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Research Report

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THE GENERAL EQUILIBRIUM STRUCTURE OF BARGAINING MODELS AND MARKET EXPERIMENTS

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Abstract

We map induced supply and demand environments to equivalent general equilibrium economies. This general equilibrium structure facilitates reinterpretation of bargaining models and market experiments, typically formulated as partial equilibrium environments, in the framework of implementation in general equilibrium economies. We reinterpret results of and relationships among several bargaining models and market experiments to illustrate advantages of the delineation between environment, institution, and behavior that our formulation provides. This leads to a clearer understanding of those environments in which various forms of boundedly rational behavior will achieve efficient outcomes under several alternative exchange mechanisms.

1 Introduction

Assigned unit values and costs is a standard technique for analysis of a wide range of models of pricing and bargaining, including adverse selection (such as Akerloff [1970]), bargaining (Rubinstein [1982]), and auctions (e.g., Wilson [1985] and Rusticini, Satterthwaite, and Williams [1993]). Induced supply and demand is a powerful tool used to establish preferences in experimental markets. Smith [1982] provides a thorough description of the theory and techniques of induced supply and demand. Through use of this tool, the vast literature on experimental markets has empirically established the performance properties of different market institutions under a variety of economic environments and information structures.[1] The robustness with which the double auction generates competitive and efficient outcomes under incomplete information is striking. However, these important results have only been on the periphery of the extensive literature on informationally decentralized systems and implementation in general equilibrium economies.

There are several possible reasons why little effort has been made in the informational decentralization literature to build models that rationalize data from market experiments or that incorporate laboratory institutions. One is that induced supply and demand experiments are usually viewed as tests of partial equilibrium theory. Another likely reason is a reluctance to build models that interpret induced supply and demand as economic primitives rather than as *behavioral rules* derived from primitives such as endowments, preferences, and technologies. We construct a map from induced supply and demand environments to general equilibrium production economies to demonstrate that neither of these reasons is compelling. This general equilibrium perspective clarifies the roles of market institution, economic environment (i.e., preferences, endowments, and technologies), and individual behavior in induced supply and demand experiments. In addition, this perspective permits application (and empirical evaluation) of the extensive body of theory on decentralized resource allocation processes in private good economies to induced supply and demand experiments.

[1] For extensive surveys of the literature and results on these issues, see Plott [1982] and Smith [1982].

As a demonstration of these applications, we develop an analytic underpinning for the results by Gode and Sunder [1993] (henceforth GS) on attainment of efficient outcomes in double auctions. Through this underpinning we establish a larger class of environments in which the GS result holds, and (perhaps more importantly) we identify conditions under which their result does not hold. The focus of GS is to assess efficiency of double auction (DA) markets when buyers and sellers exhibit limited rationality. They address this question by simulating DA experiments in which computerized buyers and sellers randomly propose terms of trade. GS refer to these computerized economic agents as zero-intelligence (ZI) traders. In their most noteworthy set of simulations, buyers with induced demand schedules make random bids below their unit valuations and sellers with induced supply schedules randomly make offers above their marginal costs. In the induced supply and demand environments that GS examine, ZI behavior results in high allocative efficiency when the exchange mechanism employed is the double auction. With our general equilibrium interpretation of these environments, we show that in the GS model, a buyer randomly proposes trades in the upper contour set of her utility function and current commodity holding. Similarly, a seller only proposes trades that result in increased profits.

Under this interpretation, the result of the GS model is strikingly similar to an analytic result obtained by Hurwicz, Radner, and Reiter [1975] (henceforth HRR). The main result of HRR is that individual rationality[2] leads to Pareto optimality for a wide class of exchange and production economies in an institution called the "B-process" (for *bid* process). We then assess whether the GS results for the DA institution are as robust across economic environments as the HRR results. First, we identify three crucial differences between the mechanisms of the GS and HRR models: namely the GS model prohibits retrading, only permits contracts that satisfy a single-unit quantity restriction, and only facilitates bilateral exchange. We demonstrate that with these differences there are a variety of settings under which high efficiencies will not be attained by ZI traders in the double auctions adopted by GS, while high efficiency is guaranteed by HRR. One of these problematic settings is when

[2] Our operating definition of individual rationality, following Luce and Raiffa [1957], pp. 192-3, is that no agent attempts to take part in a trade that fails to increase, or at least leave constant, his own utility.

there are non-convexities in preferences or technologies.

We then consider the non-convexity issue in more detail in our second application by examining experiments by Van Boening and Wilcox [1996], which study behavior in a DA for environments with avoidable costs. In this second example we demonstrate that economies of scale (non-convexities) in production combined with the bilateral trading requirement of the DA institution can prevent ZI traders from achieving Pareto optimal allocations. Also, we use the HRR framework to suggest trading institution improvements that increase efficiency. It is our hope that the introduction of our map of the induced cost and value environments to general equilibrium economies and these demonstrations of its applicability spur more interaction between the experimental markets literature and theoretical work on implementation in private good economies.

2 Induced supply and demand as GE environments

In this section, we describe induced cost and value environments and then present the construction of a map from these environments to general equilibrium production economies. We begin with a demonstration that the induced supply schedule is a marginal cost curve derived from cost minimization of a well-defined production function. Following this we show that the induced demand schedule characterizes the solution to the constrained maximization of a particular quasi-linear utility function. To complete the description of the general equilibrium environment, we describe endowments for this economy which are consistent with sellers' supply schedules and buyers' demand schedules. We follow our description with an example of a map from an induced cost and value environment to a GE economy that integrates these elements.

2.1 Induced cost and valuation environments

Consider an environment that consists of a set I of sellers and a set J of buyers. Each seller's supply curve is given by his marginal cost schedule. The marginal cost schedule for seller $i \in I$ is represented by the finite vector $c_i = (c_i^1, c_i^2, c_i^3, \ldots, c_i^{m_i})$. Notice that seller i has finite selling capacity and only sells integer quantities, so that the marginal cost of any unit beyond m_i is infinite. Element c_i^k is interpreted as the marginal cost incurred by seller i when he produces his k^{th} unit. Assume that seller i makes k' transactions at prices $p_i^1, p_i^2, \ldots, p_i^{k'}$. The amount of currency received from the sale of these units is $r_i(k') = \sum_{\iota=1}^{k'} p_i^{\iota}$; the currency received from the sale of 0 units is $r_i(0) = 0$. For $k' \in \{1, 2, \ldots, m_i\}$, the total cost of selling k' units is $c_i(k') = \sum_{\iota=1}^{k'} c_i^{\iota}$; the cost of producing 0 units is $c_i(0) = 0$. Sellers' payoffs are determined as the difference between the revenue received from sales of units and the cost of producing these units, i.e.,

$$\pi_{s,i}(k') = \sum_{\iota=1}^{k'} p_i^{\iota} - \sum_{\iota=1}^{k'} c_i^{\iota} = \sum_{\iota=1}^{k'} (p_i^{\iota} - c_i^{\iota})$$

and $\pi_{s,i}(0) = 0$.

Each buyer $j \in J$ has a vector of positive valuations $v_j = (v_j^1, v_j^2, \ldots, v_j^{n_j})$ for units of the commodity Y, where $v_j^1 \geq v_j^2 \geq v_j^3 \geq \cdots \geq v_j^{n_j} > 0$. Assume that buyer j makes l' transactions at prices $p_j^1, p_j^2, \ldots, p_j^{l'}$. The value to buyer j of purchasing l' units is $v_j(l') = \sum_{\gamma=1}^{l'} v_j^{\gamma}$ for $l' \in \{1, 2, \ldots, n_j\}$. The value of purchasing $l' = 0$ units is $v_j(0) = 0$, and the value of purchasing $l' > n_j$ units is $v_j(l') = \sum_{\gamma=1}^{n_j} v_j^{\gamma}$. The amount of currency spent to obtain these units is $e_j(l') = \sum_{\gamma=1}^{l'} p_j^{\gamma}$ for $l' \in \{1, 2, 3, \ldots\}$. The expenditure on $l' = 0$ units is $e_j(0) = 0$. Buyers' payoffs are determined as the difference between the sum of the valuations for units purchased and the total expenditure on these units. Therefore, the payoff buyer j receives from the purchase of l' units is

$$\pi_{b,j}(l') = \sum_{\gamma=1}^{l'} v_j^{\gamma} - \sum_{\gamma=1}^{l'} p_j^{\gamma} = \sum_{\gamma=1}^{l'} (v_j^{\gamma} - p_j^{\gamma})$$

for $l' \in \{1, 2, 3, \ldots\}$, and $\pi_{b,j}(0) = 0$.

Example 1 Figure 1 shows a simple example of an induced supply and demand environment. In the example there is one seller and one buyer. The seller's vector of costs is $c_i = (1, 2, 3, 4)$, and the buyer's values are $v_j = (4, 3, 2, 1)$. If there is a market with N buyers and N sellers, each with the value or cost vector above, then in a competitive equilibrium two units are traded per agent at a price p^* in the range $[2, 3]$.

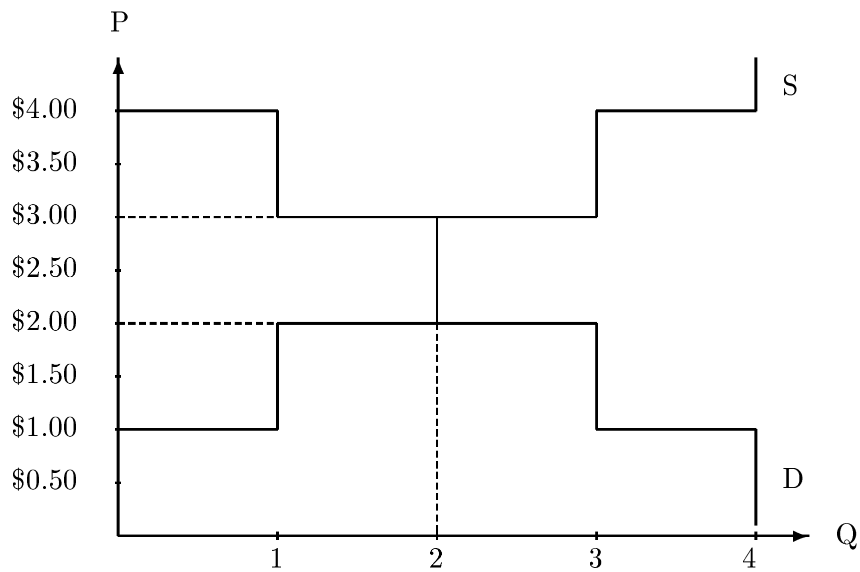

Figure 1: Supply and demand conditions for market of Example 1.

It is important to note that there are actually two goods in this economy: a currency (X) and a commodity (Y). This observation is the starting point for our construction of production and utility functions, to which we now turn.

2.1.1 Relationship between induced supply and production functions

The induced supply curve for seller i (as in figure 1, for example) is derived as the solution to the seller's profit maximization problem for a production function f_i which depends on the cost vector c_i. This production function describes the technology of seller i for transforming units of X (the currency) into units of Y (the commodity). The production function f_i, which in Lemma 1 we show is dual to the cost vector c_i, is identified as follows. For seller i with the cost vector $c_i = (c_i^1, c_i^2, c_i^3, \ldots, c_i^{m_i})$, recall that the total cost function is $c_i(y) = \sum_{\iota=1}^{y} c_i^{\iota}$ when output is $y \in \{1, 2, 3 \ldots, m_i\}$, $c_i(0) = 0$, and $c_i(y) = \infty$ for $y > m_i$. Define the production function as

$$f_i(x) \equiv \max\{k \in Y_i : c_i(k) \leq x\}, \tag{1}$$

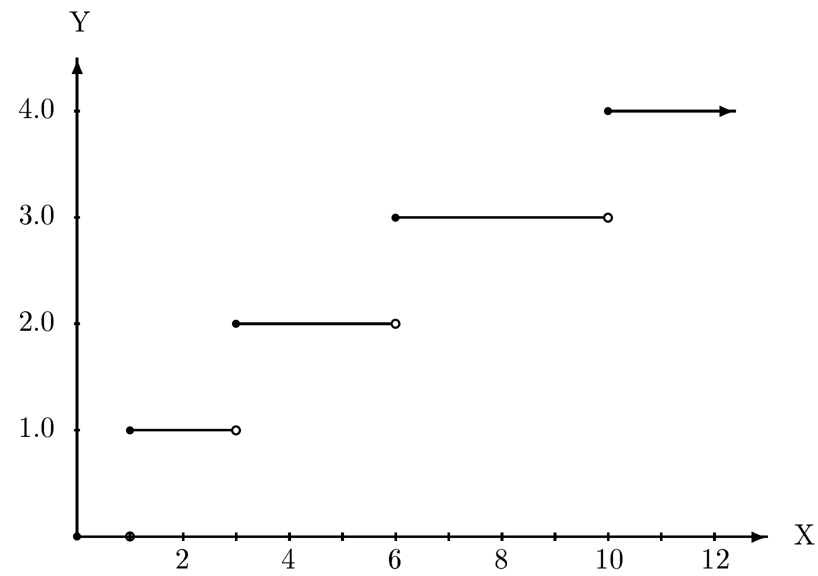

Figure 2: Production function $y = f_i(x)$ for seller i.

where $Y_i \equiv \{0, 1, 2, 3, \ldots\}$. Figure 2 shows the production function $f_i(x)$ dual to the cost vector $c_i = (1, 2, 3, 4)$ from example 1. If, for example, $x = 4.2$, then $\{k \in Y_i : c_i(k) \leq 4.2\} = \{0, 1, 2\}$, so $f_i(4.2) = 2$. This construction can be carried out for any cost vector c_i with positive elements c_i^k.

Lemma 1 If $f_i(x)$ is derived from $c_i(y)$ as in equation (1), then $c_i(y)$ is dual to $f_i(x)$.

Proof The proof is carried out in two steps. First, for the production function $f_i(x)$, define for every $y \in Y_i$, $\tilde{c}_i(y) = \min\{x \in X : f_i(x) \geq y\}$. By definition, $\tilde{c}_i(y)$ is the solution to the cost minimization problem min x subject to $f_i(x) \geq y$ for every $y \in N$. Hence, $\tilde{c}_i(y)$ is the dual of $f_i(x)$. The proof is completed by showing that $\tilde{c}_i(y) = c_i(y)$.

Suppose $x \in [c_i(y), c_i(y+1))$. Then $f_i(x) = y$. So $\{x \in X : f_i(x) \geq y\} = [c_i(y), \infty)$. Since $\tilde{c}_i(y) = \min\{x \in X : f_i(x) \geq y\}$, we get $\tilde{c}_i(y) = c_i(y)$. ■

As an example of this, consider the supply curve in figure 1. The production function $f_i(x)$ in figure 2 is obtained from the marginal cost array c_i (or supply function) in figure 1.

2.1.2 Relationship between induced demand and quasi-linear utility

In this subsection we show that induced demand functions are derived from constrained maximization of quasi-linear utility functions. In these derivations, we assume that a buyer's endowment is large enough so that her maximization problem has an interior solution.

Recall that buyer $j \in J$ has a positive vector of valuations $v_j^1 \geq v_j^2 \geq v_j^3 \geq \cdots \geq v_j^{n_j} > 0$. We assume that buyer j values any units after n_j at zero. Let y_j denote the number of units of the commodity that buyer j holds once all contracts are executed. Also, let $x_j \in X$ be the quantity of currency that buyer j holds. Define the consumption space of buyer j as $X \times Y$. Recall that the total value of purchasing y units is $v_j(y) = \sum_{\gamma=1}^{y} v_j^{\gamma}$ for $y \in \{1,\, 2\, \ldots,\, n_j\}$, $v_j(0) = 0$, and $v_j(y) = \sum_{\gamma=1}^{n_j} v_j^{\gamma}$ for $y = \{n_{j+1}, n_{j+2}, ...\}$. Finally, consider the following utility function $u_j : X \times Y \to R$:

$$u_j(x,\, y) = x + M_j + v_j(y) \tag{2}$$

where M_j is a constant. Notice that equation (2) is linear in the currency (X) and is additively separable in the two commodities.

We show that constrained maximization of equation (2) generates the demand vector v_j. Figure 3 shows three indifference curves for the utility function $u_j(x,\, y)$ that generates the demand $v_j = (4,\, 3,\, 2,\, 1)$ from the example in figure 1. Notice that the indifference curves $u_j(x,\, y) = 3$ and $u_j(x,\, y) = 5$ are horizontal translations of the indifference curve $u_j(x,\, y) = 0$, i.e., preferences are *quasi-linear.*

Lemma 2 Buyer j's demand for Y – derived from maximization of equation (2) for a sufficiently large endowment – is characterized by v_j.

Proof The vector v_j of valuations is non-increasing. By Lemma 3 of the appendix, the total valuation function $v_j(y)$ is (weakly) concave for $y \in Y_j$. Therefore the utility function $u_j(x,\, y) = x + M_j + v_j(y)$ is (weakly) quasi-concave. The theorem of the maximum therefore implies that for any given price p of good Y, the set of values that maximize $u_j(\cdot)$ is convex.

Let $y_j(p)$ be the demand of buyer j at price p, i.e., the solution to the maximization problem for $u_j(x,\, y)$. We complete the proof of the lemma by showing that the demand

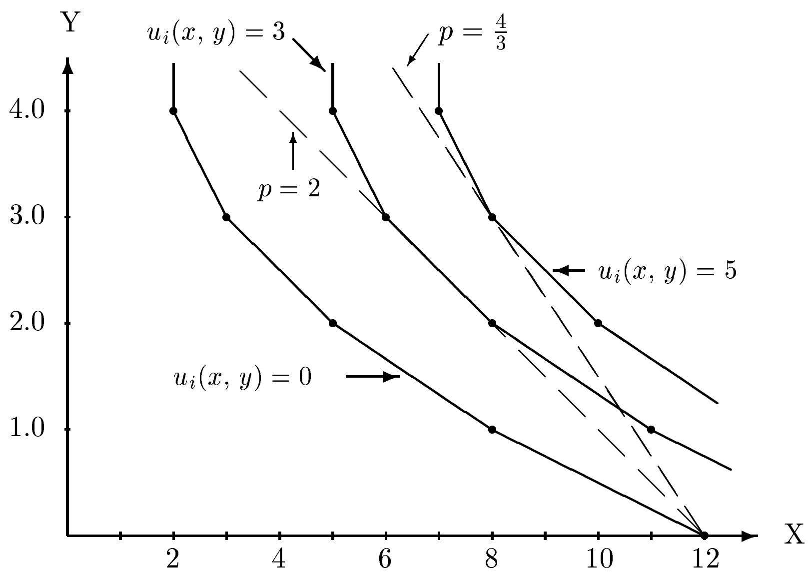

Figure 3: Indifference curves $u_i(x,y) = 0$, $u_i(x,y) = 3$, and $u_i(x,y) = 5$ for buyer in example 1.

$y_j(p)$ has the same graph as the vector v_j of values. If $p = v_j^k$, then $y_j(p) \in \{k-1, k\}$. If $p \in (v_j^k, v_j^{k+1})$, then $y_j(p) = k$, for a sufficiently large endowment. ∎

Figure 3 shows an example of both these cases[3] for a consumer with the endowment $(x^0, y^0) = (12, 0)$, the utility function dual to the vector of valuations $v_j = (4, 3, 2, 1)$ and the constant $M_j = -12$. When the price is $p = 2$ (which is equal to v_j^3) the set of utility maximizing choices of the commodity (Y) is $y_j(2) \in \{2, 3\}$. If $p = \frac{4}{3}$ then $p \in (2, 1) = (v_j^3, v_j^4)$, so the demand is $y_j(\frac{4}{3}) = 3$. The budget sets generated by these two prices are depicted in figure 3, along with the utility maximizing choice sets associated with these prices.

Our rationalization of the induced demand schedule as the solution to the constrained maximization of a quasi-linear utility function is similar to the construction by Smith [1982,

[3]In this paper the commodity subspace Y is discrete. However, in the illustrations we show a continuous extension in order to indicate which points result in equal utility levels.

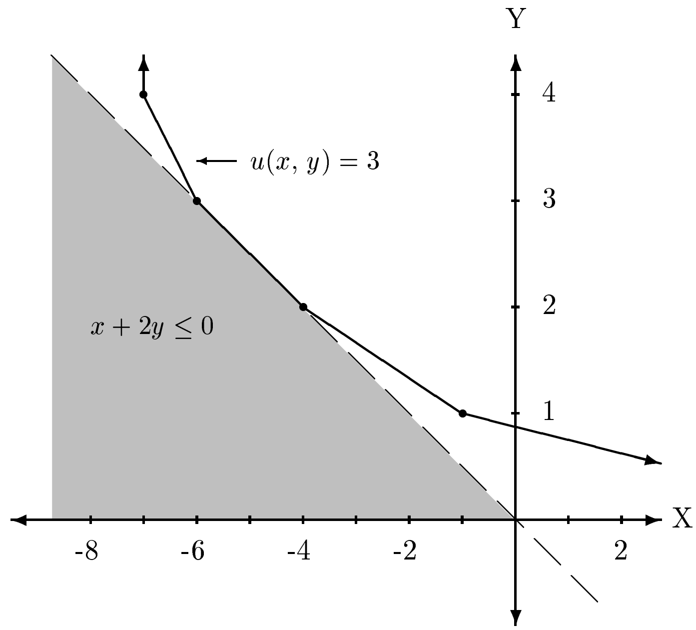

Figure 4: Smith's construction for example 1 and $p = 2$. Note the unbounded choice set.

p. 932]. Smith derives the induced demand curve by maximizing the utility function

$$u_j(x_j, y) = x_j + v_j(y_j)$$

subject to the budget constraint

$$x_j + p\,y_j \leq 0, \text{ where } x_j \leq 0 \text{ and } y_j \geq 0. \tag{3}$$

The graphical depiction of the interpretation proposed by Smith is given in figure 4. Notice that the feasibility constraints in equation (3) do not bound buyer j's choice of x_j. As a result, the choice set for buyer j is not compact and this allows buyer j to make purchases of good y at arbitrarily large prices. In contrast to Smith [1982], who takes the origin as the endowment point, we define finite positive endowments of X for buyers and sellers that are internally consistent with the manner in which economies are specified in general equilibrium models.

2.2 Endowments

Specification of initial endowments of X completes our map from induced supply and demand environments to general equilibrium economies.[4] Given the absence of income effects in preferences, initial endowments of X only need to be large enough to avoid corner solutions that would invalidate our previous derivations of induced supply and demand. However, since we have not specified an institution that governs the reallocation process it is not clear at what level of wealth such corner solutions occur. For example, in a call market (in which there is single market price) a buyer only needs to have a currency endowment equal to $x_j^0 = \max p \cdot D(p)$, where $D(p)$ is the induced demand schedule. In contrast, in a posted offer institution in which a seller could potentially employ perfect price discrimination, the currency endowment $x_j^0 = \sum_{\gamma=1}^{n_j} v_j^\gamma$ is required to guarantee that buyer j would be be able to purchase each unit for which buyer j has a positive valuation.

Sellers may also have positive endowments of X in addition to a technology for converting units of X into units of Y. A seller's endowment of X is the minimum amount of X required in addition to the amount received from the buyers that is necessary to fulfill any contract that is admissable in the adopted trading institution. For example, if the seller is restricted from selling any unit of Y for less than its marginal cost, he may have an endowment of zero units of X. On the other hand, if a seller is restricted only to sell units at a positive price then he must have an endowment of X equal to the sum of the marginal costs for each unit in order to be able to produce the units.[5] Hence, a seller can sell a unit of Y at a loss as long as he has a quantity of X large enough to cover the difference between the quantity of X received from the sale of the unit and that necessary to produce the unit.

We make one final adjustment to our specification that has no theoretical implications but is relevant to experimental studies. To make our description consistent with the typical

[4] In these production economies we always consider the case where the endowment of Y is zero.

[5] Restrictions on seller behavior based upon inherently private endowment and cost information violates a premise of informational decentralization (see Hurwicz, [1972]); however, one can easily view this enforcement as an abstract feature of the institution that would correspond to real world phenomenon such as bonding and letters of credit.

implementation of an experiment, we adjust traders' utility and profit functions according to the size of their endowments. Subjects' payments in these experiments are proportional to their utility level at the final allocation. Hence the induced utility functions of the buyers are adjusted so that payments are zero in the autarky outcome. For buyer j the constant M_j is set equal to the negative of her endowment of X, and sellers i's profit is adjusted by the negative of his endowment of X. Thus, the endowment has a utility of zero, and any participant who does not trade away from their endowment receives a payoff of zero.

2.3 Production, utility, and equilibrium

The elements described in the previous subsections combined create a general equilibrium environment that generates its dual partial equilibrium environment. In order to obtain a graphical depiction of a general equilibrium environment that corresponds to the partial equilibrium environment of figure 1, we consider the case where a single consumer owns the shares of the firm. For this situation, the general equilibrium environment is depicted in figure 5. The firm chooses its input level to maximize profit given the output price $p = p_y$. The consumer then chooses consumption of Y to maximize utility.

Note that there is a range of output prices that are consistent with competitive equilibrium, just as there is a range of CE prices when the problem is represented as an induced cost and value environment, as in example 1, which is depicted in figure 1. The lower price ratio in this CE price range ($p = \frac{p_y}{p_x} = 2$) results in a profit of $\pi = 1$ for the firm. This is easily seen as the producer surplus in figure 1. In figure 5, at the equilibrium price $\frac{p_y}{p_x} = 2$, the supporting price intersects the X axis at $x = 13$, so that the firm can achieve a profit of $\pi = 1$ by producing 2 units at a total cost of 3 and selling these units at $p = 2$. When $\frac{p_y}{p_x} = 3$, the supporting price in figure 5 intersects the X axis at $x = 15$, so that the firm can achieve a profit of $\pi = 3$. In either case, since the consumer owns the firm, the consumer's utility level is $u(x, y) = 4$: in figure 1 we see that this is the sum of producers' and consumers' surplus.

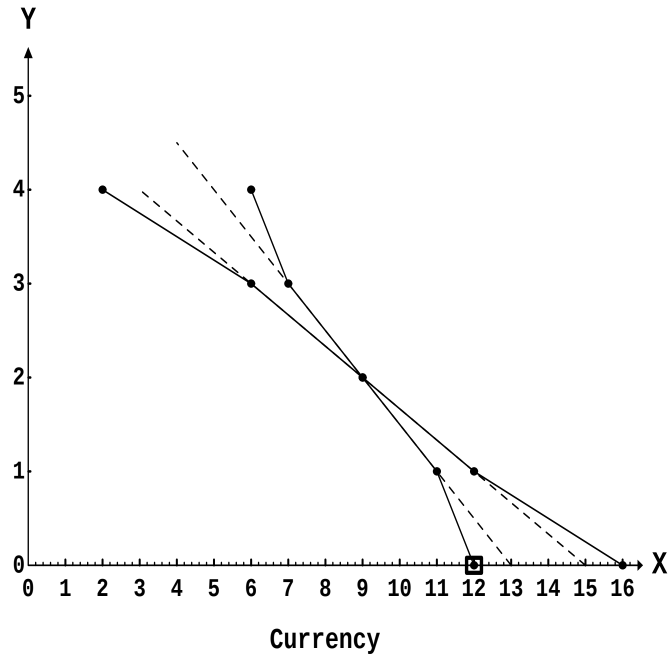

Figure 5: General equilibrium environment for example 1

3 Applications

The framework that we developed in Section 2 can be used to clarify aspects of several models of bargaining, as well as their relationships to one another. As examples of this, we examine two bargaining models, and one set of market experiments. In each of these examples, we relate the work to the other two examples in light of the GE representation discussed in the previous section.

The first application we discuss is the model by Gode and Sunder [1993] (GS) of "zero-intelligence" (ZI) traders. This model shows that the bargaining behavior of these traders, whose actions are random bids and offers in a double auction trading institution, leads to efficient outcomes. This is often considered surprising, since (according to GS) ZI agents "do not maximize or *seek* profits."[6] There are three main conclusions that we draw from the GE interpretation of the GS simulations. First, their agents exhibit individual rationality (according to the definition of Luce and Raiffa [1957]) and as a result, they do in fact seek

[6]See GS [1993, p. 120]. Emphasis added.

profits. Secondly, we compare agent behavior and the market institution that GS employ in their simulations to the results in Hurwicz, Radner, and Reiter [1975] (HRR) and show that it is in fact this profit seeking behavior that produces efficient outcomes in the GS model. Finally, we examine performance of ZI agents in environments with non-convexities and show that their result is not as general as the HRR result.

As background for development of the argument outlined in the previous paragraph, we describe more completely the double auction bargaining institution that GS employ in their simulations.

3.1 The DA institution and experimental commodities

The double auction is a decentralized trading institution in which buyers propose publicly observable bids to purchase units of a commodity and similarly sellers propose publicly observable offers. A contract occurs when either a seller accepts a buyer's bid or when a buyer accepts a seller's offer. Once time expires in the auction, each contract is executed as follows: first, the buyer transfers payment to the seller, and then the seller produces the unit and transfers it to the buyer. Then, after the execution of all contracts, buyers' and sellers' payoffs are adjusted to reflect the reallocation of resources. A buyer's payoff is adjusted by the difference between the sum of her purchased units' valuations and her total expenditure. Similarly, a seller's payoff is adjusted by the difference between the revenue he receives and the total cost of producing all of the units he sells.

After the experiment, these payoffs (in experimental currency) are converted into the national currency at a predetermined exchange rate. We assume that the utility for seller i is an increasing function of this monetary payoff.

3.2 A General Equilibrium Perspective on the "ZI" Model

The DA market simulations in GS fit perfectly into the GE structure developed in Section 2. GS report results of simulations with two primary treatment variables. We focus our attention on the treatment in which high allocative efficiency is observed: the treatment that they refer to as "budget constrained." In this treatment, each buyer has a positive valuation for a single unit, and each seller has the capacity to produce a single unit with some positive marginal cost. We define endowments for each agent after we describe the institution employed in their simulations. The institution adopted is a double auction, described above. At each moment in a market period each seller (and buyer) who has not already sold (purchased) a unit submits a random ask (bid). If an ask is submitted that is less than or equal to the highest bid, then the seller who submitted the ask sells a unit to the corresponding buyer at a price that is equal to the buyer's bid. Since each seller has a finite marginal cost for only one unit, and each buyer has a positive valuation for a single unit, this implies that once a buyer and seller become parties to a contract, neither participate in any further bargaining or contracts for that period.

Buyers in the GS model are restricted to purchase at a price at or below their valuation. Likewise, sellers are restricted in the GS model to sell at prices at or above their unit cost. According to GS, "the market *forbade* traders to buy or sell at a loss because then they would not have been able to settle their accounts." A natural interpretation of not settling accounts is that buyers will not have the endowments to purchase at the agreed upon terms, or sellers will not have the endowment and technology to complete an agreement. Viewed within the framework that we described in Section 2.2 above, the constraint that GS impose implies that each buyer has a currency endowment (commodity X) exactly equal to her unit valuation. Similarly, each seller has no endowment of X, so that a seller can only produce a unit if he receives revenue greater than or equal to his unit cost. Therefore, for each agent the set of feasible trades[7] is strictly contained within the set of individually rational trades. This ambiguity allows one to interpret the behavioral rules in either of the following ways: (1) each seller (buyer) submits random offers (bids) from their feasible sets of trades or; (2) each seller randomly proposes trades that increase his profits and each buyer proposes only those trades that lie in her upper contour sets. We adopt the second interpretation

[7] 'Feasible' trade here means that a trade is feasible for both parties given their endowments and also that the trade is permissible under the rules of the institution. For example it could be jointly feasible for a buyer to purchase two units but this would not be permissible under the rules of the adopted institution.

as it permits generalizations of the results of these simulations to other institutions and environments.

We now have a complete description of a stochastic process and a microeconomic system in which performance can be evaluated. The result of these simulations is that very high allocative efficiency is observed in all markets. Efficiency losses occur when there are trades of extra-marginal units and the institution prevents buyers from reselling a unit to another buyer with a higher valuation, or prevents a seller from sub-contracting the production of a unit to a seller with a lower marginal cost.[8] When this result was first introduced, it was considered quite surprising. However, when viewed from the general equilibrium perspective, the observed high allocative efficiencies that result when agents randomly propose trades in their upper contour sets are not new to the readers of the informationally decentralized systems literature.

3.3 The ZI model and the *B*-process

The *B*-process is a simple non-tatonnement trading institution. In a discrete world, such as the environment of a typical market experiment, random sequences of proposed trades submitted from each agent result in a sequence of net trades. An element of the sequence of net trades differs from the previous element if the corresponding realized proposals form a compatible trade (i.e. for each commodity, the net sum of proposed trades across individuals is zero). HRR show under weak conditions on preferences and technologies that if at every iteration of the bargaining process, each individual only submits trade proposals from their individually feasible and rational choice set, conditional on the current state of agreed net exchanges, then the process converges to a Pareto optimal allocation in finite time.

This result applies to a wide class of environments that includes (but is not limited to) the one we described earlier, and that GS consider. Recall that we earlier demonstrated that the GS "budget constrained" traders generate all proposed trades randomly from their individually rational and feasible choice sets. The strong similarity between the HRR and

[8] In a sequel to GS, Gode and Sunder [1996] present an analysis of how great these departures can be in their simulations.

the GS models generates optimism that the GS results are robust. For example, it would be interesting to know whether ZI behavior in a single unit sequential double auction generates Pareto optimal outcomes for any private good economy without externalities. Unfortunately this is not the case: there are many environments for which ZI behavior does not generate Pareto optimal outcomes in the DA. In fact, we have already seen that the prohibition on retrading within the DA can lead to non-Pareto optimal outcomes even in standard quasi-linear environments. Even if one wants to dismiss this scenario as unrealistic since most markets allow one to act as a buyer and seller, there are still classes of environments for which Pareto optimal outcomes are not guaranteed. These classes of environments can be identified by examining differences between the double auction adopted by GS and the B-process.

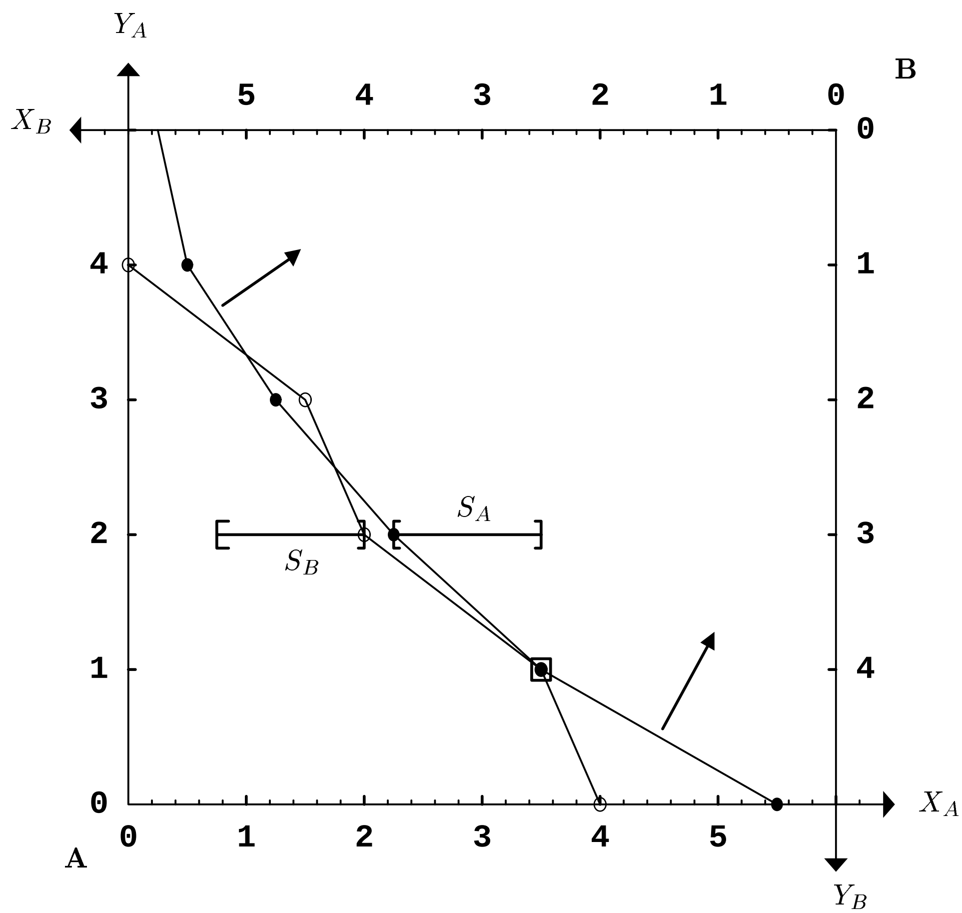

Figure 6: Edgeworth diagram with nonconvexity in preferences.

In the double auctions GS consider, each contract is for a fixed quantity of one commodity, Y, while in the B-process contracts do not have this quantity restriction. This quantity restriction can prevent convergence to a Pareto optimal allocation. An illustrative example is presented in figure 6. In this example we have a simple Edgeworth box representation of an exchange economy in which agent A (the buyer of commodity Y) has the endowment $(3.5, 1)$ and agent B has the endowment $(2.5, 4)$. These endowments are indicated by the square box in the diagram. The agents' indifference curves are presented for the endowments: A's indifference curve is marked by filled circles at its kinks and B's indifference curve is marked by the empty circles at its kinks. Notice that A's indifference curve is convex while B's has a nonconvexity. If A and B bargain through a successive single unit double auction adopting "ZI" behavior we can depict their random bid supports on the Edgeworth diagram. The set of bids for one unit of commodity Y that would increase the utility of agent A are depicted by the set S_A. The set of offers that would increase the utility of agent B are represented by the set S_B. (In the representation for consumer B, we assume, as in the GS model, that there is an upper bound on the offers that are made by a seller, although the seller would benefit from offers above this upper bound.) Since there is no overlap in these two supports no Pareto improving trade will be realized. However, a Pareto improving trade would occur if B could sell two units of Y to A for a price between $\frac{p_y}{p_x} = 1\frac{1}{8}$ and $\frac{p_y}{p_x} = 1$. Nonconvexities are common economic phenomena: for example, they are an inherent feature of the avoidable cost environments to which we now turn our attention.

3.4 Avoidable Cost Environments and Institutions

Van Boening and Wilcox [1996] (VBW hereafter) report a set of experiments with an elegant and innovative design which pairs an avoidable cost environment with a multiple unit double auction institution. Avoidable costs differ from fixed costs. A fixed cost is incurred whether or not any output is produced. An avoidable cost is zero when output is zero; the cost is incurred when there is any positive level of output. The most interesting cases of avoidable cost are when the avoidable cost is large and subsequent marginal costs are small. A simple example is a commercial airline flight. The marginal cost of the first passenger is extremely high and the marginal cost of successive passengers is very small relative to the cost of the first passenger. However, the large marginal cost of the first passenger is clearly avoided if no air passages are produced. The VBW experiments capture the spirit of this example by adopting producers with heterogeneous avoidable costs and capacity constraints, and zero marginal costs for each unit up to capacity.

These experiments provide one of the tougher "boundary tests" of the DA institution. In their experiments, efficiencies were significantly below the typical full efficiency observed in DA market experiments. Furthermore, VBW note that for certain sets of parameters ZI traders will only generate the autarky outcome. Next, we provide an example which highlights the fact that it is not only non-convexity of the production technology but also the restriction to bilateral exchange which prevents ZI traders from achieving Pareto efficient outcomes.

Consider an economy with two commodities (X and Y), two sellers (agents 1 and 2), and two buyers (agents 3 and 4). The sellers produce good Y using good X as the input. Both sellers derive utility from their respective holdings of X. The first seller has an avoidable cost of six, a production capacity of four, and zero marginal cost up to her capacity. The second seller has an avoidable cost of three, a production capacity of two, and marginal cost of zero up to his capacity. Each seller has the endowment $(x_i^0, y_i^0) = (0,0)$. Each buyer has the utility function $u_i(x,y) = x + 2y$ and the endowment $(x_i^0, y_i^0) = (4,0)$.

In this economy there are three sets of Pareto optimal allocations. In one set of Pareto optimal allocations, $\sum_{i=1}^{4} y_i = 0$ (i.e., there is no production), $x_3 + x_4 < 3$ (i.e., the buyers' combined currency aloocations is less than 3), and $\sum_{i=1}^{4} x_i = 8$. In the second set of Pareto optimal allocations seller 2 produces two units of Y, $y_3 + y_4 = 2$, $x_3 + x_4 < 3$, and $\sum_{i=1}^{4} x_i = 5$. In the final set of Pareto optimal allocations seller 1 produces four units of Y, $y_3 + y_4 = 4$, and $\sum_{i=1}^{4} x_i = 2$.

For this economy, the B-process converges to a Pareto optimal allocation that is individually rational for every agent. Of course the only set of Pareto optimal allocations that is individually rational for all agents given the endowment points is the one with a total production of four units of Y. In contrast to the performance of the B-process in this economy, ZI behavior in a single unit DA (where X is currency and Y is the commodity) results in the autarky allocation since no trade of a single unit can simultaneously increase both a seller's and buyer's utility.

As we demonstrated in the previous subsection, this result can occur with nonconvexities because of the single unit quantity restriction of the DA. This begs the following question: If we relax the single unit quantity restriction in the DA, will ZI behavior generate a Pareto optimal allocation for this economy? As long as only bilateral trade is permitted, we would have convergence to an allocation reached through a single trade of two units of Y between seller 2 and either buyer for between three and four units of X. This allocation is pairwise optimal but not Pareto optimal since the buyers' final holdings of X exceeds three.[9] Thus, the bilateral trade feature of the DA can prevent convergence to a Pareto efficient outcome. In view of these insights and those provided by HRR, we see that institutions may be identified or constructed so that efficient outcomes are achieved in avoidable cost environments that are robust to behavioral departures from full rationality. Clearly, it is desirable to have institutions which eliminate quantity constrained contracts and facilitate multilateral exchange.

4 Conclusions

The method of induced costs and values is a powerful and effective tool for *conducting* market experiments and defining bargaining models. However, our general equilibrium description is a potent tool for *interpreting* and *understanding* these bargaining models and market experiments.

Broadly viewed, there are three significant issues with models of bargaining and market

[9]For a detailed discussion on pairwise optimality and when it does imply Pareto optimality we refer the reader to Feldman [1973].

experiments addressed by the research we report. These issues fall under each of the three elements of microeconomic systems, as defined in the decentralized mechanisms literature: environment, behavior, and institution.

Interpretation of the role of *environment* is enhanced by our demonstration that a typical induced supply and demand experiment is derived from a general equilibrium production economy in which consumers have quasi-linear preferences and sellers have concave production functions.[10] This illuminates two important aspects of those models of bargaining and market experiments that employ the induced cost and value framework. First, these models and experiments implicitly treat general equilibrium environments, so that economists actually have more knowledge of the performance of microeconomic systems in general equilibrium environments than we had recognized. For example, models such as Gjerstad and Dickhaut [1998], Rustichini, Satterthwaite, and Williams [1994], and Wilson [1987] and experiments such as those described by Plott [1982] and Smith [1982] which consider buyers with unit values, and sellers with unit costs, can be viewed as tests of general equilibrium exchange, albeit in the restrictive context of quasi-linear preferences. Yet once we recognize that these bargaining models and experiments can actually be viewed as general equilibrium environments, we also realize that most of the experimental tests of bargaining and almost all of the bargaining literature treat the case of quasi-linear preferences, which exhibit no income effects. As a consequence of this observation, we are left to wonder which of the conclusions of these literatures will survive generalization to broader classes of general equilibrium environments, such as those with income effects, and non-convexities. For example, in an exchange economy with the gross substitutes property, Walrasian equilibrium is unique and globally stable (see, for example, Arrow, Block, and Hurwicz [1959]), yet we have no comparable result based on models in which prices form based on the interactions of the agents in the economy, such as a bargaining model or a market experiment. A similar situation prevails in markets with multiple Walrasian equilibria (such as Gale [1963], Gjerstad [1996], and Shapley and Shubik [1977]) and in markets with globally unstable Walrasian

[10]The production functions are only concave if their are no fixed costs and if marginal costs are increasing. We can of course use the same construction if marginal costs are not monotonic.

equilibria (as in Scarf [1960]).

Our construction clarifies aspects of agent *behavior* in bargaining models and market experiments. Examples of this include the role of individual rationality in the bid and offer choices in the ZI model, and the problems that can arise in a bilateral matching institution (such as the double auction) in economies with non-convexities. Viewed from the perspective of utility functions, it is apparent that buyers in the ZI model choose from their upper contour set. In the case of non-convexities, it is clear from examination of these same upper contour sets that Pareto optimal points may be unreachable if each trade is bilateral and needs to be utility improving. This last example is of particular interest, since it illustrates that an alternative specification of the *institution* may facilitate convergence to a Pareto optimal outcome. Specifically, if exchange involves multilateral matching and allows minimum quantities, then buyers and sellers may propose trades that permit them to "step across" a non-convex portion of their utility or production set, such as the one exhibited by consumer B in Figure 6.

We believe that our general equilibrium description provides a vital link that will facilitate dialog between those investigating behavior in market institutions via experimental techniques and theoretical models of bargaining and those studying theoretical implementation in informationally decentralized institutions. Smith [1982] noted that these independent research efforts address similar problems from different perspectives. Our new interpretation of induced supply and demand should encourage experimentalists to discuss their results in the context of well established theoretical models as well as new models of implementation within informationally decentralized institutions. Also, the theorist can verify the behavioral accuracy of their implementation models via controlled laboratory experiments and can use existing experimental data to guide the design of future models and institutions. These efforts should increase widespread interests in these fundamental topics, generating for the relevant parties, dare we say, gains from exchange.

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APPENDIX

Lemma 3 *Let $f(i)$ be a non-increasing function on $\mathcal{N} = \{1, 2, \ldots, N\}$. Let $F(k)$ be the partial sums of $f(i)$, i.e. $F(k) = \sum_{i=1}^{k} f(i)$. Then F is a concave function on $\mathcal{N}$.*

Proof Let $m + 1 \in \mathcal{N}$ and $m + n \in \mathcal{N}$. For any k with $m + 1 \leq k \leq m + n$, the average of f on $m + 1, m + 2, \ldots, m + k$ is greater than or equal to the average of f on $m + 1, m + 2, \ldots, m + n$. That is,

$$\frac{1}{k} \sum_{i=m+1}^{m+k} f(i) \geq \frac{1}{n} \sum_{i=m+1}^{m+n} f(i),$$

so

$$\sum_{i=m+1}^{m+k} f(i) \geq \frac{k}{n} \sum_{i=m+1}^{m+n} f(i).$$

From this it follows that

$$\begin{aligned} \sum_{i=1}^{m} f(i) + \sum_{i=m+1}^{m+k} f(i) &\geq \sum_{i=1}^{m} f(i) + \frac{k}{n} \sum_{i=m+1}^{m+n} f(i) \\ \sum_{i=1}^{m+k} f(i) &\geq \frac{n-k}{n} \sum_{i=1}^{m} f(i) + \frac{k}{n} \sum_{i=1}^{m+n} f(i) \\ F(m+k) &\geq \frac{n-k}{n} F(m) + \frac{k}{n} F(m+n). \end{aligned}$$

The last inequality shows that the function F is concave on $\mathcal{N}$. ■