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# Research Report

## Learning Non-Equilibrium Beliefs, and Non-Pecuniary Payoffs in an Experimental Game

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# Learning, Non-equilibrium Beliefs, and Non-pecuniary Payoffs in an Experimental Game *

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**Abstract**

We present a parametric learning model of players' dynamic and possibly out-of-equilibrium beliefs about other players' preferences that also incorporates random utility (noise). We estimate the model using the data from the four-country ultimatum game experiments of Roth et al. (1991). We find evidence that in the US and in Israel, the estimated beliefs of proposers are stationary and out-of-equilibrium, that in Slovenia, they are in equilibrium, and that in Japan, they are out-of-equilibrium, change from period to period and move away from equilibrium over time. In Japan and in the US, the estimated proposers' beliefs are further away from the uniform prior than the estimated equilibrium beliefs. The results seem to provide support for a non-pecuniary payoff explanation in all countries.


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# 1 Introduction

Subjects' initial responses in experimental games almost always deviate from Nash equilibrium. In experiments where subjects play the same stage game repeatedly with varying opponents (to suppress repeated-game effects), such deviations often persist. Two leading approaches to modeling such behavioral patterns have emerged.

Models such as reinforcement learning, Roth and Erev (1995), belief-based learning models, e.g. Crawford (1995) and Fudenberg and Levine (1998) among others, or hybrids of the two, such as experience-weighted attraction (EWA) learning, Camerer and Ho (1999), typically impose a parametric structure on subjects' (possibly) noisy decisions. This structure depends on *observable* variables, such as past payoffs and players' own and opponents' past decisions. The *observable* variables are then used to infer the values of *unobservables*, such as propensities in reinforcement-based, beliefs in belief-based, or attractions in experience-weighted attraction models.[1] The dynamics of decisions or strategies are modeled without imposing equilibrium (neither in the stage game nor in the entire game that describes the entire learning process), though in many cases the models imply that behavior converges to equilibrium with sufficiently many repetitions, as subjects' behavior often does as well. In belief-based models, the dynamics of subjects' deviation from equilibrium are explained by differences in their beliefs about each others' responses.

Quantal response equilibrium models, McKelvey and Palfrey (1995, 1998), by contrast, assume that subjects' behavior in each stage game is a noisy but approximately payoff-maximizing response to the distribution of others' behavior, imposing equilibrium in a game in which players take others' decision

[1] In belief-based models, common priors and beliefs in every period are backed out from the data, once one imposes a dynamic specification (fictitious-play, Cournot, etc.) of how beliefs evolve in response to other players' decisions. In reinforcement learning models, common initial propensities and propensities in every period are backed out from the data given the parametric structure (cumulative or averaged reinforcements, etc.) of how propensities evolve in response to players' own past payoffs. The parameters of the dynamic specification are then estimated from the data.

noise into account.[2] The idea is to find out what beliefs players would need to have so that the *estimated* frequencies of their decisions (and not the *empirical* frequencies themselves) would be in equilibrium. In such models, the dynamics of subjects' decisions are explained by variations of the level of noise in players' decisions, always imposing equilibrium.[3] This approach is agnostic about how *observable* variables influence players' beliefs in future periods. Although players' learning obviously depends on *observable* variables, it might also be influenced by other factors which are unobservable, such as enhanced reasoning about the game that is gathered along the way. Moreover, the information on observables might be used in a forward-looking rather than a backward-looking way, where it is assumed that history is the best predictor of the future, as the learning models mentioned above assume.

In this paper, we present a model that combines features of the quantal response equilibrium models, the dynamics of belief-based learning models, and some non-equilibrium approaches. We focus on the (dynamic) social utility non-equilibrium belief model (SUNB), a specification which allows for the different effects of non-pecuniary payoffs, random utility (heterogeneity), non-equilibrium beliefs, and dynamics. This allows us to compare the learning and quantal response equilibrium approaches evaluating the extent to which subjects' chronic deviations from equilibrium or some of its refinements in experimental games are due to deviations from equilibrium beliefs, random utility, or non-pecuniary payoffs.[4] It also allows us to test for the existence of learning under weaker hypotheses than have so far been maintained.

We re-analyze the data from Roth et al.'s (1991) four-country ultima-

[2]These models are also known as models of random *expected* utility. The decision noise is usually seen as a result of computational errors of the expected utility associated with each strategy. Anderson, Goeree, and Holt (1999) present a justification for logit equilibrium (a particular parametric specification of McKelvey and Palfrey's (1995) quantal response equilibrium) in the sense that the continuous strategies of a dynamic noisy directional learning model converge to a (continuous strategy) logit equilibrium as its steady state.

[3]McKelvey and Palfrey (1995) analyze players' decisions across subsets of periods and across all periods.

[4]Mookherjee and Sopher (1997) test different learning models with logistic choice functions in experimental constant sum games. Chen, Friedman, and Thisse (1997) present a theoretical analysis of a model where players make decisions which are probabilistic best-responses (i.e., subject to noise as in QRE) to beliefs formed according to fictitious play.

tum bargaining experiments, since this game and its data havc important advantages for our purposes. First, the ultimatum game is a two-player, two-move, perfect-information extensive-form game.[5] This allows us to focus our analysis only on the beliefs of the first mover, since modeling the beliefs of the second-mover is redundant.[6] Second, statistical tests suggest that responders' behavior is stationary, which allows us to attribute any changes in proposers' beliefs to proposers' attempts to learn responders' conditional acceptance-rejection rates and to attribute any non-equilibrium behavior of proposers to the incorrect beliefs of proposers.[7]

We extend the model in Costa-Gomes and Zauner (1999) (CG&Z) allowing for both non-equilibrium and dynamic behavior. First, as in CG&Z we keep the assumption that players are rational, that is, their *estimated* decisions are best responses to some beliefs that they have, and allow for a more general utility function of players. Each players' utility function al-

[5]In this game, the first mover, the proposer, proposes how to divide a sum of money with the second mover, the responder. The responder has the choice of either accepting or rejecting the proposed division of the sum of money (the pie). If the responder accepts the proposed division, then the division is as proposed by the proposer. If the responder rejects the proposed division, both players earn zero. If players only care about their own monetary earnings, and if they prefer more to less, then the proposer should offer the responder the smallest strictly positive amount of money and the responder should accept this proposed division. In contrast, results from ultimatum game experiments show that proposers offer positive amounts of money almost always averaging more than 40% of the pie, and that responders reject significant amounts of money, although less frequently as the percentage of the pie offered goes up. These findings are not very sensitive to changes of the basic experimental design (Hoffman, McCabe, and Smith (1996), and Slonim and Roth (1998)). See Guth and Tietz (1990) for a survey of the ultimatum game literature, and Crawford (1997) for a survey of game theoretic motivated experimental work.

[6]In normal-form games with two or more players, we would have to model the dynamic interaction of the non-equilibrium beliefs of players which complicates the analysis to a great extent. In this case, it would be much more difficult to determine and disentangle the effects of dynamic beliefs, non-equilibrium, and non-pecuniary payoffs. The current focus avoids these issues altogether.

[7]See the analysis of responders' behavior using logit regressions in Costa-Gomes and Zauner (1999). The results of Cooper, Feltovich, Roth, and Zwick (2000) suggest that in long repetitions of the ultimatum game (50 periods), it is possible to detect some evidence of responders' learning using a random effects model. If the behavior of the last mover changes a lot over time, the task for the first-mover is much harder because he needs to predict a "moving target" rather than a "fixed target". In such situations it would be very difficult to disentangle the roles of dynamic beliefs and decisions.

lows players' preferences to depend on whole monetary payoff profiles (or monetary allocations) and not just on players' own monetary payoffs. In addition, players' preferences contain an added noise term to account for heterogeneity as well as any stochastic elements in choice. Second, we allow for non-equilibrium and dynamic beliefs of proposers about responders' preferences, which is the main difference to CG&Z. Proposers have noisy beliefs about responders' preferences. The noise in proposers' beliefs evolves exogenously and parametrically over time. This specification allows for a wide class of dynamics of proposers' beliefs about responders' conditional acceptance probabilities over time and, at the same time, for CG&Z's interpretation. The model therefore incorporates dynamic behavior as well as non-pecuniary considerations that Abbink et al. (1998) and Binmore (1999) advocate.[8]

CG&Z study and estimate the Bayesian Nash equilibrium of a typical stage game with the preferences given above. Here, in contrast, we allow for dynamic and non-equilibrium behavior. This means, we study the deviation from the stage-game Bayesian-Nash equilibrium and the dynamics of the possibly non-equilibrium beliefs and its implied behavior. Proposers' strategies are not necessarily equilibrium strategies, since proposers are allowed to have noisy and dynamic non-equilibrium beliefs about the behavior of responders. The equilibrium beliefs of proposers are the beliefs of the stage game, and the time dependency of proposers' beliefs models the attempt of proposers to learn the behavior of responders over time. The beliefs of the proposers are dynamic and are not in equilibrium whenever the beliefs have not settled down to the stage game equilibrium beliefs. The dynamic behavior in the model stems only from the dynamics of the non-equilibrium beliefs of proposers.

We use a dynamic aggregate rather than a dynamic individual approach.[9]

[8]Note that proposers' beliefs are modeled on the social preferences of responders directly, instead of players' strategies. In the context of non-pecuniary payoffs, this is more natural and also simpler to implement than other modeling choices. Since beliefs about opponents' preferences map into beliefs about opponents' strategies, this difference is immaterial.

[9]In principle, we could use panel data techniques, for example, fixed or random effects that are applied in labor econometrics. The small data set and the matching procedure (where each proposer played only once with each responder) make it infeasible to derive and estimate the model and the equilibrium in such a case. In this paper, we content

We predict a common probability distribution over strategies for all subjects in the same role, allowing it to change from period to period in a way that is not pre-determined by history-dependent observables like, for example, past payoffs or opponents' past decisions. This approach allows us to study non-equilibrium beliefs and dynamic behavior when the number of periods in the data set is small.[10]

We apply maximum likelihood methods to estimate the different behavioral parameters of several nested models that come out of imposing restrictions on the dynamic social utility non-equilibrium belief ($SUNB$) model, using the data from Roth et al. (1991) four-country ultimatum bargaining experiments. The estimates and subsequent hypotheses tests seem to suggest that there is strong support for a non-pecuniary payoff explanation in all countries, support for dynamics in Japan, support for equilibrium behavior in Slovenia, and support for non-equilibrium behavior in the other countries (Israel, Japan, and the US). As in CG&Z, there is severe heterogeneity between countries that shows up in the estimation. However, we are able to use the estimation from pooling the data of Israel, Slovenia, and the US to predict behavior in Japan.

Other authors have explained the divergence between the standard game-theoretic solution and the observed behavior in such ultimatum game experiments using both static and dynamic models. Examples of static approaches include Bolton (1991) (social comparisons), Kirchsteiger (1994) (envy), Fudenberg and Levine (1997) (self-confirming equilibrium), Levine (1998) (spitefulness), and Engle-Warnick (2000) (binary classification trees) among others.[11] Dynamic explanations include Roth and Erev (1995) (reinforcement learning processes), Gale, Binmore, and Samuelson (1995) (evolutionary (replicator) dynamics), Prasnikar (1997) (machine learning with

ourselves with this simpler aggregate model.

[10]Reinforcement and belief-based learning models, as well as hybrids of the two are usually models of individual learning where each subject's decision in each period is represented and predicted by a probability distribution over all possible decisions, according to his history of *observable* variables that the specific model at hand considers. The parameters of the model are then estimated across the learning paths predicted for each subject.

[11]Bolton and Ockenfels (1999) and Fehr and Schmidt (1999) are static general decision theoretic approaches to self-centered and non-self-centered inequality aversion, which aim at explaining the qualitative features of a variety of experimental results.

learning direction theory).

The plan of the paper is as follows. In section 2, we describe the experimental design of Roth et al. (1991), as well as its results. In section 3, we describe the dynamics of proposers' behavior, the different models, and the hypotheses considered. In section 4, we present the estimation and hypotheses test results. In section 5, we provide the goodness-of-fit and the robustness test results of our models. In section 6, we conclude.

# 2 Roth, Prasnikar, Okuno-Fujiwara, and Zamir's (1991) Experimental Design and Results

Roth et al. (1991) designed the experiment so as to systematically address each of the main problems from conducting multinational experiments: different experimenters, different languages, and different currencies.

In this experiment, players were first assigned a role: proposers or responders. Then they played the game in their respective roles for 10 rounds, each time with a different, anonymous, and randomly selected opponent, so as to preserve the one-shot nature of the game. The pie (worth the equivalent of \$10 in terms of purchasing power in all four countries) was represented as 1000 tokens, and all offers were made in multiples of 5 tokens. Subjects were paid according to their monetary payoffs in one randomly selected round. There were three sessions in each country, which produced roughly an equal number of observations (between 270 and 300) for each country.

Although the experimental setting provided room to observe many different offers (corresponding to multiples of 5 tokens), most of them were concentrated on even hundreds (i.e., 100, 200, etc.). This characteristic of the data led us to group the observed offers in 11 categories corresponding to hundreds of tokens, a procedure also used by Roth and Erev (1995), who used 9 categories. Therefore, we discretize the ultimatum game and assume in our analysis that offers cannot be made in units smaller than 100 tokens. To be exact, we group all the offers between $Y - 49$ and $Y + 50$, where $Y = 0, 100, 200, \ldots, 1000$ and consider them to be offers of $\$Y/100$ (with the obvious change for the largest and smallest possible offers).

The possible strategies for the proposer are offering \$X, X = 0, 1, 2, ..., 10, to the responder and demanding \$(10 - X) for herself. The responder can accept or reject the offer. If the responder accepts the offer, the proposer receives \$(10 - X) and the responder \$X. If the responder rejects the offer, both earn \$0. The corresponding (discretized) extensive form game has 22 terminal nodes, with the first node (offer \$0, accept), the second (offer \$0, reject), the third (offer \$1, accept), and so on.

Figures 1a), b) c) and d) display the relative frequencies of proposers' offers period by period in each of the four countries. Table I displays the observed conditional frequencies of acceptance of particular offers (over all ten periods) by the responder, with the absolute frequencies indicated in parentheses.

Roth et al. summarized the principal patterns in the data as follows:

1. The frequencies of the offers implied by the subgame perfect equilibrium of the (discretized) game, i. e., \$0 or \$1, ranged from 1% of the time in Slovenia to 9.3% in Israel.

2. Offers were highest in the United States and Slovenia, then in Japan, and lowest in Israel.

3. The conditional frequency of rejected offers was inversely related to the fraction of the pie offered, i. e., low offers were rejected more frequently than high offers. The conditional frequencies of acceptance of offers were lowest in Slovenia, then in the United States, and highest in Israel and Japan.

4. The conditional frequency a given offer is rejected was lower in countries where lower offers were observed.

5. The differences between the empirical offer distributions between any two countries statistically increase from round 1 to round 10 (Roth et al. (1991), pp. 1086-88).

# 3 Players' Preferences and Dynamic Beliefs

We now formalize the model in detail. The utility function in CG&Z has two components: a social utility component and a stochastic component. The first component is the portion of the partners' monetary payoffs that is added or subtracted to each player's own monetary payoffs, and is called the social utility parameter. This parameter allows us to model players' altruistic, selfish, or spiteful preferences. The purpose of the second component is to account for heterogeneity in subjects' preferences and stochastic elements of subjects' choices.

More precisely, players' preferences (player $i$, $i = p$ for proposers, $i = r$ for responders), assumed to be time-independent, in the stage game, at the terminal node $k$, $k = 1, \ldots, 22$, denoted $v_{(i,k)}$, are of the form

$$v_{(i,k)} = u_{(i,k)} + a_i u_{(j,k)} + \epsilon_{(i,k)} \ , \tag{1}$$

where $u_{(i,k)}, u_{(j,k)}$ are the players' monetary payoffs, $a_i \in \Re$ is the social utility parameter, $\epsilon_{(i,k)}$ are independent (across players and across terminal nodes) random variables distributed according to normal distributions with mean 0 and variance $\sigma^2$, and $i \neq j$.[12] [13] [14]

Since the payoff function entails randomness, a Bayesian Nash analysis

[12]Our model is a model of random utility (like Zauner (1999)) and not of random *expected* utility as in McKelvey and Palfrey's (1995, 1998) quantal response equilibrium model. Anderson, Goeree, and Holt (1998) derive qualitative predictions in public goods games in a static model with altruism and random *expected* utility.

[13]We do not explicitly model intentions, which might influence players' decisions. The role of intentions is presently a matter of debate. See Bolton, Brandts, and Ockenfels (1998) and Falk, Fehr, and Fischbacher (1999) for different results.

[14]A first reading of our model would suggest that one player's regard for her opponent's monetary payoff does not depend on whether she receives more or less than her opponent, unlike the models of unfairness aversion (e.g. Fehr and Schmidt (1999)). It turns out that the four-country ultimatum data basically prevent us from using a more general formulation. In fact, proposers almost always asked for at least 50% of the pie (93.8% of the time in Japan, 97% in the US, and 100% in Israel and Slovenia), and therefore responders made their decisions in situations where they were offered less or equal than 50% of the pie. Thus, it would be impossible to estimate two values of the social utility parameter for proposers, one value corresponding to the scenario in which proposers earn less than responders and one value corresponding to the scenario in which proposers earn more than responders. A similar argument applies for responders.

is used in the game with the modified payoffs.[15] Under the assumptions of stationarity and equilibrium behavior of proposers and responders in a typical stage game, CG&Z's results suggest that while responders have negative regard for their opponents' (monetary) payoffs in all countries, proposers have negative regard for responders' (monetary) payoffs only in countries where responders are very "spiteful".

Here we propose a model that allows for out-of-equilibrium and dynamic beliefs of proposers to account for the observed behavior in the ultimatum game experiments. Instead of modeling the dynamics of the choices of players directly, we model the evolution of proposers' beliefs and investigate what those beliefs entail for the dynamics of players' behavior in the game. We jointly estimate the parameters of a dynamic specification of the motion of proposers' beliefs and the parameters of a specification of players' preferences.

## 3.1 Proposers' Beliefs

There are at least two ways to derive players' beliefs in game experiments: by having players state beliefs about their opponents' behavior when conducting the experiment or by uncovering beliefs from players' decisions. The first method has not been used extensively primarily out of the concern that asking players to state beliefs might change their decisions. Moreover, players' decisions are often not consistent with players' stated beliefs which leads to the question of the extent to which players' stated beliefs correspond to players' true beliefs.[16] An alternative method, the one we will adopt here, consists of using players' decisions *(observables)* to indirectly (and not directly as in fictitious play, or in Cournot dynamics) infer players' beliefs *(unobservables)* with the help of a theoretical model and statistical techniques. Unlike the usual belief-based models we do not model how opponents' past decisions

[15]Existence of a Bayesian Nash Equilibrium in such an extensive-form game can be shown by routine methods (see Stinchcombe and Zauner (1999)).

[16]Recent studies about belief elicitation are Nyarko and Schotter (1998) and Offerman, Sonnemans, and Schram (1996). In particular, Nyarko and Schotter provide some evidence that players are far more likely to optimize against their stated beliefs than against fictitious play or Cournot beliefs. This seems to suggest that subjects' stated beliefs are different from the beliefs that are generated by the models of belief formation currently available.

*(observables)* influence players' beliefs *(unobservables)* in the next period.

Moreover, there are several difficulties of using some belief models encountered in the literature. For example, the standard models of ficticious play or of the Cournot best response dynamics lead to the zero likelihood problem, since these models are deterministic. Adding noise parameters to these models may lead to over-fitting. Non-nested models may involve difficulties when it comes to model selection and hypothesis testing. Non-parametric tests may not be powerful enough to separate different effects or hypotheses (see section 4.2). Here, we try to avoid all those difficulties and present a belief-based model with nested restrictions that is particularly powerful in that it allows us to separate different effects that are at work in these experiments, namely, dynamics, non-equilibrium beliefs, and non-pecuniary preferences.

More specifically, we assume that proposers have beliefs about responders' preferences that are given by responders' preferences augmented by time-dependent, but not history-dependent noise. This allows the proposers to predict the responders' conditional acceptance rates for the possible offers, so that they can choose the offer that maximizes their utility given their beliefs. We assume that proposers perceive responders to behave according to

$$V_{(p,k)} = u_{(r,k)} + a_r u_{(p,k)} + \gamma_{(r,k,t)}, \tag{2}$$

where $u_{(r,k)}$ ($u_{(p,k)}$) are the responder's (proposer's) monetary payoffs at terminal node $k = 1, 2, \ldots 22$, $a_r \in \Re$ is the social utility parameter of responders, $\gamma_{(r,k,t)}$ are (across terminal nodes and time periods) independent random variables distributed according to normal distributions with mean 0 and variance $(\sigma + \theta/t^{\lambda})^2$, and $t = 1, 2, \ldots 10$ are the time periods.

Proposers' beliefs about responders' preferences differ from the true preferences of responders to the extent that proposers' beliefs about responders' preferences have a level of randomness that is higher or lower than the true level. The standard deviation of proposers' beliefs, $\sigma + \theta/t^{\lambda}$, is different from the standard deviation of responders' true preferences, $\sigma$, and this difference depends on the non-equilibrium-beliefs parameter, $\theta$, the dynamics parameter, $\lambda$, and the time period, $t$.

It is important to note the following features about this specification of beliefs. First, the dynamics parameter, $\lambda$, different from zero ($\lambda \neq 0$) implies that proposers' beliefs change with repeated play, as long as the non-equilibrium beliefs parameter, $\theta$, is different from zero ($\theta \neq 0$). Second, the non-equilibrium-beliefs parameter, $\theta$, not equal to zero ($\theta \neq 0$) implies that beliefs are not equilibrium beliefs. Third, beliefs can be out of equilibrium even if beliefs are stationary, i.e. the dynamics parameter, $\lambda$, is equal to zero ($\lambda = 0$). In other words, beliefs are out of equilibrium whenever the non-equilibrium beliefs parameter, $\theta$, is not equal to zero ($\theta \neq 0$). The term $\theta/t^{\lambda}$ measures the deviation from the equilibrium beliefs.[17]

The proposers' beliefs about the acceptance of an offer of \$X in period $t$, $Q_{(X,t)}$, is the probability that proposers' beliefs about the responders' preferences (as given above) for accepting the offer are greater than proposers' beliefs about responders' preferences for rejecting the offer, or $Pr\{X + a_r(10 - X) + \gamma_{(r,k,t)} > 0 + \gamma_{(r,k+1,t)}\}$. This can be computed as $\Phi_{2(\sigma+\theta/t^{\lambda})^2}(X + a_r(10 - X))$, where $\Phi_s(\cdot)$ is the cumulative distribution function of a normal random variable with mean 0 and variance $s$.

Results from comparative statics exercises of the parameters $\sigma$, $\theta$, and $\lambda$ show how they influence proposers' beliefs about the acceptance rates of responders.

**Proposition 1** *An increase in the variance of the noise of proposers' beliefs, $(\sigma + \theta/t^{\lambda})^2$, leads to an increased (decreased) belief in the acceptance of an offer of $X$ in time period $t$, $Q_{(X,t)}$, when the expected utility of responders from accepting it, $X + a_r(10 - X)$, is negative (positive), or equivalently, the initial belief of acceptance is below (above) 50%.*

Proof: Let $s = \sqrt{2}(\sigma + \theta/t^{\lambda}) > 0$ and $D = X + a_r(10 - X)$. Now, applying Leibnitz's rule, we have

$$d\Phi_{s^2}(D)/ds = d/ds \int_{-\infty}^{D} (1/\sqrt{2\pi s^2}) \exp[-x^2/(2s^2)]dx$$

[17] The model does not encompass fully rational Bayesian beliefs under learning that may not be stationary. To see this, note that the specification allows the noise in the beliefs of proposers to (monotonically) decline, stay constant, or to (monotonically) grow, but never to fluctuate up and down. If responders' acceptance rates fluctuate a lot over over time, then, according to the specification, proposers will not be able to learn the acceptance rates over time. Fortunately, as we have seen above, the empirical acceptance-rejection rates appear to be stationary , so that this issue does not arise with these data.

$$= \int_{-\infty}^{D} d/ds(1/\sqrt{2\pi s^2}) \exp[-x^2/(2s^2)]dx$$
$$= \int_{-\infty}^{D} \{-1/(\sqrt{2\pi}s^4)\} \exp[-x^2/(2s^2)](s^2 - x^2)dx$$
$$= -\{1/(\sqrt{2\pi}s^2)\} \exp[-D^2/(2s^2)]D$$

The derivatives can now be obtained by substitution and the chain rule. They are given in Table II.□

Proposition 1 does not depend on the normal distribution. If the random variable $\gamma_{(r,k+1,t)} - \gamma_{(r,k,t)}$ is distributed according to a logistic distribution (with mean 0 and variance parameter $\sigma + \theta/t^{\lambda}$), double exponential distribution (with mean 0 and variance parameter $\sigma + \theta/t^{\lambda}$), or a uniform distribution on $[-(\sigma + \theta/t^{\lambda}), \sigma + \theta/t^{\lambda}]$, then Proposition 1 continues to hold. Proposition 1 also holds for more general forms of mean-preserving increases in the noise of beliefs. To see this, let $f(x)$ be the density function of a random variable with mean 0 and variance $\sigma^2$ and let the distribution function $F(x)$ be differentiable. If the random variable $\gamma_{(r,k+1,t)} - \gamma_{(r,k,t)}$ is distributed according to the density function $f_{\alpha}(x) \equiv (1/\alpha)f(x/\alpha)$, where $\alpha$ is a scaling parameter, then it has mean 0 and variance $\alpha^2\sigma^2$. Since $F_{\alpha}(x) = F(x/\alpha)$, the derivative $dF_{\alpha}(x)/d\alpha = (-x/\alpha^2)f(x/\alpha)$, and Proposition 1 holds in this case. On the other hand, it is easy to construct counterexamples to Proposition 1, if *any* mean-preserving spread in the sense of Rothschild and Stiglitz (1970) in the noise of beliefs is allowed. The hyperbolic form of decay in the noise of beliefs is not essential to the results. Exponential decay would work similarly.

Increases (Decreases) of the term $\theta/t^{\lambda}$, lead proposers to believe that the acceptance rate of an offer for which the responders' expected utility $(X + a_r(10 - X))$ is negative is *higher* (*lower*) than in equilibrium. Increases (Decreases) of the term $\theta/t^{\lambda}$, lead proposers to believe that the acceptance rate of an offer for which the responders' expected utility $(X + a_r(10 - X))$ is positive is *lower* (*higher*) than in equilibrium. In summary, whenever the term $\theta/t^{\lambda}$ is positive, the acceptance rates of all offers "move closer" to random behavior (where offers are accepted or rejected with equal probability). Whenever the term $\theta/t^{\lambda}$ is negative (but in absolute value smaller than $\sigma$), proposers' beliefs "move closer" to the other extreme where they believe that responders' acceptance rates are close to 0 (1) for offers with negative (positive) expected utility of responders.

This can be easily seen as follows (a proof is given in Proposition 1). Increases in the variance of the level of noise of beliefs increase the area in the two symmetric tails of the normal distribution and decreases the area in the center (between these tails) of the normal distribution. The belief of acceptance is the area to the left of the expected utility (or cut-off level), $X + a_r X$, and below the normal distribution density function. If expected utility, $X + a_r(10 - X)$, is negative, this "acceptance" region is the area in the left tail of the normal distribution. By assumption, this area increases, the symmetric area of the right tail increases, and the area in the center decreases. In total, the beliefs in the acceptance of such offers increase. A similar argument applies for the case when the expected utility from accepting an offer is positive.

## 3.2 Responders' Decision

In this section, we derive the responders' probabilities of acceptance and rejection of proposers' offers. Since responders' preferences are given by

$$v_{(r,k)} = u_{(r,k)} + a_r u_{(p,k)} + \epsilon_{(r,k)}, \tag{3}$$

where $\epsilon_{(r,k)}$ are independent (across terminal nodes) random variables distributed according to normal distributions with mean 0 and variance $\sigma^2$, the probability of acceptance of an offer of \$X in period $t$, $q_{(X,t)}$, $Pr\{X + a_r(10 - X) + \epsilon_{(r,k)} > \epsilon_{(r,k+1)}\}$, can be computed as $\Phi_{2\sigma^2}(X + a_r(10 - X))$.

## 3.3 Proposers' Decision

The beliefs about the acceptance of each offer, $Q_{(X,t)}$ (as derived in section 3.1), allow us to compute the proposers' probabilities of offering \$X, $X = 0, 1, \ldots, 10$, in period $t$. Offering \$X in period $t$ to responders, denoted $p_{((10-X),t)}$,[18] leads to a random utility of $Q_{(X,t)}[(10 - X) + a_p X +$

[18]We suppress the dependence on $\sigma$, $\theta$, $\lambda$, $a_r$, and $a_p$ whenever convenient to save notation.

$\epsilon_{(p,\cdot)}] + (1 - Q_{(X,t)})[0 + \epsilon_{(p,\cdot)}]$, denoted $N_{((10-X),t)}$. Since $\epsilon_{(p,\cdot)}$ are i.i.d. normal random variables, the random variables $N_{((10-X),t)}$ are distributed according to normal distributions with mean $((10 - X) + a_p X)Q_{(X,t)}$ and variance $(Q^2_{(X,t)} + (1 - Q^2_{(X,t)}))\sigma^2$. The corresponding distribution (density) function is denoted by $F_{((10-X),t)}$ $(f_{((10-X),t)})$.

The probability of offering $\$X$ in period $t$, is the probability that its perturbed utility in period $t$ is higher than that of all other offers in period $t$, or

$$Pr\{N_{((10-X),t)} > \max\{N_{(10,t)}, \ldots, N_{((10-X+1),t)}, N_{((10-X-1),t)}, \ldots, N_{(0,t)}\}\} = \int_{-\infty}^{\infty} F_{(10,t)}(y) \cdots F_{((10-X+1),t)}(y) F_{((10-X-1),t)}(y) \cdots F_{(0,t)}(y) f_{((10-X),t)}(y) dy.$$

In a similar fashion, we can compute the offer probabilities for each offer $X$ and for each time period $t$, given proposers' beliefs $Q_{(X,t)}$. Whenever proposers' beliefs about the responders' acceptance rates, $Q_{(X,t)}$, are equal to responders' acceptance rates, $q_{(X,t)}$, then we have equilibrium play according to our model.

If we interpret the model along the lines of Harsanyi's (1973) randomly perturbed payoffs model, a proposer solves a (computationally) *much simpler* problem than the order statistic given here. In Harsanyi's (1973) model, players know their own payoffs, i. e. they know their own utilities in the (unperturbed) ultimatum game and the *realization of the random terms* that correspond to their *own* utilities, but they are uncertain about the other players' utilities, i. e. they know the utilities of the other players in the (unperturbed) ultimatum game and only know the *distribution of the random terms* corresponding to the utilities of the *other* players. In this case, the proposer computes the (expected) payoff of each of her strategies given the beliefs about the behavior of the responder and chooses the strategy with the highest payoff. Similarly, the responder compares the payoff of the decision "accept the offer" with the payoff of the decision "reject the offer" which are both known to her. Since the analyst of the game experiments does not observe players' utilities, he is uncertain about their payoffs, and thus has to take into account the distribution of the noise structure or heterogeneity and compute the order statistic given above, when studying their behavior.

## 3.4 Likelihood Function

The offer probabilities, $p_{(10-X,t)}$, and the acceptance [rejection] rates, $q_{(X,t)}$ $[1-q_{(X,t)}]$, induce a distribution over acceptance [rejection] terminal nodes denoted by $s_{(t,k)}(\sigma,\theta,\lambda,a_p,a_r)$, where $s_{(t,k)}$ $(\sigma,\theta,\lambda,a_p,a_r) = p_{(10-X,t)}$ $(\sigma,\theta,\lambda,a_p,a_r)$ $q_{(X,t)}$ $(\sigma,a_r)$ $[= p_{(10-X,t)}$ $(\sigma,\theta,\lambda,a_p,a_r)$ $(1-q_{(X,t)}$ $(\sigma,a_r))]$ for $k = 1,3,5,\ldots,21$ $[k = 2,4,6,\ldots 22]$, and $t = 1,2,\ldots 10$.

Given a vector of observed outcomes $n_t = (n_{(t,k)}) = (n_{(1,1)}, n_{(1,2)}, \ldots, n_{(1,22)}, n_{(2,1)}, \ldots, n_{(10,22)})$ the likelihood function is given by

$$L(\sigma,\theta,\lambda,a_p,a_r) = \Pi_{t=1}^{10}\Pi_{k=1}^{22}(s_{(t,k)}(\sigma,\theta,\lambda,a_p,a_r))^{n_{(t,k)}} \tag{4}$$

and the log-likelihood function by

$$\mathcal{L}(\sigma,\theta,\lambda,a_p,a_r) = \sum_{t=1}^{10}\sum_{k=1}^{22} n_{(t,k)} \ln s_{(t,k)}(\sigma,\theta,\lambda,a_p,a_r) \tag{5}$$

## 3.5 Estimated Models

Our benchmark model, the social utility non-equilibrium beliefs model ($SUNB$) has 5 parameters: two (average) social utility parameters, one for proposers ($a_p$), another for responders ($a_r$); a common heterogeneity or random utility parameter ($\sigma$);[19] the non-equilibrium-beliefs parameter ($\theta$) which is associated with the deviation of proposers' beliefs from equilibrium in the first period; and the dynamics parameter ($\lambda$) which influences this deviation over time. This model gives rise to several nested models once we impose restrictions on the values of some of the parameters.

1. The restriction that the dynamics parameter $\lambda = 0$, implies that proposers' beliefs about responders' behavior do not change as the game is repeated, even if these beliefs may be incorrect. It implies that the distribution of proposers' offers is the same across periods. Henceforth

[19] We could allow the heterogeneity or random utility parameter to be player-specific (see McKelvey and Palfrey (1995) p. 31). A formal test in CG&Z shows that there is no need to allow for player specific noise.

we will refer to this as the static social utility non-equilibrium beliefs model ($S - SUNB$). If the restriction $\lambda = 0$ *is rejected* the $SUNB$ model is assumed to be the benchmark model, and we test the following restrictions on it:

(a) The value of the social utility parameter does not depend on players' roles ($a_p = a_r$). This restriction leads to the common social utility non-equilibrium beliefs ($C - SUNB$) model.

(b) On average, the proposers do not care about the responders' monetary payoff ($a_p = 0$). This restriction leads to the restricted social utility non-equilibrium beliefs ($R - SUNB$) model.

2. If the restriction that the dynamics parameter $\lambda = 0$ is *not rejected*, we take the static social utility non-equilibrium beliefs model ($S - SUNB$) as the benchmark and test the following restrictions:

   (a) The value of the social utility parameter does not depend on players' roles ($a_p = a_r$). This restriction leads to the static common social utility non-equilibrium beliefs ($SC - SUNB$) model.

   (b) On average, the proposers do not care about responders' monetary payoffs $a_p = 0$. This restriction leads to the static restricted social utility non-equilibrium beliefs ($SR - SUNB$) model.[20]

   (c) Imposing the restriction $\theta = 0$, we obtain another static model, the differing social utility ($D - SU$) model, since the value of the social utility parameter depends on players' roles. In this model, proposers have equilibrium beliefs about responders' behavior, starting in the first round. In this model $\lambda$ is not identified.

In all models where $a_p = 0$, proposers are assumed to maximize their expected monetary payoff given the (either correct or incorrect) beliefs about responders' behavior. Note that it is usually assumed in the literature that

[20]In case the restriction $a_p = 0$ is not rejected either for the $SUNB$ model or in the $S - SUNB$ model, we could test whether the social utility parameters play a role in describing the data by imposing the restrictions $a_p = 0$, $a_r = 0$. This leads us to the payoff uncertainty non-equilibrium beliefs model ($PUNB$), similar to Harsanyi's (1973) model with dynamic (if the benchmark model is the $SUNB$ model) or static (if the benchmark is the $S - SUNB$ model) non-equilibrium beliefs on the side of proposers.

proposers, but not responders, maximize their expected monetary payoffs (see Roth et al. (1991) for example).

# 4 Estimation and Results

Since we consider the possibility that a dynamic model can explain the experimental data, we conduct statistical tests using ten observations per subject. We estimated the different models through grid searches using Mathematica 3.0. The estimation results are contained in Tables III-VI. Hypotheses are tested according to likelihood ratio tests which are asymptotically distributed according to chi-square distributions with degrees of freedom equal to the difference of the number of restrictions imposed on the models under comparison. The values of the test statistics as well as the corresponding $p-$values are reported in Table VII. These statistics allow us to test the hypotheses described above.[21]

## 4.1 Selected Models

The hypotheses tests detailed below select the following models as the best explanation for each country: the static common social utility non-equilibrium beliefs model ($SC-SUNB$) for Israel and the US, the (dynamic) social utility non-equilibrium beliefs model ($SUNB$) for Japan, and the common social utility model ($C-SU$) for Slovenia. There seems to be strong support for a non-pecuniary payoff explanation in all countries, support for dynamics in Japan, support for equilibrium behavior in Slovenia, and support for non-equilibrium behavior in the other countries (Israel, Japan, and the US). As in CG&Z, there is severe heterogeneity between countries that shows up in the estimation and that may be due to differences in culture, expectations or other focal phenomena.

[21] Our reported significance levels assume that observations are i.i.d.

## 4.2 Dynamics vs. Stationarity

Given the stationarity of responders' behavior in all countries, we find evidence that in Japan, proposers' behavior is dynamic whereas in Israel, Slovenia, and the US, proposers' behavior is *not* dynamic. Except for Japan (with a $p-$value of 0.00003), we cannot reject the hypothesis that proposers' beliefs about responders' behavior do not change from period to period ($H_0 : \lambda = 0$).

Note that non-parametric tests (multiple two-sample Kolmogorov-Smirnov, Page, and Friedman tests) show some evidence of changes in the empirical offer distribution over time in Israel, inconclusive evidence in the US, and no evidence of changes in Slovenia and Japan. Ordinary least squares regressions of offers on time do not reveal any dynamics (see CG&Z). The reason why the specifications above do not pick up any dynamics in Israel might be due to the fact that only the first-and-ninth and first-and-tenth period comparisons show strong significant differences in the Israeli offer distributions, and these differences might not be strong enough to show up in the (overall) estimation results.

Since we can reject the null hypothesis that the dynamics parameter $\lambda = 0$ for the Japanese data, we need to test whether proposers and responders have the same preferences regarding each others' monetary payoffs ($a_p = a_r$) and whether proposers do not care about responders' monetary payoffs ($a_p = 0$). These hypotheses are strongly rejected with $p-$values of 0.01184 and 0.00859, respectively. Thus, we cannot reject the null hypothesis that the $SUNB$ model provides the (statistically) best explanation for the Japanese data among the models considered.[22]

## 4.3 Equilibrium Beliefs vs. Non-equilibrium Beliefs

We find evidence that proposers have equilibrium beliefs in Slovenia and non-equilibrium beliefs in Israel, Japan, and the US. Using the $S-SUNB$ model as the benchmark model for Israel, Slovenia, and the US, we can reject the null hypothesis that proposers have equilibrium beliefs about responders' be-

[22]Note that for the other three countries, the same is not true. If $\lambda = 0$ had been rejected in those countries, we could not have rejected the null hypothesis that proposers and responders have the same regard for each others' monetary payoffs. However, we could always reject the null that proposers do not care about responders' monetary payoffs.

havior ($H_0 : \theta = 0$) in Israel and the US, but not in Slovenia, with $p-$values of 0.00578, 0.02847, and 0.05123, respectively. The same hypothesis is rejected for Japan, with a $p-$value of 0.01450.

The fact that we cannot reject the null hypothesis that proposers have equilibrium beliefs about responders' behavior for one of the countries is reassuring, since it shows that there is a sense in which the non-equilibrium beliefs explanation is not a result of misspecification. However, our findings that proposers have non-equilibrium beliefs in other countries does not rule out the hypothesis that players might be playing an equilibrium of an augmented game, whose form our description of the game failed to capture.

Since we can reject the null hypothesis that the non-equilibrium beliefs parameter $\theta = 0$ for Israel and the US, we need to test whether proposers and responders have the same preferences regarding each others' monetary payoffs and whether proposers do not care about responders' monetary payoffs. The first hypothesis cannot be rejected with $p-$values of 0.86939 and 0.08574. The second hypothesis is rejected with $p-$values of $1.3 \times 10^{-6}$ and $5.6 \times 10^{-30}$. Thus, among our models the $SC - SUNB$ model is the model that provides the best explanation of players' decisions in Israel and the US.[23]

Using the $D - SU$ model as the baseline for Slovenia, we test the null hypothesis that proposers and responders have the same preferences regarding each others' monetary payoffs. We cannot reject the null hypothesis with a $p-$value of 0.51932. Thus, the $C - SU$ model is selected to explain players' decisions in Slovenia.[24]

## 4.4 Model Selection and Summary

To deal with the potential issue of over-fitting when comparing nested models we employ two model selection criteria, the Bayesian Information Criterion (BIC), and the Akaike's Information Criterion (AIC). The BIC selects the same models as the log-likelihood ratio tests. The AIC selects the same

[23]Considering $SC - SUNB$ as the baseline model for Israel and the US, we can reject the null that $\theta = 0$ with $p-$values of 0.00071 and 0.00137.

[24]For all countries, we tested the hypothesis that the payoff uncertainty parameter might be player specific using the $D - SU$ model as the baseline model. We could not reject the null that they are player specific with $p-$values of 0.36, 0.11, 0.43, and 0.49, for Israel, Japan, Slovenia, and US.

models as the log-likelihood ratio tests for Israel and Japan, but selects different models for Slovenia ($SC - SUNB$) and the US ($S - SUNB$). In summary, model selection criteria yield roughly the same conclusions as the log-likelihood ratio tests. When model selection criteria and log-likelihood ratio tests lead to different models, the models selected with the latter method are more restrictive, and thus more conservative with regard to potential over-fitting issues than the model selection criteria.

In summary, the results of our estimations suggest that proposers' *estimated* beliefs about the responders' conditional acceptance rates are different from the *estimated* equilibrium conditional acceptance rates in Israel, Japan, and in the US, but not in Slovenia. These beliefs are presented in Figures 3a), b), c), and d). For Japan the proposers' beliefs about the conditional acceptance rates are shown period by period. In Japan, proposers' beliefs about responders' conditional acceptance rates move away from the *estimated* equilibrium acceptance rates as the game is repeated. They move towards beliefs that can be represented by a cut-off point below which offers would always be rejected, and above which offers would always be accepted.

In Japan and the US, proposers' beliefs about responders' conditional acceptance rates differ from the *estimated* equilibrium beliefs in the direction of a more clear-cut threshold, while in Israel they differ in the direction of a uniform 50% conditional acceptance rate.[25]

# 5 Goodness-of-Fit-Tests and Robustness

In this section, we provide goodness-of-fit and robustness tests of the selected social utility model for each country. The selected models are the $SC - SUNB$ model for Israel and the US, the $SUNB$ model for Japan, and the

[25]Weizsäcker (2000) performs an analysis using data from normal-form games where players might have incorrect beliefs about the level of decision noise of their opponents. His framework assumes that players believe that their opponents are best responding to them. The result of such fixed point argument is that each player might incorrectly believe to be playing an equilibrium of the extended game that takes into account the player's own noise and the belief about the opponent's noise level. He finds that players' beliefs about their opponents are usually closer to the uniform prior than what their opponents' decisions would suggest.

$C - SU$ model for Slovenia.

### 5.1 Goodness-of-Fit-Tests

We perform goodness-of-fit tests on the offer distributions and the expected conditional acceptance/rejection rates separately. This allows us to control for the behavior of proposers when we perform goodness-of-fit tests on the behavior of the responders. Overall, the selected models fit the observed behavior of proposers and responders very well.

We use Kolmogorov-Smirnov exact tests to assess the goodness-of-fit of the estimated offer distributions. The estimated frequencies of offers are displayed in Figures 2a), b), c), and d). The tests are carried out period by period for each country. Since the models we selected for Israel, Slovenia, and the US predict the same offer distributions across periods, we also perform (more powerful) tests on the pooled-across-periods data. The results are presented in Table VIII.

In what regards Japan and Slovenia, we are unable to reject the null hypothesis that the estimated and the empirical offer distributions in each period are equal with the lowest $p - values$ for the first and second periods, with $p-$values of 0.0777 for Japan and 0.1890 for Slovenia, respectively. For Israel and the US, we only reject the null hypothesis for the first period, with $p-$values of 0.0081, and 0.0436, respectively. For the pooled-across-periods data, we only reject the null hypothesis at a significance level of 5% for US, with a $p-$value of 0.0173.

In summary, the selected models predict proposers' behavior across the different periods well. However, in the first period the model does less well in predicting the distribution of offers. It seems that both static and dynamic models (e.g., like the ones we use, or learning models such as Roth and Erev (1995) or Camerer and Ho (1999)) have a difficulty in fitting initial play.

Next, we perform goodness-of-fit tests on the predicted conditional acceptance/rejection rates. For each possible offer, we perform an exact chi-square test for the null hypothesis that the observed and predicted acceptance rate and the observed and predicted rejection rate are equal. The alternative hypothesis is that these rates are different.

We conduct these tests for the available conditional acceptance frequency for each period. At the 5% significance level, we can only reject the null hypothesis that the predicted and observed conditional acceptance rates are the same in 5 out of 200 cases. These cases are in Israel, (period 1, offer of \$5 - $p-$value of 0.0176), Japan (period 3, offer of \$3 - $p-$value of 0.0265), and USA (period 1, offer of \$6; period 2, offer of \$1; period 5, offer of \$6 with $p-$values of 0.0352, 0.0421, and 0.0352, respectively). Apart from these cases where $p-$values are lower than 5%, the immediately higher $p-$values in each country are 0.0701 (offer of \$3, period 3) in Israel; 0.1065 (offer of \$2, period 3) in Japan; 0.0658 (offer of \$3, period 1) in Slovenia; 0.1503 (offer of \$2, period 8) in the US.

We also conduct these tests for each available conditional acceptance rate for the pooled-across-periods data. The results are presented in Table IX. The entries of the table are the $p-$values of the exact chi square tests. The predicted behavior of responders fits the data well. At the 5% significance level, there are two cases for which we can reject the null hypothesis that the predicted and observed behavior of responders are the same. These cases are in USA (offer of \$6) and in Israel (offer of \$3). In summary, the predicted behavior of responders fits the data well.

## 5.2 Robustness

To investigate the robustness of our models and to determine to what extent are the results country specific, we pool the data for the three countries with a static selected model (Israel, Slovenia, and the US), re-estimate the model, select the best model for the pooled data using log-likelihood ratio tests, and check whether the estimates for the pooled data can predict behavior in the fourth country, Japan, for which we had originally selected a dynamic model, ($SUNB$). Using the pooled data of Israel, Slovenia, and the US we select the $C-SUNB$ model.[26]

We assess the ability to predict Japan's empirical offer distributions with

[26]The estimated log-likelihoods for the models $SUNB$, $C-SUNB$, $S-SUNB$, $SC-SUNB$, $D-SU$, and $C-SU$ are $-1791.685010$, $-1792.151448$, $-1793.932373$, $-1794.323766$, $-1794.318788$, and $-1794.373005$, respectively. Log-likelihood ratio tests reject the null hypothesis that $\lambda=0$ with a $p-$value of 0.03400, but not the null hypothesis that $a_p=a_r$, with a $p-$value of 0.33412. The AIC also selects the $C-SUNB$ as the best model among the models we consider. However, the BIC selects the $SUNB$ model, which has one additional parameter.

the estimated offer distributions of the selected $C - SUNB$ model using the 3-countries-pooled data. Using Kolgomorov-Smirnov exact tests, we can only reject the null hypothesis that they are equal to the Japanese empirical offer distributions at a significance level of 5% in the first period, with a $p-$value of 0.01376.

The results for the acceptance-rejection rates are similar. Using exact chi-square tests, we fail to reject the null hypothesis that the estimated conditional acceptance-rejection rates are equal to the observed frequencies in the Japanese data at the 5 % significance level in all but 2 cases. These cases are (period 3, offer of \$2 - $p-$value of 0.0222), and (period 7, offer of \$2 - $p-$value of 0.0222). Our tests show that even though subjects' populations seem to be heterogeneous across countries, we are able to use the estimation from pooling the data of Israel, Slovenia, and the US to predict behavior in Japan.[27]

# 6 Conclusions

In this paper, we use several nested models to analyze the roles of dynamic beliefs, equilibrium, and non-pecuniary preferences in the experimental results of the four-country ultimatum bargaining experiments of Roth et al. (1991). The results seem to provide support for a non-pecuniary payoff explanation in all countries, support for dynamics in Japan, support for equilibrium behavior in Slovenia, and support for non-equilibrium behavior in the other countries (Israel, Japan, and the US). The model allow us therefore to separate the different effects of non-pecuniary payoffs, dynamics, and non-equilibrium, or other focal phenomena.

By taking into account the data from all ten periods of play, and by allowing for the possibility that proposers might have dynamic non-equilibrium beliefs about responders' conditional acceptance rates, our results add to the previous findings in CG&Z. There we found that responders have negative regard for proposers' monetary payoffs in all countries, and proposers have negative regard for responders' monetary payoffs in countries with very

[27]We thank an anonymous referee for the suggestion of performing these robustness tests.

"spiteful" responders (Slovenia and the US).

The results of this paper suggest that, except for Slovenia, proposers' estimated beliefs about responders' conditional acceptance rates are different from responders' *estimated* conditional acceptance rates, and that repeated play does not seem to change those beliefs in a statistically significant way. Although proposers' beliefs are non-equilibrium beliefs in Israel, Japan, and the US, only in Japan we find evidence that they change across periods. In the case of Japan, proposers' beliefs about responders' conditional acceptance rates move away from the *estimated* equilibrium beliefs, in the direction of a threshold below which offers are assumed to be always rejected, and above which they are always accepted.[28] [29] As a result of allowing for dynamic and non-equilibrium behavior, we have a better way of estimating players' regard for each others' monetary payoffs. We find that in all four countries both players have negative regard for each others' monetary payoffs.[30]

Our empirical approach to studying out-of-equilibrium beliefs and learning is applicable to extensive-form games in general, and in particular to games in which non-pecuniary preferences do not play a role. Put differently, our approach to modeling learning should be seen separately and independently from the modeling choice of non-pecuniary preferences. The application of our learning model to other games is left for future research.

[28]This result might indicate that our model is misspecified. However, the fact that for one of the countries we cannot reject the equilibrium beliefs hypothesis suggests that non-equilibrium beliefs are not an immediate consequence of our formulation.

[29]For such beliefs the task of proposers, with estimated negative or no regard for responders' monetary payoffs is easy: always offer the minimum acceptable offer. In this case, proposers do not have to compute the expected utility of every possible offer and select the offer with the highest expected utility.

[30]If one considers that the social utility parameters of players' $a_i$ depends on their opponents' behavior as in Segal and Sobel (1999), one cannot conclude whether players' level of regard for each other's monetary payoff is different across player-roles, and across countries or whether it is different because players are responding to their opponents' different behavior as well as to behavior that is different across countries.

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Table I: Implied (Conditional) "Acceptance" Frequencies

| Offers | USA | Slovenia | Japan | Israel |
|---|---|---|---|---|
| \$0 | 0.0 (0/2) | 0.0 (0/2) | 0.0 (0/1) | 0.0 (0/1) |
| \$1 | 20.0 (1/5) | 0.0 (0/1) | 33.3 (2/6) | 40.7 (11/27) |
| \$2 | 25.0 (3/12) | 7.7 (1/14) | 40.0 (16/40) | 61.4 (35/57) |
| \$3 | 25.0 (6/24) | 33.3 (11/33) | 50.0 (22/44) | 68.6 (48/70) |
| \$4 | 70.1 (75/107) | 68.2 (90/132) | 76.6 (85/112) | 95.5 (88/92) |
| \$5 | 92.0 (103/112) | 94.1 (111/118) | 94.2 (65/69) | 96.2 (51/53) |
| \$6 | 60.0 (3/5) | – (–) | 100.0 (8/8) | – (–) |
| \$7 | 100.0 (2/2) | – (–) | 100.0 (2/2) | – (–) |
| \$8 | – (–) | – (–) | 100.0 (1/1) | – (–) |
| \$9 | – (–) | – (–) | 100.0 (1/1) | – (–) |
| \$10 | 100.0 (1/1) | – (–) | 100.0 (6/6) | – (–) |

Table II: Comparative Statics - Proposer's Beliefs about Acceptance of Offers

| Change in Beliefs | Derivative | $D<0$ | $D>0$ |
|---|---|---|---|
| $d\Phi/d\sigma$ | $-AD$ | + | - |
| $d\Phi/d\theta$ | $-ADt^{-\lambda}$ | + | - |
| $d\Phi/d\lambda$ | $ADt^{-\lambda}\theta\ln t$ | | |
| $\theta>0$ | | - | + |
| $\theta<0$ | | + | - |
| $d\Phi/dt$ | $ADt^{-1-\lambda}\theta\lambda$ | | |
| $\theta>0,\lambda>0$ | | - | + |
| $\theta<0,\lambda>0$ | | + | - |
| $\theta>0,\lambda<0$ | | + | - |
| $\theta<0,\lambda<0$ | | - | + |

Note: $D = X + a_r(10-X)$ and $A = \exp[-D^2/(4(\sigma+\theta/t^\lambda)^2)]/\{2\sqrt{\pi}(\sigma+\theta/t^\lambda)^2\} > 0$. $\Phi$ is shorthand for the proposer's belief about the acceptance rate $\Phi_{2(\sigma+\theta/t^\lambda)^2}(D)$.

Table III: Social Utility Models: Estimation Results for Israel

| Model | SUNB | C-SUNB | R-SUNB | S-SUNB | SR-SUNB | SC-SUNB | D-SU | C-SU |
|---|---|---|---|---|---|---|---|---|
| LL | -618.49852 | -618.59300 | -629.94350 | -619.48443 | -631.22061 | -619.49795 | -623.29288 | -625.22274 |
| LL/n | -2.06166 | -2.06198 | -2.09981 | -2.06495 | -2.10407 | -2.06499 | -2.07764 | -2.08408 |
| $\hat{\sigma}$ | 1.345 | 1.305 | 1.521 | 1.291 | 1.526 | 1.278 | 1.672 | 1.670 |
| $\hat{a}_p$ | -0.232 | -0.213 | 0.0 | -0.232 | 0.0 | -0.224 | -0.282 | -0.152 |
| $\hat{a}_r$ | -0.211 | -0.213 | -0.105 | -0.224 | -0.105 | -0.224 | -0.190 | -0.152 |
| $\hat{\theta}$ | 1.091 | 1.194 | 1.134 | 0.928 | 0.297 | 0.970 | — | — |
| $\hat{\lambda}$ | 0.260 | 0.215 | 1.343 | 0.0 | — | — | — | — |

Table IV: Social Utility Models: Estimation Results for Japan

| Model | SUNB | C-SUNB | R-SUNB | S-SUNB | SR-SUNB | SC-SUNB | D-SU | C-SU |
|---|---|---|---|---|---|---|---|---|
| LL | -651.13476 | -654.30180 | -654.58800 | -659.82076 | -662.56296 | -661.74951 | -662.80921 | -673.02408 |
| LL/n | -2.24529 | -2.25621 | -2.25720 | -2.27524 | -2.28470 | -2.28190 | -2.28555 | -2.32078 |
| $\hat{\sigma}$ | 2.174 | 2.322 | 2.152 | 2.249 | 2.257 | 2.392 | 1.861 | 1.848 |
| $\hat{a}_p$ | -0.162 | -0.303 | 0.0 | -0.167 | 0.0 | -0.288 | - 0.153 | -0.334 |
| $\hat{a}_r$ | -0.275 | -0.303 | -0.217 | -0.292 | -0.199 | -0.288 | - 0.296 | -0.334 |
| $\hat{\theta}$ | -0.144 | -0.375 | -0.145 | -0.856 | -0.916 | -1.108 | — | — |
| $\hat{\lambda}$ | -0.949 | -0.593 | -0.940 | 0.0 | — | — | — | — |

Table V: Social Utility Models: Estimation Results for Slovenia

| Model | SUNB | C-SUNB | R-SUNB | S-SUNB | SR-SUNB | SC-SUNB | D-SU | C-SU |
|---|---|---|---|---|---|---|---|---|
| LL | -505.65334 | -505.85533 | -574.36165 | -505.65658 | -577.48681 | -505.86193 | -507.55702 | -507.76464 |
| LL/n | -1.68551 | -1.68618 | -1.91454 | -1.68552 | -1.92496 | -1.68621 | -1.69186 | -1.69265 |
| $\hat{\sigma}$ | 0.967 | 0.981 | 1.522 | 0.967 | 1.587 | 0.981 | 1.087 | 1.103 |
| $\hat{a}_p$ | -0.556 | -0.577 | 0.0 | -0.555 | 0.0 | -0.577 | -0.583 | -0.562 |
| $\hat{a}_r$ | -0.578 | -0.577 | -0.337 | -0.578 | -0.329 | -0.577 | -0.565 | -0.562 |
| $\hat{\theta}$ | 0.360 | 0.285 | -0.002 | 0.352 | -0.365 | 0.270 | — | — |
| $\hat{\lambda}$ | 0.021 | 0.037 | -2.534 | 0.0 | — | — | — | — |

Table VI: Social Utility Models: Estimation Results for USA

| Model | SUNB | C-SUNB | R-SUNB | S-SUNB | SR-SUNB | SC-SUNB | D-SU | C-SU |
|---|---|---|---|---|---|---|---|---|
| LL | -513.40619 | -515.17541 | -553.06863 | -514.15925 | -556.04272 | -515.63557 | -516.55884 | -522.90544 |
| LL/n | -1.90150 | -1.90806 | -2.04840 | -1.90429 | -2.05942 | -1.90976 | -1.91318 | -1.93669 |
| $\hat{\sigma}$ | 1.455 | 1.518 | 1.754 | 1.489 | 1.818 | 1.519 | 1.314 | 1.257 |
| $\hat{a}_p$ | -0.456 | -0.526 | 0.0 | -0.471 | 0.0 | -0.528 | -0.439 | -0.544 |
| $\hat{a}_r$ | -0.527 | -0.526 | -0.343 | -0.529 | -0.338 | -0.528 | -0.540 | -0.544 |
| $\hat{\theta}$ | -0.105 | -0.419 | -0.044 | 0.478 | -0.536 | -0.599 | — | — |
| $\hat{\lambda}$ | -0.752 | -0.217 | -1.257 | 0.0 | — | — | — | — |

Table VII: Hypotheses Testing - Log-Likelihood Ratio Tests

| Hypothesis | Israel | Japan | Slovenia | USA |
|---|---|---|---|---|
| $SUNB$ | | | | |
| . $H_0 : \lambda = 0$ | 1.9728 | 17.372 | 0.00648 | 1.50612 |
| $(p - value)$ | 0.16025 | 0.00003 | 0.9352 | 0.21973 |
| $SUNB$ | | | | |
| . $H_0 : a_p = a_r$ | 0.18896 | 6.33412 | 0.40398 | 3.53844 |
| $(p - value)$ | 0.66378 | 0.01184 | 0.52504 | 0.05996 |
| $SUNB$ | | | | |
| . $H_0 : a_p = 0$ | 22.88996 | 6.90650 | 137.41661 | 79.32488 |
| $(p - value)$ | $1.7 \times 10^{-6}$ | 0.00859 | $9.7 \times 10^{-32}$ | $5.3 \times 10^{-6}$ |
| $SUNB$ | | | | |
| . $H_0 : \theta = \lambda = 0$ | 9.58872 | 23.34880 | 3.80736 | 6.30532 |
| $(p - value)$ | 0.00828 | 0.00001 | 0.14902 | 0.04274 |
| $S - SUNB$ | | | | |
| . $H_0 : \theta = 0$ | 7.61690 | 5.97680 | 3.80088 | 4.79920 |
| $(p - value)$ | 0.00578 | 0.01450 | 0.05123 | 0.02847 |
| $S - SUNB$ | | | | |
| . $H_0 : a_p = a_r$ | 0.02704 | 3.85750 | 0.41070 | 2.95264 |
| $(p - value)$ | 0.86939 | 0.04952 | 0.52161 | 0.08574 |
| $S - SUNB$ | | | | |
| . $H_0 : a_p = 0$ | 23.47236 | 5.48440 | 143.66046 | 83.76694 |
| $(p - value)$ | $1.3 \times 10^{-6}$ | 0.01919 | $4.2 \times 10^{-33}$ | $5.6 \times 10^{-20}$ |
| $SC - SUNB$ | | | | |
| . $H_0 : \theta = 0$ | 11.449573 | 106.813 | 3.80534 | 14.53974 |
| $(p - value)$ | 0.00071 | $4.9 \times 10^{-25}$ | 0.051089 | 0.000137 |

Table VIII: Exact Kolmogorov-Smirnov Tests of Offer Distributions

| Periods | Israel | Japan | Slovenia | USA |
|---|---|---|---|---|
| 1 | 0.2764 (0.0081) | 0.2040 (0.0777) | 0.1613 (0.1890) | 0.2342 (0.0436) |
| 2 | 0.1531 (0.2219) | 0.1386 (0.3006) | 0.1613 (0.1890) | 0.1601 (0.2259) |
| 3 | 0.0764 (0.6712) | 0.0867 (0.6122) | 0.1279 (0.3452) | 0.0826 (0.6569) |
| 4 | 0.0846 (0.6167) | 0.1214 (0.3940) | 0.0946 (0.5503) | 0.1566 (0.2404) |
| 5 | 0.0362 (0.9018) | 0.0855 (0.6198) | 0.0510 (0.8284) | 0.0860 (0.6350) |
| 6 | 0.0955 (0.5448) | 0.1031 (0.5059) | 0.1177 (0.4043) | 0.1972 (0.1073) |
| 7 | 0.0955 (0.5448) | 0.1027 (0.5081) | 0.0400 (0.8850) | 0.0860 (0.6350) |
| 8 | 0.0513 (0.8267) | 0.0945 (0.5609) | 0.0490 (0.8393) | 0.1397 (0.3190) |
| 9 | 0.0903 (0.5791) | 0.1216 (0.3927) | 0.0400 (0.8850) | 0.0727 (0.7173) |
| 10 | 0.1236 (0.3696) | 0.1704 (0.1659) | 0.0490 (0.8393) | 0.1601 (0.2258) |
| All | 0.0680 (0.0597) | – ( – ) | 0.0546 (0.1612) | 0.0860 (0.0173) |

K-S statistic ($p - value$) .

Note: P-values are computed from equation (2), Miller (1956, p.115).

Table IX: P-Values of Exact Chi-Square Tests of Conditional Acceptance-Rejection Rates

| Offers | Israel | Japan | Slovenia | USA |
|---|---|---|---|---|
| $0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $1 | 0.2003 | 1.0000 | 1.0000 | 0.1861 |
| $2 | 0.3522 | 0.4290 | 1.0000 | 0.4068 |
| $3 | 0.0429 | 0.0618 | 0.5588 | 0.2914 |
| $4 | 0.3277 | 0.6494 | 0.5830 | 0.3110 |
| $5 | 0.2063 | 0.1369 | 0.4988 | 0.0972 |
| $6 | – | 1.0000 | – | 0.0115 |
| $7 | – | 1.0000 | – | 1.0000 |
| $8 | – | 1.0000 | – | – |
| $9 | – | 1.0000 | – | – |
| $10 | – | 1.0000 | – | 1.0000 |

Note: P-values are computed following Pierce (1970), sections 11.1-11.6.

**Figure 1a) - Empirical Distributions of Offers in Israel**

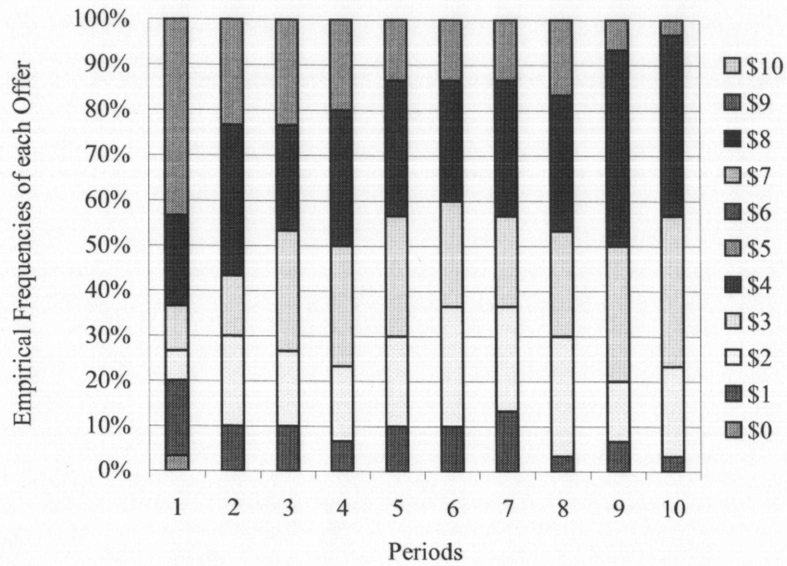


**Figure 1b) - Empirical Distributions of Offers in Japan**

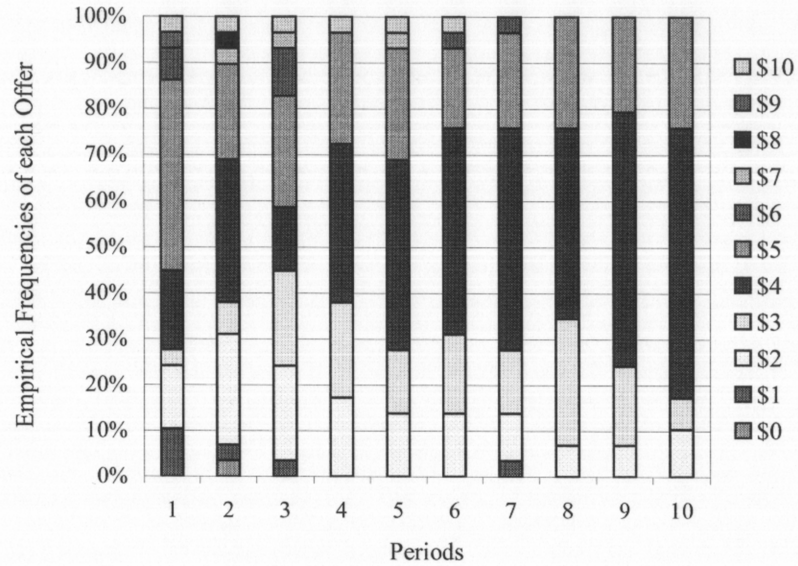


**Figure 1c) - Empirical Distribution of Offers in Slovenia**

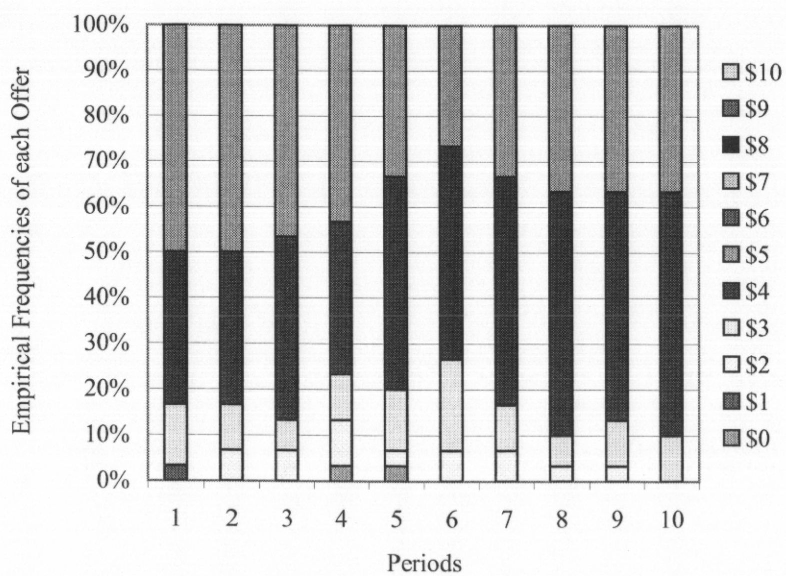


**Figure 1d) - Empirical Distributions of Offers in US**

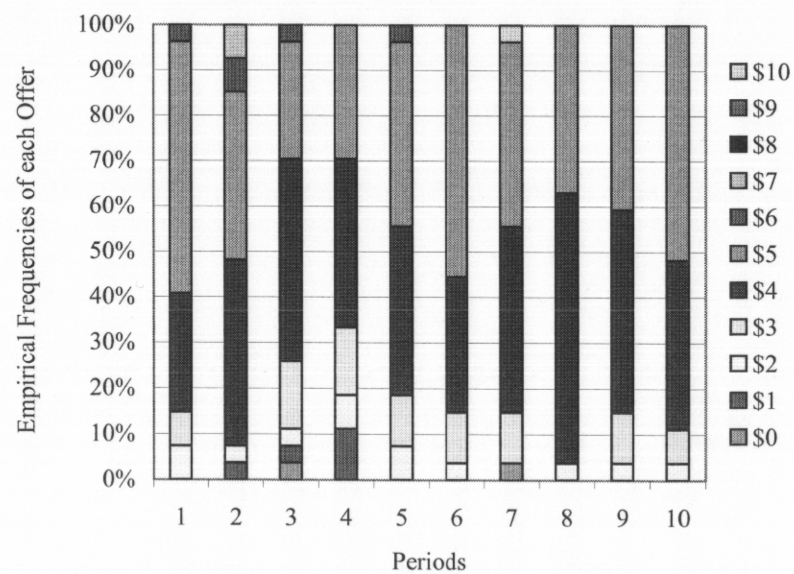

**Figure 2a) - Estimated Distribution of Offers in Israel**

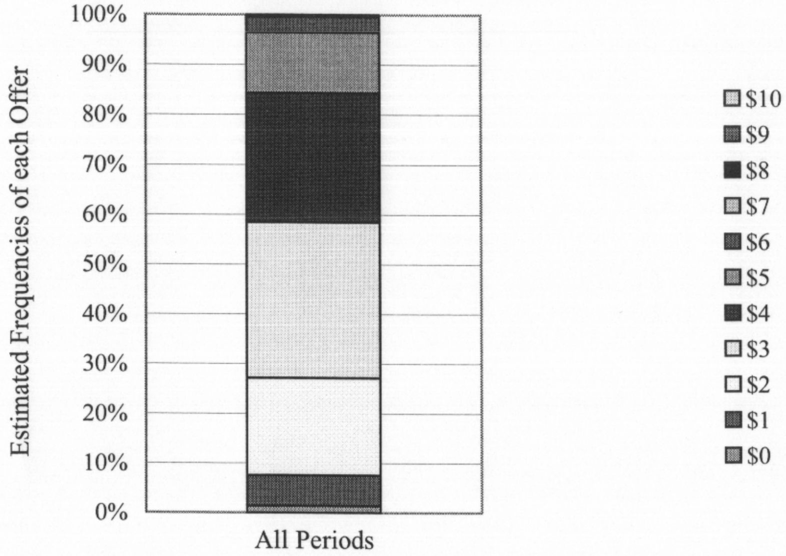


**Figure 2b) - Estimated Distribution of Offers in Japan**

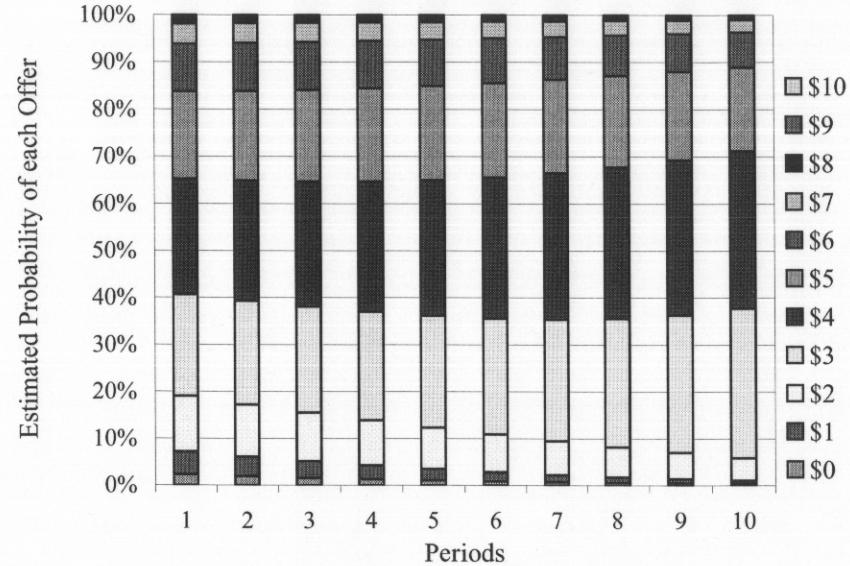


**Figure 2c) - Estimated Distribution of Offers in Slovenia**

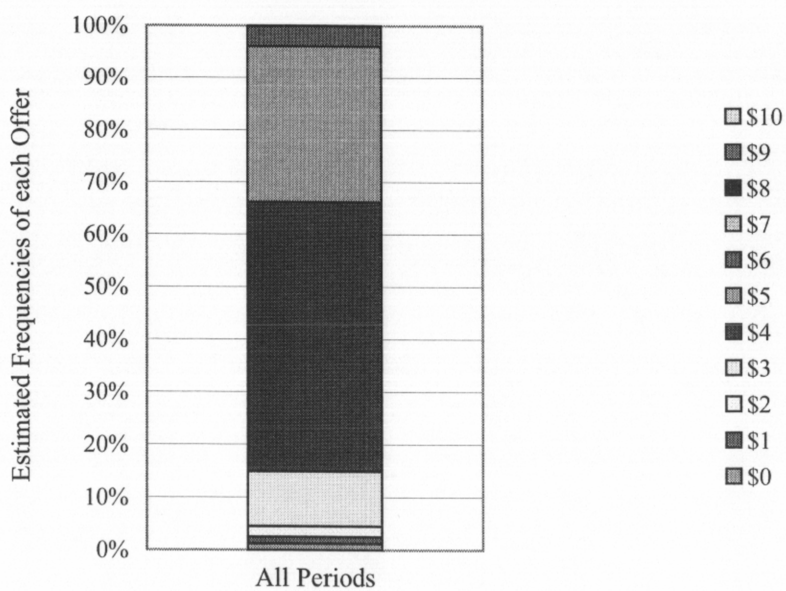


**Figure 2d) - Estimated Distribution of Offers in US**

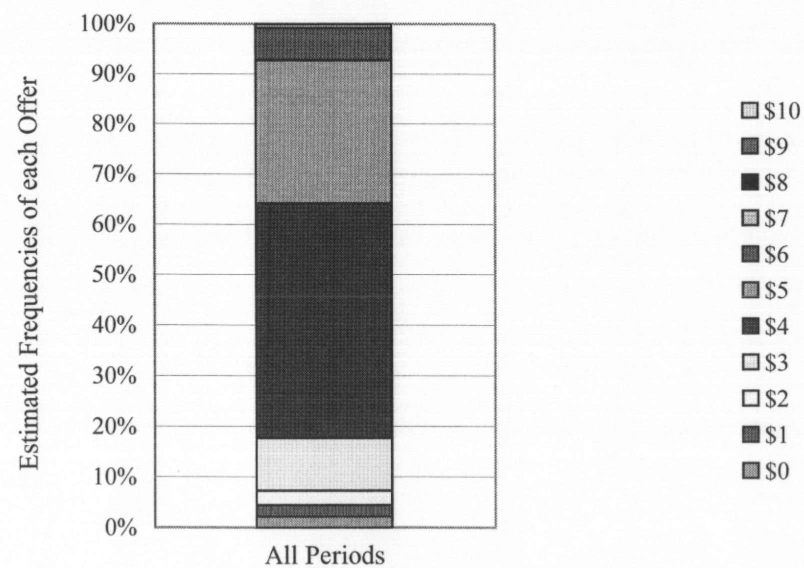

**Figure 3a) - Estimated Probabilities of Acceptance in Israel**

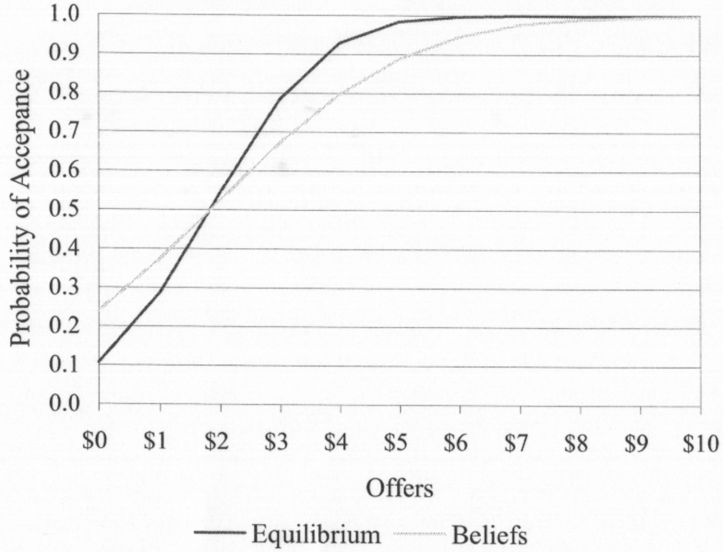


**Figure 3b) - Estimated Probabilities of Acceptance in Japan**

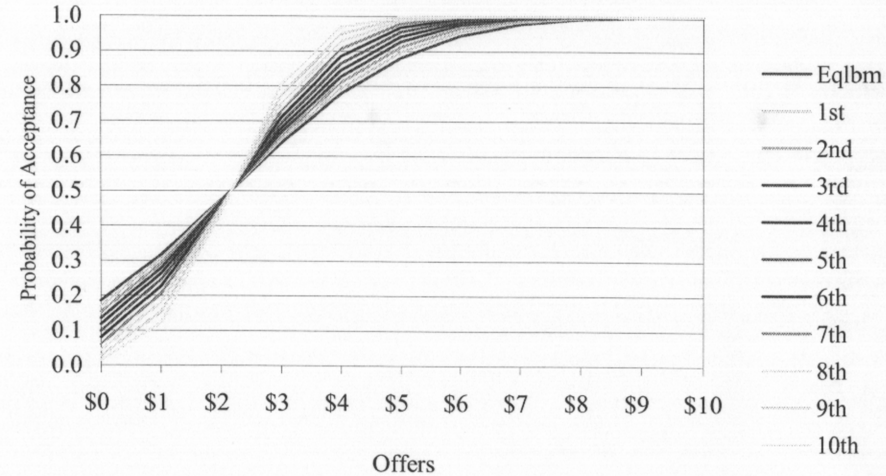


**Figure 3c) Estimated Probabilities of Acceptance in Slovenia**

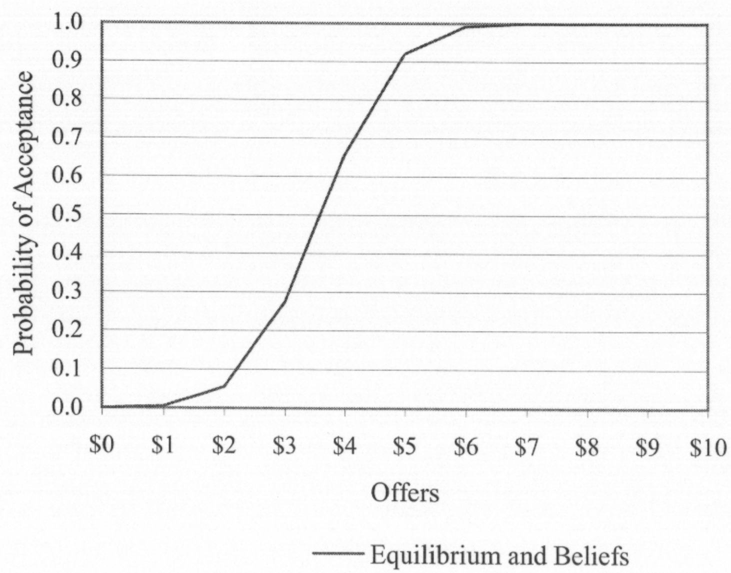


**Figure 3d) - Estimated Probabilities of Acceptance in US**