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Research Report

Supplementary Material for *A Deep Choice Model*

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Abstract

This manuscript is the supplementary material for the paper titled, "A deep choice model", to appear in *Proceedings of the 30th AAAI Conference on Artificial Intelligence* (Otsuka and Osogami 2016).

Additional experiments

Here, we use an experimental setting inspired by the task used in Shenoy and Yu (2013). In this setting, an agent selects an item from a given choice set containing either two or three items. Each item is characterized by a two-dimensional vector of attributes, $\mathbf{z} \equiv (z_1, z_2) \in [0, 100]^2$, where each attribute takes an integer value between 0 and 100. Specifically, items shown in Fig. 1a are represented by the following attribute vectors: $\mathbf{z}^{(\mathrm{A})} \equiv (40, 60)$, $\mathbf{z}^{(\mathrm{B})} \equiv (60, 40)$, $\mathbf{z}^{(\mathrm{S})} \equiv (65, 35)$, $\mathbf{z}^{(\mathrm{C})} \equiv (80, 20)$, and $\mathbf{z}^{(\mathrm{D})} \equiv (50, 30)$.

Training examples are generated from the target distribution shown in Fig. 1b that reflects three typical choice phenomena investigated in prior studies (Shenoy and Yu 2013; Osogami and Otsuka 2014). For example, when the choice set is $\mathcal{X} = \{\mathrm{A}, \mathrm{B}\}$, the choice probability is $p(\{A\}|\mathcal{X}) = p(\{B\}|\mathcal{X}) = 0.5$. For each of the five choice sets shown in Fig. 1b, we create 20 instances of a pair of a choice set and a selected item, in such a way that the frequency of each selected item given a choice set exactly matches the corresponding probability given by the target distribution. These 100 instances (20 instances $\times$ 5 choice sets) are randomly shuffled and used as the training data.

Binarization

We convert the integer-valued attributes to binary features that can be given as input to the DCM. Here, we set the Hamming distance (or equivalent L1 distance) in the binary feature space proportional to the L1 distance in the attribute space. Specifically, we convert an attribute vector, $\mathbf{z} = (z_1, z_2)$, into a binary feature vector, $\mathbf{x} \in \{0, 1\}^{400}$, in such a way that

$$\mathbf{x} = \left(0^{z_1}, 1^{100}, 0^{100-z_1}, 0^{z_2}, 1^{100}, 0^{100-z_2}\right), \tag{1}$$

where i^n denotes the n-dimensional vector whose elements are i for $i \in \{0, 1\}$, and $(\mathbf{u}, \mathbf{v})$ denotes the vector that concatenates vectors $\mathbf{u}$ and $\mathbf{v}$. Then the L1 distance between attribute vectors $\mathbf{z}$ and $\mathbf{z}'$ is twice as short as the Hamming distance between the corresponding binary feature vectors $\mathbf{x}$ and $\mathbf{x}'$:

$$||\mathbf{z} - \mathbf{z}'||_1 = |z_1 - z_1'| + |z_2 - z_2'| = \frac{1}{2}||\mathbf{x} - \mathbf{x}'||_1 \tag{2}$$

Experimental results

We train the DCM with $K = 400$ and $L = 200$ by updating the weights and biases 1,000 times with a small learning rate fixed to 0.001. For comparison, we also train the MLM (equivalently, the DCM with $K = 400$ and $L = 0$) in an analogous manner.

Our first set of experimental results demonstrates that the DCM learns the similarity effect and the compromise effect from the training data and generalizes these phenomena to choices from unseen items. Observe that the choice probabilities for the choice sets, $\{$A, B$\}$ and $\{$A, B, S$\}$, involve the similarity effect. Specifically, when item S is added into the choice set, $\{$A, B$\}$, it steals a larger share from B than from A. Likewise, the choice probabilities for $\{$A, B$\}$ and $\{$A, B, C$\}$ involve the compromise effect in that the share of B relative to A is high when C is in the choice set, making B a compromise between A and C.

Here, we evaluate how the trained DCM predicts the choice probability of selecting an item from a choice set (A, B, Z), where Z is an item with attributes $\mathbf{z} = (60{+}n, 40{-}n)$, which vary linearly from (60, 40) to (100, 0), for $n \in [0, 40]$. The lines in Fig. 2a show the choice probabilities predicted by the trained DCM. Those predicted by the trained MLM are shown in Fig. 2b. The dots in the figures denote the target distribution for the choice set included in the training data: $\{$A, B, B$\}$, $\{$A, B, S$\}$, and $\{$A, B, C$\}$. In Fig. 2a (DCM), all three lines come near each dot, while the lines do not meet most of the dots in Fig. 2b (MLM). This means that the trained DCM well fits the target distribution in the training data, while the MLM is incapable of representing the target distribution.

The results in Fig. 2 show how the DCM and the MLM generalize the choice probabilities over an unseen choice set from those learned from the training data. In particular, the training data involve the similarity effect and the compromise effect, and here we study how these effects are generalized. Observe that the choice probabilities predicted by the trained DCM (Fig. 2a) show that Z steals a larger share from

B than from A when Z is similar to B (i.e., the similarity effect). Indeed, A is more likely to be selected than B when Z has the attribute $\mathbf{z} = (60 + n, 40 - n)$ for $0 \leq n \leq 6$ (Z is similar to B), but the preference between A and B is reversed otherwise. Moreover, the trained DCM predicts that the choice probability of B is particularly high when Z has the attribute $\mathbf{z} = (60 + n, 40 - n)$ for $n \geq 10$, which makes B a compromise between A and C (i.e., the compromise effect). In the MLM (Fig. 2b), the relative choice probabilities between A and B are necessarily preserved for any Z and show neither the similarity effect nor the compromise effect.

In Fig. 3, we move the attribute vector, $(z_1, z_2) \equiv (60 - n, 40 - n)$, of item Z linearly from (60, 40) to (20, 0) by varying $n \in [0, 40]$. Here, we study how the attraction effect appearing in the training data is generalized to unseen choice sets. In Fig. 3a (DCM), A is predicted to be more popular than B when the choice set is {A, B, B}, but the preference between A and B is reversed when the choice set is {A, B, D} (i.e., the attraction effect), as is the case in the training data. The trained DCM generalizes this attraction effect to unseen choice sets, {A, B, Z} for Z $\notin \{\mathrm{B}, \mathrm{D}\}$. Specifically, A is predicted to be more popular than B when Z has the attribute $\mathbf{z} \equiv (60 - n, 40 - n)$ for $n \leq 3$, while B is predicted to be more popular than A for $n > 4$. In the MLM (Fig. 3b), the relative choice probabilities between A and B are necessarily preserved for any Z and do not show the attraction effect.

Acknowledgements

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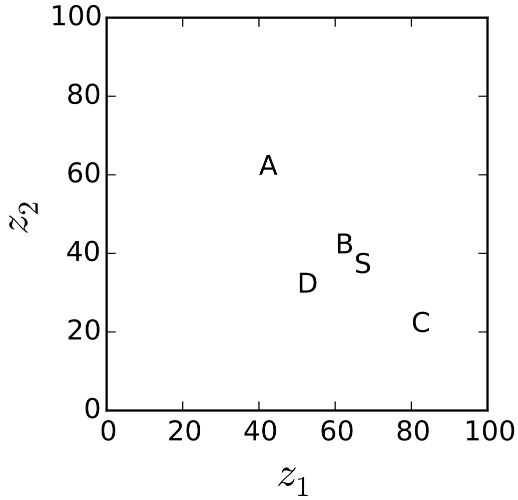
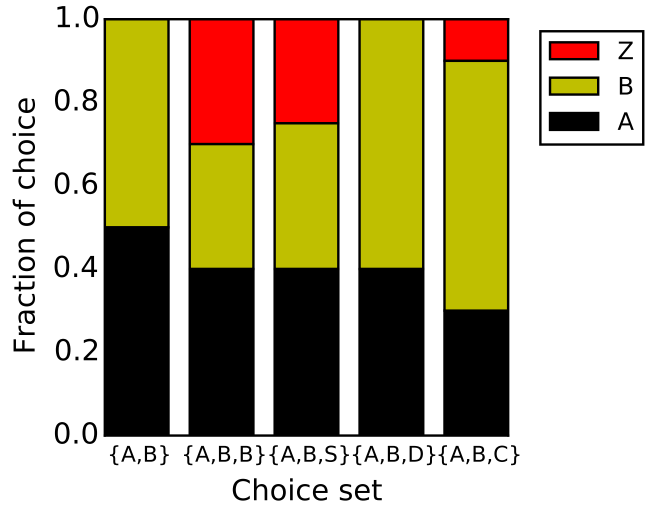
Shenoy, P., and Yu, A. J. 2013. Rational preference shifts in multi-attribute choice: What is fair? In *Proceedings of the Annual Meeting of the Cognitive Science Society (CogSci 2013)*, 1300–1305.

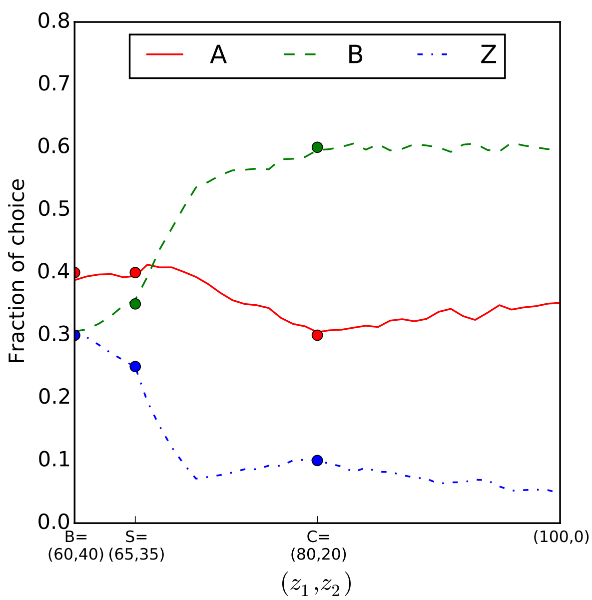


(a) Attributes of items

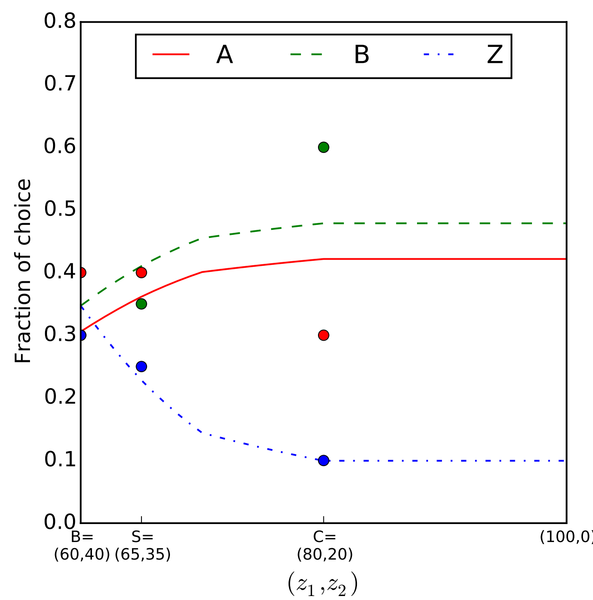


(b) Target distribution

Figure 1: Attributes of items and target distribution used in the additional experiments.

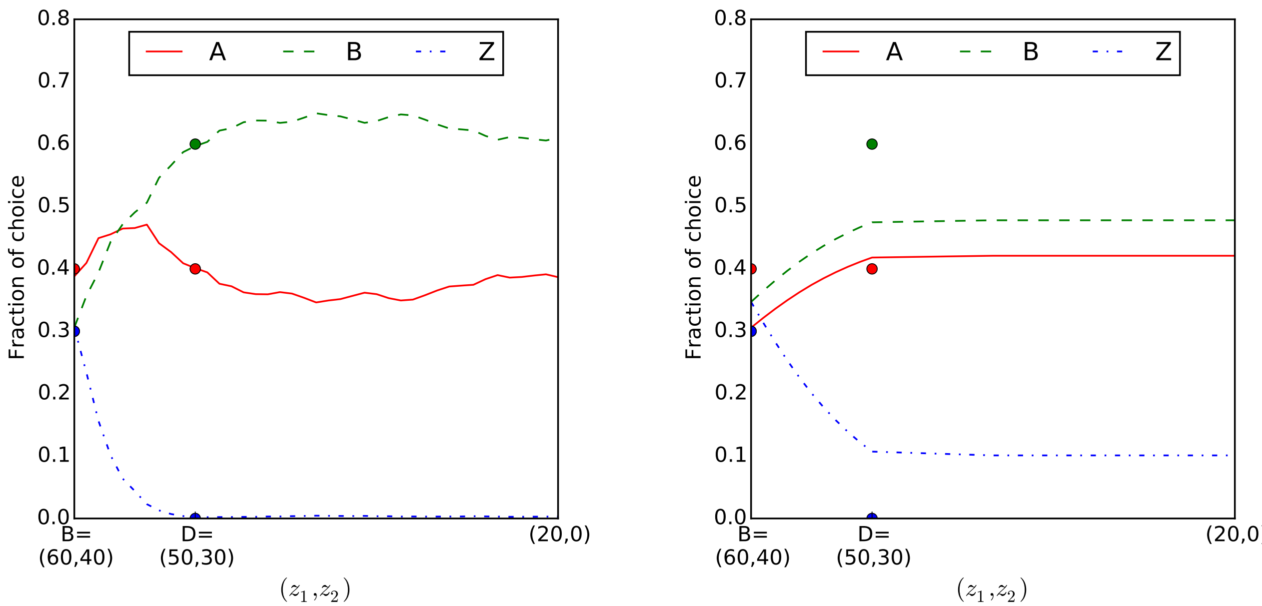


(a) DCM



(b) MLM

Figure 2: Evaluation of how the DCM and MLM generalize the similarity effect and the compromise effect to unseen items. The lines show the choice probabilities predicted (a) by the DCM and (b) by the MLM, where the choice set is $\{A, B, Z\}$, and where the attributes of Z, (z_1, z_2), vary linearly from (60,40) to (100,0). The dots show the target distribution in the training data.



(a) DCM

(b) MLM

Figure 3: Evaluation of how the DCM and the MLM generalize the attraction effect to unseen items. The lines show the choice probabilities predicted (a) by the DCM and (b) by the MLM, where the choice set is $\{$A, B, Z$\}$, and where the attributes of Z, (z_1, z_2), vary linearly from (60,40) to (20,0). The dots show the target distribution in the training data.