. We characterize the equilibrium market price at which options should trade, and demonstrate that this trading increases the expected profits of all retailers. Additional insights into the value of a market for trading supply chain options are derived through numerical exploration.

Keywords: Supply Chain Options, Options Trading, Options Price, Risk Management, Replenishment Flexibility, Channel Coordination, Risk Sharing, Information Sharing.

1. Introduction

Supply chain risks can have a significant impact on a firm's operating and financial performance. Demand uncertainty can result in under- or over-production, leading to either lost sales or excess inventory. Shortages of critical inputs can lead to expedited purchases at higher prices, or even cause major production or supply chain disruptions. Insufficient capacity can result in lost sales, while excess capacity can yield uncompetitive production costs.

Numerous examples illustrate how unexpected events can affect the smooth functioning of supply chains. In March 2000, lightning struck a Philips Electronics facility, causing a fire that shut down the plant for several weeks (Latour 2001). Because Philips was the sole source for critical components used by Ericsson to produce mobile phone handsets, and because Ericsson had no contingency plans to manage supply disruptions, the shutdown caused a reported $400 million in lost revenue. Poor demand forecasting and rigid procurement contracts at Cisco Systems precipitated $2.5 billion in inventory write-offs in 2001 (Berinato 2001). Difficulties implementing supply chain management software at Nike led to severe inventory difficulties in 2001, decreasing third quarter revenue by $100 million and reducing the firm's market capitalization by almost 20% (Piller 2001 and Wilson 2001). Quality problems with Firestone tires on the Ford Explorer resulted in over one hundred highway fatalities and forced massive tire recalls (Bradsher 2001 and Kashiwagi 2001), not only creating a potential multi-billion dollar legal exposure for the two firms, but also leading to significant loss of brand valuation.

With technological advancements and new business models, managing supply chain risks has become increasingly difficult. As supply chains continue to get leaner, they become far more dependent on the carefully orchestrated coordination of a complex network of supply chain partners. This new business environment is characterized by intense, global competition, short product life cycles, increased technological innovation, time-sensitive customer demand, and greater use of outsourcing. Operating in this environment reduces a firm's margin for error.

The consequences of failing to effectively manage supply chain risk have also become more severe. In addition to the direct impact on revenue and profit, disruptions in supply or demand can also hurt a firm's trading partners (*e.g.,* customers and suppliers), since the interconnectedness of supply chain causes a ripple effect that affects the entire supply ecosystem. The equity markets can be equally unforgiving. Hendricks and Singhal (2003) demonstrated that firms reporting supply chain difficulties typically lose about 10% of their market capitalization in the two days following announcement of the event.

However, supply chain risks can be managed, both operationally and financially. Operationally, a firm can reduce risk by changing its degree of vertical integration, product strategy, procurement practices, and inventory policies. Financially, a firm can carry insurance, modify supply contract terms, and hedge with derivatives such as futures contracts, options, and swaps. While considerable research has focused on how to operationally manage supply chain risks (see, *e.g.,* Tayur *et al.* 1999), there has been only limited study of the use of financial instruments to manage supply chain risk.

Derivative instruments have consistently proven their value as a means for managing risk (see, *e.g.,* Crouhy *et al.* 2001), and financial futures and options are actively traded on many exchanges. Derivatives are routinely used to manage financial risks, *e.g.,* exposure to security price fluctuations, foreign exchange rate movements, and changes in interest rates (Hull 1997). Within a more limited scope, a few industries have also used derivatives to manage risk (see, *e.g.,* Pilipovic 1998 on the use of options in energy markets, Bassok *et al.* 1997 on the practice at IBM printer division, and Farlow *et al.* 1995 on the practice at Sun Microsystems).

Supply contract terms and conditions often have characteristics that make them behave much like financial derivatives. Option-like contract arrangements explored in the literature include buy back policies (Pasternack 1985, Emmons and Gilbert 1998), backup agreements (Eppen and Iyer 1997), pay-to-delay capacity reservation (Brown and Lee 1998), and quantity flexibility (Tsay 1999, Tsay and Lovejoy 1999). Barnes-Schuster *et al.* (2002) explored the impact of contractual real options in a buyer-supplier system.

Note that in the supply arrangements listed above the options are embedded in the contract itself. In contrast, the options studied in this paper are completely separate from any supply chain contract, and can thus be traded in an open market. This structure offers a number of advantages. First, option holders can adjust the quantity of options they hold as they learn more about the market and their exposure to risk. For example, a retailer facing lower-than-expected demand can sell options to a retailer facing higher-than-expected demand, perhaps even realizing a profit on the transaction. In contrast, embedded options are not fungible - holders cannot divide them and take a fraction of a position. As such, an embedded option is a sunk cost for its holder, since it cannot be recovered once the contract has been signed.

Separating options from supply contracts also allows organizations to align their supply chain investments with their risk preference. Supply chain partners can have very different perceptions of value and risk, differences that can prevent optimal investment in inventory and capacity and degrade overall supply chain efficiency. Tradable options allow risk to be assumed and shared among those organizations that have the inclination and capacity to bear risk. These could include other suppliers or buyers, risk intermediaries such as insurance companies or banks, or even speculators.

Finally, information about supply and demand is an important element of supply chain management (see *e.g.,* Lee *et al.* 1997). A market for supply chain options provides a rich source of data (*e.g.,* option prices and trading volumes) that can be used to facilitate capacity planning and production scheduling.

The supply chain is the nexus for a wide variety of risks. Firms face risks when buying from their suppliers, and when selling goods and services to their customers (Grey and Shi 2003). These risks can derive from several sources, such as demand uncertainty, price fluctuations, supply disruptions, inventory risk, quality problems, and complexities associated with new technologies.

The options described in this paper are derivatives whose value is linked to customer demand. However, the framework developed here can be easily modified to study derivatives whose value is dependent on other forms of supply chain uncertainty.

The remainder of the paper is organized as follows. Section 2 describes the problem setting and derives optimal policies for both a collection of retailers and a single supplier. Section 3 discusses the impact of options on the supply chain with respect to flexibility, channel coordination, and risk and information sharing. Section 4 presents a model for trading options among buyers, which facilitates the analysis of the value of options trading. Section 5 numerically investigates the effectiveness of an options market with respect to various parameters. Section 6 concludes with a summary and suggestions for further research. All mathematical details and proofs are relegated to the Appendix.

2. Problem Formulation and Solution

We consider a supply chain comprised of a supplier producing short-life-cycle products, and a set of retailers $\{1, 2, \ldots, N\}$ who order product from the supplier and then sell to end-users. Each retailer must independently decide how many units of a single product to purchase to cover a selling season, *i.e.,* a time period $[0, \tau]$. We assume that the procurement lead-time is long relative to the selling season, so that retailers cannot observe demand before placing an order. Because of the long lead-time, there is no opportunity for retailers to replenish inventory through new orders once the season has begun. However, as discussed in Section 4, retailers can adjust their positions by trading options with one another at time $t > 0$.

At time $t = 0$, retailers can obtain goods from the supplier by two means: either through a firm order or by buying and exercising call options. Before the start of the season, each retailer i places an order for Q_i units of product at unit wholesale price W. These units are physically delivered at time $t = 0$. Each retailer i also purchases q_i options at unit cost C, each of which gives the retailer the right (but not the obligation) to buy one unit of product at exercise price X after demand has been observed at time $0 < t < \tau$.

Assume that the product has unit retail price R (which is the same for all retailers, though prices that vary across retailers can be easily accommodated) and unit manufacturing cost M. After the selling season, any excess product, regardless of whether it is owned by the retailer or the supplier, can be salvaged at unit value S. Before the selling season, the demand faced by retailer i, denoted by D_i, is uncertain. Most of the analysis applies to general continuous demand distributions with density function $f(D_i)$ and cumulative distribution function $F(D_i)$.

Both the retailers and the supplier make decisions prior to the selling season. Each retailer i places orders for Q_i units of product and q_i options, and the supplier subsequently decides the number of units of product Y to produce. Clearly Y must be at least as great as $\sum_{i=1}^{N} Q_i$, since each Q_i represents a firm order. Retailer i only exercises options when $D_i > Q_i$, and the likelihood that the retailer i will not exercise all q_i options is positive. Therefore, the number of units Y produced by a rational supplier is between $\sum_{i=1}^{N} Q_i$ and $\sum_{i=1}^{N}(Q_i + q_i)$. However, when $Y < \sum_{i=1}^{N}(Q_i + q_i)$, there is a positive probability that the supplier will default on its commitment to fill all options. In such a case, the supplier incurs a unit penalty cost P for each exercised option that cannot be immediately fulfilled from inventory.

The penalty cost P can have different interpretations. P may represent the cost the supplier incurs to obtain an additional unit of product by expediting production or buying from an alternative source. It could also represent a pre-determined cash penalty specified in the option contract. However, these two mechanisms for settling option defaults result in a different set of incentives for retailers, even for the same value of P. When the supplier finds an alternative means for delivering the product, retailers only exercise options that are truly supported by actual demand. In contrast, when the supplier incurs a cash penalty for defaulting, retailers will exercise all options as soon as they learn that the supplier cannot meet the options commitment, regardless of whether or not there is actual demand. For this reason, we assume that all options are settled by physical delivery of product rather than cash settlement.

There are several natural feasibility conditions in the supply chain:

$$M < W < C + X < R \tag{1}$$

$$P > M > S \tag{2}$$

$$X > S \tag{3}$$

Conditions $M < W < R$ and $C + X < R$ must hold to ensure profit for both the retailers and the supplier. Moreover, if $W \geq C + X$, it would be advantageous for the retailers to only order options. Condition (2) states that penalty cost P is always greater than the normal production cost M, which is always larger than salvage value S. Condition (3) is necessary to prevent the retailers from exercising all of their options, even when there is no actual demand, and salvaging the purchased products.

Supply arrangements involving embedded options are special cases of this general framework (see, *e.g.,* Barnes-Schuster *et al.* 2002, and Shi *et al.* 2003). This model also differs from Barnes-Schuster *et al.* (2002), where the supplier must produce the maximum quantity possibly requested but without penalty P. Moreover, the focus in this paper is on the impact of options trading among retailers, while Barnes-Schuster *et al.* (2002) studied the role of options in a single buyer-supplier system where trading is not possible.

2.1. The Retailers' Decisions at time $t = 0$

Each retailer i has two decision variables: the number of units Q_i to order and the number of call options q_i to purchase. We introduce $T_i = Q_i + q_i$ to represent the total order quantity for retailer i. Note that determining (Q_i, q_i) is equivalent to determining (Q_i, T_i). The retailer will always first fulfill demand using firm orders Q_i. When Q_i is insufficient to meet all demand, the retailer will exercise up to q_i options. The expected profit for retailer i with options is given as:

$$E\Pi_{R_i}(Q_i, q_i) = E_{D_i}\left[R\min(D_i, T_i) + S(Q_i - D_i)^+ - WQ_i - Cq_i - X\min\left(q_i, (D_i - Q_i)^+\right)\right]. \quad (4)$$

The first term is total revenue, which reflects the fact that the retailer's sales are limited by both total demand and total supply. The second term represents the salvage value of any leftover product. The last three terms capture the cost of ordering product directly, of purchasing options, and of exercising options as required, respectively. Substituting $q_i = T_i - Q_i$ in (4), we can rewrite the expression for expected profit as a separable function in Q_i and T_i:

$$E\Pi_{R_i}(Q_i, T_i) = (X+C-W)Q_i - (R-S)\int_0^{Q_i} F(D_i)dD_i + (R-X-C)T_i - (R-X)\int_{Q_i}^{T_i} F(D_i)dD_i. \quad (5)$$

Retailer i seeks to identify Q_i and T_i that maximize $E\Pi_{R_i}(Q_i, T_i)$, subject to the constraint that $0 \le Q_i \le T_i$. It is straightforward to show that, because of (1) – (3), the expected profit function is concave with respect to Q_i and T_i, and thus has a unique maximum. The optimality conditions are given by the following result.

Proposition 1. Let Q_i^* be the optimal number of units that retailer i should order, and T_i^* the optimal total order quantity. Then:

$$F(T_i^*) = \Pr(D_i \le T_i^*) = \frac{(R-X-C)}{(R-X)} \quad (6)$$

$$F(Q_i^*) = \Pr(D_i \le Q_i^*) = \frac{(X+C-W)}{(X-S)} \quad (7)$$

and the optimal expected profit for retailer i is given by:

$$E\Pi_{R_i}(Q_i^*, T_i^*) = (X+C-W)Q_i^* - (R-S)\int_0^{Q_i^*} F(D_i)dD_i + (R-X-C)T_i^* - (R-X)\int_{Q_i^*}^{T_i^*} F(D_i)dD_i. \quad (8)$$

Note that $Q_i^* \le T_i^*$ implies that $C \le \frac{(W-S)(R-X)}{(R-S)}$. This shows that if the option cost C is too high, retailer i will not order any options. Throughout this paper, we assume that the cost parameters always satisfy this constraint.

The classic newsvendor model (Hadley and Whitin 1962) is a special case of this formulation where the retailer cannot purchase options. The optimal order quantity Q_i^{NV} and optimal expected profit $E\Pi_{R_i}^{\mathrm{NV}}$ for retailer i in the classic newsvendor formulation are represented by:

$$F(Q_i^{\mathrm{NV}}) = \Pr(D \leq Q_i^{\mathrm{NV}}) = \frac{(R-W)}{(R-S)} \tag{9}$$

$$E\Pi_{R_i}^{\mathrm{NV}}(Q_{\mathrm{NV}}^*) = (R-S)\int_0^{Q_i^{\mathrm{NV}}} Df(D)dD = (R-W)Q_i^{\mathrm{NV}} - (R-S)\int_0^{Q_i^{\mathrm{NV}}} F(D)dD. \tag{10}$$

Equation (9) can be rewritten as $F(Q_i^{\mathrm{NV}}) = \frac{C_u}{(C_u+C_o)}$, where $C_u = R-W$ is the unit underage cost of forgone profit, and $C_o = W-S$ is the unit overage cost of salvage loss. Equations (6) and (7) also take this form. In (6), $C_u = R-(X+C)$ is the forgone profit if a unit of demand is unfulfilled for lack of options to exercise, and $C_o = C$ is the cost of an unexercised option. Similarly in (7), $C_u = X+C-W$ and $C_o = W-(C+S)$. If actual demand is greater than Q_i, the retailer pays a premium $(X+C-W)$ to satisfy the demand with options instead of firm orders. On the other hand, if actual demand is less than Q_i, then the retailer incurs the cost for purchasing the product (but avoids the options cost), and salvages the product instead.

2.2. The Supplier's Decision

Before the selling season, the supplier must determine prices (W,C,X) and how many units to produce. Let Y denote the supplier's production volume. We assume the following sequence of interactions between the retailers and the supplier:

- Prices (W,C,X) are determined and announced.
- Each retailer i places orders (Q_i, q_i) according to expressions (6) and (7).
- The supplier produces Y units of product, and delivers Q_i units to each retailer i, holding the remaining $Y - \sum_{i=1}^{N} Q_i$ units in inventory.

- During the selling season, each retailer i exercises up to q_i options, and additional units of product are delivered to retailers as required.

Retailer i will procure a total of $z_i = Q_i^* + \min[q_i^*, (D_i - Q_i^*)^+]$ units of product from the supplier to meet demand D_i. Thus, the supplier has an obligation to deliver a total of $Z = \sum_{i=1}^{N} z_i$ units to all of the retailers. Let $f_Z(z)$ and $F_Z(z)$ be the density and cumulative distribution functions of Z, respectively. The supplier's expected profit over all retailers is then given by:

$$E\Pi^S(Y, W, C, X) = E_{D_1,\ldots,D_N} \sum_{i=1}^{N} \left\{ WQ_i^* + Cq_i^* + X \min\left[q_i^*, (D_i - Q_i^*)^+\right] + S[Y-Z]^+ - P[Z-Y]^+ - MY \right\}. \quad (11)$$

The first four terms in expression (11) reflect the revenues realized by the supplier from (i) the sale of product, (ii) the sale of options, (iii) options that are exercised, and (iv) salvaging unsold product, respectively. The fifth term captures the total penalty incurred when options are exercised but can't be fulfilled, and the final term is the total manufacturing cost. For a given set of prices (W, C, X), the optimal production quantity Y^* is given by the following result.

Proposition 2. Given prices (W, C, X), compute Y^{**} such that:

$$F_Z(Y^{**}) = \Pr(Z \leq Y^{**}) = \frac{(P-M)}{(P-S)}. \quad (12)$$

Then the optimal production quantity is $Y^* = \min\left[\sum_{i=1}^{N}(Q_i^* + q_i^*), \max(\sum_{i=1}^{N} Q_i^*, Y^{**})\right]$

Expression (12) also takes the form of the newsvendor model with $C_u = P - M$ and $C_o = M - S$. Thus, the underage cost to the supplier is the premium $(P - M)$ paid to supply an additional unit from an alternative source, while the overage cost is the difference between what the supplier paid to produce the unit and the amount that can be realized in salvage. Note that since the revenue realized by the supplier is independent of the chosen stock level, the production quantity is independent of W, X, and C.

In the classic newsvendor model, the supplier fills the retailers' orders by building to order, *i.e.,* the supplier always produces Q_i^{NV} units for each retailer i (note that retailers then bear all risk associated with demand uncertainty). The supplier's profit from retailer i is then $(W - M)Q_i^{\text{NV}}$.

The challenge in determining the optimal production quantity using Proposition 2 is in finding the distribution function of Z given distributions of $\{D_1, D_2, \ldots, D_N\}$. Note that the number of units of product required for retailer i, $z_i = Q_i^* + \min\left[q_i^*, (D_i - Q_i^*)^+\right]$, is the truncation of random variable D_i with range $[Q_i^*, Q_i^* + q_i^*]$ and density function:

$$f_{z_i}(x) = \begin{cases} \frac{f_{D_i}(x)}{[F_{D_i}(Q_i+q_i) - F_{D_i}(Q_i)]}, & \text{if } Q_i^* \leq x \leq Q_i^* + q_i^* \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

The following algorithm can be used to compute the supplier's optimal production quantity.

Algorithm to Determine Optimal Production Quantity Y^*

Input. A set of feasible parameters R, W, X, C, S, and P, and distribution functions of $\{D_1, D_2, \ldots, D_N\}$.

Output. Optimal production quantity, Y^*.

Step 1. Calculate retailers' optimal order quantities using expression (6) and (7) for each retailer i, $i = 1, 2, \ldots, N$.

Step 2. Determine the distribution of the number of units of product z_i required by each retailer i, $i = 1, 2, \ldots, N$, using expression (13).

Step 3. Find the distribution of the total number of units of product Z required over all retailers by solving the convolution problem $Z = \sum_{i=1}^{N} z_i$ for the distribution of Z.

Step 4. Find the optimal production quantity Y^* using Proposition 2.

3. The Impact of Options on Supplier-Retailer Interaction

Options provide retailers with an alternative mechanism for obtaining product from the supplier. For the special case where there is a single retailer, *i.e.,* $N = 1$, we discuss in this section the impact of options on the interactions between supplier and retailer, focusing on replenishment flexibility, coordination of the channel, and risk and information sharing.

3.1. Replenishment Flexibility

Options provide retailers with the flexibility to replenish product during the selling season. In markets where product life cycles are short and demand volatilities are high, this flexibility allows retailers to respond quickly to changes in demand. Barnes-Schuster *et al.* (2002) described replenishment practices in three industries (toys, apparel and electronics), and concluded that flexibility benefits the retailer while perhaps costing the supplier. As shown in Section 3.3, the flexibility afforded by options can improve performance for both retailers and the supplier.

The impact of flexibility on competitiveness is widely recognized (see, *e.g.,* Shi and Daniels 2003), and the value of flexibility for in-season replenishment can be substantial. In the apparel industry for example, because retailers must place firm, SKU-specific orders far in advance of the start of a selling season (see, *e.g.,* Nuttle *et al.* 1991), the cost of markdowns is on the order of billions of dollars annually (see, *e.g.,* Fisher *et al.* 1994). Hunter *et al.* (1996) estimated that the potential savings to retailers are so large that they should be willing to pay a 30-50% premium to any supplier who can provide in-season replenishment.

3.2. Channel Coordination

The supply chain considered in this paper consists of parties with different objectives and asymmetric information concerning demand. All parties seek to maximize their own profits, but none exercises control over the entire supply chain nor has an incentive to globally optimize performance. This dual decision-making with conflicting objectives often degrades the efficiency of the entire supply chain. Several solution have been proposed for this problem (see *e.g.,* Anupindi *et al.* 1999, Lariviere 1999, and Taylor 2001, 2002), typically involving some modification of the payment structure between the supplier and retailers beyond a simple singleton price W. These modifications provide incentive for all partners to behave in a manner that optimizes the efficiency of the entire supply chain. As shown in this section, options can also be used to coordinate the supply chain.

When supplier and retailers make up a single entity (*e.g.,* for a vertically integrated firm), conflicting objectives are less of a confounding issue. However, double marginality leads to sub-optimal solutions in a non-integrated supply chain (see, *e.g.,* Spengler 1950). If the entire chain produces Q units of product, total profit for the supply chain is $(R-M)Q$; however, this profit must be divided between the retailers and the supplier. In addition, the retailers' order quantities influence the supplier's production decision. Each retailer i chooses an order quantity Q_i based on wholesale price W, which must be larger than manufacturing cost M to guarantee all parties positive profit margins. Double marginality induces a quantity $Q_i^{\mathrm{NV}} = F^{-1}\left\{\frac{(R-W)}{(R-S)}\right\}$ as in expression (9). On the other hand, in an integrated supply chain partners coordinate their activities to maximize the total profits of the chain. Since M replaces W in the integrated supply chain, $Q_i^I = F^{-1}\left\{\frac{(R-M)}{(R-S)}\right\}$. Since $M < W$, $Q_i^{\mathrm{NV}} < Q_i^I$, and the total profit for the entire supply chain is greater in the integrated case.

In addition to the wholesale price W, options contracts provide three more degrees of freedom (X, C and P) when negotiating contract terms. This additional flexibility enables options to be used to coordinate partners' behaviors, inducing channel coordination and ensuring that a decentralized supply chain performs as well as an integrated supply chain. To induce each retailer i to order total quantity T_i up to Q_i^I, expression (6) indicates that X and C must be set such that:

$$\frac{(R-X-C)}{(R-X)} = \frac{(R-M)}{(R-S)}. \tag{14}$$

Condition (14) ensures that the total order T_i^* from each retailer i will be as large as the order quantity in an integrated supply chain. Moreover, we must also set unit penalty P so that $Y_i^* = T_i^*$ for each retailer i, where Y_I^* denotes the production quantity allocated to retailer i. This, along with expression (15) and Proposition 2 (applied to the special case where $N = 1$), yields a condition under which the supplier is motivated to coordinate:

$$\frac{(P-M)}{(P-S)} \geq \frac{(R-X-C)}{(R-X)}. \tag{15}$$

Expressions (14) and (15) together provide sufficient conditions for channel coordination. Note that (15) and (16) imply that $P \geq R$, making optimal channel coordination feasible only when $P \geq R$.

3.3. Risk Sharing

Demand uncertainty exposes both the supplier and retailers to risks associated with mismatches between supply and demand. Specifically, if supply exceeds demand, the excess must be salvaged at a loss; and if demand exceeds supply, the unmet demand is lost. We refer to these costs as overage and underage, respectively. Overage costs include price markdowns and inventory holding costs. Underage costs capture lost sales, expediting costs, and/or customer ill will.

Options provide a mechanism for sharing these risks between the supplier and retailers. Each retailer can use options to hedge against both underage and overage costs by using firm orders to cover demand levels that are relatively more likely to occur, and options for demand levels that are less likely to occur. However, the retailers pay a premium for the reduction in risk, since the cost of procuring units with options, $X + C$, is higher than the cost W of buying product directly. The supplier earns this premium in compensation for sharing the retailers' risk. By specifying the order quantities Q_i and q_i, each retailer i decides how much risk to bear, and how much to pay for the benefit of risk reduction. By sharing risk, the supplier induces each retailer i to order a larger number of total units $Q_i + q_i$, thus increasing sales. However, in so doing the supplier creates an obligation to fulfill demand for units of product needed when the retailers exercise options. As a consequence, the supplier must hold inventories for options that may not be exercised, exposing the supplier to overage costs.

Two issues need to be considered for risk sharing: (*i*) setting transaction terms to improve the combined profits of the supplier and retailers, and (*ii*) allocating total profit equitably among the parties. Section 3.2 details the conditions under which total profits are maximized as in the integrated supply chain. Since there are larger total profits to distribute, options may benefit both the supplier and retailers.

Figure 1 plots the contribution of options to the profit of both the supplier and a single retailer. In this example, R=100, W=70, S=5, M=40, C=15, X=60, and P=80. Demand is normally distributed with mean 3000, and the standard deviation of the demand is varied from 50 to 2000 in increments of 50. The Profit Gain (%) $\dfrac{(E\Pi_S - E\Pi_S^{\mathrm{NV}})}{E\Pi_S^{\mathrm{NV}}}$ for the supplier and $\dfrac{(E\Pi_{R_1} - E\Pi_{R_1}^{\mathrm{NV}})}{E\Pi_{R_1}^{\mathrm{NV}}}$ for the retailer. These quantities are multiplied by 100 and reported as a percentage. The results yield several observations: (*i*) the profit gains for both the supplier and the retailer are always positive, highlighting that options can be advantageous for both supply chain partners, (*ii*) the contribution of options to supply chain profit is substantial, *e.g.,* when $\sigma = 2000$, the profit improvements are 11.6% and 85.7% for the supplier and the retailer, respectively, and (*iii*) the gains realized by both parties increase as the demand variance increases, but the retailer's gain increases at a faster rate.

3.4. Information Sharing

Being closer to end consumers, retailers often have better information about the distribution of demand faced by the supply chain. Focusing on the mean μ and variance σ^2 of demand, we now consider how options help transfer demand information from retailers to suppliers. Let $\phi(\xi_i)$ and $\Phi(\xi_i)$ denote the density and cumulative distribution functions of the normalized distribution with $\mu_i = 0$ and $\sigma_i^2 = 1$. We assume that all contractual parameters (R, W, X, C, P, S, and M) are known. This implies that the right hand sides of the expressions for T_i^*, Q_i^* and Y_i^* are all constants – these we will denote by $C_{T_i^*}, C_{Q_i^*}$ and $C_{Y_i^*}$, respectively.

Proposition 3. The orders of each retailer i and the supplier's production quantity satisfy the following:

1. $\Gamma = \mu_i + \sigma_i \Phi^{-1}(C_Z),$ for $\Gamma = Q_i^*, T_i^*,$ or Y_i^*.
2. $\dfrac{q_i^*}{T_i^*} = \dfrac{\left[\Phi^{-1}(C_{T_i^*}) - \Phi^{-1}(C_{Q_i^*})\right]}{\left[\mu_i/\sigma_i + \Phi^{-1}(C_{T_i^*})\right]}.$
3. The implied θ in $Y_i^* = Q_i^* + \theta q_i^*$ is constant with respect to μ_i and σ_i^2.

The first result shows that both the orders from each retailer i and the supplier's production quantity increase linearly with respect to demand mean μ_i and standard deviation σ_i. The second result demonstrates that the percentage $\frac{q_i^*}{T_i^*}$ is inversely dependent on $\frac{\mu_i}{\sigma_i}$. The final result identifies an invariant in the supplier's decisions, which can simplify the decision-making process. Upon receiving the order quantities from retailer i, (Q_i, q_i), the supplier responds first by checking the penalty cost P. If P is relatively high (low), then the supplier produces T_i^* (Q_i^*) units; otherwise, the supplier always produces $Q_i^* + \theta q_i^*$ for a fixed percentage θ regardless of demand mean μ_i and variance σ_i^2.

Like product flows and financial flows, information flows are important for supply chain management. Designing incentives to foster information sharing between suppliers and retailers remains a distinct challenge (see, *e.g.,* Lee *et al.* 1997, and Cachon and Lariviere 2001). The third result shows that options can enhance information flows from retailers to suppliers. The order quantities (Q_i, q_i) completely disclose a retailer's mean and variance of demand to the supplier, and the supplier does not need to know μ_i and σ_i explicitly to determine an appropriate production quantity, so long as ordering quantities Q_i and q_i are undistorted.

4. Trading Options Among Retailers

We now consider options trading among retailers, and investigate the value of a market where such trading occurs. To reduce problem complexity, we assume that trading is allowed only at a single point in time t (where $0 < t < \tau$). Since a market for supply chain options would likely allow almost continuous trading, the value associated with options trading should be considerably higher than estimated in this paper. We also assume independent and identically-distributed demands across retailers, though the results derived hold for cases where demands are not i.i.d.

The trading time t divides the selling season $[0, \tau]$ into two periods $[0, t]$ and $[t, \tau]$. Let D^1 be the random variable representing the demand faced by retailer i in the first period, and D_i^2 the random variable representing demand in the second period (note that subscript i is dropped for D^1, since retailers are assumed to be identical). Let d_i^1 denote the actual demand realized by retailer

i in the first period. Note that though the random variables, D^1 are i.i.d., the actual demands in the first period $\{d_i^1 : i = 1, 2, \ldots, N\}$ are different across retailers. This difference in realized demands results in varying inventory and options positions among retailers, motivating options trading. Define $D_i^C = D_i^2 | d_i^1$ be the conditional distribution of demand in the second period (given the realized demand d_i^1 in the first period), and let $f_i^{2|1}$ and $F_i^{2|1}$ denote the density function and CDF of the random variable D_i^C, respectively. The distribution functions of D^1, D_i^2, and D_i^C are all assumed to be known.

Since demands are i.i.d. across retailers, by Proposition 1 each retailer begins the season with identical inventory level Q^* and options position $q^* = T^* - Q^*$ (again we drop subscript i because we assume identical retailers). After observing the realized demand in the first period, each retailer updates the information about the distribution of demand in the second period. Since the conditional distributions $\{F_i^{2|1} : i = 1, 2, \ldots, N\}$ are no longer identical, the number of options that each retailer should hold at time t will vary across retailers. Retailers optimize their options positions at time t by trading with one another.

4.1. Trading Options at Time $t > 0$

At time $t > 0$, retailers can no longer place firm orders with the supplier. If additional units of product are required, retailers can exercise options that they either already own, or secure through options trading. This section derives the optimal number of options each retailer should hold after trading, and the impact of trading on retailers' profits.

Let C_t be the average market price of an option when trading occurs at time t. Note that retailers may pay different prices for options, depending on the mechanism used to settle bids and asks in the market. However, the average price allows us to focus on the long-ran average impact of options trading on retailers' profits. Average options price C_t creates a balance between supply and demand in the market in which retailers trade options with each other, and is assumed known. This market price is different from the option price in Section 2, where C is determined and announced by the supplier.

Given realized demand d_i^1 in the first period, the number of units of product retailer i holds in inventory at time t is $\tilde{Q}_i = (Q^* - d_i^1)^+$, and its options position before trading is the number of unexercised options, $u_i = [q^* - (d_i^1 - Q^*)^+]^+$. Let $\tilde{q}_i$ denote the options position held by retailer i after trading, and $\tilde{T}_i = \tilde{Q}_i + \tilde{q}_i$ the total position (inventory + options) after trading. Note that the unit cost of the u_i options is the supplier's announced price C at time zero, while the unit cost of $(\tilde{q}_i - u_i)$ options bought or sold to change the options position is the market price C_t (note that when $(\tilde{q}_i - u_i)$ is negative, retailer i actually sells options and thus realizes new revenue). The expected profit realized by retailer i in the second period (*i.e.,* after options trading) is then:

$$\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i) = E_{D_i^C}\left[R\min(D_i^C, \tilde{T}_i) + S(\tilde{Q}_i - D_i^C)^+ - W\tilde{Q}_i - Cu_i - C_t(\tilde{q}_i - u_i) - X\min\left(\tilde{q}_i, (D_i^C - \tilde{Q}_i)^+\right)\right]. \tag{16}$$

Similar to the expected profit in expression (4), the first term represents the total revenue realized from product sales in the second period. The second term captures the salvage value of any leftover product. The third term is the cost of purchasing product directly from the supplier. The fourth term is the cost of purchasing options from the supplier at time zero, while the fifth term is the cost (or revenue) of acquiring (selling) options from (to) other retailers at time t. The last term is the cost of exercising options to satisfy demand in the second period as required.

Since $X - R < 0$, the expected profit function $\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i)$ is concave with respect to $\tilde{q}_i$. This leads to the following result.

Proposition 4. There is a unique optimal solution $\tilde{q}_i^*$ to $\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i)$ such that:

$$F_i^{2|1}(\tilde{Q}_i + \tilde{q}_i^*) = \frac{(R - X - C_t)}{(R - X)}. \tag{17}$$

Note that expressions (17) and (6) are similar, with market price C_t replacing the supplier's announced price C, and conditional distribution function $F_i^{2|1}$ replacing distribution function F. Given the market price C_t, the right hand side of expression (17) is identical for all retailers.

Retailers can be partitioned into three disjoint sets based on their participation in options trading. $B = \{i : \tilde{q}_i > u_i, i = 1, 2, \ldots, N\}$ represents the set of retailers who buy options, $S = \{i : \tilde{q}_i < u_i, i = 1, 2, \ldots, N\}$ the set of retailers who sell options, and $I = \{i : \tilde{q}_i = u_i, i = 1, 2, \ldots, N\}$ the set of retailers who neither buy nor sell options.

Since cumulative distribution functions $F_i^{2|1}(x)$ are non-decreasing with respect to x, and the optimal options position $\tilde{q}_i^*$ in expression (17) satisfies $\dfrac{d\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i = \tilde{q}_i^*)}{d\tilde{q}_i} = 0$, we obtain the following result.

Lemma 1. The disjoint sets B, S, and I satisfy:

$$B = \arg\left\{\frac{d\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i = u_i)}{d\tilde{q}_i} > 0\right\}, S = \arg\left\{\frac{d\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i = u_i)}{d\tilde{q}_i} < 0\right\}, \text{ and } I = \arg\left\{\frac{d\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i = u_i)}{d\tilde{q}_i} = 0\right\}.$$

Based on updated information concerning demand in the second period, a rational retailer would trade options at time t only to increase expected profit in the second period. This insight is formally stated and verified in the following result.

Theorem 1. The net effect of options trading on the expected profit in the second period is positive for both buyers (B) and sellers (S), and zero for those who are inactive (I) in the options market.

This result indicates that options trading has a positive or non-negative effect on the total profits of all participants in the market, leading to improved overall performance for supply chain as a whole. Numerical results in Section 5 provide further insight into the magnitude of this effect, and how environmental parameters affect the impact of options trading on supply chain performance.

4.2. The Market Price of Options

We now analyze the average market price for options, C_t. At time $t = 0$, all retailers face the same demand distributions, and thus all enter the selling season with the same inventories, Q^*

and the same access to product T^* (inventory plus options). As time advances to t, retailer i sells d_i^1 units of product, reducing access to product to $T_i' = (T^* - d_i^1)^+$. Note that this is the supply position of retailer i prior to options trading (thus T_i' is distinct from $\tilde{T}_i$).

The marginal benefit to any retailer of fulfilling one unit of demand using options is $(R-X)$, since the cost of the option is a sunk cost when the option is exercised. Therefore, the marginal value of an option to retailer i (*i.e.,* the payout function if retailer i can secure an additional option) is given by:

$$v_i = (R-X)\Pr(D_i^C > T_i') = (R-X)\left[1 - F_i^{2|1}(T_i')\right]. \tag{18}$$

Thus, retailer i is willing to pay up to v_i in order to obtain an additional option at time t. By definition, the average market price C_t is the average of the prices that all retailers in the market are willing to pay, or:

$$C_t = \frac{\sum_{i=1}^N v_i}{N}. \tag{19}$$

C_t can be interpreted as follows. Options prices may fluctuate across buyers (or over time if continuous trading is allowed), but the fluctuation will center around C_t, *i.e.,* the price will move toward C_t in the long ran. Knowledge about C_t has significant value to market participants. For example, a retailer who wants to buy options when the current ask price is higher than C_t should wait for the price to drop; conversely, if the current ask price is lower than C_t, the retailer should buy immediately and take advantage of the discount.

Unfortunately, determining market price C_t using expression (19) requires information about the demand realized by each retailer i in the first period. Since this proprietary information need not be shared among retailers, it is unlikely that any individual retailer would have enough information to directly compute C_t using (19). However, the market price for options can be determined indirectly by assuming an infinite number of retailers and using information about an average retailer. Let:

$$\bar{C} = \lim_{N\to\infty} \frac{\sum\limits_{i=1}^{N} v_i}{N}. \tag{20}$$

The demand realized in the first period by the average retailer, as well as the associated total holdings at time t (before trading) are simply the average of all market participants:

$$d^1_{\text{AVG}} = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} d^1_i}{N}, \tag{21}$$

$$T'_{\text{AVG}} = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} T'_i}{N}. \tag{22}$$

Note that the demands realized by every retailer i in the first period, $\{d^1_i : i = 1, 2, \ldots, N\}$, are samples from the same distribution, D^1 (note that the subscript i is omitted because this distribution is assumed to be the same across retailers). Direct application of the strong law of large numbers yields:

$$d^1_{\text{AVG}} = E[D^1], \tag{23}$$

$$T'_{\text{AVG}} = E_{D^1}\left[T^* - D^1\right]^+ = \int_{-\infty}^{T^*} (T^* - x)d[F^1(x)] + \int_{T^*}^{\infty} 0d[F^1(x)] = \int_{-\infty}^{T^*} F^1(x)dx. \tag{24}$$

Since each retailer has the same information on random variable D^1 and its cumulative distribution function $F^1(x)$, the average demand realized in the first period, as well as the average access to product at time t, are known. The average retailer can then update estimates of the demand faced in the second period to yield conditional distribution $F^{2|1}$, given that the demand realized in the first period is d^1_{AVG}. Note that all retailers update the second period demand the same way, so the function $F^{2|1}$ is also known.

Whenever a retailer sells an option in the market, there must be a corresponding retailer to buy it. The total number of options taken over all retailers thus will not change with options trading, which implies $\sum_{i=1}^{N} T'_i = \sum_{i=1}^{N} \tilde{T}_i$. Therefore T'_{AVG} remains the same before and after options trading, and this must represent the optimal supply position for the average retailer at time t. Applying Proposition 4 to the average retailer, we obtain the implied market price:

$$C_{\text{AVG}} = (R - X)[1 - F^{2|1}(T'_{\text{AVG}})]. \tag{25}$$

Because the right hand sides of (18) and (25) have the same functional form, we can apply the strong law of large numbers to yield the following result:

Theorem 2. The average market price is $\bar{C} = C_{\text{AVG}}$.

Using C_{AVG} as the market average options price, the numerical experiments in the next section provide further insight into the value of options trading.

5. Numerical Experiments

This section reports the results of a series of numerical experiments. The experiments are designed to test the validity of the theoretical framework for options trading detailed in Section 4, and to explore how the benefits of supply chain options trading are affected by various environmental parameters. Specifically, we test the impact of market size, demand volatility, demand correlation, and trading time on the value of options trading, as well as on the option price.

Default values are set at $R = 100$, $W = 70$, $S = 5$, $M = 40$, $C = 15$, $X = 60$, $P = 80$, and $N = 40$. The time horizon is finite and normalized to one, and the default trading time is $t = 0.5$. The entire selling season is divided by t into two periods, $[0, t]$ and $[t, 1]$.

The demands faced by all retailers are independent and identically distributed with the following characteristics. At time zero, the forecasted demands for the first period $[0, t]$ and the second period $[t, 1]$ are correlated and normally distributed with parameters (μ_1, σ_1) and (μ_2, σ_2), respectively. Let ρ denote the correlation between the demands for the two periods. The demand for the entire selling season is also a normally-distributed random variable with mean $\mu = \mu_1 + \mu_2$ and standard deviation $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}$. The mean demand for any given time period is assumed to be proportional to the length of that time period, i.e., $\mu_1 = \alpha t$ and $\mu_2 = \alpha(1 - t)$, where α is the proportionality constant. The default values for the distribution parameters are set at $\mu_1 = \mu_2 = 2500$, $\sigma_1 = \sigma_2 = 500$, and $\rho = 0.5$. To prevent negative demand, we only consider values of σ less than $\mu/2$. Following the model used in Barnes-Schuster *et al.* (2002), given the realized demand d^1 in the first period $[0, t]$, at time t the conditional distribution of demand for the second period is normally distributed with mean and standard deviation given as follows:

$$\mu_{D_i^2|d_i^1} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(d_i^1 - \mu_1), \text{ and } \sigma_{D_i^2|d_i^1} = \sigma_2\sqrt{1-\rho^2}. \tag{26}$$

The MATLAB programming environment was used to develop the numerical simulation. All reported results are averaged over 10,000 realizations of the simulation.

Figure 1 in Section 3.3 showed how options can benefit both parties by providing a mechanism for sharing risk between supplier and retailers. Trading options among retailers at time t creates a new opportunity for risk sharing among retailers. Figure 2 plots the improvement in retailers' expected profits attributable to options, where profit gain for the retailers (%) is defined as in Section 3.3 for the case where no trading occurs. In Figure 2, the profit without trading is given by $E\Pi_{R_i}$ for the entire selling season, and the profit with trading is found by taking $E\Pi_{R_i}$ for the first period only and adding to it $\Pi_{R_i}^{\text{OT}}$ (for the second period). While $N = 1$ in Figure 1, the results reported in Figure 2 represent an average of $N = 40$ retailers.

Figure 2 clearly shows that the value to retailers of tradable options can be decomposed into two parts: (*i*) from sharing risk with the supplier as reflected in the positive profit improvement observed without trading, and (*ii*) from sharing risk with other retailers as measured by the difference between profit improvement realized with and without options trading. Figure 2 thus not only numerically supports the intuition developed in Theorem 1, but also highlights the additional benefits tradable options offer over embedded options (which are tied to the supply contract, and can thus not be traded).

The difference in Figure 2 between the profit gain realized with and without trading is a measure of the net value of options trading. We define $\text{VOT} = \frac{(\Pi_R^{\text{OT}} - E\Pi_R)}{\Pi_R}$, where Π_R and Π_R^{OT} are computed using expressions (4) and (16), respectively (note that the subscript i is dropped in expressions (4) and (16) because all retailers are identical). The ratio VOT is then multiplied by

100 and reported as a percentage. Since Π_R^{OT} is defined for the second period only, let Π_R in the calculation of VOT be the corresponding profit realized in the second period. The average VOT taken over N retailers is reported in the upcoming sections. We also compute the market average price (MAP) to gain insight into how option prices are affected by environmental parameters.

5.1. The Impact of Market Size

The number of participants in an options market affects both VOT and MAP. In Section 4, we assumed that the number of participants is sufficiently large to ensure that every options seller can be matched with a buyer. According to expression (19), the average price is a function of the number of retailers in the market, which we denote here by MAP(N). Theorem 2 shows that the average price converges to C_{AVG} as N goes to infinity.

Figure 3, which plots MAP versus the number of retailers N, shows clearly how MAP(N) converges to C_{AVG} as defined by expression (25). Henceforth, we use C_{AVG} to approximate MAP(N) in the remaining numerical experiments.

Figure 4 plots VOT as a function of the number of retailers N. We see that VOT varies only slightly from 12.2% to 12.4% as N varies from 2 to 100. Note that the assumption that every retailer can always buy (or sell) options at price C_{AVG} may not hold when N is small. However, our computational experience indicates that the benefits shown in Figure 4 apply as a close approximation for $N \geq 20$.

5.2. The Impact of Demand Volitility

Demand uncertainty typically has an adverse effect on retailer operations. In this section, we use the standard deviations σ_1 and σ_2 to represent demand volatility in the first and second periods, repectively, and explore the effectiveness of options trading in reducing the impact of demand volatility on retailers' profit.

Let $\sigma_1 = \sigma_2 = \sigma$. Figure 5 shows how VOT changes as a function of σ. We observe that VOT is uniformly positive and increases with respect to the demand volatility as measured by the standard deviation of the demand.

As discussed previously, one benefit attributable to options trading is the ability to share the risks associated with demand uncertainty with other retailers. The next set of experiments attempt to determine how much this risk is reduced through options trading. Figure 6 plots the cumulative distribution functions, with and without options trading, constructed from 100,000 realizations of profits with $\sigma_1 = \sigma_2 = 500$. This provides information on the likelihood that retailers will miss a given target profit level if they engage ($\Pr(\Pi_R \leq x)$), for a given profit target x) or do not engage ($\Pr(\Pi_R^{\mathrm{OT}} \leq x)$, for a given profit level x) in options trading. From Figure 6, we see that $\Pr(\Pi_R \leq x) \geq \Pr(\Pi_R^{\mathrm{OT}} \leq x)$, for all x, which implies that the likelihood of missing a target profit level is always lower with options trading than without. In addition to VOT, this reduction in risk illustrates another advantage of options trading over embedded options that are tied to a particular supply contract.

5.3. The Impact of Demand Correlation

In this section the impact of demand correlation on VOT and MAP is examined. The correlation parameter ρ controls the impact of demand in the first period on the mean and standard deviation of demand in the second period, as given by expression (26). When $0 < \rho < 1$, the mean demand in the second period increases beyond μ_2 whenever the actual demand in the first period demand is larger than the associated mean, *i.e.*, $d_i^1 > \mu_1$. Analogously, when $-1 < \rho < 0$, if the actual demand in the first period d_i^1 is larger than the average demand μ_1, then the demand in the second period on average will be smaller than the associated mean μ_2.

For the special cases where $|\rho| = 1$, demand in the second period is deterministic (because $\sigma_{D_i^2|d_i^1} = 0$), and takes on the value $\mu_{D_i^2|d_i^1} = \mu_2 + \frac{\sigma_2}{\sigma_1}(d_i^1 - \mu_1)$ when $\rho = 1$, and $\mu_{D_i^2|d_i^1} = \mu_2 + \frac{\sigma_2}{\sigma_1}(\mu_1 - d_i^1)$

when $\rho = -1$. Since $\sigma_1 = \sigma_2$ and $\mu_1 = \mu_2$ by design, $\mu_{D_i^2|d_i^1} = d_i^1$ for $\rho = 1$ and $\mu_{D_i^2|d_i^1} = 2\mu_1 - d_i^1$ for $\rho = -1$. Note that the actual demand in the first period takes on values larger than $2\mu_1$ with probability $1 - F^1(2\mu_1) \approx 2.3\%$. The demand in the second period may be negative with a small but positive probability when $\rho = -1$. To avoid this situation, we restrict ρ to take on values between -0.75 and 0.75 in this set of experiments.

Figures 7 and 8 show how demand correlation affects MAP and VOT, respectively. In Figure 7, the shape of the relationship between MAP and ρ indicates that higher option prices are observed as $|\rho|$ increases. For example, as ρ increases from 0 to 1, the mean demand in the second period becomes more sensitive to the difference $\mu_1 - d_i^1$, creating additional incentive to buy or sell options. Moreover, as ρ approaches 1, the standard deviation $\sigma_{D_i^2|d_i^1}$ approaches 0, which allows retailers to more accurately estimate how many options should be bought or sold. Similar reasoning holds as ρ decreases from 0 to -1, explaining the U-shaped relationship between MAP and ρ in Figure 7. In contrast, in Figure 8 we see that VOT generally increases with increasing demand correlation. This is because with negative demand correlation, options needed in period 1 become unnecessary in period 2, and options not needed in period 1 tend to be required in period 2. This results in less options trading, and thus lower value associated with options trading. On the other hand, when demands are highly correlated, an incorrect options position in period 1 becomes magnified in period 2, motivating options trading and increasing its value.

5.4. The Impact of Trading Time

The model presented in this paper assumes that options trading occurs at a single point in time that is known to all retailers. In this section, we explore the relationship between the trading time t and both MAP and VOT. Trading close to the end of the selling season (*i.e., t* close to 1) provides retailers with good information about how much product is needed for the entire selling season, but does not allow much time to compensate (through options trading) from any deviations between expected and actual demand. Similarly, trading early in the selling season (*i.e., t* close to

0) provides retailers with a large amount of flexibility to adjust their supply position, but not a lot of information on which to base any modification.

Just as the mean demand in the first and second periods depends linearly on the length of those periods, we assume that the standard deviation of demand in periods 1 and 2 is also a function of the length of the period. Specifically, we assume that $\sigma_1 = \frac{\mu_1}{3} = \frac{\alpha t}{3}$ and $\sigma_2 = \frac{\mu_2}{3} = \frac{\alpha(1-t)}{3}$.

Figure 9 shows that MAP decreases monotonically as a function of t. This is because holding options becomes more risky as trading time increases, since the likelihood that an options holder can recover the premium paid for an option decreases as t approaches 1. Thus, as trading time increases retailers are less inclined to hold options, reducing average market price.

Figure 10 plots VOT as a function of trading time t. We see that profits attributable to options trading are maximized at a point in time interior to [0,1]. As discussed above, if the trading time is close to zero, the demand forecast closely resembles the original forecast at time $t = 0$, and thus the optimal number of options to hold is approximately equal to the number without trading. Similarly, if the trading time to too close to the end of the period, then the additional profit realized by trading options is limited. For this set of experiments the optimal trading time is approximately $t = 0.8$.

6. Conclusion and Suggestions for Further Research

Recent research has established that under certain conditions the introduction of options or option-like contract arrangements can increase the expected profits of both suppliers and retailers. This paper extended this notion by considering how options trading among retailers at one predetermined point in time can further improve supply chain performance. The main contribution of this paper is in demonstrating benefits of options trading in managing supply chain risk. We showed that options trading increases the expected profits of all participants. We also derived the average market price of options to guide market participants in trading options.

The existence of a supply chain options market would create several secondary benefits. For example, a market for supply chain options would generate invaluable information flows. Suppliers could monitor options trading data from retailers to hone estimates of demand. Trading volumes and market prices could also serve as economic indicators of the vitality of the industry as a whole.

Additional insights on the trading of supply chain options were developed through numerical experimentation. We observed that the value of trading options increases as demand becomes more volatile, and the consequent reduction in risk was quantified using simulation. The results also suggested that the value of trading increases as the correlation between the first and second period demand grows. Finally, we showed that there exists a unique time at which to allow options trading in order to maximize the value of trading.

We hope that this paper stimulates further research on supply chain options and options trading. Additional work can be directed toward developing techniques to determine price parameters (X, C, W, P) to facilitate risk reduction and sharing, understanding how market structure and trading infrastructure can motivate or hinder options trading, and the impact of supply chain options markets on operational decision making, *e.g.,* on inventory and capacity management policies for both suppliers and retailers.

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Appendix A. Expressions for Expected Profit

1. Retailer's Expected Profit for the Entire Selling Season

According to expression (4), the expected profit for retailer i with options (but without options trading) is given by:

$$E\Pi_{R_i}(Q_i, q_i) = E_{D_i}\left[R\min(D_i, T_i) + S(Q_i - D_i)^+ - WQ_i - Cq_i - X\min\left(q_i, (D_i - Q_i)^+\right)\right].$$

Substituting $q_i = T_i - Q_i$, the expected profit can be expressed as:

$$\begin{aligned} E\Pi_{R_i}(Q_i, q_i) = R&\left[\int_0^{T_i} D_i f(D_i)dD_i + T_i\int_{T_i}^{\infty} f(D_i)dD_i\right] + S\int_0^{Q_i}(Q_i - D_i)f(D_i)dD_i \\ &- WQ_i - Cq_i - X\left[\int_{Q_i}^{T_i}(D_i - Q_i)f(D_i)dD_i + q_i\int_{T_i}^{\infty} f(D_i)dD_i\right]. \end{aligned}$$

Since $\Pr(D_i \le Q_i) = F(Q_i)$, $\Pr(Q_i < D_i \le T_i) = F(T_i) - F(Q_i)$, $\Pr(T_i < D_i) = 1 - F(T_i)$, and $dF(D_i) = f(D_i)dD_i$, we can integrate the expected profit function by parts to yield the following simplified form:

$$E\Pi_{R_i}(Q_i, T_i) = (X + C - W)Q_i - (R - S)\int_0^{Q_i} F(D_i)dD_i + (R - X - C)T_i - (R - X)\int_{Q_i}^{T_i} F(D_i)dD_i. \qquad (A1)$$

2. Suppler's Expected Profit as a Function of Production Quantity

Given retailers' demands $\{D_1, D_2, \ldots, D_N\}$ and optimal order quantities $\{(Q_i^*, q_i^*) : i = 1, 2, \ldots, i\}$, the supplier must deliver $Z = \sum_{i=1}^{N} z_i = \sum_{i=1}^{N}\left\{Q_i^* + \min\left[q_i^*, (D_i - Q_i^*)^+\right]\right\}$ units of product. The supplier's expected profit is then given by:

$$E\Pi^S(Y) = E_{D_1,\ldots,D_N}\left\{\sum_{i=1}^{N}\left[WQ_i^* + Cq_i^* + X\min\left(q_i^*, (D_i - Q_i^*)^+\right)\right] + S[Y - Z]^+ - P[Z - Y]^+ - MY\right\}.$$

The first four terms in the expression above reflect the revenues realized by the supplier from (*i*) the sale of product, (*ii*) the sale of options, (*iii*) options that are exercised, and (*iv*) salvaging unsold product, respectively. The fifth term captures the total penalty incurred when options are exercised but can't be fulfilled, and the final term is the total manufacturing cost. Manipulation of this expression yields:

$$\begin{aligned} E\Pi^S(Y) &= \sum_{i=1}^{N}\left\{WQ_i^* + Cq_i^* + X\int_{Q_i^*}^{Q_i^*+q_i^*}(D_i - Q_i^*)f(D_i)dD_i + Xq_i^*[1 - F(Q_i^* + q_i^*)]\right\} \\ &\quad + S\int_{-\infty}^{Y}(Y - z)f_Z(z)dz - P\int_{Y}^{\infty}(z - Y)f_Z(z)dz - MY \\ &= \text{Con1} + S\int_{-\infty}^{Y}(Y - z)f_Z(z)dz - P\int_{Y}^{\infty}(z - Y)f_Z(z)dz - MY \\ &= \text{Con2} + (P - M)Y + S\int_{-\infty}^{Y} F_Z(z)dz + P\int_{Y}^{\infty} F_Z(z)dz \qquad (A2) \end{aligned}$$

where Con1 and Con2 are constants with respect to Y.

3. Expected Profit for Retailer i After Options Trading

The expected profit for retailer i in the second period after options trading can be expressed as:

$$\Pi_{R_i}^{\text{OT}}(\tilde{q}_i) = E_{D_i^C}\left[R\min(D_i^C, \tilde{T}_i) + S(\tilde{Q}_i - D_i^C)^+ - W\tilde{Q}_i - Cu_i - C_t(\tilde{q}_i - u_i) - X\min\left(\tilde{q}_i, (D_i^C - \tilde{Q}_i)^+\right)\right]. \quad (A3)$$

The first term represents the total revenue realized from product sales in the second period. The second term captures the salvage value of any leftover product. The third term is the cost of purchasing product directly from the supplier. The fourth term is the cost of purchasing options from the supplier at time zero, while the fifth term is the cost (or revenue) of acquiring (selling) options from (to) other retailers at time t. The last term is the cost of exercising options to satisfy demand in the second period as required. Expression (A3) can be partitioned into four terms.

$$\begin{aligned}
I &= E\left[R\min(D_i^C, \tilde{Q}_i)\right] = R\left[\int_{-\infty}^{\tilde{T}_i} x dF_i^{2|1}(x) + \tilde{T}_i \Pr(D_i^C > \tilde{T}_i)\right] \\
&= R\tilde{T}_i F^{2|1}(\tilde{T}_i) - R\int_{-\infty}^{\tilde{T}_i} F_i^{2|1}(x)dx + R\tilde{T} - I[1 - F_i^{2|1}(\tilde{T}_i)] = R\tilde{T}_i - R\int_{-\infty}^{\tilde{T}_i} F_i^{2|1}(x)dx \\
II &= E\left[S(\tilde{Q}_i - D_i^C)^+\right] = S\left[\int_{-\infty}^{\tilde{Q}_i} (\tilde{T}_i - x)dF_i^{2|1}(x) + \int_{\tilde{Q}_i}^{\infty} 0dF_i^{2|1}(x)\right] \\
&= S\tilde{T}_i F_i^{2|1}(\tilde{T}_i) - S\left[\tilde{T}_i F_i^{2|1}(\tilde{T}_i) - \int_{-\infty}^{\tilde{Q}_i} F_i^{2|1}(x)dx\right] + 0 = S\int_{-\infty}^{\tilde{Q}_i} F_i^{2|1}(x)dx \\
III &= E\left[-W\tilde{Q}_i - Cu_i - C_t(\tilde{q}_i - u_i)\right] = (C_t - C)u_i - C_t\tilde{q}_i - W\tilde{T}_i \\
IV &= E\left[-X\min\left(\tilde{q}_i, (D_i^C - \tilde{Q}_i)^+\right)\right] = -X\int_{-\infty}^{\tilde{Q}_i} 0dF_i^{2|1}(x) + \int_{\tilde{Q}_i}^{\tilde{T}_i} (x - \tilde{Q}_i)dF_i^{2|1}(x) + \int_{\tilde{T}_i}^{\infty} \tilde{q}_i dF_i^{2|1}(x) \\
&= -X\left\{0 + \tilde{T}_i F_i^{2|1}(\tilde{T}_i) - \tilde{Q}_i F_i^{2|1}(\tilde{Q}_i) - \int_{\tilde{Q}_i}^{\tilde{T}_i} F_i^{2|1}(x)dx - \tilde{Q}_i\left[F_i^{2|1}(\tilde{T}_i) - F_i^{2|1}(\tilde{Q}_i)\right] + \tilde{q}_i\left[1 - F_i^{2|1}(\tilde{T}_i)\right]\right\} \\
&= -X\tilde{q}_i + X\int_{\tilde{Q}_i}^{\tilde{T}_i} F_i^{2|1}(x)dx.
\end{aligned}$$

Therefore, we obtain the following:

$$\begin{aligned}
\Pi_{R_i}^{\text{OT}}(\tilde{q}_i) &= I + II + III + IV \\
&= R\tilde{T}_i - R\int_{-\infty}^{\tilde{T}_i} F_i^{2|1}(x)dx + S\int_{-\infty}^{\tilde{Q}_i} F_i^{2|1}(x)dx + (C_t - C)u_i - C_t\tilde{q}_i - W\tilde{T}_i + X\int_{\tilde{Q}_i}^{\tilde{T}_i} F_i^{2|1}(x)dx \\
&= (R - W)\tilde{Q}_i + (C_t - C)u_i + (R - X - C_t)\tilde{q}_i + (X - R)\int_{-infty}^{\tilde{Q}_i + \tilde{q}_i} F_i^{2|1}(x)dx \\
&\quad + (S - X)\int_{-\infty}^{\tilde{Q}_i} F_i^{2|1}(x)dx.
\end{aligned} \quad (A4)$$

Appendix B. Proofs

Proof of Proposition 1.

Using Leibniz's rule for differentiating integrals, we obtain the partial derivatives of the expected profit function $\Pi_{R_i}(Q_i, T_i)$ in expression (A1) with respect to Q_i and T_i:

$$\frac{\delta E\Pi_{R_i}(Q_i,T_i)}{\delta Q_i} = X + C - W - (X - S)F(Q_i)$$
$$\frac{\delta E\Pi_{R_i}(Q_i,T_i)}{\delta T_i} = R - X - C - (R - X)F(T_i)$$

Differentiating the right-hand sides of these expressions again, since both (S-X) and (X-R) are negative, it follows that the Hessian matrix for $E\Pi_{R_i}(Q_i, T_i)$ is always negative. The expected profit function is thus concave with a unique maximum. Setting the partial derivatives above to zero, we have the sufficient and necessary conditions for the optimal quantities Q_i^* and T_i^*:

$$F(T_i^*) = \Pr(D_i \le T_i^*) = \frac{(R - X - C)}{(R - X)}$$
$$F(Q_i^*) = \Pr(D_i \le Q_i^*) = \frac{(X + C - W)}{(X - S)}$$

Substituting Q_i^* and T_i^* back into expression (A1) yields:

$$E\Pi_{R_i}(Q_i^*, T_i^*) = (X + C - W)Q_i^* - (R - S)\int_0^{Q_i^*} F(D_i)dD_i + (R - X - C)T_i^* - (R - X)\int_{Q_i^*}^{T_i^*} F(D_i)dD_i.$$

Proof of Proposition 2.

Given a set of prices (W, C, X), differentiating the supplier's expected profit function yields:

$$\frac{d\Pi^S(Y)}{dY} = (P - M) + (S - P)F_Z(Y).$$

Since $(S - P)$ is negative, the expected profit function is strictly concave, and thus has a unique maximum. Setting the right hand side of the expression above equal to 0, we obtain the necessary and sufficient conditions for optimal production quantity Y^{**}:

$$F_Z(Y^{**}) = \Pr(Z \le Y^{**}) = \frac{(P - M)}{(P - S)}. \tag{B1}$$

In expression (B1) Y^{**} was derived without considering the constraint $\sum_{i=1}^{N} Q_i^* \le Y \le \sum_{i=1}^{N}(Q_i^* + q_i^*)$. Because the expected profit function is strictly concave, combining expression (B1) and $\sum_{i=1}^{N} Q_i^* \le Y \le \sum_{i=1}^{N}(Q_i^* + q_i^*)$ yields the following optimal production volume Y^*:

$$Y^* = \begin{cases} \sum_{i=1}^{N} Q_i^*, & \text{if } Y^{**} \le \sum_{i=1}^{N} Q_i^* \\ Y^{**}, & \text{if } \sum_{i=1}^{N} Q_i^* \le Y^{**} \le \sum_{i=1}^{N} (Q_i^* + q_i^*) \\ \sum_{i=1}^{N} (Q_i^* + q_i^*), & \text{if } \sum_{i=1}^{N} (Q_i^* + q_i^*) \le Y^{**}, \end{cases} \tag{B2}$$

which is precisely the value of Y^* given in Proposition 2.

Expression (B2) can be validated by marginal analysis. If $Y < \sum_{i=1}^{N} Q_i^*$, then the profit associated with one more unit of product is $(W - M) > 0$. The expected profit is thus larger when $Y = \sum_{i=1}^{N} Q_i^*$. If $Y > \sum_{i=1}^{N}(Q_i^* + q_i^*)$, then an additional unit of product contributes a profit of $S - M < 0$; thus, a production quantity greater than $\sum_{i=1}^{N}(Q_i^* + q_i^*)$ cannot be optimal.

Proof of Proposition 3

Note that when $N = 1$, $F(Y_i^{**}) = \Pr(D_i \le Y_i^{**})$ by Proposition 2. By change of variable $\Gamma = \mu_i + \sigma_i \xi$, we can show that $F(\Gamma) = \Phi\left[\frac{(\Gamma-\mu_i)}{\sigma_i}\right]$, which yields the first result. By applying result 1, we can derive result 2 since $\frac{q_i^*}{T_i^*} = \frac{(T_i^* - Q_i^*)}{T_i^*}$. Applying result 1 also yields result 3 because $\theta = \frac{(Y_i^* - Q_i^*)}{(T_i^* - Q_i^*)}$, which is 0 when $Y_i^* = Q_i^*$, 1 when $Y_i^* = T_i^*$, and $\frac{\left[\Phi^{-1}(C_{Y_i^{**}}) - \Phi^{-1}(C_{Q_i^*})\right]}{\left[\Phi^{-1}(C_{T_i^*}) - \Phi^{-1}(C_{Q_i^*})\right]}$ when $Y_i^* = Y_i^{**}$. All are constants with respect to the mean and variance of demand.

Proof of Proposition 4

Differentiating $\Pi_{R_i}^{\text{OT}}(\tilde{q}_i)$ in expression (A3) with respect to $\tilde{q}_i$ yields:

$$\frac{d\Pi_{R_i}^{\text{OT}}(\tilde{q}_i)}{d\tilde{q}_i} = (R - X - C_t) + (X - R)F_i^{2|1}(\tilde{Q}_i + \tilde{q}_i). \tag{B3}$$

Because $X - R$ is negative, the expected profit function $\Pi_{R_i}^{\text{OT}}(\tilde{q}_i)$ is strictly concave with respect to $\tilde{q}_i$, and thus has a unique maximum. Setting the right hand side of (B3) to zero yields expression (17).

Proof of Theorem 1

We assume that the expected profit function in expression (A3) is differentiable with respect to $\tilde{q}_i$, and that its derivative is uniformly continuous. The Taylor expansion of $\Pi_{R_i}^{\text{OT}}(\tilde{q}_i)$ at $\tilde{q}_i = \mu_i$ yields:

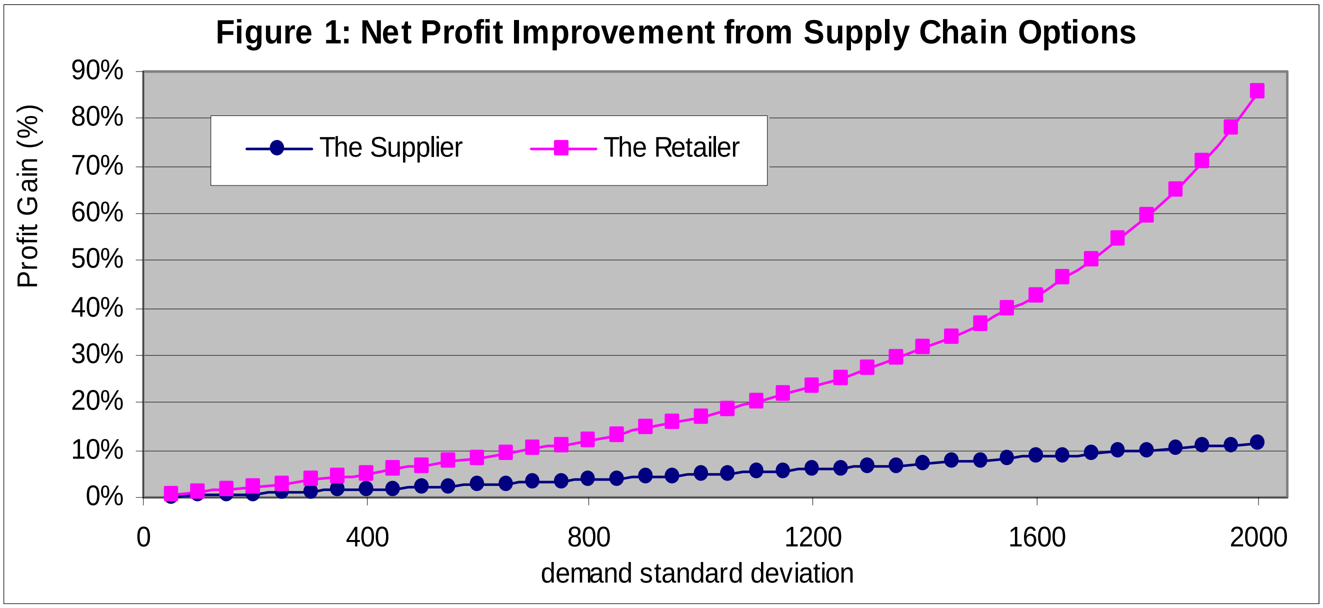
$$\Pi_{R_i}^{\text{OT}}(\tilde{q}_i) = \Pi_{R_i}^{\text{OT}}(\mu_i) + \frac{d\Pi_{R_i}^{\text{OT}}(\tilde{q}_i = \mu_i)}{d\tilde{q}_i}(\tilde{q}_i - \mu_i) + O(\tilde{q}_i - \mu_i)^2.$$

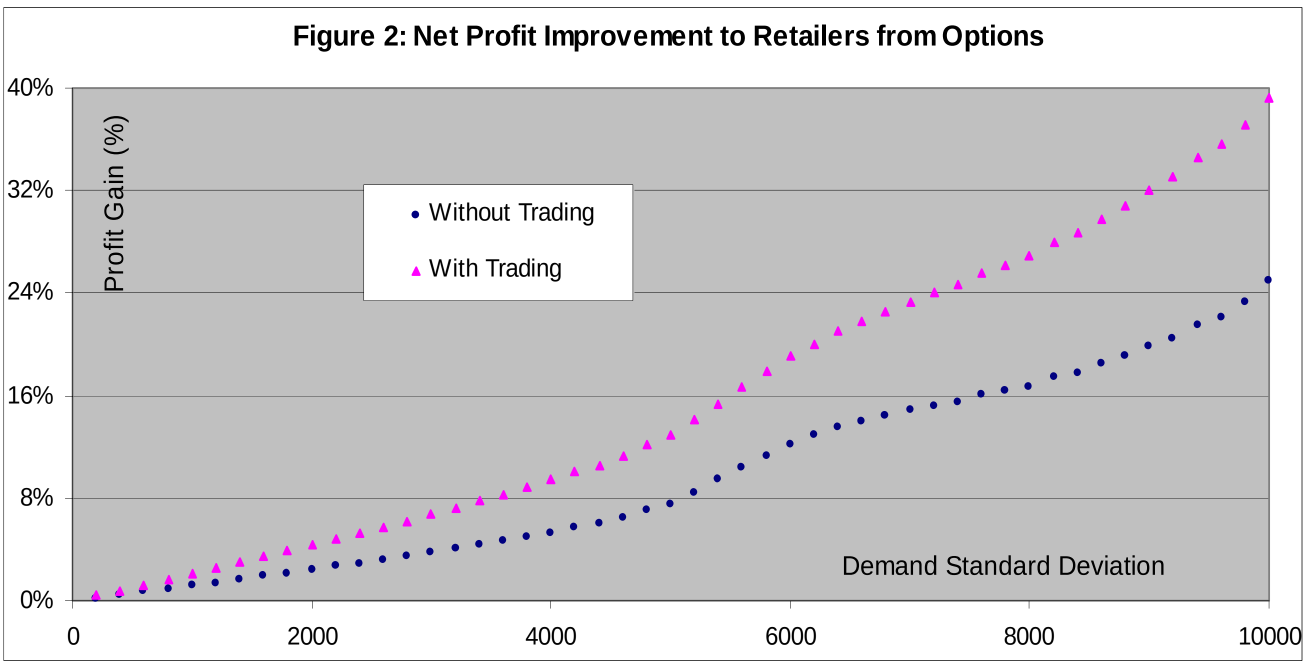
Therefore, the net effect of options trading to the retailer i is given by:

$$\Delta_i = \Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i) - \Pi_{R_i}^{\mathrm{OT}}(\mu_i) = \frac{d\Pi_{R_i}^{\mathrm{OT}}(\tilde{q}_i = \mu_i)}{d\tilde{q}_i}(\tilde{q}_i - \mu_i) + O(\tilde{q}_i - \mu_i)^2.$$

By the Lemma 1 and the definition of the sets B, S, and I, Δ_i is positive for $i \in B$ or S, but zero for $i \in I$ as long as $\tilde{q}_i \approx \mu_i$. We then use the uniformly continuous assumption to expand this local statement to a global statement to obtain Theorem 1.

Appendix C. Figures





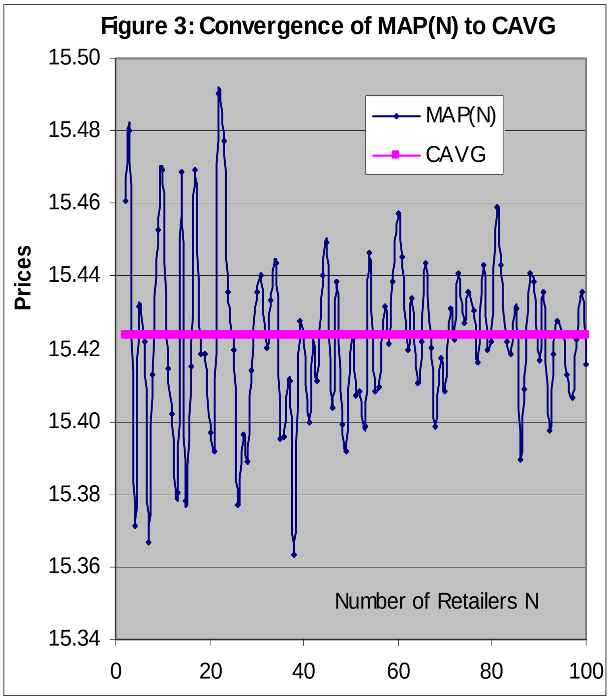


Figure 3: Convergence of MAP(N) to CAVG
Prices
15.50
15.48
15.46
15.44
15.42
15.40
15.38
15.36
15.34
0
20
40
60
80
100
MAP(N)
CAVG
Number of Retailers N

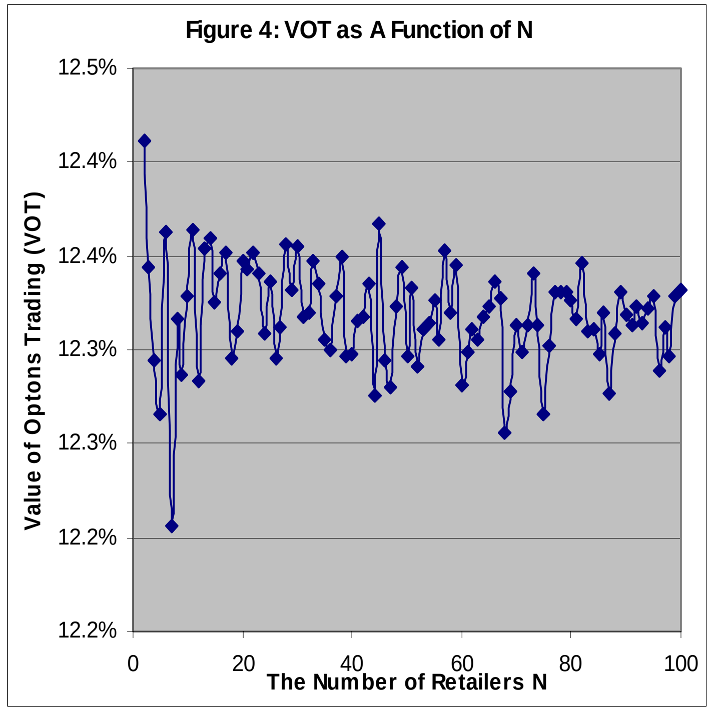


Figure 4: VOT as A Function of N
Value of Optons Trading (VOT)
12.5%
12.4%
12.4%
12.3%
12.3%
12.2%
12.2%
0
20
40
60
80
100
The Number of Retailers N

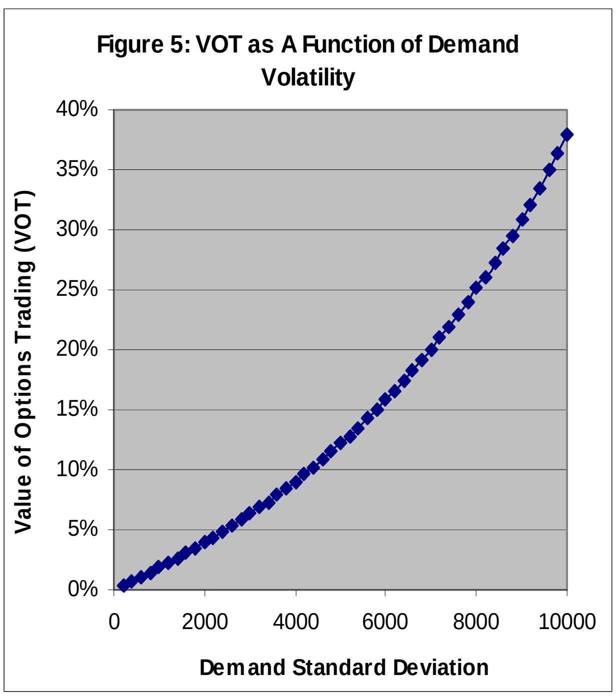


Figure 5: VOT as A Function of Demand Volatility
Value of Options Trading (VOT)
40%
35%
30%
25%
20%
15%
10%
5%
0%
0
2000
4000
6000
8000
10000
Demand Standard Deviation

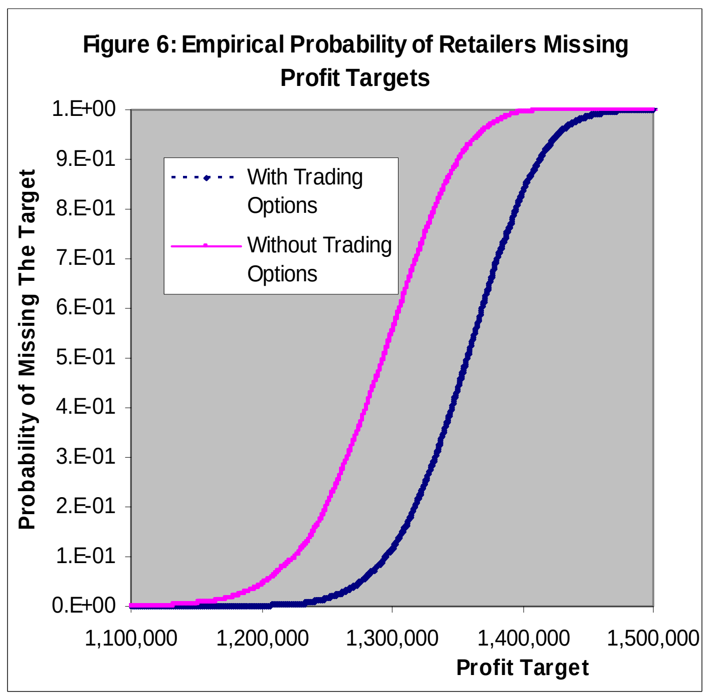


Figure 6: Empirical Probability of Retailers Missing Profit Targets
Probability of Missing The Target
1.E+00
9.E-01
8.E-01
7.E-01
6.E-01
5.E-01
4.E-01
3.E-01
2.E-01
1.E-01
0.E+00
With Trading Options
Without Trading Options
1,100,000
1,200,000
1,300,000
1,400,000
1,500,000
Profit Target

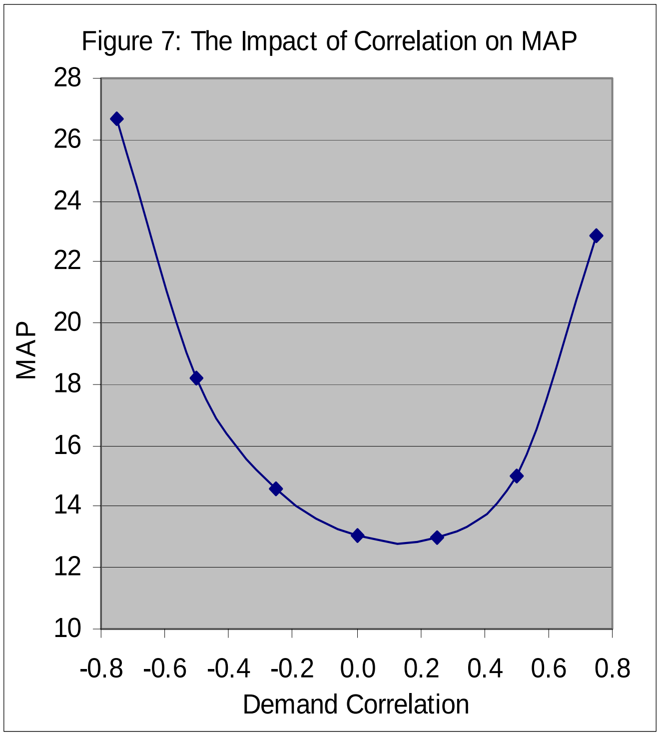


Figure 7: The Impact of Correlation on MAP
MAP
28
26
24
22
20
18
16
14
12
10
-0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8
Demand Correlation

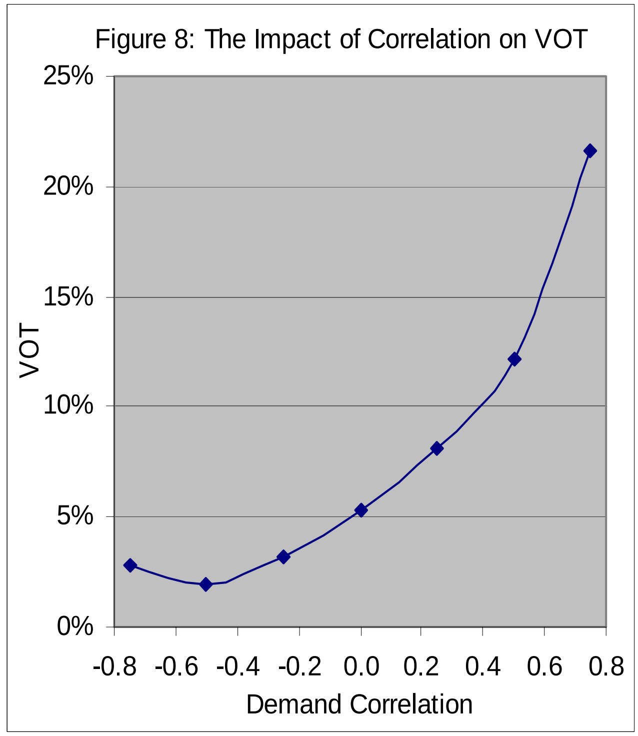


Figure 8: The Impact of Correlation on VOT
VOT
25%
20%
15%
10%
5%
0%
-0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8
Demand Correlation

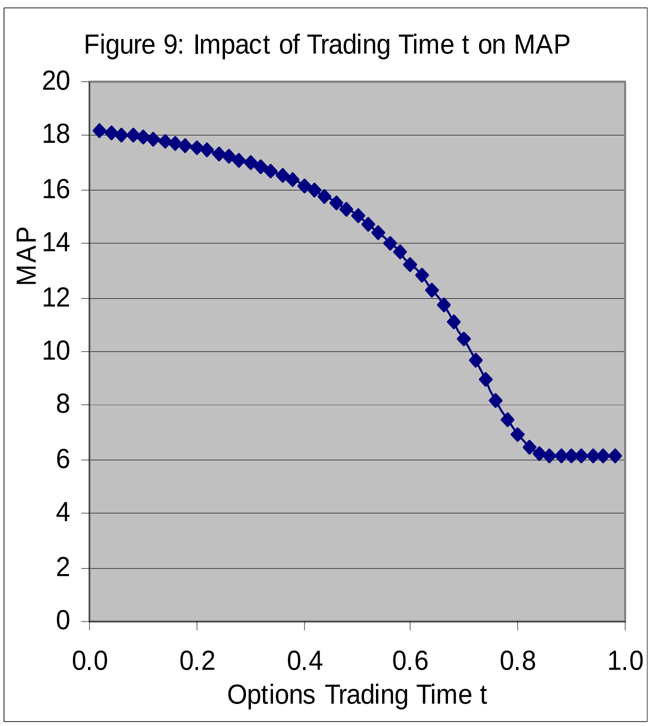


Figure 9: Impact of Trading Time t on MAP
MAP
20
18
16
14
12
10
8
6
4
2
0
0.0 0.2 0.4 0.6 0.8 1.0
Options Trading Time t

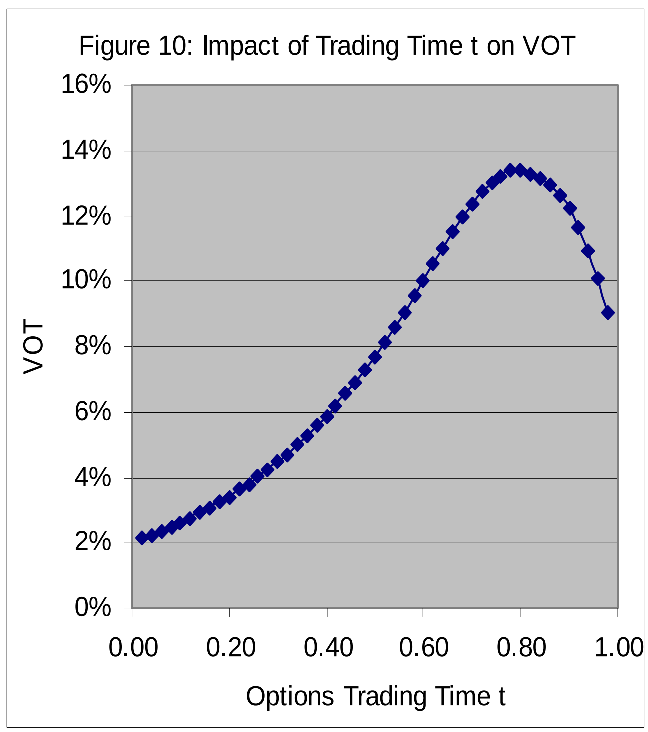


Figure 10: Impact of Trading Time t on VOT
VOT
16%
14%
12%
10%
8%
6%
4%
2%
0%
0.00 0.20 0.40 0.60 0.80 1.00
Options Trading Time t