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Research Report

The Role of Options in Managing Supply Chain Risks

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This paper studies a two-stage supply chain in which a retailer can either buy products directly, or purchase options on product. Under this scenario, we derive optimal replenishment policies for the retailer, optimal production policies for the supplier, as well as an expression for optimal expected profit. We demonstrate that options can enable supply chain partners to effectively share the risk associated with demand uncertainty. We also show the value of options for improving supply chain efficiency.

§ 1 Problem Formulation and Solution

We consider a simple two-party supply chain comprised of a supplier producing short-life-cycle products, and a retailer who orders products from the supplier, then sells to end-users. Before the selling season, the retailer must decide how many units of each product to purchase. We assume that the procurement lead-time is long relative to the selling season, so the buyer cannot observe demand before placing the order. Because of the long lead-time, there is no opportunity to replenish inventory once the season has begun. Demand uncertainty exposes the parties to risks associated with mismatches between supply and demand. Specifically, if supply exceeds demand, the excess must be salvaged at a discount, and if demand exceeds supply, the unmet demand is lost. We refer to these costs as overage and underage. Overage costs include markdowns, and inventory holding costs in cases where it is possible to carry stock over to the next year. Underage costs capture lost sales, expediting costs, the cost of buying stock from a competitor to meet demand, and customer ill will.

The retailer can obtain goods by two means: either by purchasing from the supplier with a firm order, or by exercising call options. Before the start of the season, the retailer places an order for Q units of the product at unit wholesale price W. The retailer can also purchase q units of options at unit cost C at that time. Each call option gives the retailer the right (but not the obligation) to buy one unit of the product at exercise price X after demand has been observed. Introducing options thus provides a mechanism for sharing risk between the two parties.

In this setting, the retailer can use options to manage demand uncertainty. The risk of underage can be hedged by purchasing options, then exercising them only if actual demand is high. The risk of overage can be addressed by purchasing fewer units directly, then letting purchased options expire if realized demand is below expectations. However, the benefits of reducing risk are not free, since the retailer pays a premium to purchase options. The supplier keeps the premium in compensation for sharing the retailer's risk. By specifying the order quantities Q and q, the retailer decides how much risk to bear, and how much to pay for the benefit of reducing risk.

By sharing risk, the supplier induces the retailer to purchase more units, thus increasing unit sales. However, in so doing the supplier creates an obligation to fulfill demand for product when the retailer chooses to exercise an option. As a consequence, the supplier must hold inventories for possibly unexercised options, exposing the supplier to overage costs.

Assume the product has unit retail price R and unit manufacturing cost M. After the selling season, any excess product held by either the retailer or the supplier can be salvaged at unit salvage value S. Before the selling season, demand, denoted by D, is uncertain. Most of our analysis allows for a general, continuous demand distribution with density function f(D) and cumulative distribution function F(D).

Both the supplier and the retailer make their production and purchasing decisions before the start of the selling season. The retailer places an order for Q units of product and q options, and the supplier decides the number of units of product, Y, to produce. Clearly $Y \geq Q$, since Q represents a firm order. The retailer will only exercise options when $D>Q$, and the likelihood that the retailer will not exercise all q options is positive. Therefore, the number of units Y produced by a rational supplier will be between Q and (q+Q). However, when $Y<Q+q$, there is a positive possibility that the supplier will default on his commitment to fill the options. In this case, we assume that the supplier incurs a unit penalty cost P for each exercised option that is unfilled.

There are several possible interpretations for penalty cost P. It can represent the cost for the supplier to obtain an additional unit of product by either expediting production, or buying from an alternative source. It can also represent a pre-determined cash penalty specified in the option contracts. However, the two option-default settlement mechanisms result in different behavior by the retailer, even for the same value of P. In the case where the supplier finds an alternative means for delivering the product, the retailer will only excise options that are truly supported by actual demand. In the case where the supplier incurs a cash penalty, however, whenever the retailer learns that the supplier cannot honor the options, the retailer will exercise all of its options, regardless of whether or not there is actual demand. For simplicity, this paper focuses on the case where options are settled by finding an alternative means for delivering the product, rather than cash settlement.

The model must satisfy several feasibility conditions:

$$M < W < C+X < R \tag{1}$$

$$P \geq M \tag{2}$$

$$X > S \tag{3}$$

Condition (1) holds since M must be less than W for the supplier to make a profit, and W must be less than R for the retailer to make a profit. Moreover, if the wholesale price W were larger than the sum of the option's cost and its exercise price (C+X), the retailer would find it advantageous to only order options. Condition (2) states that product expediting cost P is always greater than the normal production cost. Condition (3) is necessary to prevent the retailer from exercising all of its options, even when there is no actual demand, and salvaging the excess product.

§ 1.1 The Retailer's Decisions

The retailer has two decision variables: the number of units Q to order and the number of call options q to purchase. We introduce T to represent the retailer's total order quantity, Q + q. Note that determining (Q, q) is equivalent to determining (Q, T). The retailer will always first fulfill demand using firm orders Q. When Q is insufficient to meet all demand, the retailer will exercise up to q options to satisfy demand. There are three possible scenarios that depend on the relationship between demand and the retailer's ordering decision:

1. If D ≤ Q, then all demand will be met and no call options will be exercised. The retailer's profit is RD + S(Q-D) –WQ – Cq = (R-S)D + (S-W)Q – Cq.
2. If Q < D ≤ T, then all demand will be met with a combination of Q units purchased through firm orders, and (D-Q) units obtained by exercising options. The retailer's profit is RD –WQ – Cq – X(D-Q) = (R-X)D + (X-W)Q – Cq.
3. If T < D, then all options will be exercised, leaving (D-T) units of demand unmet. The retailer's profit is R(Q+q) – WQ – (X+C)q = (R-W)Q + (R-X-C)q.

Therefore, the profit of the retailer is summarized by:

$$\Pi(Q,q)=\begin{cases}(R-S)D+(S-W)Q-Cq, & \text{if } D\le Q\\ (R-X)D+(X-W)Q-Cq, & \text{if } Q<D\le(Q+q)\\ (R-W)Q+(R-X-C)q, & \text{if } (Q+q)<D\end{cases}\tag{4}$$

The expected profit is thus:

$$E\Pi(Q,q)=(R-S)\int_0^Q Df(D)dD+[(S-W)Q-Cq]\Pr(D\le Q)+(R-X)\int_Q^{Q+q}Df(D)dD$$
$$+[(X-W)Q-Cq]\Pr(Q<D\le(Q+q))+[(R-W)Q+(R-X-C)q]\Pr((Q+q)<D)$$

Using the equations q=T-Q, $\Pr(D\le Q)=F(Q)$, $\Pr(Q<D\le T)=F(T)-F(Q)$, $\Pr(T<D)=1-F(T)$ and f(D)dD = dF(D), and integrating by parts, the expected profit function can be simplified to:

$$E\prod(Q,T)=(X+C-W)Q+(R-X-C)T-(R-S)\int_0^Q F(D)dD-(R-X)\int_Q^T F(D)dD$$

Using Leibniz's rule for differentiating integrals, we get the partial derivatives of the expected profit function $E\Pi(Q,T)$ with respect to Q and T:

$$\begin{aligned} \partial E\Pi(Q,T)/\partial Q &= X + C - W + (S - X)F(Q) \\ \partial E\Pi(Q,T)/\partial T &= R - X - C + (X - R)F(T) \end{aligned} \tag{5}$$

Differentiating the right hand side of (5) again, and observing that both (S-X) and (X-R) are negative, it follows that the Hessian matrix for the expected profit function $E\Pi(Q,T)$ is always negative. Therefore, the expected profit function is concave, with one unique maximum. Setting the partial derivatives in (5) to zero, we have the sufficient and necessary conditions for the optimal quantities Q* and T*:

$$F(T^*) = \Pr(D \le T^*) = (R - X - C)/(R - X) \tag{6}$$

$$F(Q^*) = \Pr(D \le Q^*) = (X + C - W)/(X - S) \tag{7}$$

The optimal expected profit for the retailer is given by:

$$\begin{aligned} E\prod(Q^*,T^*) &= (X + C - W)Q^* + (R - X - C)T^* \\ &\quad - (R - S)\int_0^{Q^*} F(D)dD - (R - X)\int_{Q^*}^{T^*} F(D)dD \end{aligned} \tag{8}$$

Note that Q* ≤ T* implies $\frac{R - X - C}{R - C} \ge \frac{X + C - W}{X - S}$ (if and only if $C \le \frac{(W - S)(R - X)}{R - S}$). This expression shows that if the option cost C is too high, the retailer will not order any options.

The classic newsvendor model is a special case of this formulation, where the retailer has no opportunity to order options. In the newsvendor model (see, e.g., Hadley and Whitin, 1962), the optimal order quantity $\overline{Q}^*$ and optimal expected profit $\pi^*(\overline{Q}^*)$ are represented by:

$$F(\overline{Q}^*) = \Pr(D \le \overline{Q}^*) = (R - W)/(R - S) \tag{9}$$

$$\pi^*(\overline{Q}^*) = (R - S)\int_0^{\overline{Q}^*} Df(D)dD = (R - W)\overline{Q}^* - (R - S)\int_0^{\overline{Q}^*} F(D)dD \tag{10}$$

Expression (9) can be rewritten as $F(\overline{Q}^*) = C_u/(C_u + C_o)$, where C_u is the unit underage cost R-W representing foregone profit, and C_o is the unit overage cost W-S representing salvage loss. Expressions (6) and (7) also take on this form. In expression (6), C_u = R-(X+C), which is the forgone profit if a unit of demand cannot be satisfied for lack of an option to exercise, and C_o = C, which is the cost of an unexercised option when there is overage. Similarly, in expression (7)

C_u = X+C –W and C_o = W – (C+S). If actual demand is greater than Q, the retailer pays a premium (X+C-W) to instead satisfy an excess unit of demand using options. On the other hand, if actual demand is less than Q, then the retailer incurs the unit wholesale cost for purchasing the product (but avoids the options cost), and salvages the product instead.

§ 1.2 The Supplier's Decision

Before the selling season, the supplier decides how many units Y of the product to produce, basing his decisions on the retailer's order pair (Q, T). The sequence of events is as follows. The transaction terms (i.e., W, C, and X) are first determined, then the retailer places a pair of orders (Q, T). The supplier produces Y units of the product, then immediately delivers Q units to the retailer, holding the remaining Y-Q units in inventory. After demand is observed, the retailer exercises an appropriate number of options, and additional units of the product are delivered to the retailer. This sequence of events induces a logical constraint that $Q^* \leq Y \leq T^*$. Depending on realized customer demand, there are four possible scenarios for the supplier:

1. If $D \leq Q^*$, then the retailer will exercise no options, and the supplier will salvage (Y-Q*) units of the product. The supplier's profit is WQ* + Cq* + S(Y –Q*) – MY = (W-S)Q*+Cq*+ (S-M)Y. Note that the retailer salvages Q*-D units of the product, illustrating that risk sharing does indeed occur.
2. If $Q^* < D \leq Y$, then the retailer will exercise (D-Q*) options. The supplier will honor every exercised option, and will salvage (Y-D) units of the product. The supplier's profit is given by WQ* + Cq* + X(D - Q*) + S(Y - D) – MY = (W-X)Q*+Cq* +(S-M)Y + (X-S)D. Note that overage occurs for the entire supply chain in this scenario. Though the retailer salvages no product, it pays a premium of (C+X-W)(D-Q*)+C(T*-D) for the supplier to bear the risk.
3. If $Y < D \leq T^*$, then the retailer will exercise (D-Q*) options. (Y-Q*) units will be settled by delivering product manufactured for cost M, and (D-Y) units will be delivered at penalty cost P (representing the unit expediting cost, or the cost of obtaining the product from an alternative source). The supplier's profit is given by WQ* + Cq* + X(D-Q*) - P(D-Y) – MY = (W-X)Q*+Cq*+(P-M)Y+(X-P)D. Note that underage occurs in this scenario, but the

system doesn't lose sales. There is risk sharing for underage, since the retailer pays a premium of (X+C-W) on (Y-Q*) units.

4. If T* ≤ D, then the retailer will exercise (T*-Q*) options. (Y-Q*) units will be settled by delivering products manufactured for cost M, and (T*-Y) units will be delivered at penalty cost P. The supplier earns a profit of WQ* + Cq* + Xq* - P(T*-Y) – MY = WQ* + (C+X)q* + (P - M)Y – PT*. Note that underage occurs in this scenario, and the system has unmet demand. However, the introduction of options reduces the total lost sales by q* units more than in the case without options.

In summary, the supplier's payoff function is given by:

$$\Sigma(Y) = \begin{cases} (W-S)Q^* + Cq^* + (S-M)Y, & if\ D \le Q^* \\ (W-X)Q^* + Cq^* + (S-M)Y + (X-S)D & if\ Q^* < D \le Y \\ (W-X)Q^* + Cq^* + (P-M)Y + (X-P)D & if\ Y < D \le T^* \\ WQ^* + (C+X)q^* + (P-M)Y - PT^* & if\ T^* \le D \end{cases} \tag{11}$$

The expected value of the profit function is:

$$\begin{aligned} E\Sigma(Y) = {} & [(W-S)Q^* + Cq^* + (S-M)Y]F(Q^*) + [(W-X)Q^* + Cq^* + (S-M)Y][F(Y) - F(Q^*)] \\ & + (X-S)\int_{Q^*}^{Y} Df(D)dD + [(W-X)Q^* + Cq^* + (P-M)Y][F(T^*) - F(Y)] \\ & + (X-P)\int_{Y}^{T^*} Df(D)dD + [WQ^* + (C+X)q^* + (P-M)Y - PT^*][1 - F(T^*)] \end{aligned}$$

Simplifying the above expression and integrating by parts, we have:

$$\begin{aligned} E\Sigma(Y) = {} & (W-P)Q^* + (X+C-P)q^* + (P-M)Y \\ & - (X-S)\int_{Q^*}^{Y} F(D)dD - (X-P)\int_{Y}^{T^*} F(D)dD \end{aligned} \tag{12}$$

Leibniz's rule gives us the derivative of the expected profit function:

$$\frac{dE\Sigma(Y)}{dY} = (P-M) + (S-P)F(Y) \tag{13}$$

Since (S-P)<0, the expected profit function is strictly concave, and there is a unique maximum. Setting the right hand side of (13) equal to 0, we obtain the necessary and sufficient conditions for optimal production quantity Y**:

$$F(Y^{**}) = \Pr(D \le Y^{**}) = (P-M)/(P-S) \tag{14}$$

and the corresponding supplier's profit:

$$E\Sigma(Y^{**}) = (W - X - C)Q^* + (X + C - P)T^* + (P - M)Y^{**} \\ - (X - S)\int_{Q^*}^{Y^{**}} F(D)dD - (X - P)\int_{Y^{**}}^{T^*} F(D)dD \tag{12'}$$

As observed for the retailer, expression (14) is consistent with the newsvendor model, with C_u=P-M and C_o=M-S. Thus, to the supplier, the cost of having one fewer unit of the product on hand than needed is the premium paid to supply that unit from an alternative source; while the cost of having one more unit on hand than needed is the difference between what the supplier paid to produce the unit and what amount can be realized in salvage. Note that since the revenue realized by the supplier is independent of the chosen stock level, the production quantity is independent of W, X, and C.

Notice that we use Y** in (14) instead of Y*, because Y** is not the optimal production quantity, since expression (14) was derived without considering the constraint $Q^* \le Y \le T^*$. Because the expected profit function in (12) is strictly concave, combining (14) and the constraint ($Q^* \le Y \le T^*$) gives the following optimal production volume Y*:

$$Y^* = \begin{cases} Q^*, & \text{if } Y^{**} \le Q^* \\ Y^{**}, & \text{if } Q^* < Y^{**} < T^* \\ T^*, & \text{if } T^* \le Y^{**} \end{cases} \tag{15}$$

Expression (15) can be validated by marginal analysis as follows. If Y<Q*, the marginal profit of one more unit of product is (W-M). Since W-M>0, the expected profit is larger when Y=Q*. If Y>T*, every marginal unit of product contributes (S-M), which is negative, thus decreasing total profit. Therefore, a production quantity greater than T* cannot be optimal.

In the newsvendor model, the supplier fills the retailer's order by building to order, i.e., the supplier always produces $\overline{Q}^*$ units of the product. (Note that the retailer then bears all risk associated with demand uncertainty.) The supplier's optimal profit in this case is then:

$$\varepsilon^*(Y^*) = \varepsilon^*(\overline{Q}^*) = (W - M)\overline{Q}^* \tag{16}$$

§ 2 Value of Options in Improving Supply Chain Efficiency

The supply chain discussed above consists of two parties with conflicting objectives, and possibly different information about customer demand. Both the supplier and the retailer, who have private information, seek to maximize their profits. Neither has the power to optimize the entire supply chain. This dual decision-making process with conflicting objectives degrades the efficiency of the supply chain as a whole. Several researchers have recognized this problem, and have proposed solutions (see, e.g., Anupindi and Bassok 1999, Barnes-Schuster etc. 2000, Lariviere 1999, and Tsay etc. 1999) that involve modifying transfer payments between supply chain participants (instead of using a singleton wholesale price W) to change supply chain partners' behaviors. These modified payment mechanisms provide incentives for all partners to behave in a manner that optimizes supply chain efficiency. Note that when the supplier and the retailer are a single entity, conflicting objectives are a less important issue. In this case, supply chain partners act as if the objective is to maximize total supply chain performance. Such a structure is referred to as an integrated supply chain.

Double marginality is a key source of suboptimal replenishment decisions in a non-integrated supply chain (see, e.g., Spengler 1950). If the entire supply chain produces Q units of the product, total profit for the supply chain is (R-M)Q. But this profit must be divided between the retailer and the supplier, and the retailer's order quantity influences the supplier's production decision. The retailer chooses an order quantity Q based on wholesale price W, which must be larger than manufacturing cost M to guarantee both parties positive profit margins. Double marginality induces a quantity $\overline{Q}^* = F^{-1}[(R-W)/(R-S)]$ given by (9). Since M replaces W in the integrated supply chain, $Q_I^* = F^{-1}[(R-M)/(R-S)]$. Because M<W, $\overline{Q}^* < Q_I^*$ and the total profit for the entire supply chain is greater in the integrated case.

In addition to the wholesale price W, options contracts provide three degrees of freedom (X, C and P) when negotiating contract terms. This additional flexibility makes options very attractive as a lever for supply chain contracts. For example, they can easily be used to induce effective channel coordination, ensuring that a decentralized supply chain will perform as well as an integrated supply chain. To induce the retailer to order total quantity T up to Q_I^*, (6) indicates that we must set X and C such that:

$$(R - X - C)/(R - X) = (R - M)/(R - S) \quad (17)$$

Condition (17) ensures that the retailer's total order T* will reach the order quantity obtained in an integrated supply chain. Moreover, we must also set unit penalty P so that Y* = T*. This, along with (6), (14) and (15), yields a condition to motivate the supplier to coordinate:

$$(P - M)/(P - S) \geq (R - X - C)/(R - X) \quad (18)$$

Expressions (17) and (18) together provide sufficient conditions for optimal channel coordination. Note that (17) and (18) imply that $P \geq R$, making optimal channel coordination feasible only when $P \geq R$.

There are two fundamental issues in supply chain optimization using options: (*i*) maximizing the combined profits of the retailer and the supplier, and (*ii*) allocating the profit equitably between the two parties. Equations (17) and (18) ensure that the combined profits are the same as for an integrated supply chain, but offer no insight into how the profits should be distributed between the parties. Note that since the wholesale price W is not a term in equations (17) or (18), neither the total amount ordered T*, the supplier quantity Y**, or total supply chain profits are affected by W. This allows W to be used as a parameter to control the distribution of profit between the supplier and the retailer. Therefore, option contracts can be used not only to maximize total supply chain profitability (by specifying X, C and P), but also to ensure an equitable distribution of the profits between the two supply chain partners, by specifying an appropriate W.

There are a number of ways to consider how supply chain profits should be distributed. Since the use of options increases the total profit available for distribution, at a minimum X, C, P and W should be specified so that both the retailer and the supplier have higher profits than in the no-options case. Using this approach, the distribution of incremental profits between the retailer and supplier could vary widely, perhaps depending on their relative market power, or their skill at negotiating contract terms. Another possibility is to distribute profit based on risk-adjusted return. Using this approach, each party's increase in profit (compared to the no-options case) would be proportional to the amount of risk borne, as discussed in Shi, Daniels, and Grey (2001).

When designing and managing a supply chain, it is important to understand the impact of environmental factors. In the simple setting described in this paper, demand mean μ and variance σ^2 are good indicators of market conditions. In the rest of this section, we illustrate the impact of demand mean and variance on the behavior of both the supplier and the retailer, as well as on overall supply chain performance. For this purpose, the demand is assumed normally distributed with mean μ and variance σ^2. Let $\phi(\xi)$ and $\Phi(\xi)$ be the density and cumulative distribution functions of the standard normal distribution. Since we are interested in the impact of environmental factors, all contract terms (R, W, X, C, P, S, M) are assumed to be constant. Notice that when those parameters are fixed, the right hand sides of the equations for T*, Q* and Y** are constant (denote them C_{T*}, C_{Q*} and C_{Y**} respectively). We then have the following results:

1. $Z = \mu + \sigma\Phi^{-1}(C_Z)$ for Z = T*, Q*, or Y**.
2. The percentage of options q* in total order T* is given by $\dfrac{q^*}{T^*} = \dfrac{\Phi^{-1}(C_{T^*}) - \Phi^{-1}(C_{Q^*})}{\mu/\sigma + \Phi^{-1}(C_{T^*})}$.
3. The implied θ in Y* = Q* + θq* is constant with respect to demand parameters μ and σ^2.

Proof: By change of variables $Z = \mu + \sigma\xi$, we can show that $F(Z) = \Phi[(Z-\mu)/\sigma)]$, which gives us the equations in the first result. By applying result 1, we can derive result 2, since $q^*/T^* = (T^* - Q^*)/T^*$. Result 1 also yields result 3, because $\theta = (Y^* - Q^*)/(T^* - Q^*)$, which is 0 if Y* = Q*; 1 if Y* = T*; and $[\Phi^{-1}(C_{Y**}) - \Phi^{-1}(C_{Q*})]/[\Phi^{-1}(C_{T*}) - \Phi^{-1}(C_{Q*})]$ if Y* = Y**. All are constants with respect to demand mean and variance. **End of Proof.**

These results specify the impact of the underlying demand distribution on the retailer's replenishment decision and the supplier's production planning. The first result shows that both order and production quantities increase linearly with μ and σ. The second result demonstrates that the percentage q*/T* is inversely dependent on μ/σ. The final result identifies an invariant in production planning, which can significantly simplify the supplier's decision-making process. Upon receiving the retailer's order quantities (Q*, q*), the supplier can respond first by checking the penalty cost P. If P is relatively expensive (or cheap), the supplier then produces T* (or Q*). Otherwise, the supplier always produces Q* + θq* with a fixed percentage θ, regardless of demand mean and variance (i.e., independent of market conditions).

In general, it is challenging to design incentives to foster effective information sharing in a decentralized supply chain (see, e.g., Lee et al. 1997). In our simple setting, the retailer usually has better information about the distribution of customer demand, since the retailer is closer to the final customer. Result 3 shows that options can induce the retailer to reveal more information to the supplier. When demand is normally distributed, the pair of order quantities (Q^*, T^*) completely reveals all of the retailer's known demand information (represented by two parameters μ and σ). Result 3 thus confirms that the supplier needn't worry about μ and σ, as long as Q^* and T^* are undistorted.

In summary, this paper explored the implications of introducing options into supply chain contracts as instruments to manage the risks associated with demand uncertainty. We showed that options improve supply chain efficiency, and provide incentives for supply chain partners to bear risk and to faithfully share demand information. A more detailed treatment of this research can be found in Shi, Daniels and Grey (2001). Many questions remain to be addressed, including extensions of this work to more complex supply chain topologies and multiple decision periods. We are currently investigating some of these issues.

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