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Akshat Verma, Rahul Garg
{vakshat, grahul}@in.ibm.com
IBM Research Division
IBM India Research Lab
Block I, I.I.T. Campus, Hauz Khas
New Delhi - 110016. India.

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Akshat Verma
{vakshat@in.ibm.com}

Rahul Garg
{grahul@in.ibm.com}

Abstract

Simultaneous online auctions can lead to greater efficiency in the market than isolated ones as the global valuations could be maximized. We present a case for the bundling problem where the items a bidder desires can be grouped into independent bundles. We show that such valuations can be efficiently allocated by a local policy and present a formulation for it which can be statically verified. Finally, we also show that why a general multi-item auction can not be efficiently held.

1 Introduction

Auctions are a means to transfer goods from one entity to another where the price of the good is not easily determined and depends on the perception of the customer. Traditionally, auctions are carried on in an independent fashion where items are auctioned one after another. As such, a buyer can only participate in the auctioning of only one item at any given time. With auctions now being carried on the net, a buyer can actually participate in simultaneous online auctions at the same time. However, in this case the bids he places on an item in one auction is dependent on other bids. This is because the goods may not be independent of each other i.e the goods may be substitute or complementary goods. In such a case, his valuations of an item is dependent on the other items he is currently bidding. Any market is aimed to reach efficiency and efficiency is achieved by increasing the value of the market i.e increasing the valuation of all the items in the market. Hence, the auctioning strategy should be efficient, profitable to the buyer and fair. It has been shown that for any general set of goods, it is not possible to have an efficient allocation by a local policy where the buyer only focusses on his valuations of the items. This restriction is critical and intuitive because as a buyer, one would like to bid based on one's own valuations of the items. Also, this implies scalability because the bidding is independent of the number of bidders in the auction, which can be pretty large for multiple online auctions.

Shapley and Shubik [5] considered an assignment market where each participant wants one item and places a monetary value on each item in the market. It was shown that if the valuations of all participants for all items are known then the model always has an equilibrium and there is a unique smallest price vector P such that P is at least as small in every component as any other equilibrium price vector. Leonard [6] and Demange and Gale [7] showed that in a sealed bid multi-item auction of substitutes, where each bidder wants one item, is incentive compatible if the smallest equilibrium price vector is used as the price setting rule. This implies that submitting their true valuations is the dominant strategy for bidders. This allowed the results derived by Shapley and Shubik [5] to be directly applicable to multi-item sealed bid auction of substitutes. Bikhchandani and Mamer [4] extended the results of Shapley and Shubik [5] to an exchange economy of indivisible items where a participant may want a subset of items instead of a single item, the only condition being that the cost of disposal of goods is zero. We note that this leads to a very general setting of goods.

In this paper, we look at the specific auctioning problem where the buyer intend to buy from a bundle of goods, such that the bundles are independent of each other and inside a bundle he has an ordering on the items present. A common example could be assembling a PC where the bundles are sets of microprocessors, printers, monitors and other devices. If all the items are compatible with each other, as is generally the case, his choice of which printer he is going to select is independent of his choice of monitor. Also, in a set of monitors, he has a unique preference hierarrchy which changes only as the price of monitors change and not as he adds items other than printers to his kitty. We present a defintion of surplus equivalent substitutes which covers this particular problem (It can cover items which are not really in the bundled category as well. An example could be where he may even buy more than one item from any bundle. Also, he may choose not to buy any item from some bundles. The only restriction we impose is that the set of items we are auctioning can be partitioned into equivalence classes such that inside a class the items have an ordering which does not change on adding or removing any items from the kitty) and show that a locally greedy policy leads to efficient allocation of the items in the market. Later on we also show that for the general defintion of substitute goods, even a locally optimal policy fails to find efficient allocation and hence we suggest that our definition of surplus equivalent substitutes presents a border to which we can carry simultaneous auctions with a greedy policy and get efficient allocation. The other frontier is also defined in the sense that we show that general substitutes supplemented with some of the common economic principles like diminishing returns fail to achieve efficient allocation even if we follow a locally optimal strategy instead of locally greedy policy.

We note that Milgram [2] give a condition for achieving efficiency but they need a locally optimal policy. More importantly, the condition is not in terms of only the valuations of the items but involves the concept of market prices as well. The condition is a dynamic one and cannnot be statically verified. These problems are not present in our solution. Locally greedy policy has also been used previously to achieve efficient allocation in systems where the valuations are additive(both multi-unit and single unit). Demange [3] had shown the equilibria of the price vector in case of single-unit demand and Bansal and Garg [1] give the proof for efficiency. However, we present a result in which the valuations may not be additive but are more close to the actual definition of substitutes. In this sense, we suggest our formulation to be more realistic to the actual market.

2 Locally Greedy Bidding for Surplus Equivalent substitutes

Definition 1 *Locally Greedy Bidding is the bidding in which at any given time, a bid is a made for only one item. The item is chosen which maximizes the incremental gain with the current winning set.*
$\arg\max_i V(S+i) - V(S) - P_i$

Definition 2 *Locally optimal bidding involves bidding on a set of item at any given time where the set is identified by* $\arg\max_S V(S) - P(S)$

The significant improvement in having a locally greedy policy over locally optimal policy is the ease with which a bidder can make the next bid. The bidder just needs to make as many calculations as the number of items he wants to bid on, which is the least effort required.

Definition 3 *Surplus preservation:*
For any P(a),P(b)
If V(a) - P(a) > V(b) - P(b)
then at that P(a), P(b)
V(S+a) - P(a) > V(S+b) - P(b) for all S. $S \subset U - \{a, b\}$ U *= set of all the items*

Now this means
$V(a) - V(b) > P(a) - P(b) \Longrightarrow V(S + a) - V(S + b) > P(a) - P(b)$
i.e for any constant $C = P(a) - P(b)$
$V(a) - V(b) \geq C \Longrightarrow V(S + a) - V(S + b) \geq C$
$i.eV(S + a) - V(S + b) \geq V(a) - V(b)$
But my diminishing marginal utility $V(S + a) - V(S + b) \leq V(a) - V(b) hence$

Definition 4 *V(S+a) - V(S+b) = V(a) - V(b) (Differential preservation) For all* $S \subset U - \{a, b\}$ $, a, b$

Theorem 1 *Local bidding on the optimal set where items satisfy differential preservation leads to efficient allocation.*

Lemma 1 *If a bidder always bids on the optimal set at any given price, at the end of the auction he ends up with a set of items which are in his optimal set at the final price.*

Proof: The proof is by contradiction.
We take an item Prev which was in the optimal set at time T but not at time T+1, where P(Prev) remains same. Now, this means there are some new items which have come into the set decreasing the incremental utility of Prev. Let us say there are k such items whose surplus has now grown above the surplus of Prev. For k = 1 the surplus of new item was previously less than Prev, hence we will again choose Prev. For k = i+1; Let us first take i items in the group. Now we put the i+1 th item. Its surplus was previously less than Prev, as Prev was chosen. So, even now we are going to choose Prev. Hence, we have the condition that if a bidder bids on an item and he is not outbid, then the item is profitable to him. So, we can say that if any item is desirable for a bidder at any price vector P then it is going to be desirable at any price vector P_1 s.t the price of the item remains same and for all the other items the price does not fall i.e it either increases or remains the same. Hence, all the items he has in his kitty are in the desirable set. Also, if there are items existing which are desirable but he has not made a bid on it. He will make the bid on the remaining items as well. Hence, the proof. ■

Lemma 2 *If a set of items are such that Lemma 1 is satisfied for all the bidders, the corresponding allocation is going to be approximately efficient. i.e the difference of the total valuations from the optimal allcation is going to be bounded by bid increment * the number of items.*

Proof:
For any bidder i let S^i be the optimal allcation. By following the strategy of bidding on the optimal set, he ends up with a set $S^i - S^i_o + S^i_g$.As this is the most desirable set (by lemma 1) we have the following condition (S^i_o = element in the optimal set but not in our set, S^i_g = elements in our set but not in optimal)
$V(S^i - S^i_o + S^i_g) - P(S^i - S^i_o + S^i_g) \geq V(S^i) - P(S^i) - \epsilon * S^i_g (\epsilon =$the bid increment)
$V(S^i - S^i_o + S^i_g) - P(S^i) - P(S^i_o - S^i_g) \geq V(S^i) - P(S^i) - \epsilon * S^i_g$
$V(S^i - S^i_o + S^i_g) - P(S^i_o - S^i_g) \geq V(S^i) - \epsilon * S^i_g$
$V(S^i) - V(S^i - S^i_o + S^i_g) \leq P(S^i_o - S^i_g) + \epsilon * S^i_g$
Summing up over all the i we have,
$V(S_{optimal}) - V(S_{greedy}) \leq \epsilon * S_g$ (because the items S^i_o and S^i_gwill cancel each other)
$V(S_{optimal}) - V(S_{greedy}) \leq \epsilon *$(number of items)

But this definition is very restrictive as there is nothing similar to the actual sense of substitutes. So, we can make a more restrictive definition which is

Definition 5 *Surplus equivalent substitute:*
We define a relation R s.t
a R b iff V(a) - V(b) = V(S+a) - V(S+b) for all $S \subset U - \{a, b\}$ ....(1)
We can easily verify that this is an equivalence relation, hence this should partition all the items into equivalence classes. Also, these partitions should satisfy the following property:
V(a + S_1) = V(a) + V(S_1) for all $S_1 \subset U - U_a$(U_a =equivalence set of a) ....(2)

Theorem 2 *Surplus equivalent substitues gives efficient allocation if the bidding strategy is locally optimum.*

Proof:
Now the conditions (1) and (2) together make the allocation efficient. This is because an item Prev which has been bid on at time t will also remain in the optimal set at time t+1, if its price remains same. This is because the items which can decrease its valuation are in the same partition as it and they follow the surplus preservation property. So, we are again going to add this item over all the items in its class. As for items outside this set, tbey do not decrease its valuation, so if it was profitable at time t it should be profitable at time t+1. Hence we again satisfy Lemma 1 and we will be able to achieve efficient allocation.

Theorem 3 *Locally Greedy Bidding leads to efficient allocation in surplus equivalent substitute goods.*

Lemma 3 *For surplus equivalent substitutes, if an item is selected in the optimal set, it remains in the optimal set if the price of that item does not increase. This is similar to Lemma 1.*

Lemma 4 *The item on which we make the next bid using LGB is in the optimal set at that price.*

Proof of Lemma 4:
Assume we make a bid on an item which is not in the optimal set at that time. Then, by the defintion of LGB this item increases the incremental gain the most. So, assume other items which are in the optimal set and have not been chosen till now. (If there are no such items then we can simply add the new LGB item to the optimal set and get a more profitable set)
So, we have items which have not been chosen. Next, we take an element which belongs to the same class to which the LGB item belongs. (There exists such an item because if all the items belong to other classes then we just add those items in the set and then add our LGB item to get a set which is more profitable than the optimal set.)
So, we have an item belonging to the same class. Now, we take the optimal set and remove that item and add our LGB item. Since, the LGB item has more surplus initially, it will again have more surplus. So, we again have a set more profitable than the optimal set. Hence, the proof.

Proof of Theorem 3
By Lemma 3 and Lemma 4, we are always going to bid on items in the optimal set using LGB and as these items will always remain in the optimal set if there price does not increase, we again have satisfied condition for Lemma 1. Hence, it leads to efficient allocation.

We also note here that this defintion can also take care of valuations like k max which are more of a capacity constraint than substitutes if we introduce the notion of desirable set and check the conditions above where all the sets should be a subset of any desirable set. However, as finding the desirable set is not possible statically we omit its discussion. However, for the sake of completeness, we give the defintion of desirable set.

Definition 6 *Desirable Set :*
A set of items S is said to be a desirable set if there exists a price vector P at which S maximizes V(S)- P(S).

3 The Problem with Substitute Goods

We would now show that why conditions such as surplus preservation are required to achieve an efficient allocation. We first argue out that if we have a general definition of substitutes we would be unable to achieve optimal allocation using either LGB or locally optimal policy.

Lemma 5 *A locally optimal policy may not lead to efficient allocation if the goods follow only the substitute property.*

Proof:
We have a set which has element **a** and **b** in its desirable set at time T. There is an element **c** which is a substitute of **a** and **b** but **a** and **b** are independent. These satisfies the substitute criteria. Now we fix the price of **a**, **b** and **c** such that **a** and **b** are profitable but **a**, **b** and **c** together are not. Also, **c** independently is profitable and **c** is more profitable than **b**. We note that there exists such a price vector because

(i) V(**b**+**c**) $<$ V(**b**) + V(**c**) and
(ii) V(**a**+**b**) $>$ V(**b**)

Now set the price of a as infinite making c the desirable set. So, we are stuck with b whose incremental valuation is less than the price i.e less than the valuation somebody else had of it. So, if the item had been allocated to the first loser we would have a higher valuation sum. Hence, the existing valuation is inefficient.

Lemma 6 *Substitutes following diminishing marginal returns also do not lead to efficient allocation using a local policy*

Proof:
We just make a note that in the above argument, even if we add the constraint of diminishing marginal returns, we would be able to find the violating price vectors. Hence, the proof.

4 Conclusions

This work extends the strategy which has been used for additive utilities to a more general setting. In this sense, it approximates the market place more closely. The system can be implemented as simultaneous independent auctions where traditionally the auctions have to be carried one after the other if efficiency is the goal of the auctions. Also, we present a very common example of goods where this setting holds i.e a bundle of goods which have an ordering.

We also show that the general market place of substitute goods do not lead to an efficient allocation, where the policy followed is local. We also present the pitfall involved in a local policy, even if the items follow some of the most commom economic principle viz the principle of diminishing marginal returns. We show that if we add the additional restriction of surplus preservation, we have a decentralized algorithm which gives us approximately efficient allocation. Also, we show that the approximation error is no more than what we achieve if the valuations are additive.

We also claim that our setting covers the set of items where the count of items play a decisive part, if we introduce the concept of Desirable Set. So, instead of the more strict condition of surplus preservation holding for all classes, if we have surplus preservation holding for desirable sets we can cover a more wider set of items in our discussion. A very obvious example which comes to mind is the case of constant unit demand where the valuations are the sum of k max valuation items. However, finding the desirable set is non-trivial which needs to be dynamically computed. But, for a particular type of valuation there may exist short-cuts to compute this set. As an example, for constant unit demand, the desirable set is any subset of size k of the set of all items. Similar approached could be tried out for other cases, to see if they work. The problem of finding the desirable set easily is though a separate problem in its own right and a field on which further work may be carried out.

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