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Research Report

Channel Precoding and Low-Density Parity-Check Codes for Magnetic Recording at High Linear Densities

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Abstract Based on the Lorentzian model of the magnetic recording channel the effect of increasing linear density is investigated. It is observed that for a fixed turbo equalization scheme, which consists of a 16-state NPML detector and a decoder for a fixed LDPC code, the gap between the performance of the turbo equalization scheme and the Shamai-Laroia bound of the channel increases substantially with increasing linear density. Two complementary methods for reducing this gap to capacity at high linear densities are considered. First, without increasing the complexity, noticeable gains can be obtained by using a $1/(1 \oplus D)$ precoder for the partial response channel. Second, selecting a longer partial response target provides substantial gains at high densities. Moreover, when both methods are applied, the two individual gains add up.

1 Introduction

Low-density parity-check (LDPC) codes are currently recognized to be the best class of codes to efficiently approach the Shannon limit on the additive white Gaussian noise (AWGN) channel [1]. LDPC codes have also been proposed for magnetic recording as outer codes in a turbo equalization scheme, where the partial response channel plays the role of the “inner code”. At low and medium linear densities, the performance of such iterative detection/decoding schemes is close to the information-theoretical limits as given by the Shamai-Laroia bound for channel capacity. Specifically, the performance of high-rate LDPC codes of moderate lengths is only at about 1.5 dB from the capacity bound [2], [3], [4].

At low and medium linear densities, the gap to capacity of the turbo equalization scheme considered is small. It is observed that this gap opens up considerably for high linear densities. Two independent methods are presented to reduce this gap at high linear densities.

2 Channel Model and System Parameters

We will use the widely adopted Lorentzian model for the magnetic recording channel. Thus, we will assume that the read-back pulse of a single transition is well approximated by a Lorentzian pulse

$$s(t) = \frac{1}{1+(\frac{2t}{\mathrm{PW}_{50}})^2},$$

where PW_{50} denotes the pulse width at half of its amplitude. The read-back signal of a sequence of recorded antipodal data symbols $x_k \in \{\pm 1\}$ is then given by

$$r(t) = \sum_k x_k[s(t-kT) - s(t-(k-1)T)] + n(t), \tag{1}$$

where T denotes the symbol duration and $n(t)$ is white noise with two-sided spectral density $N_0/2$. The *signal-to-noise ratio* (SNR) is defined as $\mathrm{SNR} = E_s/(N_0/2)$, where E_s denotes the energy of the Lorentzian pulse $s(t)$.

The unitless parameter $D_c = \mathrm{PW}_{50}/T$ is called the *channel normalized linear density*. Similarly, one defines the *user normalized linear density* $D_u = \mathrm{PW}_{50}/T_u = D_c R$ for a coded system with code rate R and user symbol duration $T_u = T/R$.

The noise-predictive maximum likelihood (NPML) detection method is based on the selection of an optimized detector target $f(D) = f_0 + f_1 D + \ldots + f_L D^L$ with spectral nulls at dc and optionally also at the Nyquist frequency [5]. In particular, detector target polynomials of the form $f(D) = (1-D)p(D)$ and $f(D) = (1-D^2)p(D)$, where $p(D) = 1 + p_1 D + \ldots + p_{L'} D^{L'}$ is a noise-whitening filter, are commonly used in practical systems and have also been considered in this paper.

To obtain a discrete-time channel the read-back signal (1) is suitably low-pass filtered and sampled, which results in a discrete-time read-back sequence

$$y_k = \sum_{i=0}^{L} x_{k-i} f_i + n'_k, \tag{2}$$

where the n'_k are Gaussian noise samples, which in general are slightly correlated.

To assess the performance of the turbo equalization scheme, we consider an information-theoretic approach as in [2], [4]. A good approximation for the uniform-input information rate of the recording channel is given by the Shamai-Laroia (SL) bound [6], [7]. Given the spectrum of the channel and the spectrum of the additive Gaussian noise, the SL bound can be computed in a straight forward manner [4],[6].

The focus of this paper is on precoding and enhanced whitening methods and, therefore, we want to minimize the effects of misequalization. Thus, we have chosen long zero forcing equalizers with 20 taps.

At the moderate $D_u = 2.6$, we have used a 5-pole Butterworth low-pass filter with cutoff frequency at Nyquist. At $D_u = 3.6$, we have used a slightly lower cutoff, namely, $f_c = 0.4$.

We have considered three high-rate LDPC codes having a code length that approximately matches the sector size. To obtain a code of rate about 8/9, we have constructed a shortened array code of dimension $K = 4131$ and length $N = 4652$ as described in [8]. It has column weight $j = 4$ and row weights $k = 35$ and 36. The second code has rate about 16/17 and is also an array code with parameters $K = 4096$, $N = 4361$, $j = 4$, and $k = 65$ and 66. The third code was provided by MacKay [9]: it is a randomly generated code of rate about 16/17 with parameters $K = 4095$, $N = 4376$, $j = 4$, and $k = 61, 62$ and 63.

The schedule for the iterative detector/decoder was selected as follows. The channel output is passed to the detector, which produces a soft-output vector. This soft-output vector is then passed to the decoder to obtain reliability information about the channel input. These reliability values are passed back to the detector as apriori information. This loop from the detector to the decoder and back to the detector will be counted as one turbo iteration. Unless specified otherwise, we have always limited the number of such turbo iterations to at most 10.

3 Precoding

At low normalized linear densities, where the dominant error event[1] is $+$, precoding does not improve but deteriorates performance as predicted in [10]. This effect is illustrated in Fig. 1, where a $1/(1 \oplus D)$ precoder has been used together with a rate-4096/4361 LDPC code. It is therefore surprising that at high linear densities a noticeable gain can be achieved by precoding the partial-response channel with $1/(1 \oplus D)$. For $D_u = 3.6$ and high-rate LDPC codes a gain of 0.25 dB is obtained by precoding. In Fig. 2, the two LDPC codes both have rates close to 16/17. The precoding gain is essentially the same for the randomly constructed code of rate 4095/4376 as well as for the array-type code of rate 4096/4361. This shows that the precoding gain does not depend on the way the LDPC code has been constructed. Note that the observed precoding gain is not in contradiction to the results in [10], because in that paper no statements are made for the case where $+ - +$ is the dominant error event.

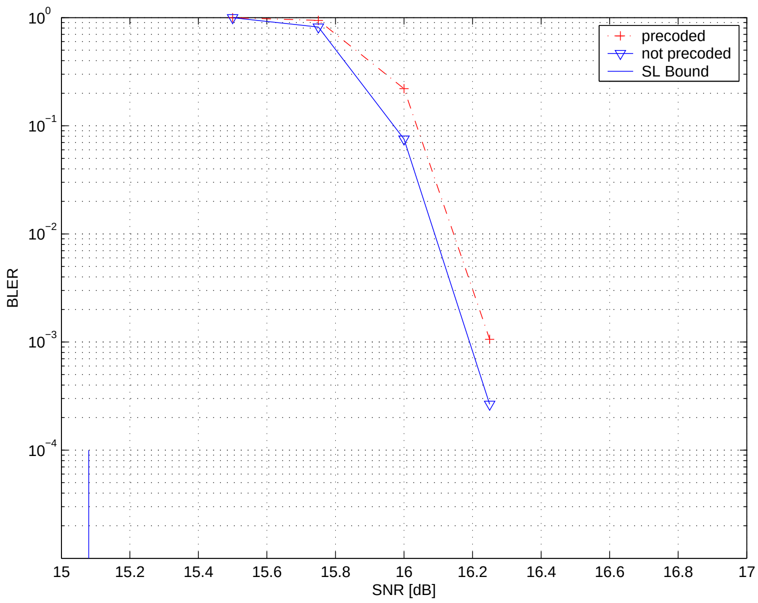

Figure 1: Block-error rate of a rate-4096/4361 LDPC code at $D_u = 2.6$ with and without precoding.

[1] For notational convenience, we only indicate error events that start with $+$. For example, $+ - +$ stands for both $+ - +$ and $- + -$.

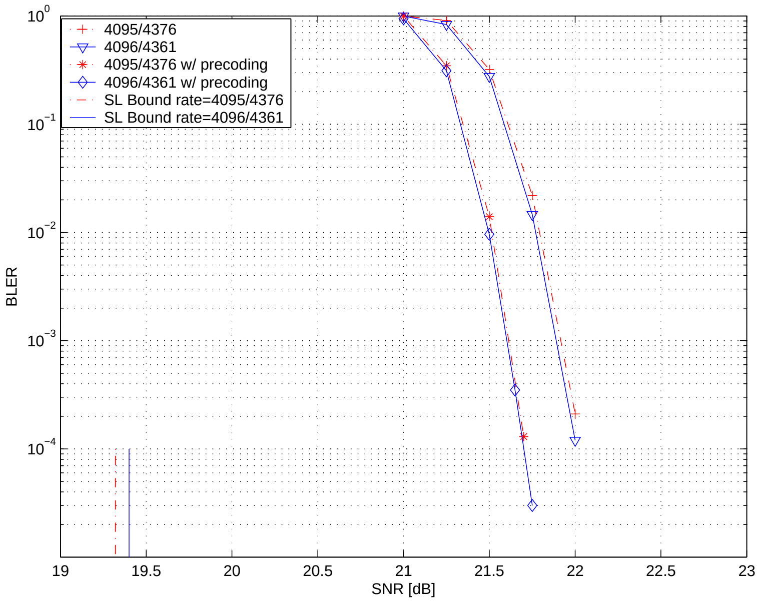

Figure 2: Block-error rate of two LDPC codes of rate 4096/4361 and 4095/4376 at $D_u = 3.6$ with and without precoding.

Fig. 2 also shows the SL bounds. At the high linear density $D_u = 3.6$, the performance of the precoded scheme at a block-error rate (BLER) of 10^{-4} is at 2.3 dB away from the SL bound; thus, this gap is much larger than the corresponding gap in Fig. 1, which is only 1.2 dB. This gap increases substantially if the code rate R is reduced and the channel density D_c increased to maintain a fixed user density D_u. For example, at $D_u = 3.6$, the gap to the SL bound is 3.1 and 3.4 dB with and without precoding, resp., for the rate-4131/4652 LDPC code with at most 10 turbo iterations (see Fig. 3).

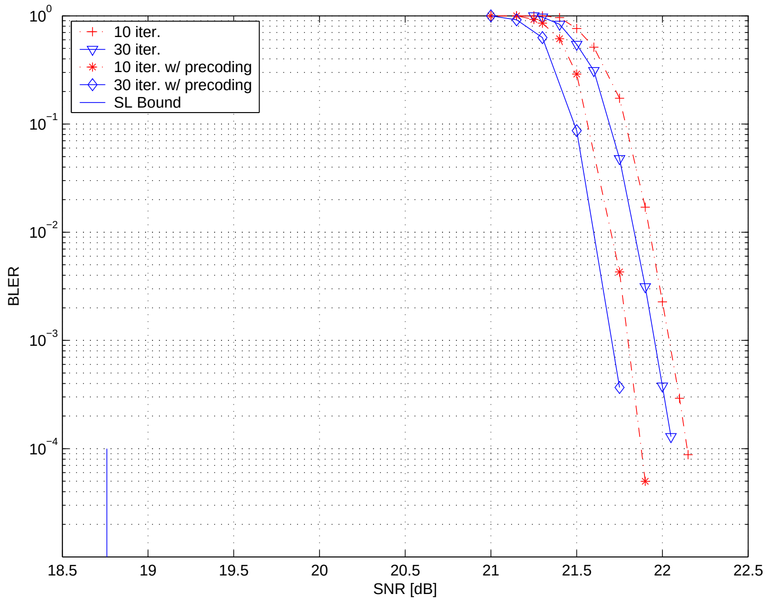

Figure 3: Block-error rate of a rate-4131/4652 LDPC code at $D_u = 3.6$ with and without precoding.

The potential precoding gain depends on the normalized linear density. For a fixed D_u, one observes larger precoding gains for LDPC codes of lower rate R because they run a higher normalized linear channel density $D_c = D_u/R$. When comparing the simulation results of the rate-4096/4361 and the rate-4131/4652 LDPC codes for the same user density $D_u = 3.6$, it is apparent that the lower-rate code has a slightly larger precoding gain, which is 0.3 dB (compare Figs. 2 and 3). Moreover, as shown in Fig. 3, increasing the maximum number of iterations from 10 to 30 in the turbo equalizer slightly improves performance by a similar amount both with and without precoding.

A heuristic explanation for the effectiveness of the precoder $1/(1 \oplus D)$ can be given in terms of dominant error events of the detector. In the case of a single error event $+$, the correct binary channel input sequence $\mathbf{b}$ and the erroneously decoded binary sequence from the detector, $\tilde{\mathbf{b}}$, differ in one bit position. When passed through the precoder inverse $1 \oplus D$, the two resulting sequences, say, $\mathbf{b}^{\text{inv}}$ and $\tilde{\mathbf{b}}^{\text{inv}}$, will differ in two consecutive bit positions. Thus, with precoding the LDPC decoder has to deal with an error pattern of two consecutive bit errors compared with a single bit error when no precoding is used.

In the case of the error event $+-+$, the binary sequences $\mathbf{b}$ and $\tilde{\mathbf{b}}$ differ in three consecutive positions and, after the precoder inverse, $\mathbf{b}^{\text{inv}}$ and $\tilde{\mathbf{b}}^{\text{inv}}$ differ in only two bit positions, which are three bits apart. Thus, with precoding the LDPC decoder has to deal with a simpler error pattern of the form 1001 rather than 111 when no precoding is used.

As LDPC codes are optimized for the AWGN channel, where the error positions are uncorrelated, one expects that an LDPC code will perform the better the shorter the number of correlated error positions. A shortcoming of this heuristic explanation is that it does not take the iterations between decoder and detector into account. However, it seems to provide a qualitative explanation for the precoder loss and gain at medium and high normalized linear density, respectively. Further investigations using the methods of [10] for the error event $+-+$ should allow one to obtain quantitative insights.

4 Enhanced Whitening

By using longer predictors $p(D)$ one can achieve better whitening of the noise at the equalizer output. The corresponding partial-response target $f(D)$ will become longer and, thus, the soft-input/soft-output detector will require a larger number of states, i.e., increased complexity. At medium linear normalized densities, increasing the detector complexity results only in minimal performance improvements. As illustrated in Fig. 4, for $D_u = 2.6$ and a rate-4131/4652 LDPC code, a SNR gain of about 0.15 dB is achieved by increasing the degree of the partial response target from 4 to 5.

However, at high normalized linear densities, the performance improvement is substantial. Specifically, using the same rate-4131/4652 LDPC code, a gain of 0.75 dB is obtained at $D_u = 3.6$ with a 32-state detector compared to a 16-state detector (see again Fig. 4). The performance of a 32-state detector together with a channel precoder (shown as dashed curve with circles in Fig. 4) gains an additional 0.3 dB over the 32-state detector scheme without precoding.

Note that the two methods to improve performance are complementary because the individual gains from precoding and enhanced whitening add up to a total gain of about 1.1 dB over the 16-state detector scheme without precoding. Combining the two methods, the initial gap to the SL bound of 3.4 dB can be reduced to 2.3 dB.

5 Conclusions and Outlook

For the high-linear-density regime, where $+-+$ is the dominant error event of the detector, two complementary methods have been considered to improve the performance of iterative detection/decoding schemes based on LDPC codes. On the one hand, precoding of the partial-response channel can achieve noticeable gains of about 0.25 to 0.3 dB at essentially no increase in complexity. On the other hand,

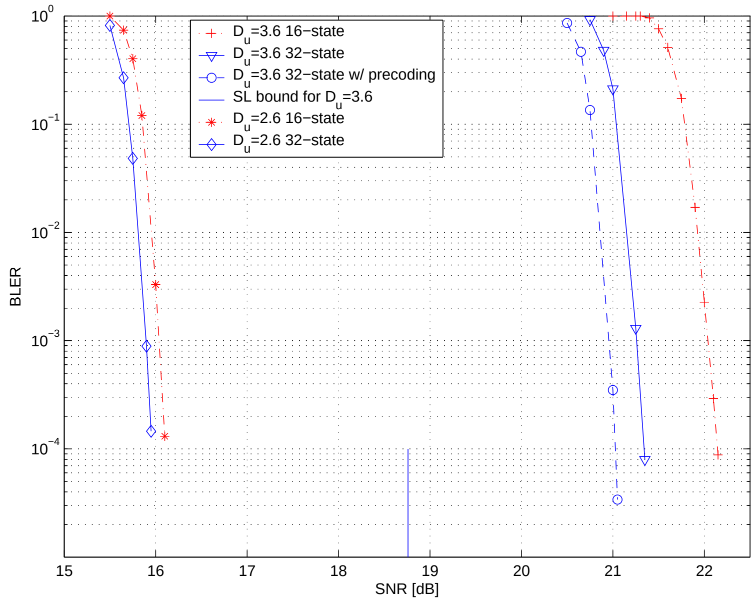

Figure 4: Block-error rate of a rate-4131/4652 LDPC code at $D_u = 2.6$ and 3.6 with 16- and 32-state detector.

substantial gains of 0.75 dB can be obtained using longer predictor filters and doubling the complexity of the detector.

It was observed that information-theoretical considerations regarding the optimum rate of a coding scheme cannot be easily transformed into practical gains in the high density regime. Capacity-based arguments, for example, show that codes of rate 8/9 should perform better than rate-16/17 codes [2]. However, when comparing the performance of the 16-state detector schemes without precoding, the two higher rate codes outperform the lower rate code (see Figs. 2 and 3) in terms of SNR. Thus, the promised gains of the lower-rate code might only be realizable if one is willing to increase the complexity of the detection/decoding scheme, as suggested by the findings shown in Fig. 4.

Future investigations might focus on alternative approaches to reducing the gap to capacity employing other or complementary methods than enhanced whitening and precoding. A possible direction could be the construction of LDPC codes that are matched to the partial-response channel rather than the AWGN channel [11].

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