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A Gradiometer-Based Superconducting Flux Qubit

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A gradiometer-based superconducting flux qubit

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Abstract

A successful superconducting flux qubit has not yet been demonstrated. We show theoretically that a flux qubit containing one or more gradiometer loops has major advantages in terms of noise immunity, independent controllability in S_x and S_z, and analyzability, which should facilitate progress towards a successful demonstration of the flux qubit.

In the drive to develop a scalable superconducting qubit for Quantum Computation, nanoscopic Josephson junction-based superconducting circuits, which can be produced in thousands on a chip by lithographic techniques, are highly promising. Despite demonstration of superconducting qubits based on a single electronic charge [1],[2], and on a mixed charge/magnetic flux state representation [3], operation of a pure flux-based qubit has not been successfully demonstrated, even though this, the closest to a "classical" implementation, might ultimately prove the most stable and satisfactory in engineering terms.

In a flux qubit the $|\uparrow\rangle$ and $|\downarrow\rangle$ basis states are represented by quantized flux of sense $\uparrow$ or $\downarrow$ threading a micron-scale superconducting ring which contains, in its canonical form, a single Josephson junction. In terms of the orthonormal $|\uparrow\rangle$ and $|\downarrow\rangle$ states, we can define the operator $S_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$ as a unit tunneling matrix between the $\uparrow$ and $\downarrow$ states, while the operator $S_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ is a unit shift in the relative energy of the $\uparrow$ and $\downarrow$ states. Single qubit operation [4] involves acting on the initialized qubit with a sequence of external *controls* $f_x(t)S_x$ and $f_z(t)S_z$, where the $f's$ are time-dependent fields, followed by a *measurement* of the qubit state; during these processes a high level of *quantum coherence* [5] must be maintained.

There is a physical realization of the z-field $f_z(t)$, which is just proportional to the external magnetic flux ϕ_z through the superconducting ring. Implementing the x-field $f_x(t)$ requires replacement of the single Josephson junction in the main superconducting ring by an interferometric ring [6] containing two Josephson junctions (see Fig. 1a). Now $f_x(t)$ is controlled (nonlinearly) by the external magnetic flux $2\phi_x(t)$ threading the interferometric ring. A DC f_x-field is also required to *tune* the time-averaged interwell tunneling matrix f_xS_x to

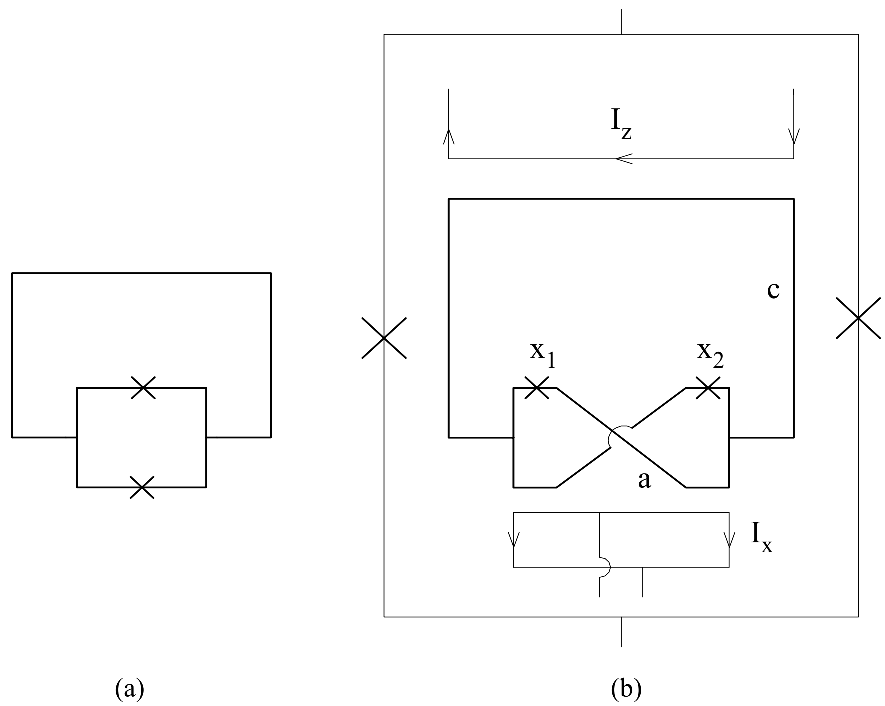

Figure 1: (a) Superconducting loop with interferometric ring containing two Josephson junctions. (b) Gradiometer qubit, showing main loop **c** and gradiometer interferometric loop **a**, containing two junctions with Josephson phases x_1 and x_2. External flux threading the main loop is controlled by current I_z in the upper z-drive coil, while external flux threading the gradiometer loop is independently controlled by current I_x in the lower x-drive coil. The flux in the main ring is weakly coupled inductively to the surrounding classical SQUID, which measures the qubit state.

an appropriate operating value. Successful operation of the flux qubit will then require (a) independent control of the flux in the interferometric and main rings, and (b) minimization in both rings of external flux noise, which acts to detroy quantum coherence (c) theoretical modelling to predict the correct operating parameters. Attention to these issues in the qubit design will be key to a cleanly operable flux qubit.

In this letter we demonstrate a simple design philosophy for flux qubits, which simultaneously ameliorates the foregoing requirements, by introducing a "gradiometer twist" into one or both rings [7] (Fig. 1b). The qubit in Fig. 1b consists of a main ring **c**, (which for clarity, is shown as untwisted), an interferometric ring **a**, containing two Josephson junctions and embodying the gradiometer twist, and a surrounding classical SQUID [8] which is coupled only to the flux in the main ring and serves to measure the final state of the qubit. In addition there are control coils carrying currents I_x and I_z, which, because of the symmetric gradiometer design (analogous to the differential and common mode electrical circuit concept) *independently* control the external interferometer flux, ϕ_x, and the main coil external flux ϕ_z, respectively. Also, the twist in the interferometer ring eliminates the effect on it of the spatially averaged external noise flux, and introduction of a similar twist into the main ring would also eliminate its sensitivity to spatially averaged noise flux. Thus both points (a) and (b) above are addressed by the gradiometer twist concept. By inductively decoupling loops **a** and **c**, the symmetry-based gradiometer design also ameliorates theoretical analysis, point (c), to which aspect we now turn.

Starting from a continuous Hamiltonian, it is necessary to solve it and hence to define the conditions required to obtain the type of spectrum useful for a qubit, i.e. a doublet well-separated from all other levels. The Hamiltonian is expressed in terms of the phase jumps x_1 and x_2 across the two junctions (Fig. 1b)

$$\begin{aligned}\mathcal{H} = & -\frac{(2e)^2}{2C}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right) - E_J\left(\cos x_1 + \cos x_2\right) \\ & + \frac{(x_1 - x_2 - 2\phi_x)^2}{2L_a}\phi_1^2 + \frac{(x_1 + x_2 - 2\phi_z)^2}{8L_c}\phi_1^2.\end{aligned} \tag{1}$$

In this representation [9], [10], the conjugate variable to phase is charge, enabling the capacitative charging energy to be expressed as the first term in Eq. (1), where C is junction capacitance. The second term in Eq. (1) is the coupling energy of the two junctions, each of Josephson energy E_J. Use of gauge invariance around a closed current loop enables expression of the inductive energies in terms of the Josephson phase jumps (third and fourth terms in Eq. (1)), where $\phi_1 = \hbar/2e$ is a quantum of flux, and L_a and L_c are the respective loop self-inductances, threaded by external fluxes $2\phi_x$ and ϕ_z respectively; there is no cross term by symmetry.

Usually L_a is much smaller than L_c (e.g. by a factor of 20), making $1/L_a$ large, effectively forcing $x_1 - x_2 \to 2\phi_x$. Then, in this approximation, we can rewrite $\mathcal{H}$ (in units $E_{L_c} = \phi_1^2/L_c$) in terms of a single variable $v = (x_1 + x_2)/2 - \phi_z$ as

$$\mathcal{H}/E_{L_c} = -\frac{1}{2M}\frac{\partial^2}{\partial v^2} + \beta_x \cos(v + \phi_z) + \frac{v^2}{2}, \tag{2}$$

where the dimensionless effective mass $M = 2r_Q^2 C/L_c$, $r_Q = \hbar/(2e)^2 = 1.03\,\mathrm{k\Omega}$, and $\beta_x = -2\left(E_J/E_{L_c}\right)\cos\phi_x$is the dimensionless, tunable, Josephson coupling.

With $\beta_x > 1$, the potential energy $V(v)$ ($V(v)$ = sum of last 2 terms in Eq.(2)) describes a quantum mechanical twin-well system whose potential energy is plotted in Fig. 2. In this model, Eq. (2), interwell tunneling is inhibited by the large mass M, which in practical realizations is hard to make smaller than $M \sim 100$; the cause is the impedance mismatch between the *resistance quantum* $r_Q \sim 1\,\mathrm{k\Omega}$ and the *low impedance* $Z_0 \simeq \sqrt{L_c/C} \sim 100\,\Omega$ of the simple LC qubit circuits, M depending on $(r_Q/Z_0)^2$. The solution is to use a DC

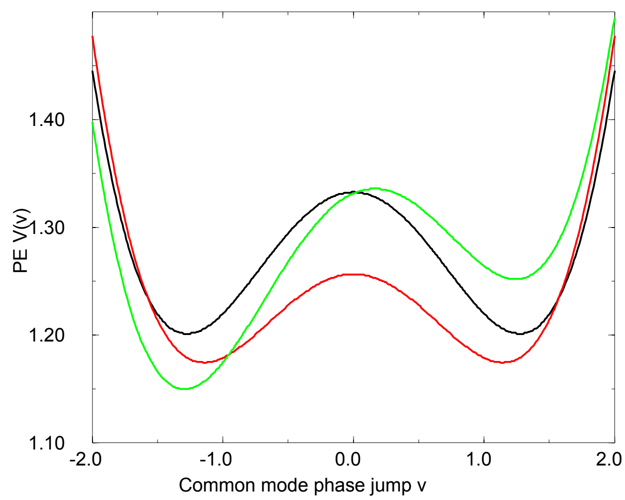

Figure 2: Potential energy $V(v)$ (sum of second 2 terms in Eq.(2)) plotted vs. common mode phase jump v, with parameters: black curve, $\phi_x = 2.3$, $\phi_z = 0$; red curve, $\phi_x = 2.25$, $\phi_z = 0$; green curve, $\phi_x = 2.3$, $\phi_z = 0.04$.

control flux ϕ_x to tune the interwell barrier in Fig. 2 to an acceptable value. Assuming that ϕ_z is small, this will require that ϕ_x lie just above $\pi - \theta_c$, where θ_c, defined by $\cos\theta_c = E_{L_c}/2E_J$, is the critical value of ϕ_x at which the interwell barrier vanishes.

Further analytic understanding in the barrier-tuning region is obtained by expanding the cosine in Eq. (2) and changing variables, when $\mathcal{H}$ is expressed in terms of a reduced, dimensionless hamiltonian $h(m^*)$, $\mathcal{H}/E_{L_c} = 3h(m^*)/2M^{2/3}$, defined for $\phi_z = 0$ as

$$h(m^*) = \left(\frac{m^*}{9}\right)^{2/3}\left[-\frac{1}{2m^*}\frac{\partial^2}{\partial s^2} - 2s^2 + s^4\right], \tag{3}$$

containing a reduced mass $m^* = 9(\beta_x - 1)^3 M$. Now in Eq. (3) the potential is always a quartic one with a unit barrier height, while the reduced mass absorbs both the effect of the original mass M and of the barrier tuning. It is seen explicitly that a large mass M can be counteracted by tuning β_x close to unity, e.g. if $M = 100$, then taking $\beta_x = 1.1$ will bring the reduced mass to $m^* = 0.9$, a relatively low value.

We found that a fit to the splitting E_{01} of the lowest doublet is obtained from the empirical tunneling formula

$$E_{01}/E_{L_c} \simeq 9(\beta_x - 1)^2 e^{-2\sqrt{m^*}}, \tag{4}$$

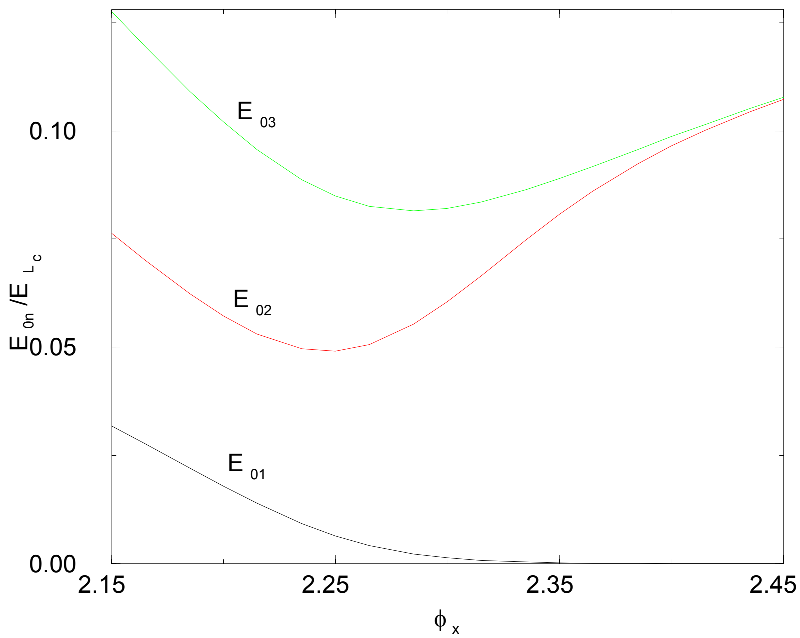

Figure 3: Eigenvalue separations E_{0n}, in units E_{L_c}, between ground state and n^{th} eigenvalue, for 2-variable Hamiltonian of Eq. (1), plotted vs. ϕ_x. Parameters as Table I.

with m^* in the exponential, explicitly demonstrating that, in flux qubit design, the reduced mass m^* is the key dimensionless parameter controlling the tunneling rate.

In addition to the stationary current I_x required for tuning the barrier, time-dependent control currents $I_x(t)$ and $I_z(t)$ will be required to operate the qubit. The gradiometer design ensures that these currents independently control the S_x and S_z operations on the qubit.

Table I: Parameters used in Eigenvalue Calculation

Par.	L_c	L_c/L_a	C	M	$I_c = E_J/\phi_1$	E_J/E_{L_c}	θ_c	φ_z
Value	750 pH	10	23 fF	64	0.44 μA	1	2.094	0

A quantitative feeling for gradiometric flux qubit design can be obtained from the example in Table I, and the corresponding eigenvalue spectrum, calculated from the full Eq.(1), plotted in Fig. 3. It is seen that if the value of ϕ_x is too close to the critical value θ_c, then the barrier is low and the spectrum is essentially harmonic, and unsuitable for qubit implementation. On the other hand, if ϕ_x is too large, then the barrier is too high and the tunneling splitting is negligible. An intermediate value of the flux ϕ_x, say $\phi_x = 2.25$, the tunneling splitting has a reasonable value $E_{01} \simeq 1.4\,\text{GHz}$, but the doublet is still well separated from the higher energy levels, indeed forming a qubit.

In summary, we have demonstrated that flux qubit design may be much improved by building in the concept of a gradiometer twist into one or more of the superconducting loops. The gradiometer design reduces external flux noise, decouples the x- and z- control fields, and inductively decouples the two rings, facilitating analysis, thus delivering significant performance improvement and designability at the cost of little design complexity. The illustrated design (Fig. 1b) would be further improved from the point of view of noise immunity by adding a gradiometer twist around the y-axis to the **c**-ring (appropriately reforming the SQUID ring in order to maintain its coupling to the **c**-ring), although some stability against ϕ_z-noise is already obtained from the symmetry of the energy surfaces at the operating point $\phi_z = 0$ [2], [3]. In addition to benefitting the most basic flux qubit design described here, more advanced designs with additional junctions [11] can also benefit by incorporation of the gradiometer design concept.

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