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Research Report

On Interpreting Key Characteristics

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Abstract

This paper examines various interpretations of key characteristics. Starting with Juran's dictum of focusing on "vital few" and not on "trivial many", it looks at a representative sample of extant definitions and examples. Causality and sensitivity are then discussed at some length. It is argued that it is the weighted sensitivities that determine what characteristics are key and that literature on design of experiments has supported this view all along. Finally, a statistical interpretation of key characteristics is offered as a unifying framework to capture many of the recent interpretations of key characteristics.

1 Introduction

> *Managers are well aware that the numerous situations and problems they face are unequal in importance….What runs through all these phenomena is the principle of the "vital few and the trivial many." A vital few members of the assortment account for most of the total effect. The bulk of the members (the trivial many) account for very little of the total effect.*
> – Joseph M. Juran, *Quality Control Handbook*

If one is willing to assume with Juran the universality of the above "Pareto principle", it follows that only a few – call them key – characteristics of a product account for most of the customer's satisfaction with it. That he has won many converts in industry is clear from the fact that leading companies such as Boeing and General Motors have been practitioners of this idea under the rubric of "key characteristics" for nearly a decade [1,2]. Others will doubtless claim that they have been using this notion all along without explicitly calling it as such. After all, didn't Juran say "no intelligent approach to quality improvement is possible without it"[3]? What is new is the attempt by many of the companies, with the help of MIT, to agree on a set of definitions for key characteristics so that common methods and tools can be developed to practice this idea more effectively [4].

This paper aims for a unifying abstraction and a framework to interpret key characteristics. Section 2 looks at some representative definitions and examples of key characteristics from leading industrial practitioners. This is followed by a brief discussion of causality in Section 3. It sets the stage for sensitivity, which is a core concept that lies at the heart of key characteristics; it is examined in detail in Section 4. A definition of key characteristics is offered in Section 5 using the idea of weighted sensitivities. A statistical viewpoint is then presented as a unifying framework to position various interpretations of key characteristics in Section 6. Main results and conclusions are summarized in Section 7. Some of the technical details are described in an Appendix.

2 Sample of Extant Definitions and Examples

Let's start with Boeing's early definition [1].

> *Key characteristics are the features of a material, part, or process whose variation has the greatest influence on the product fit, performance, service life, or manufacturability.*

Clearly Boeing's focus is on controlling variations that are inevitable in manufacturing. A similar definition comes from General Motors [2], with added emphasis on safety and regulations.

> *A key product characteristic is a product characteristic for which reasonably anticipated variation could significantly affect the product's safety or compliance with governmental standards or regulations, or is likely to significantly affect customer satisfaction with a product.*

Northrop Grumman [5] embellishes Boeing's definition further and calls key characteristic

> *a feature of a part or assembly which has a significant negative effect upon product performance (form, fit, and function) when it varies from nominal within the specified functional tolerance zone.*

In this definition we see an attempt to be more precise in dealing with variation – it is the variation from the nominal, or, in better term, the target, while staying within a specified tolerance zone. At first reading, it is difficult to discern the significance of some of the loaded terms used in these definitions. They become clearer when placed in the right context using real life examples.

Boeing provides several interesting examples of what can and cannot be called a key characteristic [1]. We will examine two such examples; one for what is deemed a key and the other that is not a key. Boeing has chosen both these characteristics from a spoiler that is attached to the wing of an aircraft. The first is a bulb seal location.

> *The bulb seals are fastened to the front and sides of the spoiler in order to provide a good fit with the mating parts and to minimize disruption to the air flow. Discussions with factory personnel at Boeing revealed that during installation it was necessary to unfasten and relocate almost every bulb seal in order to meet engineering specifications. Because of the excessive rework, this characteristic was chosen as a key.*

To explain what is not a key characteristic, Boeing chose the contour.

The contour of the spoiler's upper surface is critical to the performance of the wing. The supplier's tooling fixtures, however, supplied excellent control of the spoiler surface and there had been no recorded problems with poor contour or misfair with the adjoining surfaces. This characteristic was not chosen as key.

The last example distinguishes key characteristics from critical characteristics. To gain better understanding of these definitions and their motives we need to look at causality and sensitivity.

3 Causality

Causality refers to the "way of knowing" that one thing causes another. A simple way to capture causality relationship is by using a causality diagram as shown in Figure 1. In general, a causality diagram is a directed, acyclic graph. The tail of each arrow is a cause for the effect to which the arrow-head points. In the simple case illustrated Figure 1, y_1 is the effect of causes x_1, x_2, x_4, and x_5. Similarly, y_2 is the effect of causes x_2, x_3, and x_5.

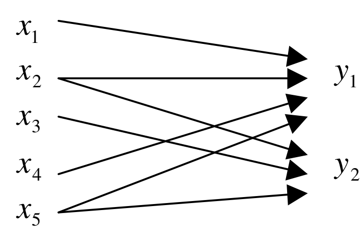


Figure 1. A simple causal diagram.

Causality is a deep scientific and philosophical concept, dating back to Aristotle. It is not easy to establish causal relationships such as the one shown in Figure 1. Often scientists define causality narrowly within the context of carefully designed and controlled experiments. In specific instances, such as an industrial product or process, considerable amount of experiments, experience, and prior knowledge may enable one to confidently claim causal relationship among various factors involved. Figure 2 is based on a Boeing example [1] for a clutch assembly and it gives a causal diagram that can be justified on the basis of that industry's knowledge. We will assume this to be our case, and thus avoid the notorious philosophical controversy associated with causality.

Directly or indirectly, designers use the notion of causality in their design process. It is debatable whether causality is necessary for all designs. It can be argued that only correlation, a weaker condition than causality, is needed for some designs. When we move from the domain of design to that of control (e.g., in manufacturing) causality assumes an important role. In quality organizations, the use of causality is very direct and prominent. Failure modes and effects analysis (FMEA) is a process of hypothesizing several potential failure scenarios and examining whether the causes for these failures have been identified and controlled. Root cause analysis (RCA) is undertaken after a failure has occurred, in the field or in the laboratory, to identify the causes so that they can be fixed in future products and processes. Causal diagrams are the commonly used representations for both FMEA and RCA.

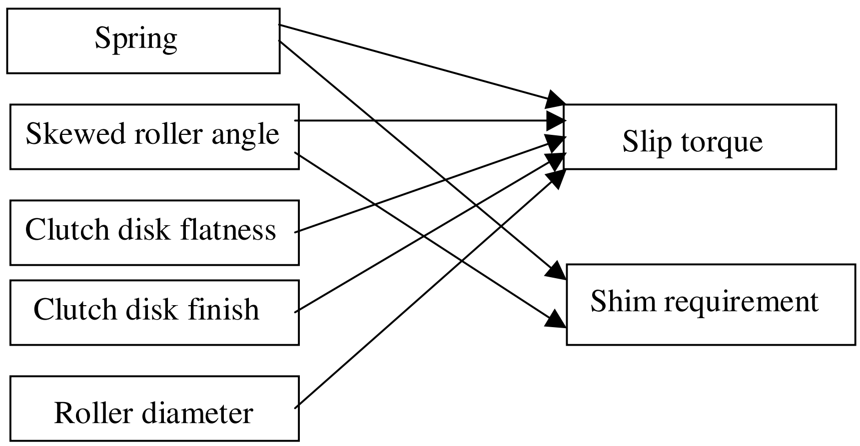


Figure 2. A causal diagram for a clutch assembly.

Causal diagrams are very effective in capturing qualitative information. We can add quantitative information to the diagram by considering the function $\mathbf{y} = F(\mathbf{x})$ that maps the vector $\mathbf{x}$ of causes to the vector $\mathbf{y}$ of effects. The actual functional form of F need not concern us here. As we will see shortly, it is only the variations in $\mathbf{x}$ and $\mathbf{y}$ that will occupy our attention. In fact, among many causes that contribute to the effects, some contribute more than others as hypothesized by Juran. This leads us directly to a consideration of sensitivity.

4 Sensitivity

Consider a variation (or, perturbation) in the cause vector $\mathbf{x}$. Our causality model indicates that this may result in a variation in the effect vector $\mathbf{y}$; let's denote this by $\Delta\mathbf{y}$. We will now try to partition each element of $\Delta\mathbf{y}$ into components, each of which is an effect due to variation in one of the causes. A simple attempt may look like the one shown in Figure 3 for one element of the vector $\Delta\mathbf{y}$. Similar "bar charts" can be constructed for other components of $\Delta\mathbf{y}$. It should be pointed out that this simple model considers only the "main effects" due to the causes - it ignores all "interactions". That is, for example, this model does not consider the interactive contribution of variations in x_2 and x_4 to Δy_1. In spite of this limitation, we will proceed with this simple model for now because of its ease of exposition. We will consider interactive effects later.

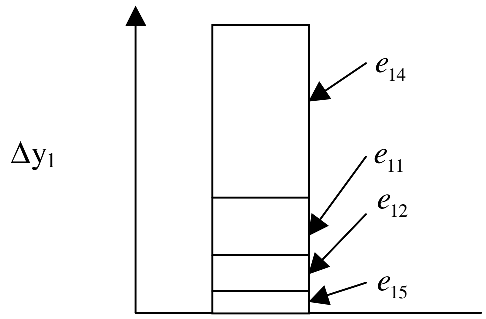


Figure 3. A simple partitioning of variation in one element of $\Delta \mathbf{y}$ into effects due to various causes. e_{ij} denotes the contribution of cause x_j to Δy_i.

A cursory look at Figure 3 seems to indicate that the cause x_4 is the major culprit, followed by others in the order indicated. Put in another way, Δy_1 seems to be most *sensitive* to variation in x_4, followed by other causes in the order shown. We can state this more precisely by considering $y_1 = f(x_1, x_2, ..., x_n)$ to be the functional relationship between the product and process characteristics x_i's (i.e., causes) and a customer satisfaction metric y_1 (i.e., an effect). Also assume that each x_i has a target value τ_i, and that f is sufficiently smooth at $(\tau_1, \tau_2, ..., \tau_n)$. Then for the variation of y_1 from its target value, we have

$$\Delta y_1 = \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + ... + \frac{\partial f}{\partial x_n}\Delta x_n + \textit{higher order terms} \qquad (1)$$

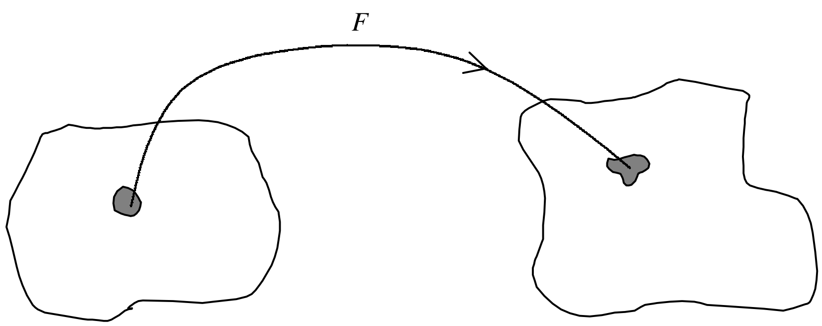
where the partial derivative $\frac{\partial f}{\partial x_i}$ is evaluated at τ_i, and $\Delta x_i = x_i - \tau_i$, $i = 1, ..., n$.

The partial derivative $\frac{\partial f}{\partial x_i}$ is the *raw* sensitivity of y_1 to x_i; that is, holding all the other characteristics constant at their target values, it is the rate of change of y_1 with respect to x_i. For many applications, such as optimization, raw sensitivity is a useful measure. It may also qualify as the measure that decides which characteristics are *critical*. In other words, a rank ordering of the *n* characteristics on the basis of the raw sensitivities will determine what are the critical characteristics. This may be the reasoning behind Boeing's consideration of the spoiler contour as being critical to wing performance. There is one caveat in this approach – the raw sensitivity is unit dependent! By changing the units of measure for the characteristics, which can be of different dimensionality, we can get vastly different results.

Problems with raw sensitivities should not concern us much because a simple observation of (1) makes it clear that it is the product $\frac{\partial f}{\partial x_i}\Delta x_i$, which equals e_{1i} in

Figure 3, that contributes to Δy_1. For a Pareto analysis, it is the *weighted* sensitivity $\frac{\partial f}{\partial x_i}\Delta x_i$ that matters. Following Juran's arguments, a rank ordering of the characteristics on the basis of their weighted sensitivities, that is $\frac{\partial f}{\partial x_i}\Delta x_i$'s, will determine what are key characteristics. Note that all the weighted sensitivities have the same unit as y_1, making their comparisons meaningful. The reasoning behind the decisions in Boeing's two examples is now clearer. In the case of the spoiler contour the variation can be controlled so well that its weighted sensitivity is not a dominant factor. On the other hand, excessive variation in the bulb location has caused its weighted sensitivity to move up in the Pareto ranking. It is also easier to see the motivation behind Northrop Grumman's focus on variation "from nominal within the specified functional tolerance zone".

Figure 4 presents an abstract view of the mapping between product and process characteristics and customer's satisfaction metrics, expressed by the functional relationship $\mathbf{y} = F(\mathbf{x})$. A finite neighborhood around the vector $\{\tau_i\}$ of target values of characteristics often maps to a finite neighborhood around a vector of customer satisfaction metrics. The nature of the mapping F at $\{\tau_i\}$, given by the partial derivatives, as well as the size of the neighborhood around $\{\tau_i\}$ determine the size of the finite neighborhood in the space of customer's satisfaction metrics.



Space of product and process characteristics Space of customer's satisfaction metrics

Figure 4. A finite neighborhood around the target values of product and process characteristics often maps to a finite neighborhood around the target in the space of customer's satisfaction metrics.

5 More on Sensitivity

Variations indicated by Δx_i's can be either actual variations or anticipated variations. In some cases we will deal with maximum allowable variations, called tolerances, which establish bounds for deviations of the characteristics from their target values. A symmetric two-sided tolerance specification results in the constraint $|\Delta x_i| \le T_i$, where T_i

is the specified tolerance for the characteristic x_i. From a design viewpoint, it would then be more appropriate to rank order the weighed sensitivities $\frac{\partial f}{\partial x_i} T_i$'s to determine key characteristics. Similar expressions can be devised when the tolerances are asymmetric or one-sided.

Weighted sensitivities are not new. One can find them in the *design of experiments* (DOE) literature where they are referred to as the "effects" [6]. By the "effect" of a characteristic (factor, in DOE terms) x_i we mean the change in the customer satisfaction metric (response, again in DOE terms) f as x_i changes from its low to high value. If we normalize each characteristic by dividing the deviation from its target by the tolerance, assuming a symmetric two-sided tolerance specification, we get a set of *normed* characteristics $X_i = \frac{x_i - \tau_i}{T_i}$, $i = 1,...,n$. Key characteristics can then be determined using the raw sensitivities to the normed characteristics because, by chain rule,

$$\frac{\partial f}{\partial X_i} = \frac{\partial f}{\partial x_i} \cdot \frac{dx_i}{dX_i} = \frac{\partial f}{\partial x_i} T_i .$$

These are precisely the "main effects" in DOE. Studies in DOE cover broader ground. It is possible to estimate the effects of *interactions* among the characteristics and include them in the decision process for determining key characteristics. This is equivalent to retaining some of the higher order terms in (1) for key characteristics determination.

If one is interested in reducing variation in some effects, we can expect to achieve this by reducing variations in their causes. But, we may not always be able to do this. If a cause is not under our control or is too expensive to control, we may not be able to reduce its variation. However, if the cause has high weighted sensitivity (that is, a key characteristic) and its variation is controllable, then we can hope to achieve effective reduction in the variation of its effect by focusing on this characteristic and reducing its variation. Figure 5 illustrates a hypothetical variation reduction strategy in which variations in all causes are reduced by half. Due to linearity in (1) after neglecting the higher order terms, reducing each Δx_i by half will reduce Δy_1 by half. In other words, a fractional change in each Δx_i will result in the same fractional change in Δy_1. Most of the reduction comes from top contributors to the variation in Δy_1; by reducing variations in these key characteristics, we can achieve most of the desired reduction in the effects.

We may now consider a general definition for key characteristics that is based on tolerances and their fractional changes:

> *Key characteristics are those whose fractional changes in specified tolerances have large adverse effect on a customer's perception of the product.*

The advantage of this definition is that it does not speak of raw and weighted sensitivities, which may be difficult ideas to incorporate in a definition. Instead it refers to tolerances and their impact on effects that customers care about – notions that can be readily related to industrial practice. The definition also admits cases where interactive effects cannot be ignored.

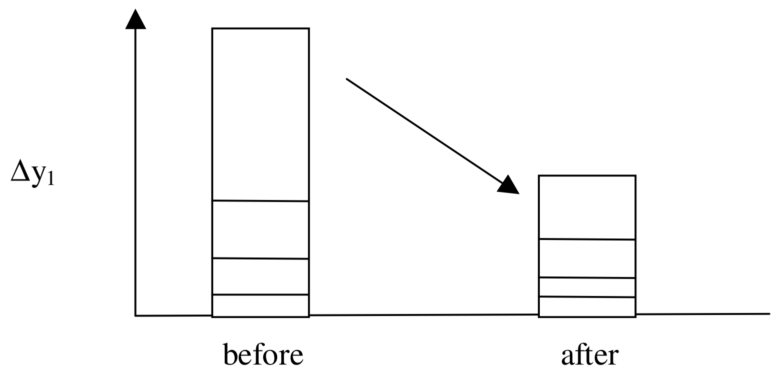


Figure 5. Reducing variation in Δy_1 by reducing variations in its causes.

6 A Statistical Viewpoint

Thus far the focus has been on what might be called the maximum deviation of a customer's satisfaction metric f from its target. From a statistical viewpoint, this is roughly equivalent to regarding only the range of variation as the relevant measure of variation. There are other statistical measures that may be more useful for some applications. Two such measures are:

1. *RMS deviation from the target*: The root-mean-square (RMS) of the deviation from the target is an interesting measure because it can be shown (see Appendix) that it is related to the average loss based on Taguchi's quadratic loss function. It has been argued [5] that this measure is preferable to the range which is sneeringly referred to as the "goal-post mentality" in some technical press. The notion of RMS deviation from the target value and the quadratic loss function is captured by the C_{pm} index currently under consideration for statistical tolerancing [7].

2. *Fraction of non-conforming items:* Another measure of interest is the fraction of the population of products for which f falls outside of some specification limits. This notion is captured by the C_{pk} index [7]. (See Appendix for some details.) This measure is popular in "six sigma" quality programs.

Either of these measures is preferable to conventional range-based measures. This leads to a further modification of our previous definition of key characteristics as follows:

> *Key characteristics are those whose fractional changes in statistical specifications have large adverse effect on a customer's perception of the product.*

7 Summary

The primary aim of this paper is to provide a unifying abstraction and a framework to position various interpretations of key characteristics. First, it is observed that it is the weighted sensitivities that determine what characteristics are key. Then it is proposed that key characteristics are those whose fractional changes in specified tolerances have large adverse effect on a customer's perception of the product. This proposal is in harmony with design of experiment studies that determine the effects of various factors on the response.

A statistical framework is offered to unify seemingly disparate interpretations of key characteristics. In this framework, one can choose (1) maximum deviation from the target, or (2) root-mean-square deviation from the target, or (3) fraction of non-conforming items, or (4) other statistical measures for the customer's satisfaction metric. Choosing the root-mean-square deviation from the target is equivalent to choosing Taguchi's quadratic loss function philosophy, while choosing the fraction of non-conforming items is in line with "six sigma" methodologies. Taking a statistical viewpoint, it is observed that key characteristics are those whose fractional changes in statistical specifications have large adverse effect on a customer's perception of the product.

Acknowledgment

The author was exposed to the current state of the art in key characteristics when he was invited by Prof. Anna Thornton to give a presentation on statistical tolerancing standards at the 2nd Key Characteristics Symposium held at the Kennedy Space Center, Florida in January, 1998. Discussions with Anna, her students, and many industrial participants in that Symposium were the prime motivators for this paper. Subsequent discussions with his colleague Michael A. O'Connor at IBM helped clarify some of the technical issues.

Appendix

A. Relationship between Taguchi's Quadratic Loss Function and C_{pm}.

Let τ be the target for a quality parameter *x*. Taguchi argues that the total "loss to society" due to deviation from the target can be captured in the cost per part as $C_0 + C_2(x-\tau)^2$ which is illustrated in Figure 6. If we have a population of parts (with probability density function *p (x),* mean μ, and standard deviation σ), the "average" cost per part is

$$\int_{-\infty}^{+\infty} \{C_0 + C_2(x-\tau)^2\} p(x) dx = C_0 + C_2\{(\mu-\tau)^2 + \sigma^2\} \qquad (2)$$

after some simple manipulation. Interestingly, this result is independent of the type of distribution of *x*. It can be seen that $(\mu-\tau)^2 + \sigma^2$, the factor multiplying C_2 in the right hand side of (2), is the mean of the square of the deviation of *x* from τ, because

$$\int_{-\infty}^{+\infty}(x-\tau)^2\,p(x)dx=(\mu-\tau)^2+\sigma^2\ .$$

Therefore, its square root is the RMS deviation from the target and hence is directly related to Taguchi's quadratic loss function.

The C_{pm} index is a non-dimensional parameter defined as

$$C_{pm}=\frac{USL-LSL}{6\sqrt{(\mu-\tau)^2+\sigma^2}} \tag{3}$$

where LSL is the lower specification limit and USL is the upper specification limit. The denominator in the right hand side of (3) is a constant multiple of the RMS deviation from the target. Therefore, setting a lower bound for C_{pm} is equivalent to setting an upper bound for the RMS deviation from the target, and hence is equivalent to setting an upper bound for the average loss to society per Taguchi's quadratic loss function.

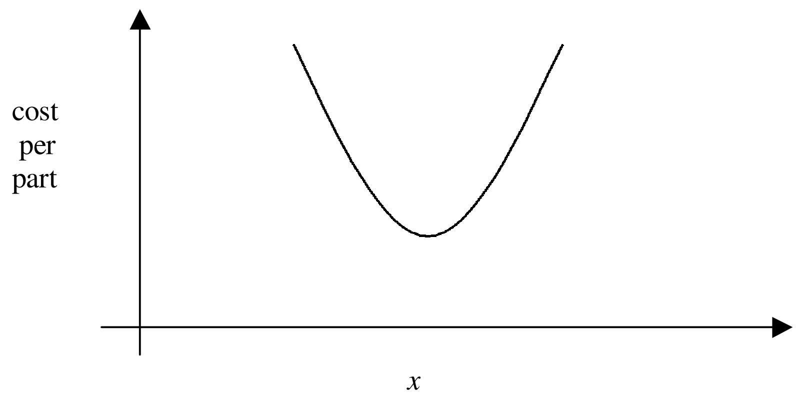


Figure 6. Cost per part according to Taguchi's quadratic loss function..

B. Relationship between Fraction of Non-conforming Items and C_{pk}.

C_{pk} is another non-dimensional parameter defined as

$$C_{pk}=\min\left(\frac{\mu-LSL}{3\sigma},\frac{USL-\mu}{3\sigma}\right). \tag{4}$$

It is a measure of "how many 3 sigmas away are the limits from the mean?" The larger this number, the less is the fraction of the population that falls outside of the interval $[LSL, USL]$ because of the powerful Chebyschev's inequality that states that the probability

$$\Pr\left[|\,x-\mu\,|>k\sigma\right]\le\frac{1}{k^2} \tag{5}$$

irrespective of the type of distribution of x. That is, the fraction of non-conforming items given by the probability

$$\Pr\left[x\notin[LSL,\ USL]\right]\le\frac{1}{9C_{pk}^2}. \tag{6}$$

For particular distributions, such as Gaussian, the bound on the right hand side of (5) and (6) can be made much smaller.

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