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Competitive Equilibrium in e-Commerce: Pricing and Outsourcing

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Abstract

The success of firms engaged in e-commerce depends on their ability to understand and exploit the dynamics of the market. One component of this is the ability to extract maximum profit and minimize costs in the face of the harsh competition that the internet provides. We present a general framework for modeling the comeptitive equilibrium across two firms, or across a firm and the market as a whole. Within this framework, we study pricing choices and analyze the decision to outsource IT capability. Our framework is novel in that it allows for any number of distributions on usage levels, price-QoS tradeoffs, and price and cost structures.

keywords: e-commerce, non-cooperative Nash equilibrium, pricing, QoS, outsourcing

1 Introduction

The area of e-commerce has evolved from the notion of internet shopping to a much wider concept, that of the virtual enterprise, and even to virtual enterprise networks. The success of firms engaged in e-commerce depends on their ability to understand the dynamics of the market and also to efficiently exploit the knowledge for improved strategic decisions. The decisions range from product pricing and capacity planning to strategic decisions such as outsourcing. Further, the introduction of an *on-demand* market paradigm has effectively blurred the distinction between different time scales in planning. Enabling this dramatic shift in the way business is conducted requires entirely new paradigms for information exchange, as well as new mathematical models to help firms make optimal decisions. In e-commerce, in particular, the market share of a firm depends not only on its own price and quality of service (QoS) but also on the price and QoS of its competitors, since the latter's offers are instantaneously available to a prospective client.

In the e-commerce sector, eliminating some of the firms' information technology (IT) infrastructure and the corresponding maintenance costs is an appealing possibility. Outsourcing *on demand* is an alternative to maintaining servers and software in-house.

Providing a framework to assess in what cases and to what extent outsourcing is profitable for a firm is important to help firms in making strategic outsourcing decisions.

To help e-commerce firms make optimal pricing, planning and outsourcing decisions, a comprehensive framework is needed. Such a framework should include models for customers' choices and the demand stochastics associated with them, models for understanding the dynamics of competition among firms with both price and QoS as parameters, and models for the firms' ability to outsource their demand to a third party service provider. The purpose of this work is to present an equilibrium framework that captures the particularities of e-commerce markets and their ability to outsource *on demand*. To do so, this framework analyzes their pricing behavior at equilibrium, taking into account the quality of service of the e-services the firms offer. The motivation is, as in traditional game-theoretic models of firms, that firms can adjust their price schedules in response to that of a competitor. Then, the question for any one firm is whether the joint setting of prices by all firms will tend towards an equilibrium, and, in the affirmative, what are the properties of the equilibrium. New to this paradigm, in addition to the stochastic demands and definitions of the goods and services offered, is the fact that firms may outsource any number of their activities. We are particularly interested in modeling the choice of outsourcing IT-related activities within e-commerce.

1.1 Related Work

The pricing of electronic goods, and, in particular, e-commerce services, such as web hosting, has received considerable attention, see, for example, [11], [13], [16], [17], [15], [19], [14], [21]. Some studies into optimal pricing have been viewed from the perspective of a single firm, or in the context of a monopoly (see, for example, [12, 20]). This approach fails to model the effect of competitions on firms' decisions. Traditional equilibrium models used for analyzing optimal decisions for firms in a competitive environment, such as [1], do not consider the particularities of e-commerce such as demand patterns induced by internet commerce, or availability of cost reduction or e-commerce quality improvements through outsourcing.

Fishburn and Odlyzko [6] explored the Nash equilibrium that would result across two firms competing in an e-service market, one charging a fixed, per-period, fee, and the other charging on a per-transaction basis, where the per-transaction fee is linear. The authors concluded that, with the exception of a few special forms of the clients' demand distribution, competitive equilibria of this type results in the trivial solution of each firm's price tending towards zero. This result may be seen, however, as natural, given that the two firms in the model of [6] were competing *solely on the basis of price, the capacity of each firm was unlimited, and no product differentiation was introduced*. In that setting, it can be seen as an instance of a classic Bertrand duopoly, which is known to result in similar economics to that of perfect competition, the latter leading clearly to zero profits for all firms in the market.

El Azouzi, Altman, and Wynter [5] analyze the two-firm price-and-QoS equilibrium in the context of internet, taking into account different forms of QoS, such as delay and loss probability. The demand for service of each firm however, was defined by deterministic, linear functions which smoothens the stochastic nature of demand resulting from heterogeneous customer preferences. Gibbens, Mason, and Steinberg

[7], studying product differentiation in the context of the Internet also consider the two parameters of price and delay, focusing on a market of two service providers. They consider in particular the choice of each provider to offer one or two QoS classes. We compare and contrast their results with ours when applicable in this work. Other work on the pricing of information goods and services has advocated differentiated services and bundling [20], or flat pricing [10, 17], due to its simplicity but have not made use of an equilibrium framework explicitly in their arguments.

Cachon and Harker [2], studied the effect of outsourcing on competition in a two-firm market. They analyze the equilibrium profits of a two-firm market, and consider the possibility of one or both firms to outsource to a common supplier. Based upon a particular model for supplier and firm profits with and without outsourcing, they conclude that both firms are better off outsourcing. We show in this paper that a very general model can be introduced that allows one to determine when the outsourcing decision is profitable, for general forms of demands, and costs.

We consider a particular characterization of quality of service (QoS), namely, response time or delay. In our model of competitive equilibrium across two firms providing e-services, each firm is characterized by the price it charges, the quality of service it offers, and a *randomly distributed trade-off parameter* that arbitrates between the two criteria of price and delay. End-users usage levels are randomly distributed parameters as well, and different forms of the distributions are analyzed and contrasted. As opposed to the model in [2, 5], we consider explicitly the stochastics associated with end-user usage levels and differentiation across users of the trade-off between price and QoS. However, unlike [5], we do not determine simultaneously the equilibrium in prices and QoS; rather the decision variable through which firms respond is their price. In [2], although terms for both price and QoS were included, all users evaluated the two parameters equivalently to each other and weighted the two parameters equally as well. It is well known, however, that the spectrum of IT users is as heterogeneous as the applications which use IT; arbitrage between high prices and low QoS vary in some stochastically quantifiable and non-constant manner across the population. Not taking into account this heterogeneity introduces a non-negligible bias into both quantitative and qualitative results (see [9] for some examples of this bias). Within this framework we also study the effect of the outsouricng decision on a two-firm market equilibrium.

While we model here customer behavior dispersion due to price and QoS differences (and their perception thereof), it is possible through our framework to isolate one or the other effect by considering, in the first case, that both providers offer the same price structure (then only QoS-dispersion effects are present). Similarly, one can set the QoS of both providers equal, thereby focusing on the price structure difference. Note, however, that in the latter case, our model reduces to that of [6].

1.2 Structure of the paper

In Section 2, we introduce the notion of a value of QoS parameter that varies continuously throughout the population of users, allowing us to model the universe of choices made by users with respect to the cost-QoS tradeoff. Depending upon whether this random variable is uniformly or exponentially distributed, we obtain qualitatively different results. We consider the case where the new entrant to the market offers a lower QoS (along with, presumably, a lower price) as well as the converse. Section 2.3 presents the case of two-tier pricing, composed of a base rate and a higher burst-rate price, where the cut-off between the two prices is defined by the user.

Section 3 introduces in the model the third-party supplier providing outsourcing services. We analyze the effect of outsourcing the IT activities to a common supplier for one or both of the firms at equilibrium. As in [2] we analyze the potential profit of the supplier given the demand it would attract from one or both of the firms; in the event that its costs exceed its revenue, no outsourcing would be offered. This is referred to in [2] as a *two-stage negotiation process:* in the first stage, an agreement is or is not reached, depending on the potential profitability for both the client firm and the outsourcing supplier. Depending on the outcome of that stage, some competitive equilibrium is achieved across the firms. A general version of the result holding for any distribution is provided in Section 3.4. Finally we conclude in Section 4 and provide recommendations for further research on this theme.

2 User differentiation and multiple service characteristics

We consider a setting in which potential customers' usage rates, or requested capacities, are defined by a probability density function g, that is, $\int_0^\infty v(x)dx = 1$, where the argument x is the desired rate of a potential user.

Contrary to the model of [6], We suppose that the e-service offered by firm $i = 1,\dots n$ is characterized by a 2-tuple, $(p_i(x), d_i)$, where $p_i : \Re_+^n \mapsto \Re_+$ is the price function charged for use of the service, which depends upon the usage level, x, and d_i the quality of service. We know from [6] as well as from classical Bertrand competition, that competition based only upon price leads to zero profits for all firms. Service differentiation, through a QoS parameter, can remedy that ruinous result.

The quality of service will be taken in the remainder of this paper to be some measure of service performance, the delay incurred on a typical e-commerce transaction. Note that we are *not* considering the situation in which $d_i = d_i(x)$; in this work, we shall restrict ourselves to the simpler setting of *usage-independent delays*.

Each user is then characterized by a particular constant w that models his willingness to pay for a higher quality of service. That is, w gives the user's own tradeoff between price and delay. Note then that in the case of multiple service characteristics, w_k would provide the user's tradeoff between price and attribute k, or how much the user is willing to pay to increase the level of QoS by one unit.

We shall suppose that the user tradeoff parameter, w, is not constant for all users, but rather is described by a random variable, distributed over the population of potential customers. This feature of our model is critical; we are in effect capturing the *universe of users' behaviors* with respect to the cost vs. quality tradeoff. For example, a user requiring low-priority service, for email or file transfer operations, would be characterized by a *low value of tradeoff, w*, whereas a job requiring more bandwidth, faster service, e.g., mission-critical applications etc. and for which the user is willing to pay for the better quality, would be characterized by a high value of w. As has been observed in internet traffic as well as in the population in general, the percentage of low values of QoS is much higher than the percentage of high values, across users. This observation has an impact on the *form* of the distribution of the tradeoff parameters, w, as we shall discuss in this paper.

Then, the probability density on user's rate levels becomes a joint distribution of rate levels and cost-QoS tradeoff parameters:

$$\int_{x=0}^{\infty}\int_{w=0}^{\infty} v(x,w)dwdx = 1.$$

Consider one potential user or, equally, one potential usage decision. Note that a user may make several decisions, one may interpret each atomic deicision as a single user, or a single choice, where a user may make several choices. Then, given each value of the tradeoff parameter, w (for each usage choice), and the desired usage level, x, he will optimize, for each choice, his choice of provider, among the n firms, by choosing the one that minimizes his combined cost:

$$i^* \in \arg\min_i \{p_i(x) + wd_i\}. \tag{1}$$

If prices are exactly equal across providers, then one may assume that the market is split equally across those providers. Note that not only does w have the behavioral represention of the user's cost-QoS tradeoff, but, as it is expressed in units of dollars per time, it permits summing the two criteria, $p_i(x)$ and d_i.

Up to now, we have not specified the forms of the prices offered by each provider, $p_i(\cdot)$. In order to specify fully the model, we must make some assumptions about the pricing structures, p_i. Suppose that $n = 2$ providers, and that g is continuously differentiable in its arguments. Let further $p_1(x) = p_1$ and $p_2(x) = p_2 x$. That is, provider 1 charges a flat (subscription-based) fee while provider 2 charges a simple (linear) usage-based fee.

In this case, a user characterized by two-tuple (x,w) chooses provider 1 if

$$p_1 + wd_1 \leq p_2 x + wd_2, \tag{2}$$

and chooses provider 2 otherwise.

Let us suppose initially that $p_1 > p_2$ and $d_1 < d_2$; that is, the supplier offering the flat rate offers a better quality of service (lower delay) as well.

It is clear then that there are thresholds in x and w for which one or the other supplier is cost-effective for a user. Specifically, for $x(w) \geq p_1/p_2$, supplier 1 is cheaper. Since in this example, supplier 1 also has a better QoS in that the delay it offers is lower, users with $x(w) \geq p_1/p_2$ will choose supplier 1 for all w. Similarly, when $x \leq p_1/p_2$ and $w \geq \hat{w} = (p_1 - p_2 x)/(d_2 - d_1)$, supplier 1 is chosen. Supplier 2 is chosen for all other values of w,x.

For succinctness, let us refer to the vector (p_1, p_2) as p, henceforth. Then, the revenues of providers 1 and 2 can be expressed by:

$$R_1(p) = p_1 \left[\int_{\frac{p_1}{p_2}}^{\infty} \int_0^{\infty} v(x,w)dwdx + \int_0^{\frac{p_1}{p_2}} \int_{\hat{w}(x)}^{\infty} v(x,w)dwdx \right], \tag{3}$$

$$R_2(p) = p_2 \int_0^{\frac{p_1}{p_2}} \int_0^{\hat{w}(x)} xv(x,w)dwdx, \tag{4}$$

where,

$$\hat{w}(x) = \frac{p_1 - p_2 x}{d_2 - d_1}. \tag{5}$$

In that case, we can write out the first order conditions for Nash equilibrium, that is, $\partial R_1(p)/\partial p_1 = 0$ and $\partial R_2(p)/\partial p_2 = 0$.

While we do not include a fixed portion of the usage-based cost for provider 2, it is clearly the case that one could add such a cost, in which case, the revenue for provider 2 would have two integrals, where the limits would be the same, but only the second one would include the variable x. The constants would be the fixed fee and p_2, respectively. The interpretation would be then that, in order to have service from provider 2, one needs to pay some upfront fee, and thereafter a usage-dependent price.

Then, the question of interest is whether this system has a nontrivial solution, that is, one in which $p_i \neq 0$, $i = 1,2$ for different assumptions on the forms of the distribution $v(x,w)$, and, if so, what are the properties of that equilibrium.

2.1 Simplified model: homogeneous usage levels and uniformly distributed values of QoS

We may first consider a simplified model in which we do not maintain a distribution of usage levels x, but rather examine the equilibrium in which all users are defined by a unique, constant, usage level, x, i.e., $v(x,w) \equiv v(w)$.

To further simplify, let the distribution of delay-cost tradeoff constants, w, be uniform on the interval $[0,1]$. Then, assuming still that $d_2 - d_1 \geq 0$, provider 1 will obtain $1 - \hat{w}(x)$ of the market when $p_1 \geq p_2 x$. Note that if $p_1 \leq p_2 x$, provider 1 obtains the entire market, since both price and delay are less than that offered by provider 2. Therefore, consider the former setting; we have that

$$R_1(p) = p_1(1 - \hat{w}(x)) = p_1 \left[1 - \frac{p_1 - p_2 x}{d} \right], \tag{6}$$

$$R_2(p) = p_2 x \hat{w}(x) = p_2 x \left[\frac{p_1 - p_2 x}{d} \right]. \tag{7}$$

Solving, we obtain that the equilibrium prices are

$$p_1^* = 2d/3, \tag{8}$$

$$p_2^* x = d/3, \tag{9}$$

where $d = d_2 - d_1$. The equilibrium threshold, $\hat{w}^*(x)$, for choosing provider 2 is

$$\hat{w}^*(x) = 1/3.$$

The two prices are equal only in the case where the two providers offer the same QoS, that is, $d_1 = d_2$, and, indeed, both would be zero. Provider 1, offering the flat, subscription-based price structure always has the larger market share, with 2/3 over provider 2's 1/3.

However, we observe here that the use of a uniformly-distributed value of QoS parameter, w, can lead to misleading conclusions. By taking the distribution, $v(.,w)$ to be uniform, we assume that there are as many economical-minded users as there are 'big-spending' users. High values of w signify a high willingness to pay for an improved QoS. Common knowledge, and empirical data, tell us otherwise, however – the proportion of users who opt for cheaper, lower QoS, service configurations is generally much larger than the proportion who pay highly for the best QoS. Typically, an exponential, or log-normal distribution should be used to model such tradeoff parameters over the population. Through the example, we see that the simplification of uniformly distributed parameter, w may lead to the misleading conclusion that the market share for provider 2, appealing to users wishing for cheaper, lower QoS, service, would always be fixed at 1/3. For more realistic non-uniform distribution, as we shall see, this conclusion may no longer be true.

Remark 1 *[Both providers offer flat, subscription-based services] Note further that this result is unchanged if both providers charge flat, subscription-based fees. In that case, the equilibrium prices are simply $p_1 = 2d/3$, $p_2 = d/3$, where provider 2's price is now flat rather than multiplied by the usage level, x, and the threshold $\hat{w}^*(x) = \hat{w}^* = 1/3$.*

This type of result contrasts with that of Gibbens, Mason, and Steinberg [7][Prop.1], who consider two service providers, each offering one type of service, defined by price and delay, and, as in this example, a uniformly-distributed value of QoS parameter, w. In their model, delay is linear in usage for both providers, and prices for both providers are flat (subscription- rather than usage-based). They conclude that the unique equilibrium in this case occurs when both providers' prices are equal. However, the assumption that both providers offer services with the same delay level is clearly a strong one; we saw from the equilibrium we computed in our above simple example, that, if delays of both providers are equal, ($d_1 = d_2$, so that $d = 0$), then prices are indeed equivalent for the two providers, but they are also zero. Furthermore, the use of a uniformly-distributed QoS parameter, as we shall confirm, leads to biased results.

2.1.1 New Entrant with Better QoS

Let us return again to a market situation in which provider 1 charges a flat fee and provider 2 a usage-based fee, $p_2 x$. We now consider the scenario where provider 2's offered QoS is better than that of provider 1, i.e., $d_2 < d_1$. Then a user (x,w) will choose provider 2 if $p_1 \geq p_2 x$, or if $p_1 < p_2 x$ and $w \geq \hat{w}(x)$, where $\hat{w}(x)$ is given by (5). In the latter setting, provider 1 will then get $\hat{w}(x)$ of the market when $p_1 < p_2 x$.

Therefore, when assuming all users have a single usage level x, the revenue of the two providers are respectively,

$$R_1(p) = p_1\hat{w}(x) = p_1\frac{p_2x - p_1}{d_1 - d_2}, \tag{10}$$

$$R_2(p) = p_2x(1-\hat{w}(x)) = p_2x\left[1 - \frac{p_2x - p_1}{d_1 - d_2}\right]. \tag{11}$$

Solving, we obtain that the equilibrium prices are

$$p_1^* = -d/3 = \frac{d_1 - d_2}{3} \geq 0, \tag{12}$$

$$p_2^*x = -2d/3 = \frac{2(d_1 - d_2)}{3} \geq 0, \tag{13}$$

since, as before, $d = d_2 - d_1$, which is now negative, and

$$\hat{w}^*(x) = 1/3.$$

That is, provider 2, now having a better QoS, will get 2/3 of the market in equilibrium.

2.2 Exponentially-distributed usage levels and value of QoS parameters

Suppose now that both the usage levels, x, and the values of QoS, w, are distributed according to exponential distributions, each with its own mean, $1/a$ and $1/b$, respectively, where $a, b > 0$. Then, $v(x,w) = g(x)h(w)$, with $g(x) = ae^{-ax}$ and $h(w) = be^{-bw}$.

For usage levels, this hypothesis is a well-motivated one, since it represents the usual Poisson arrivals into the system. For describing the dispersion of the value of QoS parameter over the population, the exponential distribution seems also well-justified since it possesses a shape in which a higher concentration of the population has a lower willingness-to-pay.

The system (3)-(4) then simplifies due to the separability of the distributions on x and w. Evaluating the expression for $R_1(p)$, we obtain

$$R_1(p) = Qp_1e^{-\frac{bp_1}{d}} + (1-Q)p_1e^{-\frac{ap_1}{p_2}},$$

where

$$Q = \frac{a}{a - \frac{bp_2}{d}}.$$

The expression for provider 2's revenue is less compact. We obtain

$$R_2(p) = ap_2 \int_0^{p1/p2} x \left[e^{-ax} - e^{-\frac{p_1 b}{d}} e^{-\left[a-\frac{bp_2}{d}\right]x} \right] dx, \quad (14)$$

$$= \frac{p_2}{a} \int_0^{ap_1/p_2} xe^{-x} dx - \frac{ap_2 e^{-\frac{p_1 b}{d}}}{\left[a - \frac{bp_2}{d}\right]^2} \int_0^{\left[a-\frac{bp_2}{d}\right]\frac{p_1}{p_2}} xe^{-x} dx, \quad (15)$$

$$= \frac{p_2}{a} \left[1 - \left(1 + \frac{ap_1}{p_2}\right) e^{-\frac{ap_1}{p_2}} \right]$$

$$- \frac{ap_2 e^{-\frac{p_1 b}{d}}}{\left[a - \frac{bp_2}{d}\right]^2} \left[1 - \left(1 + \frac{p_1}{p_2}\left[a - \frac{bp_2}{d}\right]\right) e^{-\frac{p_1}{p_2}\left[a-\frac{bp_2}{d}\right]} \right]. \quad (16)$$

Now, defining dummy variables u and z and letting $u = ap_1/p_2$ and $z = [a - bp_2/d](p_1/p_2) = u - bp_1/d$, and then noting that $\partial/\partial p_2 = (\partial u/\partial p_2)(\partial/\partial u) = -(ap_1/p_2^2)(\partial/\partial u)$ and that $\partial z/\partial u = 1$, we obtain in terms of u and z:

$$\frac{\partial R_1}{\partial p_1} = \frac{u}{z}(1+z-u)e^{z-u} + \left(1 - \frac{u}{z}\right)(1-u)e^{-u} \quad (17)$$

$$\frac{\partial R_2}{\partial p_2} = \frac{1}{a}\left[1 + \frac{u^2}{z^2}\left(1 - \frac{2u}{z}\right)e^{z-u} - \left(1 + u + u^2 - \frac{2u^3}{z^3} + \frac{u^2}{z^2} - \frac{2u^3}{z^2} - \frac{u^3}{z} + \frac{u^2}{z}\right)e^{-u} \right]. \quad (18)$$

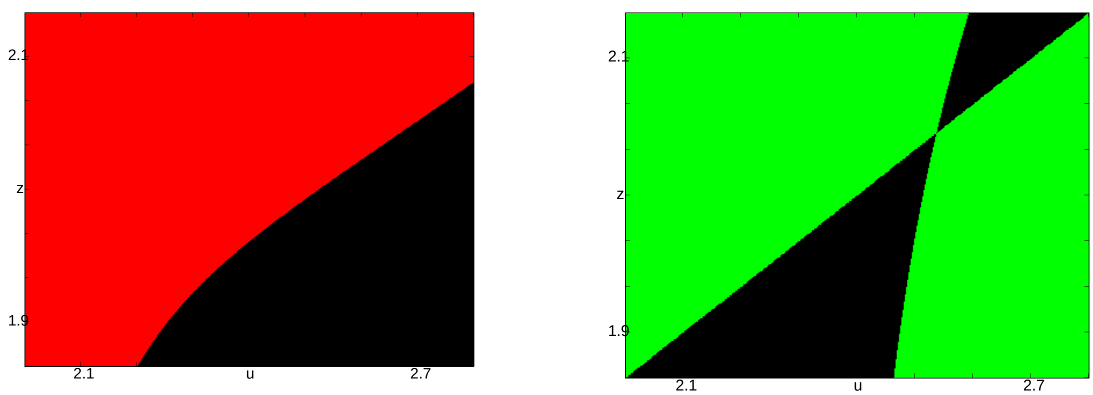

Figure 1: Zeros of each derivative function, $dR1$ and $dR2$ in the space of dummy variables (u,z)

It is not possible to solve the system (17)–(18) analytically; however, we may examine the system numerically for simultaneous solutions (i.e. interior solutions). The Figure 1 provides a graphical illustration of the zeros of each function, $\frac{\partial R_1(p)}{\partial p_1}$, which we shall denote $dR1$, and $\frac{\partial R_2(p)}{\partial p_2}$, denoted $dR2$. In the figure, when $dR1$ or $dR2$ is

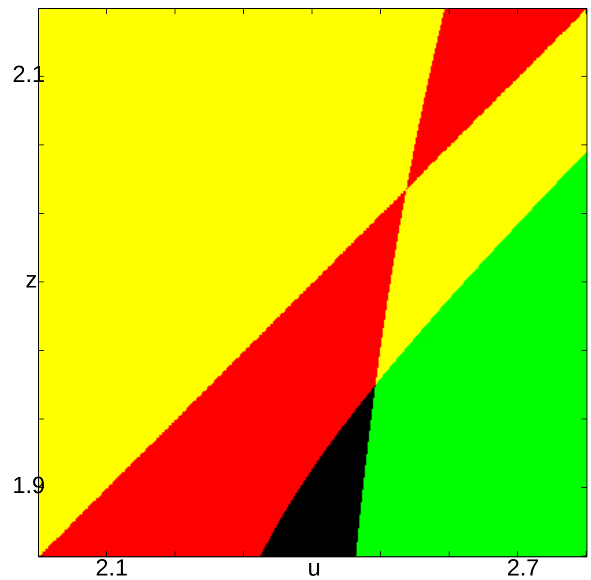

Figure 2: The two plots of Figure 1 super-imposed to illustrate the interior Nash equilibrium solutions in the space of dummy variables (u,z)

negative, at a point (u,z), the corresponding point in the space (u,z) is black. When $dR1(u,z) > 0$, then the point (u,z) is colored dark grey, and when $dR2(u,z) > 0$, the point (u,z) is colored light grey. The Figure 2 then illustrates those two graphs super-imposed, providing the simultaneous zero of the two equations.

Two equilibria were identified by exploring a large feasible region in this manner; that is, two points in the (u,z) plane were found at which black, and the two shades of grey meet (i.e., simultaneous zeros of both $dR1$ and $dR2$). The first point, shown in the figure, can be shown to exist at $(u,z) = (2.5701, 1.96)$ and another solution found by examining a different region of the space was found at $(u,z) = (2.133, -2.64)$.

In terms of extremal, solutions, we would not expect interesting solutions in which either p_1 or p_2 were zero.

Solving then for the equilibrium prices, p_1^* and p_2^*, when $(u,z) = (2.5701, 1.96)$, we obtain:

$$p_1^* = 0.61d\frac{1}{b}, \tag{19}$$

$$p_2^* = 0.24d\frac{a}{b}. \tag{20}$$

When $(u,z) = (2.133, -2.64)$, we obtain:

$$p_1^* = 4.77d\frac{1}{b}, \tag{21}$$

$$p_2^* = 2.23d\frac{a}{b}. \tag{22}$$

Analyzing the revenue of the two providers at this solution, we consider a continuum of values for the two means of the exponential distribution, a, the mean on client usage levels, and b the mean value of users values of QoS, and two possible QoS differences, $d = d_2 - d_1$. The revenues depend upon the QoS differences, and increase as d increases, as can be seen for the case of Provider 1 from Figure 3. However, the ratio of R_2 to R_1 stays constant at around 1.1; that is, provider 2, who charges a usage-based price, always has a higher revenue than that of provider 1, who charges a flat fee. This contrasts with the results obtained when the distribution governing the value of QoS parameter was uniform, illustrating that the effect of using a more realistic, non-rectangular probability distribution for the willingness-to-pay parameter is significant.

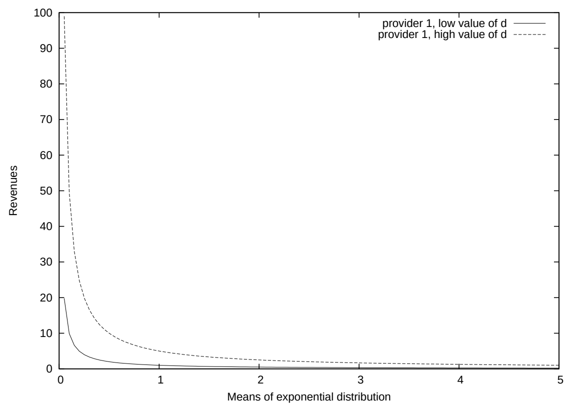

Figure 3: Revenues of provider 1 and provider 2 as d increases on the x-axis

2.2.1 Both providers charge flat fees

Suppose now that both providers 1 and 2 choose to charge flat, subscription-based fees. Then, assuming still that $d_2 > d_1$ and $p_2 \leq p_1$, the revenues of the two providers are given by

$$\begin{aligned} R_1(p) &= p_1 \int_{\hat{w}}^{\infty} v(w)dw, \\ R_2(p) &= p_2 \int_{0}^{\hat{w}} v(w)dw. \end{aligned}$$

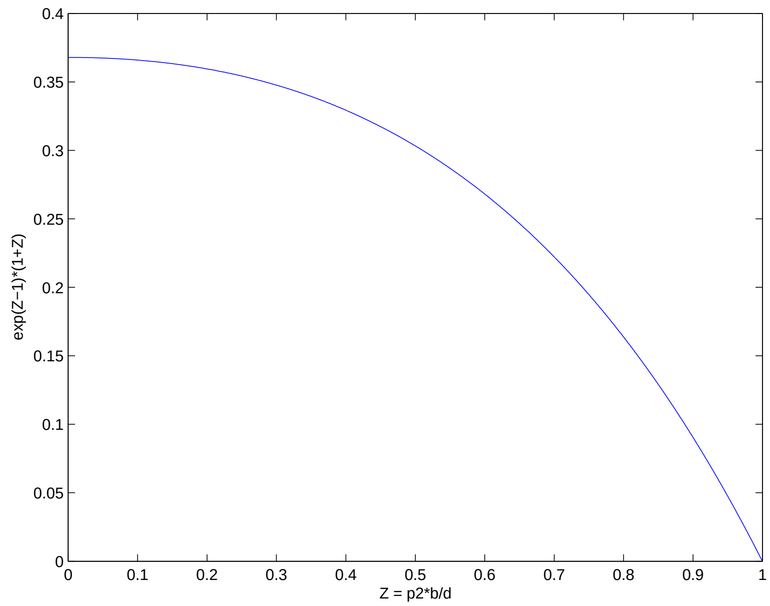

Figure 4: There is no equilibrium solution in the case of both providers offering flat prices

With exponentially distributed value of QoS parameters, we have that

$$
\begin{aligned}
R_1(p) &= p_1 e^{-b\hat{w}} = p_1 e^{-b\left(\frac{p_1-p_2}{d}\right)}.\\
R_2(p) &= p_2\left(1-e^{-b\hat{w}}\right) = p_2\left(1-e^{-b\left(\frac{p_1-p_2}{d}\right)}\right).
\end{aligned}
$$

Solving, we have

$$\frac{\partial R_1}{\partial p_1} = \left(\frac{1-bp_1}{d}\right)e^{-b\hat{w}} = 0 \tag{23}$$

$$\frac{\partial R_2}{\partial p_2} = 1 - e^{-b\hat{w}}\left(1+\frac{bp_2}{d}\right) \tag{24}$$

Then, as opposed to the case in which provider 2 chooses usage-based pricing, when both providers choose flat pricing, there is no positive solution to the system. Indeed, we have that $p_1 = d/b$ and, setting $Z = bp_2/d$, p_2 solves $e^{Z-1}(1+Z) = 1$. However, as we see from Figure 4 there is no solution to that system, as the function does not equal 1 for any set of positive parameter values. This parallels the finding of Fishburn and Odlyzko [6] that price wars would result from price competition in the sector.

2.2.2 New Entrant with Better QoS

Assume now that $d_2 < d_1$, that is, provider 2, who charges a usage-based fee, offers a better QoS. Then a user (x,w) will choose provider 2 if $p_1 \geq p_2 x$, or if $p_1 < p_2 x$ and

$w \geq \hat{w}(x)$, where $\hat{w}(x)$ is given by (5). The corresponding revenues of providers 1 and 2 can be expressed by:

$$R_1(p) = p_1 \int_{\frac{p_1}{p_2}}^{\infty} \int_0^{\hat{w}(x)} v(x,w)dwdx, \tag{25}$$

$$R_2(p) = p_2 \left[\int_0^{\frac{p_1}{p_2}} \int_0^{\infty} xv(x,w)dwdx + \int_{\frac{p_1}{p_2}}^{\infty} \int_{\hat{w}(x)}^{\infty} xv(x,w)dwdx \right], \tag{26}$$

where $\hat{w}(x)$ is given by (5).

Again, suppose that both the usage levels and the values of tradeoff are distributed according to exponential distributions with means $1/a$ and $1/b$where $a,b > 0$. Then, $v(x,w) = g(x)h(w)$, with $g(x) = ae^{-ax}$ and $h(w) = be^{-bw}$. Evaluating the system (25)-(26) at the given exponential distributions, we obtain

$$R_1(p) = (1-Q)p_1 e^{-\frac{ap_1}{p_2}}, \tag{27}$$

$$R_2(p) = \frac{p_2}{a} \left[\int_0^{\frac{ap_1}{p_2}} xe^{-x}dx + Q^2 e^{-\frac{p_1 b}{d}} \int_{\frac{ap_1}{p_2}}^{\infty} xe^{-x}dx \right], \tag{28}$$

where, as before, $d = d_1 - d_2$ (but now negative) and $Q = \frac{a}{a-\frac{bp_2}{d}}$.

Define, as before, dummy variables u and z and letting $u = ap_1/p2$ and $z = [a - bp_2/d](p_1/p_2) = u - bp_1/d$, we obtain in terms of u and z:

$$\begin{aligned} \frac{\partial R_1}{\partial p_1} &= (1-\frac{u}{z})(1-u)e^{-u}, \\ \frac{\partial R_2}{\partial p_2} &= \frac{1}{a}\left[1 - \left(1+u+u^2-\frac{u^3}{z}+\frac{u^2}{z^2}(1+z)(1-\frac{2u}{z})\right)e^{-u}\right]. \end{aligned}$$

Solving the above system then gives the equilibrium: $u^* = 1$, and $z^* = 2.5227$, which is the only real solution of $(3-e)z^3 - z - 2 = 0$.

The equilibrium prices of the two providers are thus

$$\begin{aligned} p_1^* &= -1.5227d\frac{1}{b} \geq 0, \\ p_2^* &= ap_1^*. \end{aligned}$$

Note that both equilibrium prices are therefore non-negative, since here, $d = d_2 - d_1 \leq 0$.

2.3 Two Tier Pricing

Let us now consider the question of two-tier pricing, that is, a usage-based pricing structure with two different unit costs: lower if the usage level is below a threshold, and a higher unit cost above that threshold. Suppose that provider 2 now charges using a two-tier, i.e., burst-rate, pricing structure, with

$$p_2(x) = \begin{cases} p_2 x, & \text{if } x \leq T, \\ q_2 x, & \text{otherwise,} \end{cases}$$

and $q_2 > p_2$. That is, provider 2 still charges a (linear) usage-based fee but the burst rate will be higher when the usage level is above a given threshold T.

For example, T here may represent the 95 percentile of total period usage, referred to as α for an particular user.

Assume that provider 1 still charges a flat fee, p_1, and that the delays are the same as in the previous section, i.e. $d_2 - d_1 \geq 0$.

For a user characterized by two-tuple (x, w), his strategies can be summarized by the following three cases:

1) If $T \geq \frac{p_1}{p_2}$, a user would choose provider 1 if either $x \geq \frac{p_1}{p_2}$ or $x < \frac{p_1}{p_2}$ and $w \geq \hat{w}(x)$, where $\hat{w}(x)$ is given by (5) as before. In this case, the revenues of the two providers R_1 and R_2 are the same as in (36) and (37), and are independent of q_2.

2) If $\frac{p_1}{p_2} > T \geq \frac{p_1}{q_2}$, users would choose provider 1 if either $x \geq T$ or $x < T$ and $w \geq \hat{w}(x)$. Then, the corresponding revenues are

$$R_1(p) = p_1 \left[\int_T^\infty \int_0^\infty \mu(x,w) dw dx + \int_0^T \int_{\hat{w}(x)}^\infty \mu(x,w) dw dx \right], \qquad (29)$$

$$R_2(p) = p_2 \int_0^T \int_0^{\hat{w}(x)} x \mu(x,w) dw dx, \qquad (30)$$

3) If $\frac{p_1}{q_2} > T$, users would choose provider 1 in one of the following three sub-cases: i) $x \geq \frac{p_1}{q_2}$; ii) $T \geq x$ and $w \geq \hat{w}(x)$; and iii) $T \leq x < \frac{p_1}{q_2}$ and $w \geq \hat{w}_q(x)$, where

$$\hat{w}_q(x) = \frac{p_1 - q_2 x}{d}. \qquad (31)$$

The corresponding revenues are given by

$$R_1(p) = p_1 \left[\int_{\frac{p_1}{q_2}}^\infty \int_0^\infty \mu(x,w) dw dx + \int_T^{\frac{p_1}{q_2}} \int_{\hat{w}_q(x)}^\infty \mu(x,w) dw dx \right. \\ \left. + \int_0^T \int_{\hat{w}(x)}^\infty \mu(x,w) dw dx \right], \qquad (32)$$

$$R_2(p) = p_2 \int_0^T \int_0^{\hat{w}(x)} x \mu(x,w) dw dx + q_2 \int_T^{\frac{p_1}{q_2}} \int_0^{\hat{w}_q(x)} x \mu(x,w) dw dx. \qquad (33)$$

2.4 Uniformly-distributed QoS

As before, we begin with the simplest model based on a single usage level, x, and assume that the distribution of delay-cost tradeoff constants, w, is uniform on the interval $[0, 1]$. Assume still that $d_2 - d_1 \geq 0$.

Note that when $x \leq T$, the user strategy is independent of q_2. By checking through the three cases introduced above, one can easily identify that provider 2 will obtain the portion $\hat{w}(x)$ of the market when $p_1 \geq p_2 x$, and 0% of market otherwise. In this case, the revenues of the two providers are the same as in Section 2.1, hence the same equilibrium exists as before, namely,

$$p_1^* = 2d/3, \quad \text{and} \quad p_2^* = d/3x$$

and $w^* = \frac{1}{3}$.

When $x > T$, however, the unit price charged by provider 2 is q_2. So provider 2 will obtain the portion $\hat{w}_q(x)$ of the market when $p_1 \geq q_2 x$, and 0% of the market otherwise. Hence

$$R_1 = p_1(1 - \hat{w}_q(x)) = p_1 \left[1 - \frac{p_1 - q_2 x}{d}\right], \tag{34}$$

$$R_2 = q_2 x \hat{w}_q(x) = q_2 x \left[\frac{p_1 - q_2 x}{d}\right], \tag{35}$$

which can be easily solved to obtain

$$p_1^* = 2d/3, \quad \text{and} \quad q_2^* = d/3x$$

and $w^* = \frac{1}{3}$.

That is, assuming the delay-cost tradeoff w is uniformly distributed, the multi-tier pricing structure does not change the Nash equilibrium.

This implies the following, perhaps counter-intuitive result: *it may not be worth the effort for a firm to engage in convincing users to subscribe to two-tiered prices, if customer willingless is already low, since equilibrium profits will not be higher with that more complex price structure*. Naturally, if there are other reasons for using the burst-rate structure, such as obtaining a priori estimates of customer usage levels, e.g. for capacity planning, these would have to be weighed with the simplicity gained from eliminating the two tiers. Other customer preference-revealing methods may also exist without resorting to a two-tiered structure, as well.

We have looked at the competitive equilibrium when the two firms are maintaining all the offered service capability in-house. The option of outsourcing, typically in *on-demand* business pardigm has given the firms the flexibility to outsource their loads to some outsourcing service provider (third -party) by paying some price and thus free itself from maintianing the capacity and other resources in-house to meet the demand. We next supplement our model with an outsourcing service provider and investigate the effect of outsouricng on market equilibrium and on the equilibrium revenue of the firms.

3 Third-Party Supplier: Outsourcing

Recall that the revenues of providers 1 and 2 can be expressed by:

$$\begin{aligned} R_1 &= \int_0^\infty \int_{\hat{w}(x)}^\infty p_1(x) g(x) h(w) dw dx, \\ R_2 &= \int_0^\infty \int_0^{\hat{w}(x)} p_2(x) g(x) h(w) dw dx. \end{aligned}$$

The profit functions of each firm then take into account the revenue accrued less the cost of providing the necessary IT services. Keeping very general forms of the firms' IT costs, we can express the two firms' profit functions as:

$$\Pi_1 = \int_0^\infty \int_{\hat{w}(x)}^\infty (p_1(x) - c_1(x)) g(x) h(w) dw dx, \quad (36)$$

$$\Pi_2 = \int_0^\infty \int_0^{\hat{w}(x)} (p_2(x) - c_2(x)) g(x) h(w) dw dx, \quad (37)$$

We are interested in modeling the possibility of each firm to outsource its IT needs to a third-party supplier and determining the effect that such a choice has on the firm's profit. As in [2], we suppose that the supplier gives no preferential treatment to either firm, and therefore proposes the same price structure to each.

Suppose that a single IT outsourcing supplier charges based on price function $p_s(x)$. That is, the fees may be different at different usage levels x. Here $p_s(x)$ may be a concave increasing function of x, or a discretization of a concave increasing function, as typically the case in practice.

In addition, we assume that if a firm decides to use the outsourcing supplier, it offloads *all* its IT needs to that supplier. Therefore, firm i, $i = 1, 2$, will face IT cost

$$c_i(x) = \begin{cases} p_s(x) & \text{if firm } i \text{ outsources} \\ c_i^{in}(x) & \text{if firm } i \text{ insources} \end{cases} \quad (38)$$

That is, a firm's IT cost will be either the fee it pays to a supplier s, $p_s(x)$, if it decides to outsource or its own cost $c_i^{in}(x)$ if it decides to insource.

Given above cost functions, the firms compete in this game essentially on the pricing structure, each deciding whether to insource or outsource its IT provision. The first question of interest is whether this system has a nontrivial solution, that is, one in which $p_i \neq 0$, $i = 1, 2$ for different assumptions on the forms of the distributions $g(x), h(w)$, and, if so, what are the properties of that equilibrium.

3.1 Uniformly Distributed Usage-levels and Trade-off Parameters

We first consider a simple model in which the distributions of usage levels and the price-QoS tradeoff are uniform on interval $[\underline{x}, \overline{x}]$ and $[\underline{w}, \overline{w}]$ respectively. To avoid bias due to specific pricing policies (usage based, flat etc.) we conisder the scenario where the prices as well as delays are usage-independent and cost is linear in usage level for the two firms. In other words, $p_i(x) = p_i$, $c_i(x) = c_i x$ and $d_i(x) = d_i$, $i = 1, 2$. As a result of these assumptions, $\hat{w}(x) = \hat{w}$; that is, the critical value of the price-QoS tradeoff parameter, $\hat{w}$, which determines what portion of the usage-level distribution will choose which firm, is also independent of usage. Let $w_r = \overline{w} - \underline{w}$ then $h(w) = \frac{1}{w_r}$.

Evaluating (36) and (37) with these uniformly-distributed random variables and solving for equilibrium prices, p_i^*, $i = 1, 2$, by putting $\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0$, we obtain (with

$x_m = \frac{\underline{x}+\bar{x}}{2}$):

$$
\begin{aligned}
p_1^* &= \frac{1}{3}\left[d(2\overline{w}-\underline{w})+x_m(2c_1+c_2)\right], \\
p_2^* &= \frac{1}{3}\left[d(\overline{w}-2\underline{w})+x_m(c_1+2c_2)\right].
\end{aligned}
$$

By substitution of p_i^*, $i=1,2$ in (36) and (37), we obtain the profits of the two firms at their equilibrium prices:

$$
\begin{aligned}
\Pi_1^* &= \frac{d}{9w_r}\left((2\overline{w}-\underline{w})-x_m\frac{(c_1-c_2)}{d}\right)^2, && (39)\\
\Pi_2^* &= \frac{d}{9w_r}\left((\overline{w}-2\underline{w})+x_m\frac{(c_1-c_2)}{d}\right)^2. && (40)
\end{aligned}
$$

Thus the difference of equilibrium prices is

$$
p_1^*-p_2^* = \frac{1}{3}\left[(\overline{w}+\underline{w})d+x_m(c_1-c_2)\right], \tag{41}
$$

and at the equilibrium prices, the profit difference is

$$
\Pi_1^*-\Pi_2^* = \frac{1}{3}[(\overline{w}+\underline{w})d-2x_m(c_1-c_2)].
$$

That is, the equilibrium price difference is linear in the quality difference and cost difference. Therefore, the higher firm 1's IT cost is, the higher its equilibrium price will be, which would in turn negatively impact its profit in a linear fashion, when demand and price-QoS tradeoff parameters are found to be uniformly-distributed. Clearly, a similar analysis applies to firm 2. Thus from (39), (40) the equilibrium profits under insourcing and outsourcing for two firms are:

$$
\begin{aligned}
\Pi_1^{*,in} &= \frac{d}{9w_r}\left((2\overline{w}-\underline{w})-x_m\frac{(c_1^{in}-c_2^{in})}{d}\right)^2, && (42)\\
\Pi_2^{*,in} &= \frac{d}{9w_r}\left((\overline{w}-2\underline{w})+x_m\frac{(c_1^{in}-c_2^{in})}{d}\right)^2. && (43)\\
\Pi_1^{*,out} &= \frac{d}{9w_r}(2\overline{w}-\underline{w})^2, && (44)\\
\Pi_2^{*,out} &= \frac{d}{9w_r}(\overline{w}-2\underline{w})^2. && (45)
\end{aligned}
$$

From (42)-(43) we observe that

$$
\Pi_1^{*,in}-\Pi_2^{*,in} = \frac{d}{3w_r}(\overline{w}-\underline{w})\left(\overline{w}+\underline{w}-x_m\frac{(c_1^{in}-c_2^{in})}{d}\right). \tag{46}
$$

We conclude from (46):

- If $c_1^{in} \leq c_2^{in}$, then $\Pi_1^* \geq \Pi_2^*$, i.e., *provider 1 with higher price, better quality and less in-house cost has higher profit than provider 2 with lower price, poorer quality and high in-house cost in a uniform market.*

- If $c_1^{in} > c_2^{in}$, then if $\frac{d(\overline{w}+\underline{w})}{x_m} > c_1^{in} - c_2^{in}$, provider 1 will have higher profit then provider two and vice-versa.

Further, from (44)-(45) we observe that $\Pi_1^{*,out} > \Pi_2^{*,out}$ always, i.e., *when both firms decide to outsource all their loads, the higher price, better quality firm makes more profit than the lower price, low quality firm in a uniform market.*

Next we analyze the profit of the IT supplier to determine whether or not outsourcing is profitable in this market, and in the affirmative, we determine the equilibrium profit levels of the two firms with outsourcing. This follows the approach undertaken by [2] who suppose that the provision of IT by the third-party supplier is profitable if his revenue less his costs is positive. The IT supplier, referred to by the index, s, charges using a (usage independent) price p_s and has a linear cost structure $c_s(x) = xc_s$.

If firm 1 decides to use the IT outsourcing supplier, we assume that it offloads all its IT needs to that supplier. Therefore the contribution to the supplier's profits from firm 1 would be given by

$$\Pi_s^1 = \int_0^{\infty} \int_{\hat{w}}^{\infty} (p_s - xc_s) g(x) h(w) dw dx, \tag{47}$$

and the contribution to its profits if firm 2 uses its services would be:

$$\Pi_s^2 = \int_0^{\infty} \int_0^{\hat{w}} (p_s - xc_s) g(x) h(w) dw dx. \tag{48}$$

3.1.1 Supplier-feasible price set

We need to look at those values of p_s for which Π_s^i is positive. We have from (47) and (48):

$$\Pi_s^1 = (p_s - c_s x_m) \frac{\overline{w} - \hat{w}}{w_r}, \tag{49}$$

$$\Pi_s^2 = (p_s - c_s x_m) \frac{\hat{w} - \underline{w}}{w_r}. \tag{50}$$

For $\Pi_s^1 > 0$ we need either $p_s > c_s x_m$ and $\overline{w} > \hat{w}$, or $p_s < c_s x_m$ and $\overline{w} < \hat{w}$. For $\Pi_s^2 > 0$ we need either $p_s > c_s x_m$ and $\underline{w} < \hat{w}$, or $p_s < c_s x_m$ and $\underline{w} > \hat{w}$. When $p_s < c_s x_m$, offering IT outsourcing will not be profitable for the supplier since $\hat{w} \in [\underline{w}, \overline{w}]$. That is, the supplier would have positive profit for any outsourcing service only if its cost is lower than the flat price, p_s, as expected. Now suppose $p_s > c_s x_m$. Observe that when both firms choose to outsource, at price equilibrium, from (41) the corresponding $\hat{w}$ equals $\frac{\underline{w}+\overline{w}}{3}$ as $c_1 = c_2 = p_s$. Thus for $\Pi_s^1 > 0$ we need $\overline{w} > \frac{\underline{w}}{2}$ which is always true as $\overline{w} > \underline{w}$; and for $\Pi_s^2 > 0$ we need $\overline{w} > 2\underline{w}$. Thus when $\overline{w} > 2\underline{w}$, it shall be profitable for the supplier to provide outsourcing services to both the firms and when $\overline{w} < 2\underline{w}$, it is only profitable to provide outsourcing to firm 1.

3.1.2 Firm-feasible costs for outsourcing

It will only be profitable for a firm to outsource if its profit under outsourcing is greater than that under insourcing. We shall next look at feasible parameter set of the two firms when it will be profitable for both of them to outsource. For firm 1 we need $\Pi_1^{*,in} < \Pi_1^{*,out}$ which implies that

$$(c_1^{in} - c_2^{in})\left(\frac{(c_1^{in} - c_2^{in})x_m}{2d} - (2\overline{w} - \underline{w})\right) < 0.$$

If $c_1^{in} - c_2^{in} < 0$ the inequality is never satisfied as $d > 0$. For firm 2 we need $\Pi_2^{in} < \Pi_2^{out}$ which implies, in turn, that:

$$(c_1^{in} - c_2^{in})\left(\frac{(c_1^{in} - c_2^{in})x_m}{2d} + (\overline{w} - 2\underline{w})\right) < 0.$$

3.1.3 Feasible parameter set for outsourcing

- Observe that when $p_s > c_s x_m$ and $\overline{w} > 2\underline{w}$ then it will be profitable for the supplier to outsource to both the firms. Then c_1 and c_2 should satisfy the following inequalities:
  - *Firm 1:* It will be profitable for firm 1 to outsource if $0 < c_1^{in} - c_2^{in} < \frac{2d(2\overline{w} - \underline{w})}{x_m}$.
  - *Firm 2:* It will be profitable for firm 2 to outsource if: $\frac{2d(2\underline{w} - \overline{w})}{x_m} < c_1^{in} - c_2^{in} < 0$.
- When $p_s > c_s x_m$ and $\overline{w} < 2\underline{w}$ then it will be profitable for the supplier to outsource to firm 1 and not to firm 2. Then c_1^{in} and c_2^{in} should satisfy the inequalities in the case *Firm 1* above.

3.2 Exponentially Distributed Usage-Levels and Tradeoff Parameters

We next consider the case where the usage levels and the price-QoS tradeoff have exponential distributions with means $1/a$ and $1/b$, respectively and thus $x_m = \frac{1}{a}$. For the tradeoff parameter the exponential distribution seems well-justified as it possesses a shape close to the commonly used log-normal distribution.

When both firms decide to in-source their IT needs, the Nash equilibrium profits $(\Pi_1^{*,in}, \Pi_2^{*,in})$ can be solved for using (36) and (37), which in the setting of exponentially-distributed x and w gives:

$$\Pi_1^{in} = e^{-b\hat{w}}\left(p_1 - \frac{c_1^{in}}{a}\right), \quad (51)$$

$$\Pi_2^{in} = (1 - e^{-b\hat{w}})\left(p_2 - \frac{c_2^{in}}{a}\right). \quad (52)$$

The Nash equilibrium prices can then be solved for:

$$e^{-b\hat{w}}\left(1-\frac{b}{d}\left(p_1-\frac{c_1^{in}}{a}\right)\right) = 0, \tag{53}$$

$$e^{-b\hat{w}}\left(1+\frac{b}{d}\left(p_2-\frac{c_2^{in}}{a}\right)\right) = 1. \tag{54}$$

From (53) and (54) we have:

$$p_1^* = \frac{d}{b}+\frac{c_1^{in}}{a}, \tag{55}$$

$$e^{\frac{b}{d}\left(p_2^*-\frac{c_1}{a}\right)}\left(1+\frac{b}{d}\left(p_2^*-\frac{c_2^{in}}{a}\right)\right) = e. \tag{56}$$

3.3 Equilibrium under outsourcing

When both firms decide to outsource, the profit functions are given by:

$$\Pi_1^{out} = (p_1-p_s)e^{-b\hat{w}}, \tag{57}$$

$$\Pi_2^{out} = (p_2-p_s)(1-e^{-b\hat{w}}). \tag{58}$$

The Nash equilibrium prices (p_1^*, p_2^*) can then be solved from $\frac{\partial \Pi_1^{out}}{\partial p_1} = \frac{\partial \Pi_2^{out}}{\partial p_2} = 0$:

$$e^{-b\hat{w}}\left(1-\frac{b}{d}(p_1-p_s)\right) = 0,$$

$$e^{-b\hat{w}}\left(1+\frac{b}{d}(p_2-p_s)\right) = 1,$$

which then gives us p_i^* as a function of p_s:

$$p_1^* = p_s+\frac{d}{b}, \tag{59}$$

$$e^{\frac{b}{d}(p_2^*-p_s)}\left(1+\frac{b}{d}(p_2^*-p_s)\right) = e. \tag{60}$$

Similar to the uniform case we are interested in studying the sensitivity of the equilibrium prices and profits to quality and cost. From (55) and (56) we have by rearranging:

$$2-\frac{b}{d}\left(p_1^*-p_2^*-\frac{(c_1^{in}-c_2^{in})}{a}\right)-e^{\frac{b(p_1^*-p_2^*)}{d}} = 0 \tag{61}$$

and for the outsourcing case from (59) and (60):

$$2-\frac{b}{d}(p_1^*-p_2^*)-e^{\frac{b(p_1^*-p_2^*)}{d}} = 0 \tag{62}$$

From (61) and (62), we observe that the equilibrium price difference depends only on the difference in quality and costs of two firms (when both firms insource) and only on the difference in quality of the two firms (when both firms outsource) similar to the uniform case. We next study this sensitivity through an example. Let $b = 1$ and $\frac{c_1}{a} - \frac{c_2}{a}$ varies from 0.1 to 10 and d varies from 0.1 to 10. The variations in equilibrium price and profit differences are shown in Fig. 5 for the insourcing case. We observe that the price difference increases with the increase in cost difference and with the increase in quality difference (better the quality offered by a firm more is the equilibrium price charged by the firm). Also as the cost difference increases the profit difference decreases for a given quality difference. The sensitivity is in a non-linear fashion though the general trends are same as for the uniform case.

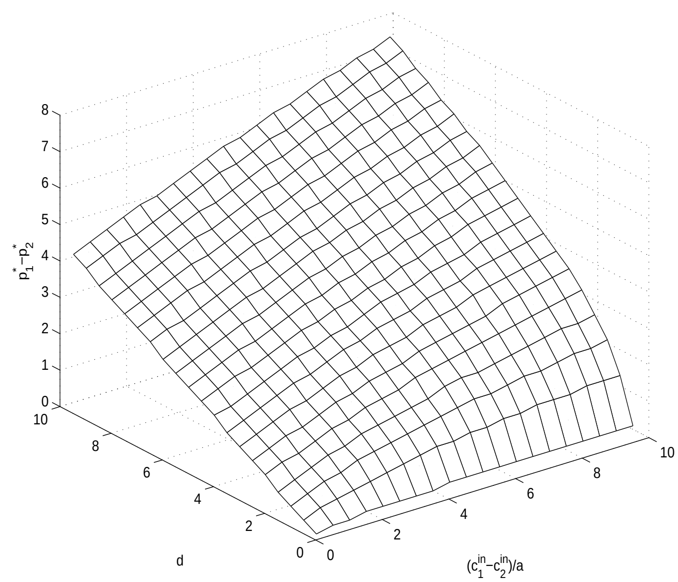

Figure 5: The sensitivity of the equilibrium price difference to quality and cost differences for the insourcing case.

3.4 Equilibrium with Outsourcing: General Results

In this section, we shall present results that summarize and generalize the results for competitive equilibrium in presence of outsourcing by deriving the formulae that hold regardless of the distributions employed. Recall that the usage levels, X, and the user's tradeoff between price and delay W, are modeled as two independent random variables, with probability density functions $g(x)$ and $h(w)$ respectively. For convenience, let $d(x) = (-d_1(x)) - (-d_2(x))$ denote the QoS difference across the two firms. The

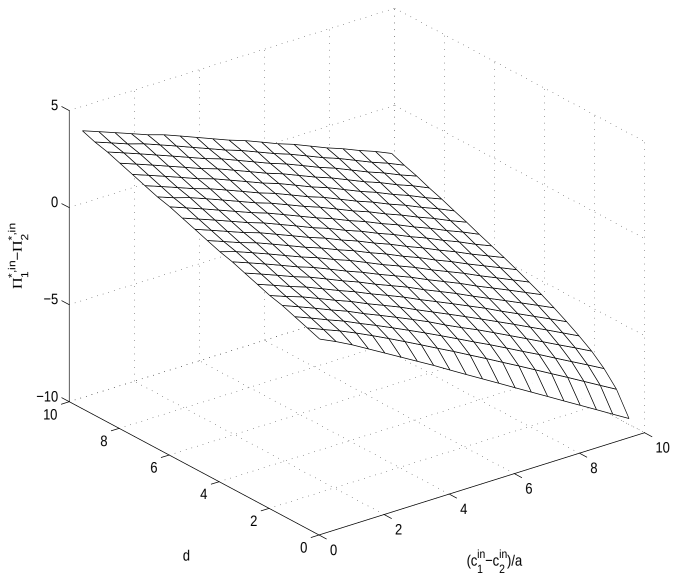

Figure 6: The sensitivity of the equilibrium profit difference to quality and cost differences for the insourcing case.

profits of the two firms are then given by

$$\Pi_1 = \int_0^\infty \int_{\hat{w}(x)}^\infty [p_1 - c_1(x)]\, g(x) h(w) dw dx, \quad (63)$$

$$\Pi_2 = \int_0^\infty \int_0^{\hat{w}(x)} [p_2 - c_2(x)]\, g(x) h(w) dw dx, \quad (64)$$

Let $H(w) = \mathbf{P}[W \le w]$, and $\overline{H}(w) = 1 - H(w)$. From (63) and (64), we have:

$$\Pi_1 = \mathbf{E}\left[(p_1 - c_1(X))\overline{H}(\hat{w}(X))\right]$$
$$\Pi_2 = \mathbf{E}\left[(p_2 - c_2(X))H(\hat{w}(X))\right]$$

We then have the following:

Proposition 1 *Suppose prices (p_1, p_2) of the two firms are given. Then firm 1 would prefer to outsource if and only if $\boldsymbol{E}\left[c_i^{in}(X)\overline{H}(\hat{w}(X))\right] < \boldsymbol{E}\left[p_s(X)\overline{H}(\hat{w}(X))\right]$. Similarly, firm 2 would prefer to outsource if and only if*

$$\boldsymbol{E}\left[c_i^{in}(X)H(\hat{w}(X))\right] < \boldsymbol{E}\left[p_s(X)H(\hat{w}(X))\right].$$

The following corollary simply serves to confirm one's intuition.

Corollary 2 *If $c_i^{in}(x) > p_s(x)$ for all x, then it is optimal for firm i to outsource.*

Recall that the price schedules and in-house costs may be complex nonlinear functions of usage. The next corollary is immediate when performance is usage independent i.e., $d_i(x) = d_i, i = 1,2$. This could be the case, e.g. some sector where quality is independent of the usage, or if the total capacity is large. Note that we then have $\hat{w}(x) = \frac{p_1 - p_2}{-d_1 + d_2}$, which is independent of x.

Corollary 3 *Suppose performance is usage independent i.e., $d_i(x) = d_i, i = 1,2$. Under arbitrary prices (p_1, p_2), firm $i, i = 1,2$ would prefer outsource if $\boldsymbol{E}\left[c_i^{in}(X)\right] > \boldsymbol{E}\left[p_s(X)\right]$, and insource otherwise. The optimal IT cost of firm i, $i = 1,2$ is given by $c_i^* = \min(\boldsymbol{E}\left[c_i^{in}(X)\right], \boldsymbol{E}\left[p_s(X)\right])$.*

Based on above proposition, once the outsourcing price structure $p_s(\cdot)$ is given, each firm will know immediately whether it should insource or outsource its IT provision, thus the optimal IT cost will also be known.

Denote c_i^* to be the optimal IT cost of firm i. The Nash equilibrium prices (p_1^*, p_2^*) can then be solved from $\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0$:

$$-\frac{1}{d} h(\hat{w}) \left[p_1 - c_1^*\right] + \overline{H}(\hat{w}) = 0 \tag{65}$$

$$-\frac{1}{d} h(\hat{w}) \left[p_2 - c_2^*\right] + H(\hat{w}) = 0 \tag{66}$$

Recall that if firm 1 decides to use the IT outsourcing supplier, we assume that it offloads *all* its IT needs to that supplier. Therefore the contribution to the supplier's profits from firm 1 would be given by

$$\Pi_s^1 = \int_0^\infty \int_{\hat{w}(x)}^\infty (p_s(x) - c_s(x)) g(x) h(w) dw dx,$$

and the contribution to its profits if firm 2 uses its services would be:

$$\Pi_s^2 = \int_0^\infty \int_0^{\hat{w}(x)} (p_s(x) - c_s(x)) g(x) h(w) dw dx.$$

Given that, it follows immediately that

$$\Pi_s^1 = \mathbf{E}\left[(p_s(X) - c_s(X)) \overline{H}(\hat{w}(X))\right] \tag{67}$$

$$\Pi_s^2 = \mathbf{E}\left[(p_s(X) - c_s(X)) H(\hat{w}(X))\right] \tag{68}$$

We then claim the following:

Proposition 4

1. *If $p_s(x) \geq c_s(x)$ for all x, then the supplier's business is always profitable, and it will be advantageous for s to provide outsourcing services.*
2. *If $c_i(x) \geq p_s(x)$ for all x, then firm i would prefer to outsource its IT provision to the supplier.*

3. *If $c_i(x) \geq p_s(x) \geq c_s(x)$, then outsourcing is advantageous both for firm i, $i = 1,2$, and for the supplier, s. If $c_i(x) \geq c_s(x)$ for $i = 1,2$, then the optimal pricing structure for the supplier is to set $p_s(x) = \min\{c_1(x), c_2(x)\}$.*

The following is for the case when performance (QoS) is usage independent i.e., $d_i(x) = d_i, i = 1,2$.

Proposition 5 *Suppose performance is usage independent i.e., $d_i(x) = d_i, i = 1,2$. The supplier will be willing to provide outsourcing service only if $\boldsymbol{E}[p_s(X)] \geq \boldsymbol{E}[c_s(X)]$. Given there is no other, different, outsourcing supplier in the competition (i.e. a monopoly, or oligopoly with price-fixing), the optimal strategy of the supplier is to set $p_s^* = min(\boldsymbol{E}\left[c_1^{in}(X)\right], \boldsymbol{E}\left[c_1^{in}(X)\right])$ in order to attract both firms.*

4 Conclusion and Perspectives

We have proposed a model for analyzing markets for electronic goods, that takes into account the stochastic nature of user demand, as well as the spread of tradeoffs between cost and quality of service across the population of end users. As a by-product of our general model of a firm selling electronic services, we demonstrated that the pricing of e-services need not result in a ruinous game, as suggested by [6]. We also demonstrated that the nature of the market equilibrium, in terms of both prices and of market share, depends heavily on the assumptions made on user behavior. The simplified model of uniformly-distributed value of QoS parameters, for example, leads to a fundamentally different conclusion than the better-motivated exponentially distributed value.

Within the frameowrk of our model we studied the effects of outsourcing on the equilibrium profits and obtain conditions on the values of parameters (in-house cost, supplier price, user tradeoff distribution, offered quality) when outsourcing is profitable for the supplier and/or the firm(s). We also studied the sensitivity of the equilibrium prices and profits of competing firms to quality and insourcing/outsourcing cost differences.

An interesting topic of future study would be incorporating usage-dependent values of QoS parameters, as the resulting model is significantly more complex, both theoretically and numerically, but would allow sensitivity analysis in terms of demand (and profitability) increases as a function of improved (or diminished) QoS.

Finally, it is of substantial interest to develop more complex definitions of QoS in this framework. One such effort in this direction is the recent work [4], which considers delay as a function of provider capacity, through explicit queueing relationships. However, the complexity introduced by the capacity-delay dependencies renders difficult the modeling of the price structure complexities studied in this work. That is, prices in the latter references are flat, or subscription-based, rather than usage-dependent. On the other hand, as suggested in [4], the additional complexity can be handled through a bilevel, or Stackelberg, framework that optimizes capacity decisions for a particular supplier, when prices are determined by Nash equilibria. Other more sophisticated definitions of QoS are envisageable as well: loss probability, reliability, delay variance (rather than expected value, etc.)

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