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BUSINESS MODELS FOR MULTI-CURRENCY AUCTIONS

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Business Models for Multi-currency Auctions

1. PROBLEM

The participants (buyers & sellers) have different currencies. This exposes at least one of the two parties of a trade to potential exchange rate fluctuations. The risk extends into 2 phases:

1. Bidding Phase Risk - From placing an order (bid / offer) to the time the match is decided.
2. Post-bidding Phase Risk - From the time the match happens to the time of payment.

The risk in post-bidding phase is easily eliminated if one of the two parties books a forward exchange rate contract (for payment date and time) for the given amount as soon as the match happens[1].

Therefore, the problem sought to be addressed is of handling the exchange rate risk during the bidding phase.

2. DISCUSSION ON ISSUES

1. Gain / loss is ambiguous: The impact of an exchange rate change on a participant cannot be easily determined by a third party. Consider for example,
   a. A bidder wants to buy X. He bids in INR, the seller's currency being USD. If INR/USD exchange rate depreciates from 40 to 50, it is not clear how the bidder's valuation of X will change:
      i. If international price of X remains constant in USD terms, he may be willing to pay upto 50/40, i.e., 1.2 times his earlier willingness in INR terms, because the price of the good in Indian market would now go up.
      ii. If international price of good were to remain constant in INR terms, the bidder may not be willing to pay any more INR than what he bid earlier, so that his willingness measured in USD terms declines.
      iii. If bidder was buying X to resell it in Germany, he would be interested in knowing the changes in INR/EUR exchange rate as well to determine his new INR bid.
   b. In general, a bidder's willingness is a function of all the exchange rates in the world.
2. The exchange rates are dynamic - can change any instant. The auction model must be able to capture the fact that a participant's valuation of an item may change over time and the participant should be able to modify his position as his valuation changes.
3. *When* to be fair? Since the exchange rates are dynamic, the question is as to when the auction model should truly represent the positions based on *current* valuations of the participants. For any auction, in general, it will be a requirement that the property should hold at least at the winner-determination time (because that is the time when a participant can lock in the forward rate he is bidding by booking a contract in the market). For an open-cry auction that can close any time, it is only appropriate that this property should hold at any instant throughout the bidding phase.

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[1] Note that since such contracts require specification of amount and are binding, they cannot be entered into before the match is actually decided.

4. *What can the market take care of and what needs to be handled* - A change in exchange rates, in general, can cause a participant's valuation to either go up or go down. Consider a bidder (buyer). If a market allows re-bidding at a higher level (as in an open-cry auction), it can take care of the increase in bidder's valuation, because the bidder can place a higher bid, if required. But the model in such a case will have to specify a way that handles a possible decline in the bidder's valuation. In the case of sealed bid auctions, the bidder can bid only once and so the market cannot take care of either an increase or a decrease. Note that a market mechanism in which participants can dynamically increase or decrease their position values will eliminate all exchange rate risk.

3. MODEL FOR A PARTICIPANT'S BEHAVIOR

Assumptions

1. There are N currencies in the world. Any currency pair can be traded freely, so that no arbitrage condition will ensure that, between any three currencies A, B and C, the exchange rate AC can be compute from the two rates AB and BC. Thus, there are (N-1) independent exchange rates from which all the C(N,2) rates can be derived.
2. The change (with time) of a participant's valuation of the item being traded depends only on the changes in the (N-1) exchange rates[2]. That is, if V_0 is the valuation at time t_0 (when the N-1 exchange rates are $\{E_{1,0},E_{2,0}, ...,E_{(N-1),0}\}$, then the valuation at time t (when the exchange rates have changed to $\{E_{1,t},E_{2,t}, ...,E_{(N-1),t}\}$ is given by some function f :

   $$V_t = f(V_0, r_1, r_2, ..., r_{(N-1)}), \quad \text{where, } r_i = E_{i,t} / E_{i,0}$$

   Additionally, $V_t = V_0$ if $r_i = 1$ for all i.
3. The values of input variables, r_i are allowed to change as time progresses. However, the function f does not change over the life of the auction.

Definition of Fairness to Participants (Single Auctions Case)

Consider the case when the auctioneer is the seller.

a. The auctioneer (seller) should get at least the amount that he would have received had the auction been conducted at constant exchange rates[3] equal to the exchange rates that existed at the time[4] of the determination of the winners.
b. The winner (buyer) should pay no more that what he may be willing, at the time when the winner is determined, to pay in his currency on the payment date.
c. A bidder should not be deprived from winning if he were willing to pay more than any of the winners in auctioneer's currency terms at the time of winner determination. (Fairness to auctioneer (a) will automatically ensure this too).

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[2] Note that nearly all auction models (for a single currency) implicitly assume that the bidder's valuation does not change over the life of the auction. Here, it is allowed to change over time as the exchange rates change.

[3] Forward exchange rates for the payment date

[4] The auctioneer can lock in the exchange rate for payment date through a forward contract as soon as he determines the winner (and the amount). Therefore, this is *the instant of time* when he ideally would have liked the entire competitive game to have been played at.

4. SPECIFIC MODELS FOR DIFFERENT AUCTION TYPES

4.1 SEALED BID AUCTION (SINGLE ROUND)

This is characterized by four times T1, T2 and Tw and Tp, where T1 < T2 < Tw < Tp. The bidders are allowed to submit bids in the time interval T1 < t < T2. The auctioneer evaluates the bids and declares the winner(s) at time Tw. The payment is made at time Tp.

Nature of Problems

1. If bids are in the auctioneer's currency, a bidder is faced with the risk of depreciation of his currenciy w.r.t. auctioneer's currency at least over the time interval (T2,Tw).
2. If bids are in local currencies of bidders[5], the outcome would at best resemble an auction conducted at time T2 (assuming the bidders place the bids as late as possible, almost at T2), and not at time Tw. The bidders' valuations may have changed from the time they placed their bids till the time Tw when the bids are evaluated. This may be unfair to both the auctioneer as well as the bidders.

Proposed Model

Assumptions:

1. The payment date, Tp, is known to the auctioneer at time Tw.

The model can be used for single / multiple quantities and for first / second price sealed bid auctions.

Auction Process

Each bidder bids in his local currency by specifying his *willingness function*[6]. This function relates his bid price at any time *t,* to the values of all (or a subset of) exchange rates at the time, *t*.
The auctioneer evaluates all bids at time Tw. He uses the forward exchange rates (for Tp) as inputs to the functions supplied by the bidders, and obtains their bids as the values of those functions. These bids are converted into his currency using the same set of exchange rates and ranked. The highest bidder(s) is the winner(s). The auctioneer books a forward contract at the time Tw for the payment date Tp for converting the receivable from the winner(s) into his currency at Tp.

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[5] While evaluating, the auctioneer compares them by converting to his currency using forward exchange rates for payment date.

[6] For the second price auction, where the *dominant strategy* of the bidder is to specify his true valuation, this function could just be the *valuation function*, *f*, described in earlier section. For a first price auction, the bidder may construct another function from *f,* which he provides.

Payment

At Tp, the winner(s) pay their local currency bid amounts to the auctioneer. The auctioneer converts it to his currency using the forward contract booked earlier. Thus, he is able to realize the same amount in his own currency which he used for ranking the bids and deciding the winner.

4.2 DUTCH AUCTION

Assumptions:

1. The payment date, Tp is known in advance to the participants.

Auction Process and Payment

The Dutch auction model requires no changes. The bidders know their own *valuation and willingness functions*. They monitor the forward exchange rates for Tp, and use them as inputs to their *willingness functions* on a dynamic basis as long as the Dutch auction is *open*. Any time, say *t*, when the result of some bidder's willingness function is below the current auction price, a bid will result. Since the bid results in a winner at the instant *t* itself, auctioneer can, at time *t* itself, book a forward contract for converting the bid amount into his currency on the payment date, Tp.

4.3 OPEN-CRY AUCTION

This is characterized by three times To, Tc and Tp, where To < Tc <= Tp. The bidders are allowed to submit bids in the time interval To < t < Tc. The auctioneer evaluates the bids and determines the winner(s) at time Tc itself. The payment is made at time Tp.

Nature of Problems

1. Each participant may look at the value of the item to him in the currency of his choice (usually the currency of the country where he transacts) and determine his bid values in his chosen currency.
2. The value of a bid (placed by one of the participants) as seen by other participants (in their own currency terms) changes with time and differently for different currencies.
3. The valuations of bidders (in their own currency terms) change with time as the exchange rates change. The valuation may go either up or down after a bidder has placed his bid.
   a. An increase in the valuation is not a problem because the bidder can now submit a higher bid, *if required for being in the winning set*.
   b. A decrease in valuation causes problems because normal open-cry auction models do not allow reduction of bid values or their withdrawal.
4. The auction model ensure that the current active bids in the system reflect the bidders' *current willingnesses* at any point of time during the bidding phase.
5. If the auction is conducted in the auctioneer's currency (as is commonly proposed), a winner is faced with the risk of depreciation of his currenciy w.r.t. auctioneer's currency from the time of his placing the bid to the time Tc.

Proposed Models

Essentially any multi-currency open cry model needs to provide a way of handling the possible reduction in bidders' willingnesses due to exchange rate changes. There can be two generic approaches:

1. The auctioneer reduces the bid values on adverse exchange rate changes
2. The auctioneer allows the bidders to reduce bid values on exchange rate changes.

These models are detailed below. The set of common assumptions are listed here:

Common Assumptions

1. The payment date, Tp is known in advance (to the auctioneer) in at least one of the following ways:
   a. Tp is a fixed date, known throughout the bidding process. In this case, the exchange rates used at any time throughout the auction will the forward rates for Tp
   b. Tp is defined relative to the auction close, as a fixed time-interval, T from the close of the bidding process. The time of close of bidding process may not be known beforehand. In this case, the exchange rates to be used during the auction are forward rates for a time-period T ahead of the current time.
2. All the bidders stay in the system throughout the auction. If the auction close time is known beforehand, this can be relaxed to say that all the bidders should be in the system at least at the closing time.

The models can be used for single unit as well as for a multi-unit auction of a single item.

Model 1 : Auctioneer reduces the bid values on adverse exchange changes

Additional Assumptions

1. Amongst the N-1 exchange rates in the world, each bidder's valuation is a function of only one rate, that of his currency verses the auctioneer's currency. The other rates do not affect his valuation.
2. Definition of a bidder's willingness (the definition is time-invariant):
   a. A bidder is willing for a bid (A,B) that he places, where B is the bid value in the bidder's currency and A was the corresponding value in the auctioneer's currency at the instant that the bid was placed.
   b. If a bidder is willing for a bid (A,B), then he is also willing for all bids given by:
      i. If bidder's currency has since appreciated: (A,B') where B'<=B*, B* being the bidder's currency equivalent of A at the new exchange rate.
      ii. If bidder's currency has since depreciated: (A',B) where A'<=A*, A* being the bidder's currency equivalent of B at the new exchange rate.

   In a simple world where a bidder's valuation is independent of all other exchange rates (except that of his currency verses auctioneer's currency), the above assumption easily holds.

Auction Process and Payment

The process (for a single unit auction) works as follows:

1. Each bidder bids in his chosen currency.
2. The auctioneer remembers only the leading bid. By default, the first bid becomes the leading bid. A new bid must be greater than the current value of the leading bid. If so, it replaces the leading bid.
3. The auctioneer adjusts the value of leading bid (on an exchange rate change) as follows:
   a. On bidder currency depreciation - a bid (A,B) is adjusted to (A*,B)
   b. On bidder currency appreciation - a bid (A,B) is adjusted to (A,B*), where A* and B* were defined earlier.

   Thus, the auctioneer makes sure that the leading bid, at any time, is acceptable to the person who placed the bid.
4. Any bidder, who wishes to take advantage of a fall in the value of the leading bid can submit a new higher bid.
5. At the auction close, the leading bid (A,B) becomes the winning bid. The auctioneer immediately books a forward contract for converting the amount B into A on payment date. On payment date, the winner pays the amount B to the auctioneer.

The process can be extended to a multi-unit case by replacing the leading bid by the *set of leading bids* (defined at any time as the set of bids that would be winners if the auction were to be closed then). The auction price for multi-unit case can be either discriminatory (each winner paying his bid amount Bi) or uniform (each bidder paying his currency equivalent of the lowest of the winning bids).

Potential Shortcomings

1. Frequent bid reductions and re-bidding: The auctioneer will adjust the leading bid(s) value even though the bidder may be actually willing to pay the higher price implied by the new set of exchange rates. Any reduction of bid value in auctioneer currency terms can invite a considerable amount of re-bidding by the participants.
2. Considerable amount of up-and-down movement of bids in any chosen frame of reference (currency).
3. Restrictive 2-currency model imposed on each bidder's valuation.

Ideally one would like a system, where the up-and-down movements of the bids are restricted to a minimum possible. This would require that bid values be adjusted only if really required (i.e., they are really unacceptable to the bidders who placed them). Also, it would be desirable to have the bids remain constant to the extent possible, in at least one frame of reference, namely the auctioneer's currency.

These motivations lead us to the second model:

Model 2: Bidders allowed to reduce bid values

Auction Process and Payment

The process (for a single unit auction) works as follows:

1. Each bidder bids in his chosen currency, represented by (B).
2. The auctioneer remembers only the leading bid. By default, the first bid becomes the leading bid. The auctioneer converts any new bid it receives to its own currency equivalents at the time when it is received. A new bid must be greater than the current value of the leading bid to the auctioneer. If so, it replaces the leading bid.
3. The auctioneer keeps the value of leading bid constant in its own currency unless a reduction is requested by the bidder. As exchange rates change, the implied value in the currency of the bidder who placed it may change from a value B to a new value B' (increase or decrease).
4. The leading bidder may request for a reduction in the value of his bid (from B' to B''). The auctioneer may grant the requested reduction if it is within the permissible reduction limits (given the extent of the change in the exchange rates that took place). The reduced bid continues to remain the leading bid.
5. Any bidder, who wishes to take advantage of a reduction in the value of the leading bid can submit a new higher bid.
6. At the auction close, the leading bid (B) becomes the winning bid. The auctioneer immediately books a forward contract for converting the amount B into its own currency equivalent on payment date. On payment date, the winner pays the amount B to the auctioneer.

The process can be extended to a multi-unit case by replacing the leading bid by the *set of leading bids* (defined at any time as the set of bids that would be winners if the auction were to be closed then). The auction price for multi-unit case can be either discriminatory (each winner paying his bid amount Bi) or uniform (each bidder paying his currency equivalent of the lowest of the winning bids).

Setting limits for bid-reduction:

Some of options are:

1. Limits based on a exchange-rate sensitivity function (along with the set of exchange rates over which it applies) provided by each bidder to the auctioneer at the start of the auction. The auctioneer may further put restrictions on maximum allowable sensitivities (say max(at most the same percent as some exchange rate's change)).
2. Limits based some function of the extent of exchange rate changes, along with a set (limited by number of rates allowed) of rates over which an auctioneer can register.

A minimum step size for reduction should also be specified to prevent frequent (and insignificant) requests for bid-reductions.

Note:

1. The provision that only the leading bid is remembered helps to eliminate problems that could have arisen because of allowing bid-reduction. Consider the case when the non-leading bids are also remembered:
    a. A bidder (acting for the auctioneer, but having no intention to actually buy) can potentially raise the bid-level to the second-highest maximum-willingness level of all other bidders and then exit. It can do so by following a strategy of consistently out-bidding any other bidder. When no bidders are left in the system that are willing

to match his bid, this bidder reduces his bid and exits from the system. If only the bid being remembered, he would not be able to exit without buying unless someone outbids him.

   b. A bidder, X places a bid in response to bid of some other bidder Y. Subsequently exchange rates changes such that Y reduced his bid value, but X could not. X would feel cheated if he saw Y's bid being reduced. If Y's bid is not remembered at all after X makes his bid, such a situation would not arise.

2. Since some form of bid reduction is allowed, the winner(s) would try to bring their bids down (within the extent possible and permitted) to levels as close as possible to the highest non-winning bid. Thus, it is obvious that the price realized in such auction models would tend towards the second price (highest losing price for multi-unit case).

Characteristics of the model:

1. In the reference frame of auctioneer's currency, it would resemble a single-currency auction, except that there may be a few infrequent bid-reductions initiated by the winning bidder.
2. The frequency of bid-value changes (in auctioneer currency reference frame) is significantly reduced than in the earlier model. The bid-value change (and possibly re-bidding initiated by that) would happen only when it is significant and actually required.

5. NON-EQUIVALENCE OF DUTCH AND SEALED-BID AUCTIONS

If all participants use the same currency, then the Descending Price Dutch Auction and the Sealed Bid First Price Auction turn out to be equivalent (under assumptions of risk-neutrality on the bidders' behavior). The only choice available to a bidder in either of the 2 cases is to choose the highest price apriori at which he would be willing to buy the item at. It is implicitly assumed that the bidder's valuation of the item does not change over the time when the bidding for Dutch auction is in progress.

The equivalence no longer holds in a multi-currency scenario because a bidder's valuation changes with time as the exchange rates change. For the equivalence to hold in a multi-currency set-up, one would require that the state (set of exchange rate values) remained constant throughout the life of the Dutch auction, and further that this constant state was identical to the state that prevailed at the time of winner determination in the sealed-bid case.