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# Redistribute White Space for Minimizing Wire Length 

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#### Abstract

Most existing floorplanning algorithms compact blocks to the left and bottom. Although the compacting obtains an optimal area, it may not be good to meet other objectives such as minimizing total wire length, routing congestion or buffer allocation. In this paper, we study the problem of distributing white space in floorplanning for minimizing total wire length. The problem can be formulated as linear programming. We propose an efficient min-cost flow based approach to solve it. Our approach guarantees to obtain the minimum of total wire length and meanwhile keep the minimum area for a given floorplan representation. We also show that the approach can be easily extended to handle constraints such as fixed-frame (fixed area), boundary pins, pre-placed blocks, boundary blocks, range placement, alignment and abutment, rectilinear blocks, cluster placement, and bounded net delay, without loss of optimality. The algorithm is so efficient in that it finishes in less than 0.4 seconds for all MCNC benchmarks of block placement. It is also very effective. Experimental results show we can further improve 4.2\% of wire length even on very compact floorplans. Thus it provides an ideal way of post-floorplanning (refine floorplanning).


## 1. Introduction

Floorplanning is to decide the positions of circuit blocks or IP blocks on a chip subject to various objectives. It is the early stage of physical design and determines the overall chip performance. Due to the enormous complexity of VLSI design with continuous scalingdown of technology, a hierarchical approach is needed for the circuit design in order to reduce runtime and improve solution quality. Also, IP (module reuse) based design methodology becomes widely adopted. This trend makes floorplanning even more important.

Floorplan can be classified into two categories, slicing and nonslicing. Among slicing representations, there are binary tree[15] and normalized Polish expression[19]. For non-slicing structure, many representations are invented recently, such as topology representation (BSG[14], sequence pair[13], TCG[11]), packing representation ( O-tree[7], B*-tree[5]), and mosaic representation (CBL[8], Q-sequence[16], twin binary tree[21], twin binary sequence[22]). All of these algorithms compact blocks to the left and bottom subject to the given topological constraints. Recently, additional constraints are addressed in floorplanning, such as fixed frame[17, 1], alignment and performance (bounded net delay)[18], buffer planning in floorplanning[12], etc.. Again, inside the approaches, the floorplan is compacted to lower-left (or upper-right) corner and then evaluated. In general, compacting implies minimum of area. However, it may be sub-optimal for other objectives, such as minimizing wire length, routing congestion, and buffer allocation. As we can see, even with the same minimum area and the same topology, there exist lots of different packings that have different distribution of white space and thus have different values on other objectives. We illustrate the problem by a simple example in Figure 1.

We observe that in floorplanning and placement, minimizing total wire length is first-order objective. If a floorplanner/placer can minimize total wire length very well, then there is much freedom and space to consider and tradeoff other concerns such as routability


Figure 1: (a) The floorplan compacting blocks to the lower-left corner. However, the wire length is not optimal. (b) The floorplan with optimal wire length, which has the same topology but different distribution of white space. The dimensions for the 3 blocks are: $b_{1}(4 \times 4), b_{2}(2 \times 2), b_{3}(4 \times 2)$.
and timing. Thus in the paper, we study the problem of minimizing total wire length in floorplanning, while leaving the consideration of routability and timing optimization (including buffer insertion) for future work.

Most floorplanning algorithms use simulated annealing to search for an optimal floorplan. The implementation of simulated annealing scheme relies on a floorplan representation where a neighbor solution is generated and examined by perturbing the representation. In the paper, we use sequence pair as representation. The reason we pick sequence pair is that the evaluation of $x$ and $y$ coordinates can be done independently in sequence pair representation, which may cause our approach faster. However, our approach is not limited to sequence pair representation. For any floorplan represented by any other presentation, we can derive a constraint graph and thus apply the approach to redistribute white space for minimizing total wire length. As we will discuss in the paper, our approach can take any input of floorplan or block placement even with a large set of additional constraints.

The problem can be formulated as linear programming. However, we find it can be solved by efficient min-cost flow implementation. The approach guarantees to obtain the minimum of total wire length for a given floorplan representation. We also show that the approach is capable of handling various constraints such as fixed-frame (fixed area), boundary pins, pre-placed blocks, boundary blocks, range placement, alignment and abutment, rectilinear blocks, cluster placement, and bounded net delay, without loss of optimality. It is an exact algorithm to minimize wire length and meanwhile keep the minimum area, as well as to optimize the composite cost of both area and wire length. The algorithm is so efficient in that it finishes in less than 0.4 seconds for all MCNC benchmarks of block placement. It is also very effective. Experimental results show we can further improve $4.2 \%$ of wire length even on very compact floorplans. Thus it is worth applying as a step of post-floorplanning (refine floorplanning). It is noted that researchers have studied the problem of allocating white space in placement for various objectives $[9,20,4,2]$. These methods are heuristics in terms of minimizing wire length. Our approach optimally minimizes wire length for a given floorplan, and may be
applicable to mixed-cell placement (behaving as a post-placement step, which is left as future work).

The rest of the paper is organized as follows. Section 2 briefly reviews sequence pair and constraint graph construction to evaluate a sequence pair. Section 3 formulates the problem of redistributing white space to minimize total wire length, and presents a mincost flow based approach to solve it. The capabilities of handling various constraints such as fixed-frame, boundary pin, pre-placed block, boundary block, range placement, alignment and abutment, rectilinear block, cluster placement, bounded net delay, etc., are discussed in Section 4. Experimental results are reported in Section 5, followed by concluding remarks in Section 6.

## 2. Preliminary

A sequence pair is a pair of sequences of $n$ elements representing a list of $n$ blocks. The two sequences specify the geometric relations (such as left-of, right-of, below, above) between each pair of blocks as follows:

$$
\begin{align*}
\left(\ldots b_{i}^{\ldots} b_{j \ldots}^{\ldots}, \ldots b_{i \ldots}^{\ldots} \ldots\right) & \Rightarrow b_{i} \text { is to the left of } b_{j}  \tag{1}\\
\left(\ldots b_{j \ldots}^{\ldots} b_{i} \ldots, \ldots b_{i} \ldots b_{j \ldots} \ldots\right. & \Rightarrow b_{i} \text { is below } b_{j} \tag{2}
\end{align*}
$$



Figure 2: Sequence pair ( $\left.b_{3} b_{1} b_{2}, b_{1} b_{2} b_{3}\right)$ specifies an oblique grid.

The sequence pair structure can be shown as an oblique grid (refer to Figure 2). The original paper which proposed sequence pair [13] presented an algorithm to translate a sequence pair to a placement by constructing two constraint graphs, $G_{h}$ and $G_{\nu}$. Both $G_{h}$ and $G_{v}$ have $n+2$ vertices representing $n$ blocks plus source node and sink node (representing boundaries). $G_{h}$ has a directed edge $\left(b_{i}, b_{j}\right)$ if block $b_{i}$ is to the left of block $b_{j}$. Similarly, if block $b_{i}$ is below block $b_{j}, G_{\nu}$ has the corresponding directed edge $\left(b_{i}, b_{j}\right)$. For any pair of blocks (e.g. $b_{i}, b_{j}$ ), there exists exactly one edge connecting the two nodes either in $G_{h}$ or in $G_{v}$. Both $G_{h}$ and $G_{v}$ are vertex weighted, directed, acyclic graphs. The weights in $G_{h}$ represent the widths of blocks, and the weights in $G_{V}$ represent the heights of blocks. Given that the coordinates of a block are the coordinates of the lower-left corner of the block, a longest path algorithm can be applied to determine the coordinates of each block and the total width and height of the bounding box. As an example, the sequence pair specifying the placement in Figure 1 is $\left(b_{3} b_{1} b_{2}\right.$, $b_{1} b_{2} b_{3}$ ).

## 3. Problem and Solution

Sequence pair specifies the topological relation between blocks. Given a sequence pair, previous algorithm compacts blocks to lowerleft corner to minimize area. Even with the same minimum area, there exist different placements of blocks satisfying the topological constraint imposed by the sequence pair. It is very common that white space exists even in the floorplan packed to minimum area. The problem is to find a floorplan that fairly allocates white space and minimizes the total wire length, as defined as follows:

Problem 1. Given a sequence pair $(X, Y)$ with a set of m macro blocks $B=\left\{b_{1}, b_{2}, \ldots, b_{m}\right\}$ where $w_{i} \times h_{i}$ specifies the dimension of block $b_{i}\left(w_{i}:\right.$ width, $h_{i}$ : height $)$, and a set of nets $N=\left\{N_{1}, N_{2}, \ldots, N_{n}\right\}$ where $N_{i}, i=1,2, \ldots, n$ describes the connection between blocks, find a placement of blocks B satisfying the topological relation imposed by the sequence pair, such that the total wire length

$$
\sum_{i=1}^{n} \lambda_{i} W\left(N_{i}\right)
$$

is minimized where $W\left(N_{i}\right)$ denotes the wire length of net $N_{i}$ and $\lambda_{i}$ is its weight.

Without loss of generality, we assume that all $\lambda_{i}$ are integers. In the following we use $x_{i} / y_{i}$ to denote the $x / y$ coordinate of block $b_{i}$ referring to the lower-left corner of the block. For simple representation and easy understanding, we assume all pins are located in the center of the block. Actually as we can see later, our approach has no restriction that pins should be in the center of the block. It is common to use half perimeter of bounding box as an estimate of wire length for a net. Let us consider a net $N_{i}$ connecting a set of $z$ blocks $\left\{b_{i_{1}}, b_{i_{2}}, \ldots, b_{i_{z}}\right\}$, and use ( $\left.L_{i}, L_{i}^{\prime}: R_{i}, R_{i}^{\prime}\right)$ as its bounding box where ( $L_{i}, L_{i}^{\prime}$ ) and ( $R_{i}, R_{i}^{\prime}$ ) refer to lower-left and upper-right corner of the bounding box respectively. Thus we have $\forall j \in\{1,2, \ldots, z\}$

$$
\begin{align*}
L_{i} & \leq x_{i_{j}}+w_{i_{j}} / 2  \tag{3}\\
R_{i} & \geq x_{i_{j}}+w_{i_{j}} / 2  \tag{4}\\
L_{i}^{\prime} & \leq y_{i_{j}}+h_{i_{j}} / 2  \tag{5}\\
R_{i}^{\prime} & \geq y_{i_{j}}+h_{i_{j}} / 2 \tag{6}
\end{align*}
$$

Note that the coordinate $\left(x_{i_{j}}+w_{i_{j}} / 2, y_{i_{j}}+h_{i_{j}} / 2\right)$ is the center of the block $b_{i_{j}}$ where pin is located. When pin is not at the center, we can use the actual pin location to substitute the coordinate. In addition, the geometric constraint imposed by sequence pair can be written as follows:

$$
\begin{align*}
\left(\ldots b_{i} \ldots b_{j \ldots}^{\ldots}, \ldots b_{i} \ldots b_{j \ldots} \ldots\right) & \Rightarrow x_{i}+w_{i} \leq x_{j}  \tag{7}\\
\left(\ldots b_{j \ldots} \ldots b_{i} \ldots, \ldots b_{i} \ldots b_{j \ldots} \ldots\right) & \Rightarrow y_{i}+h_{i} \leq y_{j} \tag{8}
\end{align*}
$$

Thus the problem can be stated as:

$$
\begin{equation*}
\min \sum_{i=1}^{n} \lambda_{i}\left(R_{i}-L_{i}+R_{i}^{\prime}-L_{i}^{\prime}\right) \tag{9}
\end{equation*}
$$

subject to the set of constraints as stated in (3) (4) (5) (6) (7) (8). Since in sequence pair representation, the evaluation of $x$ and $y$ coordinates can be done independently, the problem can be decoupled into two subproblems:

$$
\begin{equation*}
\min \sum_{i=1}^{n} \lambda_{i}\left(R_{i}-L_{i}\right) \tag{10}
\end{equation*}
$$

subject to the set of constraints as stated in (3) (4) (7), and

$$
\begin{equation*}
\min \sum_{i=1}^{n} \lambda_{i}\left(R_{i}^{\prime}-L_{i}^{\prime}{ }_{i}\right) \tag{11}
\end{equation*}
$$

subject to the set of constraints as stated in (5) (6) (8). The problems (10) and (11) can be solved separately. As we can see, all of the three problems, (9), (10) and (11), are linear programming. However, each of the problems has special property that all constraints are difference constraints[3]. Thus its dual problem is a min-cost flow problem, since in the constraint matrix of the dual problem, each column has exactly one " 1 " and " -1 ". Let us first consider the problem (10). We can construct a network graph (called horizontal network graph) $G_{H}=\left(V_{H}, E_{H}\right)$ as follows.

1. $V_{H}=\left\{s, t, x_{1}, x_{2}, \ldots, x_{m}, L_{1}, R_{1}, L_{2}, R_{2}, \ldots, L_{n}, R_{n}\right\}$, where $s$ is the source node, $t$ is the sink node, $x_{i}$ represents the $x$ coordinate of block $b_{i}$, and $L_{i}$ and $R_{i}$ represent the left and right boundary of bounding box of net $N_{i}$ as denoted above.
2. $E_{H}=\left\{\left(s, R_{i}\right) \mid i=1,2, \ldots, n\right\} \cup\left\{\left(x_{i}, x_{j}\right) \mid\right.$ block $b_{i}$ is to the right of block $\left.b_{j}\right\} \cup\left\{\left(R_{i}, x_{j}\right),\left(x_{j}, L_{i}\right) \mid\right.$ net $N_{i}$ connects block $\left.b_{j}\right\} \cup$ $\left\{\left(L_{i}, t\right) \mid i=1,2, \ldots, n\right\}$, where $\left(s, R_{i}\right)$ is the edge from source to right boundary of bounding box, $\left(x_{i}, x_{j}\right)$ is the edge imposed by the sequence pair as in constraint (7), $\left(R_{i}, x_{j}\right)$ is the edge imposed by net connection as in constraint (4), $\left(x_{j}, L_{i}\right)$ is the edge imposed by net connection as in constraint (3), and $\left(L_{i}, t\right)$ is the edge from left boundary of bounding box to sink.
3. Edge Capacity: $U_{H}\left(s, R_{i}\right)=U_{H}\left(L_{i}, t\right)=\lambda_{i}, \forall i \in\{1,2, \ldots, n\}$; for any other edge $e \in E_{H}, U_{H}(e)$ is unlimited.
4. Cost Function: $C_{H}\left(s, R_{i}\right)=0, C_{H}\left(L_{i}, t\right)=0, C_{H}\left(x_{i}, x_{j}\right)=$ $-w_{j}, C_{H}\left(R_{i}, x_{j}\right)=-w_{j} / 2$, and $C_{H}\left(x_{j}, L_{i}\right)=w_{j} / 2$.
It should be noted that the subgraph, which contains only the vertices $x_{i}, i=1,2, \ldots m$ and the edges $\left(x_{i}, x_{j}\right)$ imposed by sequence pair, is similar to the horizontal constraint graph mentioned in [13]. The difference is that the direction of edges is inverted and the edge cost is negative. Thereafter, in [13] a longest path algorithm is applied to compute the positions of blocks, while in the paper we shall use min-cost flow algorithm in the sense of shortest path. It should also be noted that the transitive edges on the subgraph can be safely omitted, which will speed up the computation considerably.

Thus we compute the min-cost flow of amount $\sum_{i=1}^{n} \lambda_{i}$ on the graph $G_{H}$, which solves the dual problem. Our goal is to compute the positions of blocks subject to the constraints and minimize the total wire length (the primal problem), which can be done as follows. We first compute the residual graph derived from the mincost flow. Then a shortest path algorithm applied on the residual graph would give the positions for all blocks. If necessary, a common source node connecting to all other nodes can be added to the residual graph for shortest path computation.

Analogously, we can construct another network graph and solve the problem (11) by min-cost flow approach. The graph (called vertical network graph) is denoted as $G_{V}=\left(V_{V}, E_{V}\right)$.

We use the example as shown in Figure 1 to illustrate the approach. The input of the problem is: sequence pair $\left(b_{3} b_{1} b_{2}, b_{1}\right.$ $b_{2} b_{3}$ ) with 3 blocks, and nets $N_{1}=\left\{b_{1}, b_{2}\right\}$ with weight $\lambda_{1}=2$, $N_{2}=\left\{b_{2}, b_{3}\right\}$ with weight $\lambda_{2}=1$. Then the problem (10) to minimize wire length in $x$ dimension can be stated as follows:

$$
\min \left\{2\left(R_{1}-L_{1}\right)+\left(R_{2}-L_{2}\right)\right\}
$$

subject to

$$
\begin{aligned}
x_{1}+4 & \leq x_{2} \\
x_{1}+2 & \geq L_{1} \\
x_{1}+2 & \leq R_{1} \\
x_{2}+1 & \geq L_{1} \\
x_{2}+1 & \leq R_{1} \\
x_{2}+1 & \geq L_{2} \\
x_{2}+1 & \leq R_{2} \\
x_{3}+2 & \geq L_{2} \\
x_{3}+2 & \leq R_{2}
\end{aligned}
$$

Then this can be transformed to min-cost flow problem in the network graph $G_{H}$ as shown in Figure 3(a). Similarly, the problem (11) to minimize wire length in $y$ dimension is transformed to min-cost flow problem in the network graph $G_{V}$ as shown in Figure


Figure 3: (a) The horizontal network graph. (b) The vertical network graph. The pair of numbers, "c,u", on the edge represent cost and capacity respectively, and " $U$ " means unlimited capacity (the same meaning on the graphs that follows).


Figure 4: (a) The min-cost flow on the horizontal network graph. (b) The min-cost flow on the vertical network graph. The highlighted lines represent flows. Their widths are proportional to the amount of flow.

3(b). Then we compute the min-cost flow of amount: 3 (because $\lambda_{1}+\lambda_{2}=3$ ) on the two graphs, $G_{H}$ and $G_{V}$. The results are illustrated in Figure 4(a) and Figure 4(b) respectively. Based on the flow results, we derive the residual graphs of $G_{H}$ and $G_{V}$, as shown in Figure 5(a) and Figure 5(b) respectively. Then we apply shortest path algorithm on the residual graphs to compute the positions of blocks by adding a common source node connecting all other nodes. Thus the results are: $x_{1}=-5, x_{2}=-1, x_{3}=-2, y_{1}=-5$, $y_{2}=-4$, and $y_{3}=-1$. The placement with minimum wire length is shown in Figure 6. The overall approach is summarized as follows.
Algorithm Min-wire

1. Construct the network graphs $G_{H}$ and $G_{V}$

Apply min-cost flow algorithm on $G_{H}$ and $G_{V}$
Derive the residual graphs of $G_{H}$ and $G_{V}$
Apply shortest path algorithm on residual graphs to compute positions of blocks

The min-cost flow based algorithm, Min-wire, optimally solves the problem of minimizing total wire length, as stated in the following theorem.

Theorem 1. The algorithm, Min-wire, generates a placement of all blocks such that the total wire length is minimized optimally for the given sequence pair.


Figure 5: (a) The horizontal residual graph. (b) The vertical residual graph.


Figure 6: The placement with minimum wire length.

Due to space limit, the proof, as well as the proof in the rest of the paper, is omitted.

The complexity of the algorithm, Min-wire, is determined by min-cost flow, since other steps are smaller portions compared to min-cost flow algorithm. Finding a min-cost flow in a network is a classical problem for which several polynomial-time optimal algorithms are available [3]. The number of vertices in either $G_{H}$ or $G_{V}$ is $O(m+n)$ where $m$ is the number of blocks and $n$ is the number of nets. The number of edges on the subgraph, which contains only the vertices representing blocks and the edges between, is $O(m \log m)$ on average[10]. The rest of edges includes the edges introduced by net connections, and the edges incident from/to source/sink. The number of edges incident from source and to sink is $O(n)$. The edges introduced by net connections in the graph is proportional to the number of pins in all nets. Typically in practice, we can assume that the number of pins is a constant on average in a net. Thus the number of edges introduced by net connections is typically $O(n)$. In total, the number of edges is $O(m \log m+n)$. Therefore, if we adopt Orlin's algorithm in [3] to compute the mincost flow, the time complexity of the algorithm Min-wire is typically $O(|E| \log |V|(|E|+|V| \log |V|))=O((m \log m+n) \log (m+$ $n)(m \log m+n+(m+n) \log (m+n)))$. Practically, we can assume that net weight $\lambda_{i}$ is $O(1)$ (for example, 1-10), which is true in most applications. We observe that too large weight is unnecessary in actual applications. When net weight is beyond some threshold, it behaves the same in minimizing wire length. Then we can apply successive shortest path augmenting algorithm in computing mincost flow[3] which is faster in the case. Thus, the complexity is $O(n S(m, n))$ where $S(m, n)$ denotes the time taken to solve a shortest path problem. If we associate each node with an adjusted weight to eliminate negative cost[3], then $S(m, n)$ is the complexity of Dijkstra algorithm. Finally the complexity is $O(n(|V| \log |V|+|E|))=$ $O(n(m \log m+n+(m+n) \log (m+n)))$.

## 4. Discussion of Capabilities

It is useful and important in applications that floorplanning handles constraints. As we can see, the approach is capable of handling various constraints without loss of optimality.

### 4.1 Fixed-frame


(a)

(b)

Figure 7: The modification of graphs to handle fixed-frame where the frame is $6 \times 6$. (a) The horizontal network graph. (b) The vertical network graph.

In some applications, floorplanning is confined in a given frame, $W \times H$, where $W$ and $H$ represent width and height respectively. In addition, if we still want to keep the minimum area in minimizing wire length, we can solve the problem with a frame of minimum area. When a frame is taken into account, we modify the graphs as follows. To horizontal network graph $G_{H}$, two nodes, $f_{L}$ and $f_{R}$, are added where $f_{L}$ and $f_{R}$ represent the left and right boundary of the frame respectively. We have $x_{i}+w_{i} \leq f_{R}, x_{i} \geq f_{L}$ and $f_{R}-f_{L} \leq W$. Accordingly, a set of edges are added, $\left(f_{R}, x_{i}\right)$ with cost $-w_{i}$ and unlimited capacity, $\left(x_{i}, f_{L}\right)$ with cost 0 and unlimited capacity, $i=$ $1,2, \ldots, m$, and $\left(f_{L}, f_{R}\right)$ with cost $W$ and unlimited capacity. Again, the transitive edges in $\left(f_{R}, x_{i}\right)$ and $\left(x_{i}, f_{L}\right)$ can be omitted. Similarly, two nodes, $f^{\prime}{ }_{L}$ and $f^{\prime}{ }_{R}$ representing the lower and upper boundary of the frame respectively, and the corresponding edges are added to vertical network graph $G_{V}$. Figure 7 illustrates the two modified graphs for the example above. The frame is $6 \times 6$ (minimum area). Thus the algorithm Min-wire can still be applied to minimize the total wire length and place blocks in the given frame. Note that the node $f_{R}\left(f^{\prime}{ }_{R}\right)$ will act as the source node in the step of shortest path computation in $G_{H}\left(G_{V}\right)$ to obtain the positions of blocks. Figure 1 (b) actually gives the optimal placement within the frame.

### 4.2 Boundary Pin

Usually there exist IO nets which connect to pins on the boundary of the frame (boundary pins). Let us consider a net $N_{i}$ connects a boundary pin at location $\left(p_{x}, p_{y}\right)$. Thus $L_{i} \leq p_{x} \leq R_{i}$ and $L_{i}^{\prime} \leq$ $p_{y} \leq R^{\prime}$. Assume the frame is $W \times H$. Then equivalently, $L_{i}-f_{R} \leq$ $p_{x}-W \leq R_{i}-f_{R}$ and $L^{\prime}{ }_{i}-f^{\prime}{ }_{R} \leq p_{y}-H \leq R_{i}^{\prime}-f^{\prime}{ }_{R}$. As a result, we add two edges, $\left(f_{R}, L_{i}\right)$ (with cost $p_{x}-W$ and unlimited capacity) and $\left(R_{i}, f_{R}\right)$ (with cost $W-p_{x}$ and unlimited capacity), to graph $G_{H}$, and add two edges, $\left(f_{R}^{\prime}, L^{\prime}{ }_{i}\right)$ (with cost $p_{y}-H$ and unlimited capacity) and $\left(R^{\prime}{ }_{i}, f^{\prime}{ }_{R}\right)$ (with cost $H-p_{y}$ and unlimited capacity), to graph $G_{V}$. Thus the algorithm Min-wire can be applied. In this way, fixed pin can be handled where the pin location may not be on the boundary of the frame.

### 4.3 Pre-placed and Boundary Blocks

In the situation where some blocks are to be placed at fixed location or on the boundary of the frame, the algorithm can still apply by adding additional edges to graphs. For example, a block $b_{i}$ is placed
at a location $\left(l_{x}, l_{y}\right)$, i.e. $x_{i}=l_{x}$ and $y_{i}=l_{y}$. Thus $x_{i}-f_{R}=l_{x}-W$ and $y_{i}-f^{\prime}{ }_{R}=l_{y}-H$. Equivalently, $x_{i}-f_{R} \leq l_{x}-W, x_{i}-f_{R} \geq l_{x}-$ $W, y_{i}-f_{R}^{\prime} \leq l_{y}-H$, and $y_{i}-f^{\prime}{ }_{R} \geq l_{y}-H$. These are transformed to edges $\left(f_{R}, x_{i}\right)$ and $\left(x_{i}, f_{R}\right)$ on graph $G_{H}$, and edges $\left(f_{R}^{\prime}, y_{i}\right)$ and $\left(y_{i}, f^{\prime}{ }_{R}\right)$ on graph $G_{V}$. Boundary blocks can be handled similarly in the sense that boundary blocks fix locations in $x$ or $y$ coordinate.

### 4.4 Range Placement

Range constraint specifies that a block is to be placed within a given range. Pre-placed constraint is a special case of range constraint. Similarly, we can add additional edges to graph $G_{H}$ and $G_{V}$ to enforce the computation of position in algorithm Min-wire such that the block is placed within the range.

### 4.5 Alignment and Abutment

Alignment constraint specifies several blocks to be aligned in a row within a range[18]. It can be transformed to a set of difference constraints that keep the relative positions between them. Thus we can add additional edges to the graphs accordingly. Abutment is a special case of alignment.

### 4.6 Rectilinear Block

Rectilinear block is partitioned into a set of rectangular subblocks. Then a set of constraints are used to keep the relative positions, which can be transformed to the additional edges in the graphs accordingly[6].

### 4.7 Cluster Placement

It is useful in applications that several blocks are placed close to each other (cluster placement). In other words, the distance between any two of the blocks should not be too far away. This can be written as a set of constraints that specify the distance bound between any two of the blocks. Thus we can solve the problem by adding the corresponding edges to the graphs.

### 4.8 Bounded Net Delay

The approach is to minimize the total wire length, which can not guarantee bounded delay for critical nets. To address bounded net delay, we use a linear function in terms of distance to estimate delay. Although interconnect delay is quadratic in terms of wire length, with appropriate buffer insertions the actual delay is close to linear in terms of source-sink distance. In this way we convert bounded net delay into bounded net wire length. Thus as in [18], we impose constraints on the bounding box of the net, which results in the additional edges in the graphs accordingly.

### 4.9 Composite Cost Function

We have talked about fixed-frame constraint that blocks are confined within a given frame. Actually, the method is an exact algorithm to optimize the composite cost function of area and wire length:

$$
\min \left\{\alpha(W+H)+\sum_{i=1}^{n} \lambda_{i} W_{i}\right\}
$$

Note that existing methods can only minimize area, and use compacted blocks' locations to compute the cost of wire length. To optimize the composite cost, we can modify the graphs, $G_{H}$ and $G_{V}$, as follows.

It is similar to the modification that handles fixed-frame constraint, except that there is no edge $\left(f_{L}, f_{R}\right)$ on graph $G_{H}$ and no edge $\left(f_{L}^{\prime}, f^{\prime}{ }_{R}\right)$ on graph $G_{V}$. Instead, we add edges, $\left(s, f_{R}\right)$ with cost 0 and capacity $\alpha$ and $\left(f_{L}, t\right)$ with cost 0 and capacity $\alpha$, to graph $G_{H}$; and we add edges, $\left(s, f_{R}^{\prime}\right)$ with cost 0 and capacity $\alpha$ and $\left(f_{L}^{\prime}, t\right)$ with cost 0 and capacity $\alpha$, to graph $G_{V}$. Figure 8 illustrates the two modified graphs for the example above to optimize

(a)

(b)

Figure 8: The modification of graphs to minimize composite cost where $\alpha=3$. (a) The horizontal network graph. (b) The vertical network graph.
the composite cost function, where $\alpha=3$. Thus the min-cost flow based algorithm can be used to optimally minimize the composite cost. It should be noted that the amount of flow is $\alpha+\sum_{i=1}^{n} \lambda_{i}$ in the case.

We have the following necessary and sufficient condition with respect to all these constraints.

Theorem 2. There exists a feasible placement that satisfies all these constraints if and only if there is no negative cycle in graphs $G_{H}$ and $G_{V}$.

When a graph has a negative cycle with unlimited capacity, there does not exist min-cost flow. As we can see, in the graph $G_{H}$ and $G_{V}$, the edges except the edges incident from source node or to sink node have unlimited capacity, and the edges incident from source node or to sink node can not be part of any cycle. Thus any negative cycle will have unlimited capacity. If there is no negative cycle, then the algorithm Min-wire can be used to compute a placement that satisfies all constraints and has the minimum of wire length.

Although the condition is similar to that in [6], there exist important differences. (i) The graphs are different. The graph in [6] contains only nodes representing blocks/subblocks. (ii) The approach in [6] operating on its graph thus does area packing only, while our approach can minimize both area and wire length. (iii) Longest path algorithm is used in [6], while longest/shortest path can not solve our problem and instead main part of our algorithm is min-cost flow.

### 4.10 Applied to Block Placement

Although we take input of sequence pair in the problem definition, the approach can be applied to any floorplan/block placement. Given a floorplan/block placement, we can first extract the topological relation for any pair of blocks and describe as "left of"/"'below". For the pair with diagonal relation, we can choose one of "left of"/"'below" based on the distance in $x / y$ dimension. Then we can construct the constraint graphs and network graphs. Thereafter, we can apply the approach to minimize wire length.

## 5. Experimental Results

We have implemented the algorithm and integrated with the floorplanner, FAST-SP[17]. Our program can also read an existing floorplan and redistribute white space to optimize wire length. Assuming that $\lambda_{i}=O(1)$, we use successive shortest path augmenting algorithm in min-cost flow computation.

We have tested the program as a post-floorplanning step with two floorplanner, FAST-SP[17] and Parquet[1]. The test problems

Table 1: Results of improving wire length in post-floorplanning of FAST-SP and Parquet.

| circuit | block | net | FAST-SP |  |  |  | Parquet |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | wire( mm ) |  | improve | time(s) | wire( mm ) |  | improve | time(s) |
|  |  |  | original | after |  |  | original | after |  |  |
| apte | 9 | 97 | 426.7 | 418.9 | 1.8\% | 0.02 | 476.3 | 458.0 | 3.8\% | 0.03 |
| xerox | 10 | 203 | 486.1 | 462.8 | 4.8\% | 0.07 | 581.6 | 550.6 | 5.3\% | 0.06 |
| hp | 11 | 83 | 170.0 | 161.5 | 5.0\% | 0.02 | 161.1 | 152.4 | 5.4\% | 0.02 |
| ami33 | 33 | 123 | 60.0 | 58.0 | 3.3\% | 0.03 | 77.2 | 74.1 | 4.0\% | 0.03 |
| ami49 | 49 | 408 | 790.1 | 760.2 | 3.8\% | 0.38 | 857.8 | 818.9 | 4.5\% | 0.36 |

are derived from MCNC benchmarks for block placement. We first run FAST-SP or Parquet to obtain a floorplan with option of "minimize wire length". Note that both FAST-SP and Parquet compact blocks to the left and bottom. Then the algorithm Min-wire is applied to further optimize wire length. For all tests, we use the center of block as pin's location, and impose a fixed frame constraint. The locations of IO pins (IO pads) are resized proportionally to the frame boundaries. Table 1 lists the experimental results for minimizing wire length, where all blocks are hard blocks. It should be noted that our program does not change the floorplan topology and the area. Thus area is omitted from the table. The experiments were carried out on a Pentium 4 Mobile( 2.4 Ghz ). As we can see, the algorithm is very efficient in that it takes less than 0.4 seconds for all of the benchmarks. It is also very effective in that it can further improve $4.2 \%$ of wire length on average even on very compact floorplans. As illustrations, Figure 9 and 10 display the placement results of original and after optimization for ami33 and ami49 in FAST-SP respectively.

## 6. Concluding Remarks

In the paper, we have presented a novel method to distribute white space in floorplanning. The method optimally distributes white space among blocks and guarantees to obtain the minimum of total wire length for a given floorplan representation. It is also an exact algorithm to optimize the composite cost function of area and wire length: $\min \left\{\alpha(W+H)+\sum_{i=1}^{n} \lambda_{i} W_{i}\right\}$. We have also shown that the method can handle various constraints such as fixed-frame, boundary pins, pre-placed blocks, boundary blocks, range placement, alignment and abutment, rectilinear blocks, cluster placement, and bounded net delay, without loss of optimality. Experimental results show it is very efficient and effective. Thus it provides an ideal way to refine floorplanning (post-floorplanning). The future work is to extend the method to consider routing congestion and buffer insertion in floorplanning and to apply it in mixed-cell placement.

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Figure 9: (a) The original ami33 placement in FAST-SP. (b) The new placement result when minimizing wire length in the same frame.


Figure 10: (a) The original ami49 placement in FAST-SP. (b) The new placement result when minimizing wire length in the same frame.


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