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# Managing Supply Chain Risks with Derivatives 

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#### Abstract

This paper explores the impact of derivatives on supply chain risks when consumer demand is uncertain. The analysis is based on a simple two-stage supply chain consisting of a single supplier and a single retailer. The retailer can either buy product directly from the supplier, or purchase options on product. An option gives the retailer the right, but not the obligation, to buy an additional unit of product. As such, the retailer will exercise an option if an only if that unit of product is needed to meet demand. The retailer must balance the reduced uncertainty that options afford against the price premium that must be paid to procure product with options.

We first derive optimal replenishment policies for the retailer, optimal production policies for the supplier, and closed-form solutions for optimal expected profits. We then show how options enhance information flows, encourage risk sharing, and improve supply chain efficiency. The paper includes a discussion of how options can be used to align the incentives of supply chain partners, and to improve supply chain responsiveness to changes in the business environment. We also provide analytical and numerical results that provide insight into how supply chain participants can most effectively utilize options to enhance profitability.


Keywords: Risk Management, Supply Chain Management, Derivatives, Options, Supply Chain Coordination, Contract Management

## 1. Introduction

Supply chain risks can have a significant impact on a firm's operating and financial performance. Demand uncertainty can result in under- or over-production, leading to either lost sales or excess inventory. Shortages of critical inputs can lead to expedited purchases at higher prices, or even cause major production or supply chain disruptions. Insufficient capacity can lead to lost sales, while excess capacity can result in uncompetitive production costs.

Supply chain risks can be managed both operationally and financially. Operationally, a firm can reduce risk by changing its inventory policies, product strategy, procurement practices, and degree of vertical integration. Financially, a firm can carry insurance, modify supply contract terms, and hedge with derivatives such as futures contracts, options and swaps. While considerable research has focused on how to operationally manage supply chain uncertainties (see e.g., Johnson and Pyke 1999), there has been only limited study of the use of financial instruments to manage supply chain risk.

Derivatives are important tools for risk management (see, e.g., Hull 1997). A derivative is a contingent claim, whose value depends on another underlying variable. For example, an option gives its holder the right, but not the obligation, to buy or sell an underlying asset at or before a certain date for a certain price. A call option is an option to buy, and gives its holder an opportunity to participate in the upside gains associated with an asset, without any exposure to losses. On the other hand, a put option gives the holder the right to sell an asset at a specified price. A put option acts like an insurance policy, limiting the losses that the holder of an asset can incur if the asset declines in value.

In the financial services industry, the variables underlying derivatives are often the prices of traded securities. An equity option, for example, is a derivative whose value is contingent on the price of a company's stock. However, derivatives can be contingent on almost any variable, from the price of gold, to the amount of snow falling in a given year.

Derivative instruments have consistently proven their value as a vehicle for managing risk (see, e.g., Crouhy et al. 2001), and financial futures and options are actively traded on many exchanges. Derivatives are used routinely to manage financial risks, such as exposures to foreign exchange rate movements and changes in interest rates. Some industries have also used derivatives to manage the risk associated with changes in the prices of commodities, such as oil, grains, and metals, which are important factors in their production processes (see, e.g., Pilipovic 1998 for an application to the energy market).

Despite extensive use of derivatives for managing financial risk, there have been only limited applications of derivatives to manage supply chain risks. Supply contract terms and conditions sometimes have characteristics that make them behave much like financial derivatives. These are sometimes referred to as "embedded options", since they are an integral part of the supply contract. One example is quantity flexibility agreements, which enable trading partners to adjust their transactions to respond to changes in demand (see, e.g., Lariviere 1999 and Corbett and Tang 1999). However, these embedded options have a number of shortcomings that limit their effectiveness for risk management.

In this paper, we explore more generally how supply chain risks can be managed using derivatives. To frame this discussion, we consider a simple two-party supply chain, involving a single retailer and supplier. We introduce a simple call option contract whose value depends on the level of customer demand realized by a retailer in a single period. In this stylized setting, we show how derivatives can increase overall supply chain performance and efficiency, and enhance supply chain coordination. In contrast to embedded options, such a derivative offers greater flexibility in that it can be decoupled from the contracting process and implemented much in the same way that financial instruments or insurance products are.

The remainder of the paper is organized as follows. Section 2 describes the problem setting and derives optimal policies for both the retailer and the supplier. We also compare this option
model with others encountered in the supply chain management literature. Section 3 discusses the impact of options on the supply chain with respect to flexibility, channel coordination, and risk and information sharing. Section 4 analyzes the impact of option contract terms on profitability for both the retailer and the supplier. Section 5 numerically illustrates the potential improvements in supply chain profitability that can be realized with options. Section 6 concludes with a summary and suggestions for future research.

## 2. Problem Formulation and Solution

We consider a simple two-party supply chain comprised of a supplier producing short-lifecycle products, and a retailer who orders products from the supplier and then sells to end-users. Before the selling season, the retailer must decide how many units of a single product to purchase. We assume that the procurement lead-time is long relative to the selling season, so that the buyer cannot observe demand before placing the order. Because of the long lead-time, there is no opportunity to replenish inventory once the season has begun. Demand uncertainty exposes the parties to risks associated with mismatches between supply and demand. Specifically, if supply exceeds demand, the excess must be salvaged at a discount, and if demand exceeds supply, unmet demand is lost. We refer to these costs as overage and underage. Overage costs capture markdowns or inventory holding costs, while underage costs include lost profit margin and perhaps the costs of expediting, buying stock from a competitor, and/or customer ill will.

The retailer can obtain goods by two means: either through a firm order or by purchasing and exercising call options. Before the start of the season, the retailer places an order for $Q$ units of the product at unit wholesale price $W$. The retailer can also purchase $q$ options at unit cost $C$ at that time. Each call option gives the retailer the right (but not the obligation) to buy one unit of the product at exercise price $X$ after demand has been observed. Introducing options thus provides a mechanism for sharing risk between the two parties.

The retailer can use options to manage demand uncertainty. The risk of overage is addressed by purchasing fewer units of product directly, and the risk of underage is reduced by purchasing options that are exercised only if demand is high. However, the reduced risk comes at a cost, since the retailer pays for the options. The supplier keeps the option premium in compensation for sharing the retailer's risk. By specifying the order quantities $Q$ and $q$, the retailer decides how much risk to bear, and how much to pay to hedge the risk associated with demand uncertainty.

By sharing risk, the supplier induces the retailer to purchase more units, thus increasing sales. However, in doing so the supplier creates an obligation to fulfill demand for product when the retailer chooses to exercise an option. As a consequence, the supplier must hold inventories for options that may not be exercised, exposing the supplier to overage costs.

Assume that the product has unit retail price $R$ and unit manufacturing cost $M$. After the selling season, any excess product, regardless of whether it is owned by the retailer or the supplier, can be salvaged at unit value $S$. Before the selling season, demand, denoted by $D$, is uncertain. Most of the analysis applies to general continuous demand distributions with density function $f(D)$ and cumulative distribution function $F(D)$. However, for simplicity the numerical computations assume normally distributed demand.

Both the supplier and the retailer make decisions prior to the selling season. The retailer first places orders for $Q$ units of product and $q$ options, and the supplier subsequently decides the number of units of product $Y$ to produce. Clearly $Y$ must be at least as great as $Q$, since $Q$ represents a firm order. The retailer only exercises options when $D>Q$, and the likelihood that the retailer will not exercise all $q$ options is positive. Therefore, the number of units $Y$ produced by a rational supplier is between $Q$ and $(Q+q)$. However, when $Y<Q+q$, there is a positive probability that the supplier will default on its commitment to fill all options. In such a case, the supplier incurs a unit penalty cost $P$ for each exercised option that cannot be immediately fulfilled from inventory.

The penalty cost $P$ can have different interpretations. $P$ may represent the cost the supplier incurs to obtain an additional unit of product by expediting production or buying from an alternative source. It could also represent a pre-determined cash penalty specified in the option contract. However, these two mechanisms for settling option defaults result in a different set of incentives for the retailer, even for the same value of $P$. When the supplier finds an alternative means for delivering the product, the retailer only exercises options that are truly supported by actual demand. In contrast, when the supplier incurs a cash penalty for defaulting, the retailer will exercise all options as soon as it learns that the supplier cannot meet the options commitment, regardless of whether or not there is actual demand. For this reason, we assume that all options are settled by physical delivery of product rather than cash settlement.

There are several natural feasibility conditions in the supply chain:

$$
\begin{align*}
& M<W<C+X<R  \tag{1}\\
& P>M>S  \tag{2}\\
& X>S \tag{3}
\end{align*}
$$

Conditions $M<W<R$ and $C+X<R$ must hold to ensure profit for both parties. Moreover, if $W \geq C+X$, it would be advantageous for the retailer to only order options. Condition (2) states that penalty cost $P$ is always at least as great as the normal production cost $M$, which is always larger than salvage value $S$. Condition (3) is necessary to prevent the retailer from exercising all of its options, even when there is no actual demand, and salvaging the purchased products.

### 2.1. The Retailer's Decisions

The retailer has two decision variables: the number of units $Q$ to order and the number of call options $q$ to purchase. We introduce $T=Q+q$ to represent the retailer's total order quantity.

Note that determining $(Q, q)$ is equivalent to determining $(Q, T)$. The retailer will always first fulfill demand using firm orders $Q$. When $Q$ is insufficient to meet all demand, the retailer will exercise up to $q$ options. The retailer's expected profit with options is given as:

$$
\begin{equation*}
E \Pi^{R}(Q, q)=E_{D}\left[R \min (D, T)+S(Q-D)^{+}-W Q-C q-X \min \left(q,(D-Q)^{+}\right)\right] \tag{4}
\end{equation*}
$$

The first term is total revenue, which reflects the fact that the retailer's sales are limited by both total demand and total supply. The second term represents the salvage value of any leftover product. The last three terms capture the cost of ordering product directly, of purchasing options, and of exercising options as required, respectively. Substituting $q=T-Q$ in (4), the expected profit can be expressed as:

$$
\begin{aligned}
E \Pi^{R}(Q, q)= & R\left[\int_{0}^{T} D f(D) d D+T \int_{T}^{\infty} f(D) d D\right]+S \int_{0}^{Q}(Q-D) f(D) d D \\
& -W Q-C q-X\left[\int_{Q}^{T}(D-Q) f(D) d D+q \int_{T}^{\infty} f(D) d D\right]
\end{aligned}
$$

Since $\operatorname{Pr}(D \leq Q)=F(Q), \operatorname{Pr}(Q<D \leq T)=F(T)-F(Q), \operatorname{Pr}(T<D)=1-F(T)$, and $d F(D)=f(D) d D$, we can integrate the expected profit function by parts to yield the following simplified form:

$$
E \Pi^{R}(Q, T)=(X+C-W) Q-(X-S) \int_{0}^{Q} F(D) d D+(R-X-C) T-(R-X) \int_{0}^{T} F(D) d D
$$

Using Leibniz's rule for differentiating integrals, we obtain the partial derivatives of the expected profit function $E \Pi^{R}(Q, T)$ with respect to $Q$ and $T$ :

$$
\begin{aligned}
& \frac{\delta E \Pi^{R}(Q, T)}{\delta Q}=X+C-W-(X-S) F(Q) \\
& \frac{\delta E \Pi^{R}(Q, T)}{\delta T}=R-X-C-(R-X) F(T)
\end{aligned}
$$

Differentiating the right-hand sides of these expressions again, since both (S-X) and (X-R) are negative, it follows that the Hessian matrix for $E \Pi^{R}(Q, T)$ is negatively definite. The expected
profit function is thus concave with a unique maximum. Setting the partial derivatives above to zero, we have the sufficient and necessary conditions for the optimal quantities $Q^{*}$ and $T^{*}$ :

$$
\begin{align*}
& F\left(T^{*}\right)=\operatorname{Pr}\left(D \leq T^{*}\right)=\frac{(R-X-C)}{(R-X)}  \tag{5}\\
& F\left(Q^{*}\right)=\operatorname{Pr}\left(D \leq Q^{*}\right)=\frac{(X+C-W)}{(X-S)} \tag{6}
\end{align*}
$$

The optimal expected profit for the retailer is given by:

$$
\begin{equation*}
E \Pi^{R}\left(Q^{*}, T^{*}\right)=(R-S) \int_{0}^{Q^{*}} D f(D) d D+(R-X) \int_{Q^{*}}^{T^{*}} D f(D) d D \tag{7}
\end{equation*}
$$

Note that $Q^{*} \leq T^{*}$ implies that $C \leq \frac{(W-S)(R-X)}{(R-S)}$. This shows that if the option cost C is too high, the retailer will not order any options. Throughout this paper, we assume that the cost parameters always satisfy this constraint.

The classic newsvendor model is a special case of this formulation where the retailer cannot purchase options. The optimal order quantity $Q_{\mathrm{NV}}^{*}$ and optimal expected profit $E \Pi_{\mathrm{NV}}^{R}$ in the classic newsvendor formation are represented by:

$$
\begin{gather*}
F\left(Q_{\mathrm{NV}}^{*}\right)=\operatorname{Pr}\left(D \leq Q_{\mathrm{NV}}^{*}\right)=\frac{(R-W)}{(R-S)}  \tag{8}\\
E \Pi_{\mathrm{NV}}^{R}\left(Q_{\mathrm{NV}}^{*}\right)=(R-S) \int_{0}^{Q_{\mathrm{NV}}^{*}} D f(D) d D=(R-W) Q_{\mathrm{NV}}^{*}-(R-S) \int_{0}^{Q_{\mathrm{NV}}^{*}} F(D) d D \tag{9}
\end{gather*}
$$

Equation (8) can be rewritten as $F\left(Q_{\mathrm{NV}}^{*}\right)=\frac{C_{u}}{\left(C_{u}+C_{o}\right)}$, where $C_{u}=R-W$ is the unit underage cost of forgone profit, and $C_{o}=W-S$ is the unit overage cost of salvage loss. Equations (5) and (6) also take this form. In (5), $C_{u}=R-(X+C)$ is the forgone profit if a unit of demand is unfulfilled for lack of options to exercise, and $C_{o}=C$ is the cost of an unexercised option. Similarly in (6), $C_{u}=X+C-W$ and $C_{o}=W-(C+S)$. If actual demand is greater than $Q$, the retailer pays a premium $(X+C-W)$ to satisfy the demand with options instead of firm orders. On the
other hand, if actual demand is less than $Q$, then the retailer incurs the cost for purchasing the product (but avoids the options cost), and salvages the product instead.

### 2.2. The Supplier's Decision

Before the selling season, the supplier must decide how many units of product $Y$ to produce. The supplier bases this decision on the retailer's order pair $(Q, q)$. Assume the following sequence of events: transaction terms (i.e., $W, C$ and $X$ ) are first determined, and then the retailer places orders $(Q, q)$. The supplier produces $Y$ units of product, and immediately delivers $Q$ units to the retailer, holding the remaining $Y-Q$ units in inventory. After demand is observed, the retailer exercises an appropriate number of options, and additional units of product are delivered to the retailer. This sequence of events imposes a logical constraint that $Q^{*} \leq Y \leq T^{*}$.

Since the retailer will require $\min \left(D, T^{*}\right)$ units of product from the supplier, the supplier's expected profit can be expressed as:

$$
\begin{align*}
E \Pi^{S}(Y)= & E_{D}\left\{W Q^{*}+C q^{*}+X \min \left[q^{*},\left(D-Q^{*}\right)^{+}\right]+S\left[Y-\min \left(D, T^{*}\right)\right]^{+}\right. \\
& \left.-M Y-P\left[\min \left(D, T^{*}\right)-Y\right]^{+}\right\} . \tag{10}
\end{align*}
$$

The first four terms in (10) are the revenues realized by the supplier from $(i)$ the sale of product, (ii) the sale of options, (iii) options that are exercised, and (iv) salvaging unsold product, respectively. The fifth term captures manufacturing costs, while the final term is the total penalty incurred when options are exercised but can't be fulfilled. Manipulation of (10) yields:

$$
\begin{align*}
E \Pi^{S}(Y)= & W Q^{*}+C q^{*}+X\left[\int_{Q^{*}}^{T^{*}}\left(D-Q^{*}\right) f(D) d D+\int_{T^{*}}^{\infty} q^{*} f(D) d D\right]+S \int_{Q^{*}}^{Y}(Y-D) f(D) d D \\
& -M Y-P\left[\int_{Y}^{T^{*}}(D-Y) f(D) d D+\int_{T^{*}}^{\infty}\left(T^{*}-Y\right) f(D) d D\right] \tag{11}
\end{align*}
$$

Integrating expression (11) by parts, the following simplified form is obtained:

$$
\begin{equation*}
E \Pi^{S}(Y)=(W-P) Q^{*}+(X+C-P) q^{*}+(P-M) Y-(X-S) \int_{Q^{*}}^{Y} F(D) d D-(X-P) \int_{Y}^{T^{*}} F(D) d D . \tag{12}
\end{equation*}
$$

Leibniz's rule yields the derivative of the supplier's expected profit function:

$$
\begin{equation*}
\frac{d E \Pi^{S}(Y)}{d Y}=(P-M)+(S-P) F(Y) \tag{13}
\end{equation*}
$$

Since $(S-P)$ is negative, the expected profit function is strictly concave, and thus has a unique maximum. Setting the right hand side of (13) equal to 0 , we obtain the necessary and sufficient conditions for optimal production quantity $Y^{* *}$ :

$$
\begin{equation*}
F\left(Y^{* *}\right)=\operatorname{Pr}\left(D \leq Y^{* *}\right)=\frac{(P-M)}{(P-S)} \tag{14}
\end{equation*}
$$

and the corresponding supplier's profit:

$$
\begin{equation*}
E \Pi^{S}\left(Y^{* *}\right)=(W-X-C) Q^{*}+(X+C-P) q^{*}+(P-M) Y^{* *}-(X-S) \int_{Q^{*}}^{Y^{* *}} F(D) d D-(X-P) \int_{Y^{* *}}^{T^{*}} F(D) d D . \tag{15}
\end{equation*}
$$

Expression (14) also takes the form of the newsvendor model with $C_{u}=P-M$ and $C_{o}=$ $M-S$. Thus, the underage cost to the supplier is the premium $(P-M)$ paid to supply an additional unit from an alternative source, while the overage cost is the difference between what the supplier paid to produce the unit and the amount that can be realized in salvage. Note that since the revenue realized by the supplier is independent of the chosen stock level, the production quantity is independent of $W, X$, and $C$.

Notice that the term $Y^{* *}$ is used in expression (14). This is because $Y^{* *}$ is not necessarily the optimal production quantity, since (14) was derived without considering the constraint $Q^{*} \leq$
$Y \leq T^{*}$. Because the expected profit function in (12) is strictly concave, combining (14) and $Q^{*} \leq Y \leq T^{*}$ yields the following optimal production volume $Y^{*}$ :

$$
Y^{*}= \begin{cases}Q^{*}, & \text { if } Y^{* *} \leq Q^{*}  \tag{16}\\ Y^{* *}, & \text { if } Q^{*}<Y^{* *}<T^{*} \\ T^{*}, & \text { if } T^{*} \leq Y^{* *}\end{cases}
$$

Expression (16) can be validated by marginal analysis. If $Y<Q^{*}$, then the profit associated with one more unit of product is $(W-M)>0$. The expected profit is thus larger when $Y=Q^{*}$. If $Y>T^{*}$, then an additional unit of product contributes a profit of $S-M<0$; thus, a production quantity greater than $\mathrm{T}^{*}$ cannot be optimal.

In the classic newsvendor model, the supplier fills the retailer's order by building to order, i.e., the supplier always produces $Q_{\mathrm{NV}}^{*}$ units (note that the retailer then bears all risk associated with demand uncertainty). The supplier's profit is then:

$$
\begin{equation*}
E \Pi_{\mathrm{NV}}^{S}\left(Y^{*}\right)=(W-M) Q_{\mathrm{NV}}^{*} . \tag{17}
\end{equation*}
$$

### 2.3. Comparison with Other Options Models

Options and option-like contract arrangements have been widely used in practice (see e.g., Farlow et al. and Bassok et al. 1997). Buy back policies (Pasternack 1985, Emmons and Gilbert 1998), backup agreements (Eppen and Iyer 1997), pay-to-delay capacity reservation (Brown and Lee 1998), and quantity flexibility (Tsay 1999, Tsay and Lovejoy 1999) have received considerable attention in the literature.

A buy back contract specifies a price $B$ at which the retailer may return unsold goods to the supplier. This is a special case of the more general options framework proposed here, in which the retailer receives embedded put options with exercise price $X=B$, and where the option premium $C$
is included in the wholesale price $W$. A quantity flexibility contract allows the retailer to purchase any number of units of product within a range of $[(1-\alpha) y,(1+\beta) y)]$ after actual demand is observed, where $y$ is the initial order quantity and $\alpha$ and $\beta$ are contract parameters between 0 and 1. The same profits for both the retailer and the supplier of a quantity flexibility contract can be achieved by setting appropriate options parameters (Lariviere 1999). Alternatively, the general options framework in this paper can replicate the cash flows of a quantity flexibility contract by generating $\alpha y$ put options and $\beta y$ call options with exercise price $X=W$.

Articles by Cachon and Lariviere (2001) and Barnes-Schuster et al. (2002) are most closely related to this work. Cachon and Lariviere (2001) studied the impact of options on information flow using a similar setting without penalty cost $P$, but within a game-theory structure for enforcing the supplier's option obligation. Barnes-Schuster et al. (2002) proposed a two-period model to study options with a focus on channel coordination.

## 3. The Impact and Benefits of Options

Options provide the retailer with an alternative mechanism for obtaining product from the supplier. This section discusses the impact of options on the interactions between supplier and retailer, focusing on replenishment flexibility, coordination of the channel, and risk and information sharing.

### 3.1. Replenishment Flexibility

Options provide the retailer with the flexibility to replenish product during the selling season. In markets where product life cycles are short and demand volatilities are high, this flexibility allows the retailer to respond quickly to changes in the demand. Barnes-Schuster et al. (2002) described replenishment practices in three industries (toys, apparel and electronics), and concluded that flexibility benefits the retailer while perhaps costing the supplier. As shown in Section 5, the flexibility afforded by options can improve performance for both the supplier and the retailer.

The impact of flexibility on competitiveness is widely recogninized, and the value of flexibility for in-season replenishment can be substantial. In the apparel industry for example, because retailers must place firm, SKU-specific orders far in advance of the start of a selling season (see, e.g., Nuttle et al. 1991), the cost of markdowns are on the order of billions of dollars annually (see, e.g., Fisher et al. 1994). Hunter et al. (1996) estimated that the potential savings to retailers are so large that they should be willing to pay a $30-50 \%$ premium to any supplier who can provide in-season replenishment.

### 3.2. Channel Coordination

The supply chain considered in this paper consists of two parties with different objectives and asymmetric information concerning demand. Both parties seek to maximize their own profits, but neither exercises control over the entire supply chain or has an incentive to globally optimize performance. This dual decision-making with conflicting objectives often degrades the efficiency of the supply chain as a whole. Several solution have been proposed for this problem (see e.g., Anupindi et al. 1999, Lariviere 1999, and Taylor 2001), typically involving some modification of the payment structure between the two parties beyond a simple singleton price $W$. These modifications provide incentive for all partners to behave in a manner that optimizes the efficiency of the entire supply chain. As shown in this section, options can also be used to coordinate the supply chain.

When supplier and retailer are a single entity (e.g., for a vertically integrated firm), conflicting objectives are less of a confounding issue. However, double marginality leads to suboptimal solutions in a non-integrated supply chain (see, e.g., Spengler 1950). If the entire chain produces $Q$ units of product, total profit for the supply chain is $(R-M) Q$; however, this profit must be divided between the retailer and the supplier. In addition, the retailer's order quantity influences the supplier's production decision. The retailer chooses an order quantity $Q$ based on wholesale price $W$, which must be larger than manufacturing cost $M$ to guarantee both parties positive profit margins. Double marginality induces a quantity $Q_{\mathrm{NV}}^{*}=F^{-1}\left\{\frac{(R-W)}{(R-S)}\right\}$ as in expression (8). On the
other hand, in an integrated supply chain partners coordinate their activities to maximize the total profits of the chain. Since $M$ replaces $W$ in the integrated supply chain, $Q_{I}^{*}=F^{-1}\left\{\frac{(R-M)}{(R-S)}\right\}$. Since $M<W, Q_{\mathrm{NV}}^{*}<Q_{I}^{*}$, and the total profit for the entire supply chain is greater in the integrated case.

In addition to the wholesale price $W$, options contracts provide three more degrees of freedom $(X, C$ and $P)$ when negotiating contract terms. This additional flexibility enables options to be used to coordinate partners' behaviors, inducing channel coordination and ensuring that a decentralized supply chain performs as well as an integrated supply chain. To induce the retailer to order total quantity $T$ up to $Q_{I}^{*}$, expression (5) indicates that $X$ and $C$ must be set such that:

$$
\begin{equation*}
\frac{(R-X-C)}{(R-X)}=\frac{(R-M)}{(R-S)} \tag{18}
\end{equation*}
$$

Condition (18) ensures that the retailer's total order $T^{*}$ will be as large as the order quantity in an integrated supply chain. Moreover, we must also set unit penalty $P$ so that $Y^{*}=T^{*}$. This, along with expressions (5), (14) and (16), yields a condition under which the supplier is motivated to coordinate:

$$
\begin{equation*}
\frac{(P-M)}{(P-S)} \geq \frac{(R-X-C)}{(R-X)} \tag{19}
\end{equation*}
$$

Expressions (18) and (19) together provide sufficient conditions for channel coordination. Note that (18) and (19) imply that $P \geq R$, making optimal channel coordination feasible only when $P \geq R$.

There are two fundamental issues in supply chain optimization using options: ( $i$ ) maximizing the combined profits of the retailer and the supplier, and (ii) allocating profits equitably between the two parties. Expressions (18) and (19) ensure that the combined profits are the same as that of the integrated supply chain, but offer no insight into how the profits should be distributed between the parties. Note that since $W$ is not a term in expressions (18) or (19), nor in the total order $T^{*}$, the supplier quantity $Y^{* *}$ and total supply chain profits are not affected by $W$. This
allows the wholesale price $W$ to be used to control the distribution of profit between supplier and retailer. Thus, option contracts can be used not only to maximize total supply chain profitability (by specifying $X, C$ and $P$ ), but also to ensure an equitable distribution of the profits between supply chain partners by specifying an appropriate value of $W$. The role $W$ plays in controlling the distribution of profits is confirmed in the numerical experiments discussed in Section 5.

Total profits can be distributed in several ways. Since options increase the total profit available for distribution, at a minimum $X, C, P$ and $W$ should be set so that both parties have profits no worse than when no options are used. Within this broad guideline, the distribution of incremental profits between the retailer and supplier could vary widely, perhaps depending on their relative market power, or their skill at negotiating contract terms. A second possibility is to distribute profits based on risk-adjusted return, with each party's incremental profit proportional to the amount of risk borne.

### 3.3. Risk Sharing

Demand uncertainty exposes both the supplier and retailer to risks associated with mismatches between supply and demand. Options provide a mechanism for sharing this risk between supplier and retailer. The retailer can use options to hedge against both underage and overage risks by using firm orders to cover demand that is relatively likely to occur, and options for demand levels that are less likely to occur. However, the retailer pays a premium to reduce risk, since the cost of procuring units with options, $X+C$, is higher than the cost of buying product directly, $W$. The supplier earns this premium in compensation for sharing the retailer's risk. By specifying the order quantities $Q$ and $q$, the retailer determines how much risk to bear, and how much to pay for the benefit of reducing risk. By sharing risk, the supplier induces the retailer to order a higher total quanitity $Q+q$, thus increasing sales. However, in so doing the supplier creates an obligation to fulfill demand for units of product needed when the retailer exercises options. As a
consequence, the supplier must hold inventory for options that may not be exercised, exposing the supplier to overage costs.

As discussed in Section 3.2, two issues need to be considered for risk sharing: (i) setting transaction terms to improve the combined profits of the retailer and the supplier, and (ii) allocating total profit equitably between the two parties. Section 3.2 details the conditions under which total profits are maximized as in the integrated supply chain. Since there are larger profits to distribute, options can benefit both suppliers and retailers, as demonstrated in Figure 2.

Figure 2 plots the contribution of options to the profit of both the supplier and the retailer, where the Profit Gain (\%) is defined as $\frac{\left(E \Pi^{S}-E \Pi_{N V}^{S}\right)}{E \Pi_{\mathrm{NV}}^{S}}$ for the supplier and $\frac{\left(E \Pi^{R}-E \Pi_{\mathrm{NV}}^{R}\right)}{E \Pi_{\mathrm{NV}}^{R}}$ for the retailer. The design of the the numerical experiments that result in Figure 2 is detailed in Section 5; for now, the chart highlights how risk sharing can be advantageous for both retailers and suppliers.

### 3.4. Information Sharing

Being closer to end consumers, retailers often have better information about the distribution of demand faced by the supply chain. Focusing on the mean $\mu$ and variance $\sigma^{2}$ of demand, we now consider how options help transfer demand information from retailers to suppliers. Let $\phi(\xi)$ and $\Phi(\xi)$ denote the density and cumulative distribution functions of the normalized distribution with $\mu=0$ and $\sigma^{2}=1$. We assume that all contractual parameters $(R, W, X, C, P, S$, and $M)$ are known. This implies that the right hand sides of the expressions for $T^{*}, Q^{*}$ and $Y^{* *}$ are all constants - these we will denote by $C_{T^{*}}, C_{Q^{*}}$ and $C_{Y^{* *}}$, respectively.

Proposition 1. The retailer's orders and the supplier's production quantity satisfy the following:

1. $Z=\mu+\sigma \Phi^{-1}\left(C_{Z}\right)$, for $Z=Q^{*}, T^{*}$, or $Y^{* *}$.
2. $\frac{q^{*}}{T^{*}}=\frac{\left[\Phi^{-1}\left(C_{T^{*}}\right)-\Phi^{-1}\left(C_{Q^{*}}\right)\right]}{\left[\mu / \sigma+\Phi^{-1}\left(C_{T^{*}}\right)\right]}$.
3. The implied $\theta$ in $Y^{*}=Q^{*}+\theta q^{*}$ is constant with respect to $\mu$ and $\sigma^{2}$.

Proof. By change of variable $Z=\mu+\sigma \xi$, we can show that $F(Z)=\Phi\left[\frac{(Z-\mu)}{\sigma}\right]$, which yields the first result. By applying result 1 , we can derive result 2 since $\frac{q^{*}}{T^{*}}=\frac{\left(T^{*}-Q^{*}\right)}{T^{*}}$. Applying result 1 also yields result 3 because $\theta=\frac{\left(Y^{*}-Q^{*}\right)}{\left(T^{*}-Q^{*}\right)}$, which is 0 when $Y^{*}=Q^{*}, 1$ when $Y^{*}=T^{*}$, and $\frac{\left[\Phi^{-1}\left(C_{Y * *}\right)-\Phi^{-1}\left(C_{Q^{*}}\right)\right]}{\left[\Phi^{-1}\left(C_{T^{*}}\right)-\Phi^{-1}\left(C_{Q^{*}}\right)\right]}$ when $Y^{*}=Y^{* *}$. All are constants with respect to the mean and variance of demand.

The first result shows that both the retailer's orders and the supplier's production quantity increase linearly with respect to demand mean $\mu$ and standard deviation $\sigma$. The second result demonstrates that the percentage $\frac{q^{*}}{T^{*}}$ is inversely dependent on $\frac{\mu}{\sigma}$. The final result identifies an invariant in the supplier's decisions, which can simplify the decision-making process. Upon receiving the retailer's order quantities $(Q, q)$, the supplier responds first by checking the penalty cost $P$. If $P$ is relatively high (low), then the supplier produces $T^{*}\left(Q^{*}\right)$ units; otherwise, the supplier always produces $Q^{*}+\theta q^{*}$ for a fixed percentage $\theta$ regardless of demand mean $\mu$ and variance $\sigma^{2}$.

Like product flows and financial flows, information flows are important for supply chain management. Designing incentives to foster information sharing between suppliers and retailers remains a distinct challenge (see, e.g., Lee et al. 1997, and Cachon and Lariviere 2001). The third result shows that options can enhance information flows from retailers to suppliers. The order quantities $(Q, q)$ completely disclose a retailer's mean and variance of demand to the supplier, and the supplier does not need to know $\mu$ and $\sigma$ explicitly to determine an appropriate production quantity, so long as ordering quantities $Q$ and $q$ are undistorted.

## 4. The Impact of Contract Terms on Profitability

Options contracts include several parameters that are subject to negotiation by suppliers and retailers. Depending on which party has leverage to specify contract terms, very different outcomes are possible. This section examines the impact of contract terms on the profitability of the retailer, the supplier, and the entire supply chain. We assume that $R, M$ and $S$ are external variables that are controlled by neither supplier nor retailer, and thus are non-negotiable.

### 4.1. Impact of Contract Terms on Retailer Profitability

If the retailer has dominant negotiating power, it will propose values of $W, X$ and $C$ that maximize its expected profit. Recall that expressions (5), (6) and (7) specify the retailer's decisions and profits; hence, we have the following:

$$
\begin{aligned}
\frac{\delta E \Pi^{R}}{\delta C}= & Q^{*}+(X+C-W) \frac{\delta Q^{*}}{\delta C}-T^{*}+(R-X-C) \frac{\delta T^{*}}{\delta C}-(R-S) F\left(Q^{*}\right) \frac{\delta Q^{*}}{\delta C} \\
& -(R-X)\left[F\left(T^{*}\right) \frac{\delta T^{*}}{\delta C}-F\left(Q^{*}\right) \frac{\delta Q^{*}}{\delta C}\right]=Q^{*}-T^{*} \\
\frac{\delta E \Pi^{R}}{\delta X}= & Q^{*}+(X+C-W) \frac{\delta Q^{*}}{\delta X}-T^{*}+(R-X-C) \frac{\delta T^{*}}{\delta X}-(R-S) F\left(Q^{*}\right) \frac{\delta Q^{*}}{\delta X} \\
& +\int_{Q^{*}}^{T^{*}} F(D) d D-(R-X)\left[F\left(T^{*}\right) \frac{\delta T^{*}}{\delta X}-F\left(Q^{*}\right) \frac{\delta Q^{*}}{\delta X}\right]=Q^{*}-T^{*}+\int_{Q^{*}}^{T^{*}} F(D) d D \\
\frac{\delta E \Pi^{R}}{\delta W}= & -Q^{*}+(X+C-W) \frac{\delta Q^{*}}{\delta W}-(R-S) F\left(Q^{*}\right) \frac{\delta Q^{*}}{\delta W}+(R-X) F\left(Q^{*}\right) \frac{\delta Q^{*}}{\delta W}=-Q^{*} .
\end{aligned}
$$

Observe that $0 \leq F(D) \leq 1$ for all $D$; therefore $0 \leq \int_{Q^{*}}^{T^{*}} F(D) d D<\int_{Q^{*}}^{T^{*}} d D=T^{*}-Q^{*}$. All three derivatives are thus negative, and show respectively the marginal impact of $C, X$, and $W$ on the retailer's profitability. If $X+C$ is fixed, the first two derivatives taken together show that every dollar transferred from option price $C$ to exercise price $X$ improves the retailer's profitability by a constant, positive amount $\int_{Q^{*}}^{T^{*}} F(D) d D$.

### 4.2. Impact of Contract Terms on Supplier Profitability

If the retailer has dominant negotiating power, it will propose values of $W, X$ and $C$ that maximize its expected profit. Because of the structure in (16), there are three possible scenarios.

Scenario I: $P \geq S+\frac{(R-X)(M-S)}{C}$. Note that $\frac{(P-M)}{(P-S)} \geq \frac{(R-X-C)}{(R-X)}$ implies that $Y^{* *} \geq T^{*}$. So under this scenario, the penalty cost is so high that the supplier always produces $T^{*}$ units. The supplier's
expected profit is:

$$
\begin{equation*}
E \Pi^{S}\left(T^{*}\right)=(C+X-M) T^{*}-(X+C-W) Q^{*}-(X-S) \int_{Q^{*}}^{T^{*}} F(D) d D \tag{20}
\end{equation*}
$$

Differentiating (20) with respect to $W, X$, and $C$ yields:

$$
\begin{align*}
& \frac{\delta E \Pi^{S}\left(T^{*}\right)}{\delta C}=T^{*}-Q^{*}+\left[C+S-M+\frac{C(X-S)}{R-X}\right] \frac{\delta T^{*}}{\delta C}  \tag{21}\\
& \frac{\delta E \Pi^{S}\left(T^{*}\right)}{\delta X}=T^{*}-Q^{*}+\left[C+S-M+\frac{C(X-S)}{R-X}\right] \frac{\delta T^{*}}{\delta X}-\int_{Q^{*}}^{T^{*}} F(D) d D  \tag{22}\\
& \frac{\delta E \Pi^{S}\left(T^{*}\right)}{\delta W}=Q^{*} \tag{23}
\end{align*}
$$

Note that $\frac{\delta T^{*}}{\delta C}=\frac{\delta T^{*}}{\delta F\left(T^{*}\right)} \frac{\delta F\left(T^{*}\right)}{\delta C}=\frac{1}{f\left(T^{*}\right)} \frac{-1}{(R-X)}<0, \frac{\delta T^{*}}{\delta X}=\frac{1}{f\left(T^{*}\right)} \frac{-C}{(R-X)^{2}}<0, \frac{\delta Q^{*}}{\delta C}=\frac{1}{f\left(Q^{*}\right)} \frac{1}{(X-S)}>0$, and $\frac{\delta Q^{*}}{\delta X}=\frac{1}{f\left(Q^{*}\right)} \frac{(W-S-C)}{(X-S)^{2}}<0$, consistent with intuition. When $C \geq \frac{(R-X)(M-S)}{(R-S)}$, let $\Delta=C+S-M+$ $\frac{C(X-S)}{(R-X)}$ to yield $\frac{\delta E \Pi^{R}}{\delta C}+\frac{\delta E \Pi^{S}}{\delta C}=\Delta \frac{\delta T^{*}}{\delta C} \leq 0$. Note that $C \geq \frac{(R-X)(W-S)}{(R-S)}$ is the condition for $Q^{*} \leq T^{*}$. Moreover, when $C$ increases, $Q^{*}$ increases and $T^{*}$ decreases; thus, $Q^{*}-T^{*}$ increases. Therefore, when $\frac{(R-X)(W-S)}{(R-S)} \geq C \geq \frac{(R-X)(M-S)}{(R-S)}$, increasing $C$ will reduce both total supply chain profit and the retailer's profitability. The sign of $T^{*}-Q^{*}+\Delta \frac{\delta T^{*}}{\delta C}$ determines whether the supplier's profit will increase or decrease. However, even when the profit of the supplier increases with $C$, the financial gain realized by the supplier is less than the loss incurred by the retailer.

Expressions (21) and (22) show that $\frac{\delta E \Pi^{s}\left(T^{*}\right)}{\delta X}=\frac{\delta E \Pi^{s}\left(T^{*}\right)}{\delta C}-\int_{Q^{*}}^{T^{*}} F(D) d D$, thereby quantifying the impact of shifting the relative magnitude of option price $C$ and exercise price $X$ (with $X+C$ remaining constant). Notice that $T^{*}$ is independent of the wholesale price $W$, while $Q^{*}$ decreases with increasing $W$. Therefore, expression (23) shows that $W$ has a diminishing marginal impact on the supplier's profitability. Moreover $\frac{\delta E \Pi^{R}}{\delta W}+\frac{\delta E \Pi^{s}\left(T^{*}\right)}{\delta W}=0$, so we see that the impact of $W$ has two dimensions: $(i)$ a cash transfer between the two parties, and (ii) an induced change in buying
behavior (reflected in $Q^{*}$ and $T^{*}$ ). The last identity shows that the behavioral impact on total profits is negligible.

Scenario II: $S+\frac{(M-S)(X-S)}{W-S-C} \leq P \leq S+\frac{(R-X)(M-S)}{C}$. Note that the condition for $Q^{*} \leq T^{*}$ implies that such values of $P$ exist. It is straightforward to show that this condition induces a production quantity $Y^{*}$ between $Q^{*}$ and $T^{*}$. Thus the supplier's profit is given by expression (12). Differentiating (12) with respect to $X, C, W$, and $P$ yields:

$$
\begin{align*}
& \frac{\delta E \Pi^{S}\left(Y^{*}\right)}{\delta C}=T^{*}-Q^{*}+\frac{C(R-P)}{R-X} \frac{\delta T^{*}}{\delta C}  \tag{24}\\
& \frac{\delta E \Pi^{S}\left(Y^{*}\right)}{\delta X}=T^{*}-Q^{*}+\frac{C(R-P)}{R-X} \frac{\delta T^{*}}{\delta X}-\int_{Q^{*}}^{T^{*}} F(D) d D  \tag{25}\\
& \frac{\delta E \Pi^{S}\left(Y^{*}\right)}{\delta W}=Q^{*}+(X+C-W) \frac{\delta Q^{*}}{\delta W}  \tag{26}\\
& \frac{\delta E \Pi^{S}\left(Y^{*}\right)}{\delta P}=Y^{*}-T^{*}+\int_{Y^{*}}^{T^{*}} F(D) d D \tag{27}
\end{align*}
$$

Expressions (24) and (25) quantify the impact of $C$ and $X$ on the supplier's profit. Again, $\frac{\delta E \Pi^{S}\left(T^{*}\right)}{\delta X}=\frac{\delta E \Pi^{s}\left(T^{*}\right)}{\delta C}-\int_{Q^{*}}^{T^{*}} F(D) d D$ (as in Scenario I), showing the impact of shifting the relative magnitude of option price $C$ and exercise price $X$ (with $X+C$ remaining constant). However, $\frac{\delta E \Pi^{R}}{\delta W}+\frac{\delta E \Pi^{S}\left(T^{*}\right)}{\delta W}=(X+C-W) \frac{\delta Q^{*}}{\delta W}$, which is non-positive because $Q^{*}$ decreases with $W$ while $X+C-W>0$. An increase in wholesale price $W$ thus decreases the total profitability of the entire chain. Though it is hard to determine the sign of the right hand side of expression (26), the profit gain to the supplier from increasing $W$ is certainly less than the profit loss for the retailer.

Scenario III: $P \leq S+\frac{(M-S)(X-S)}{W-S-C}$. It is easily verified that $Y^{* *} \leq Q^{*}$ under this condition because $\frac{P-M}{P-S} \leq \frac{X+C-W}{X-S}$. In this scenario, the penalty is so small that it is cheaper for the supplier to incur the penalty than to produce additional units to meet its options obligation, and the supplier always
produces $Q^{*}$ units. This scenario could occur in at least three situations: $(i)$ if the manufacturing cost $M$ is too large, $(i i)$ if the penalty cost $P$ is too small, or $(i i i)$ if the option is ill priced in terms of $X$ and $C$. Under this scenario, the supplier's profit function is given by:

$$
\begin{equation*}
E \Pi^{S}\left(Q^{*}\right)=(C+X-P) T^{*}+(W-M+P-X-C) Q^{*}-(X-P) \int_{Q^{*}}^{T^{*}} F(D) d D \tag{28}
\end{equation*}
$$

Differentiating expression (28) with respect to $W, X$, and $C$, we obtain:

$$
\begin{align*}
& \frac{\delta E \Pi^{S}\left(Q^{*}\right)}{\delta C}=T^{*}-Q^{*}+\left[P-M+\frac{(S-P)(X+C-W)}{X-S}\right] \frac{\delta Q^{*}}{\delta C}+C\left[1+\frac{X-P}{R-X}\right] \frac{\delta T^{*}}{\delta C}  \tag{29}\\
& \frac{\delta E \Pi^{S}\left(Q^{*}\right)}{\delta X}=T^{*}-Q^{*}+\left[P-M+\frac{(S-P)(X+C-W)}{X-S}\right] \frac{\delta Q^{*}}{\delta X}+C\left[1+\frac{X-P}{R-X}\right] \frac{\delta T^{*}}{\delta X}-\int_{Q^{*}}^{T^{*}} F(D) d D  \tag{30}\\
& \frac{\delta E \Pi^{S}\left(Q^{*}\right)}{\delta W}=Q^{*}+\frac{(X-P)(X+C-W)}{X-S} \frac{\delta Q^{*}}{\delta W} \tag{31}
\end{align*}
$$

The qualitative results obtained for this scenario are similar to those discussed previously for Scenarios I and II. We omit them here for the sake of brevity.

## 5. Numerical Experiments

This section reports numerical computations showing the impact of demand forecasting error and bias (in $\mu$ and $\sigma$ ) on financial performance, highlighting the value of options in supply chain management.

In the first numerical example, we set $R=100, W=70, S=5, M=40, C=15, X=60$, and $P=80$. The mean demand is $\mu=3000$ or 4000 . The standard deviation of the demand is varied from $\sigma=50$ to 2000 in increments of 50 . The retailer's optimal order quantities are determined using expressions (5) and (6), and the retailer's profit is computed using expression (7). Similarly, the supplier's optimal production quantities are identifed using expressions (14) and (16), and the supplier's profit is calculated using expression (15). Figure 1 and Figure 2 report the computational results.

Figure 1 plots the change in profitability as the standard deviation of demand increases from 50 to 2000 in increments of 50 . As expected, profits increase with $\mu$, and decrease with $\sigma$. However, the slopes of the retailer's curves are steeper than those of the supplier's, indicating that the impact of demand uncertainty is greater on the retailer's profits than on those of the supplier. The curves are almost linear, so linear approximation can be used to estimate the impact of forecasting bias (in $\sigma$ ) on expected profit. Though not shown in Figures 1 and 2, there is a nearly linear relationship between mean demand and profitability as well.

As stated previously, the Profit Gain (\%) is defined as $\frac{\left(E \Pi^{j}-E \Pi_{\mathrm{NV}}^{j}\right)}{E \Pi_{\mathrm{NV}}^{j}}$ for $j=$ the supplier, the retailer, or the entire supply chain. This quantity is multiplied by 100 and reported as a percentage, where the profit without options is calculated by using the classical newsvendor model. Figure 2 shows the percentage profit gain for each party with $\mu=3000$ (similar patterns result from other values of $\mu$ ). The results yield several observations.

- The contribution of options to supply chain profitability is substantial. When $\mu=3000$ and $\sigma=2000$, the percentage improvement in profit is $11.6 \%, 85.7 \%$, and $31.5 \%$ for the supplier, the retailer, and the supply chain as a whole, respectively.
- Both the retailer and the supplier realize positive gains, illustrating that options can improve performance for both supply chain partners.
- As demand variance increases, the retailer's percentage gain increases much faster than the supplier's. When $\mu=3000$ and $\sigma$ increases from 50 to 2000, the retailer's gain increases from $0.5 \%$ to $85.7 \%$, while the supplier's gain increases only from $0.2 \%$ to $11.6 \%$.

Option contract terms and conditions are typically negotiated, often based on market conditions or the nature and relative market power of the two parties. Well-negotiated contracts terms and conditions help develop and preserve hard-to-build partner relationships, a key factor for supply chain success. Negotiations should consider the efficiency of the supply chain, as well as the profit distribution between the two parties. For example, the wholesale price $W$ can be set by the
market, allowing the supplier to propose the option terms $X$ and $C$. The retailer can then set the penalty $P$ under a cash settlement agreement. The second set of numerical experiments illustrates how each party should react in such a negotiation, by numerically demonstrating the impact of $W$, $X, C$, and $P$ on the profit of each party.

In this set of experiments, we set $R=100, S=5, M=40, C=15, X=60, W=70$, and $P=80$. Demand is normally distributed with mean $\mu=3000$ and standard deviation $\sigma=1000$. When studying the impact of one parameter, all the other parameters are fixed at the base level. The retailer's and supplier's optimal order and production quantities are again determined using expressions (5), (6), (14) and (16), and the optimal expected profits are computed using expressions (7) and (15).

The constraint $X+C \geq W$ implies that $C \geq W-X=70-60=10$. On the other hand, $T^{*}=Q^{*}$ induces the constraint $C \leq \frac{(R-X)(W-S)}{R-S} \approx 27.4$. Thus, if $C>27.4$, selling options is so attractive that the retailer prefers to sell options rather than buying them, and will therefore order a negative number of options. Hence, we let $C$ vary from 10 to 27.5 in increments of 0.5 when studying the impact of option cost $C$. Similarly, we let $X$ vary between $W-C=70-15=55$ and $R-\frac{C(R-S)}{W-S} \approx 78.08$, in increments of 0.5 . When studying the impact of the wholesale price, we vary $W$ from $M=40$ to $X+C=75$. When considering the penalty cost $P$, we vary $P$ between $M=40$ and 100. As discussed in Section 3, $P>R=100$ always induces the supplier to produce $T^{*}$; therefore, values of $P$ larger than 100 yield no additional insight.

Figure 3 shows the impact of option price $C$ on profits. As derived in Section 5 , the retailer's profit is monotonically decreasing with $C$, as is total profit. In contrast, the supplier's profit increases at first, peaks at approximately $C=21$, then decreases thereafter. Figure 4 shows a similar effect for exercise price $X$. Figure 5 confirms the the impact of $W$ on the allocation of total supply chain profits between the two parties. Though the precise relationship between profits and $W$ is nonlinear, Figure 5 shows that a linear approximation is good for estimating the change in profits as $W$ varies. Figure 6 illustrates the impact of penalty cost $P$.

## 6. Conclusions and Suggestions for Future Research

This paper has developed a simple framework for studying the role of options in managing supply chain risk. We considered a supply chain in which the retailer could either buy product directly from the supplier, or purchase options to buy product. The former generates a supply of product that the retailer is relatively sure to sell, while the latter provides a potential supply of product for more uncertain demand. We derived optimal replenishment policies for both the retailer and the supplier, as well as an expression for optimal expected profits. We demonstrated that options could effectively mitigate the risk associated with demand uncertainty. We also showed the value of options for improving supply chain efficiency by providing flexibility, coordinating the channel, and as a vehicle for sharing information between supplier and retailer. We thus established the benefits and value of supply chain derivatives for supply chain management.

While this simple framework has yielded insight into the potential role of derivatives in supply chains, many issues remain to be explored, both through deeper analysis within the context of the framework presented in this paper, and by extending the framework to consider multiple trading partners, more complex supply chain topologies, and multiple periods. These extensions should prove useful in understanding how options could be effectively utilized in practice. They should also provide insight into the business and financial implications of using supply chain derivatives, especially in a multi-period setting.

Supply chain options such as buyback and quantity flexibility provisions are typically embedded in supply contracts, and are implemented using the terms and conditions of bi-lateral contracts negotiated by supply chain partners. As discussed in Section 3, the options described in this paper provide even greater flexibility than these traditional policies. They also have an added benefit: because they are not embedded in a supply contract, these options can be traded among their holders. Shi el al. (2003) examined the benefits and value of options trading. These findings are summarized below.

Separating options from supply contracts offers a number of advantages. First, it allows supply chain partners to alter the quantity of options they hold as they learn more about customer demand. With embedded options, option cost $C$ is a sunk cost for the retailer, since it cannot be recovered once the contract has been signed. However, if options are separated from supply contracts, the retailer can adjust his option position to reflect new information about customer demand as the market evolves. In cases where customer demand is not uniform across retailers, retailers with low realized demand can sell options to retailers with high realized demand, recovering at least a portion of the original cost of the options. In certain cases, retailers may even be able to sell their options for a profit.

Second, separation allows supply chain participants to better align their supply chain investments with their appetite for risk. Different parties in a supply chain may have different perceptions about value and risk, which can sometimes prevent optimal investments in inventory and capacity, thus degrading overall supply chain efficiency. Separating options from a supply contract enables risk-taking and risk-sharing by third parties with different risk preferences and different capacities to bear risk. Such third parties could include other suppliers or retailers, risk intermediaries (e.g., insurance companies and banks), or even speculators. By assuming a certain amount of risk, third parties can increase overall supply chain profitability.

In the analysis presented in this paper, all price parameters (e.g., $X, C$, and $W$ ) were assumed fixed, and the focus was on the behavior and business implications of supply chain options. A natural extension of this work is to develop techniques for pricing such options. Pricing models could be used to determine appropriate values for parameters such as $X$ and $C$ in a given supply chain environment.

If standard pricing models can be developed for supply chain options, it may be possible to develop liquid exchanges where such derivatives can be freely traded by supply chain partners, and by third parties such as investors and speculators. It should be noted that many issues need to be
addressed to support the creation of such exchanges, including the development of infrastructure to support both trading and clearing transactions. However, the development of markets for trading supply chain derivatives would offer a number of advantages. Supply chain partners could purchase options rather than holding physical inventories or investing in production capacity. Market prices for options can also provide valuable insight to supply chain participants about market expectations regarding future demand and demand variability.

The options described in this paper can also serve as the basis for a new class of insurance products for managing the risk associated with demand uncertainty. When $W=X$, option price $C$ is essentially the premium a retailer would pay for insuring product availability. Under certain circumstances, maximizing overall supply chain profitability entails reducing the profitability of either the wholesaler or the retailer. When this is the case, there is an opportunity for a third party to improve overall supply chain efficiency by offering insurance products. There may also be situations where the retailer and supplier are either unwilling or unable to bear the risk associated with demand uncertainty. In the absence of active markets for supply chain derivatives, this provides an opportunity for an insurance company to step in as an intermediary to offer insurance products. In a subsequent paper, we will describe in greater deal the benefits of using insurance to protect against demand uncertainty, the conditions under which insurance markets for demand uncertainty could be expected to develop, and the mechanics of actually implementing an insurance program from the perspective of a wholesaler, retailer, and insurer.

The supply chain is the nexus for a wide variety of risks, including demand and supply variability, and price volatility for raw materials and components. The options described in this paper are simple derivatives whose value is linked to customer demand. However, it is also possible to develop derivatives whose value is linked to other forms of supply chain uncertainty. Furthermore, the example in this paper was based solely on call options. Other derivatives products - including put options, swaps, and forward contracts - can also serve a valuable role as a vehicle for mitigating, sharing, and transforming supply chain risk.

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Figure 2: Profit Improvement from Supply Chain Options


Figure 3: Impact of C on Profits


Figue 5: Impact of W on Profits


Figure 4: Impact of $X$ on Profits


Figure 6: Impact of $P$ on Profits


