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Copenhagen: Heisenberg and von Neumann

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Abstract – Quantum measurement has relatively recently become a topic of practical concern, as well as one of prodigious growth. The need to illuminate the principles and problems of measurement in presentations that can be easily grasped within a lecture period have led to some excellent developments. A survey of modern texts reveals a diversity of treatments of the Copenhagen interpretation. Tracing these variations to the primary literature reveals differences of opinion and treatment among the individuals who comprised the Copenhagen school. Since these presentations are already in use in classrooms, they can be adapted to present a much richer picture of the development of the problem of quantum measurement within a context that fits in teachable lecture blocks. This paper reviews key discussions from Heisenberg and von Neumann. The results suggest there were indeed differences in the sensibilities concerning quantum measurement between the participants at Copenhagen. Further, the tensions not just between those participants, but between the questions they raise, reflect a true difficulty in the nature of the foundations of quantum mechanics that can be clearly identified in Feynman's treatments (which also came in lecture-sized chunks). These issues provide the foundation for almost all of the new treatments of quantum measurement that have emerged in recent years, and which still deeply affect the controversial character of the topic.

Introduction

Quantum measurement had been long relegated to obscurity in the pedagogical literature due to its controversial character and near impossibility of testing some of the more curious features of the questions addressed. However, the onset of quantum cryptography [1] and quantum teleportation [2] has brought the area into the practical experimental domain. While nearly absent from the pedagogical literature, interest in the field has been strong. A recent study seeking to construct an interpretation of quantum measurements using Hagelian dialectics has produced a significant and exhaustive review of the field, [3] a necessary by-product of producing a synthesis from theses and antitheses. The sheer size of that text reveals something of both the magnitude of the interest, and the difficultly coming to a clear answer to the problems of measurement.

Often, one seeking a clear, clean, and deep understanding of a problem might explore Richard Feynman's pedagogical literature. His presentations in his "Lectures on Physics" [4] are a standard. But his deepest treatment was presented in his lectures on gravity, [5] prepared during the same years he prepared his famous lectures on physics. He ventured into the domain of quantum measurement when he considered whether it would or would not be possible to construct a classical theory of gravity that interacted with quantum particles since there is essentially a measurement question that must be answered – specifically, that must be built into the formalism of such as theory -- to connect quantum source terms with a classical gravitational field.

Yet, it is the burden of every text and course on quantum mechanics to explore the question of measurement at least deeply enough to make sense of the connection between quantum mechanics and macroscopic experience. Quantum mechanics texts show some diversity on their approaches in a number of different dimensions. The component this paper focuses on is demonstrated most clearly in the contrast between the text by L. I. Schiff, [6] and Cohen-Tannoudji, et al, [7] or Sakurai. [8] The approach adapted by Cohen-Tannoudji and Sakurai are very closely aligned with Dirac, [9] first published in 1930. The arguments that Dirac endorsed were originally articulated by von Neumann,

[10] which was first published afterwards in 1932. These represented the first publication of the notion of a collapse of the state vector upon measurement. Schiff's treatment is much softer on the notion of such a collapse. Specifically, Schiff explores an argument proposed in a monograph by Heisenberg, [11] first published in 1930, describing the formation of a bubble chamber track, and avoids the issue of state vector collapse. Therefore, the two monographs, by von Neumann and Heisenberg, represent a dialogue between Copenhagen members on the issue of quantum measurement and state vector collapse. (On the side, David Bohm's text on quantum mechanics [12] presents a reconciliation of Heisenberg's bubble chamber argument and von Neumann's collapse picture. The experience of writing this text apparently provoked him to explore an alternative construction of quantum mechanics known as a "hidden variables theory.")

The very short review presented above highlighted a very interesting fact of history: there appears to be some differences in the treatment of measurement by the students of the Copenhagen school. A review of the arguments upon which this long standing difference rests reveals that the division involves significant questions concerning measurement. Those arguments are short, simple, and instructive in a pedagogical environment. This paper purposes to present these arguments in sizes suitable for inclusion in lectures. A study of the dialogue between von Neumann and Heisenberg, seen in publications by these authors from this same period in time, provides a unique insight still deeply relevant today. This dialogue reflects the contributions seminal to the discussions of measurement now presented in many quantum mechanics texts, together with some simple observations. Particularly, von Neumann's arguments are usually not included in the texts that cite his measurement postulates.

Background

The thread of this question is picked up following Born's Nobel Prize lecture [13] with Heisenberg's seminal paper on matrix mechanics. [14] This left open the question of what the matrices operated on. Schrödinger's solution to the quantum problem, and particularly to the hydrogen atom, [15] assumed an $e|\psi|^2$ electron charge density, but that this was interpreted as a density, not a probability that a particle would be observed.. This was considered to be objectionable by the Copenhagen group because of the particle like behavior in Geiger counters, etc. suggested that the electrostatic field should be seeing charged particles distributed by a probability computable from the quantum state vector. Born considered that, if the force involved the exchange of photons in some kind of scattering process, consistent with Einstein's argument, then $e|\psi|^2$ would reflect a probability proportional to $|\psi|^2$. [16] The notion of uncertainty as expressed in Heisenberg's paper on the uncertainty principle, [17] connected the nature of the state vector to quantum uncertainty and probabilities much more clearly.

The Heisenberg uncertainty principle was pivotal in establishing a picture of quantum measurement, and which placed measurement in the center of the discussion in Copenhagen and in the famous debates with Einstein. The argument revolved around the notion that measurement of location involved scattering by a photon, which introduces an uncertainty in the momentum. Since photon resolution depends on wavelength, and higher resolution and decreased wavelength implies higher energy and momentum transferred, the more precise you measure location, the more uncertain the resulting momentum. Heisenberg and Bohr considered uncertainty to be an aspect of measurement rather than of the underlying nature of quantum mechanics – or rather quantum mechanics takes into account in some way the effects of uncertainty in measurement, even when no such measurement method is posited. Lastly, the result of measurement was to leave the system in an essentially undetermined state.

It is worth while looking at a typical derivation of the Heisenberg uncertainty relationship, because it sheds light on the relationship between the formalism, and how the uncertainty principle was used in various arguments from the Copenhagen school. Consider a state $|\psi\rangle$ subject to the estimation of operators $\Delta A = A - \langle A \rangle$ and $\Delta B = B - \langle B \rangle$. Define vectors $|u\rangle = \Delta A|\psi\rangle$ and $|v\rangle = \Delta B|\psi\rangle$. Construct $|w\rangle = |u\rangle - |v\rangle\langle v|u\rangle / \langle v|v\rangle$. Then $\langle w|w\rangle \geq 0$ implies $\langle u|u\rangle\langle v|v\rangle = \langle \Delta A^2\rangle\langle \Delta B^2\rangle \geq |\langle u|v\rangle|^2 \geq |\mathrm{Im}\langle u|v\rangle|^2 = |\langle [A,B]\rangle|^2 / 4$. If A and B are location and momentum, this reduces to the familiar result. It is clear that there is no explicit involvement of a measurement process in the above limit. The derivation suggests that the uncertainty is a property of the operator algebra independent of any particular condition of the state vector.

For Heisenberg, the question of where the uncertainty is injected inverts the position adapted in Planck's original derivation of his black-body distribution, [18] where Planck assumed the quantization was a peculiarity of the interaction of light with any system, such as classical oscillators that the light was in thermal equilibrium with, rather than of the mechanics of light and oscillators themselves. Heisenberg argued that the structure of uncertainty in measurement, and measurement itself, was integral to the structure of mechanics, being inherent in the quantum operators themselves -- being built into their algebra. Measurement is a part of the dynamics described by quantum mechanics. [11]

Von Neumann's Argument

The focus of von Neumann's argument was the Compton effect. [19] In this experiment, an X-ray is scattered from an electron. It imparts momentum to the electron. The photon's velocity and angle of scatter are measured, which given the initial momentum of the photon, is sufficient to determine the energy and momentum of the scattered electron. The scattered electron's momentum and energy can also be measured, necessarily at a later time. The measured energy and momentum are consistent with the prior measurement. This suggested to von Neumann that subsequent measurements of the system must be consistent with the measured state at a prior time, and that the process of measurement, to make all future measurements consistent, must modify the system's state vector accordingly. That modification of the state vector $|\psi\rangle$ must require the uncertainty in the measured variable A to be zero. Then $\langle \Delta A^2 \rangle = \left\| \left(A - \langle A \rangle \right) |\psi\rangle \right\|^2 = 0$, so that $A|\psi\rangle = \langle A \rangle |\psi\rangle$. Thus, it follows that the state vector must be reduced to an eigenstate of the measured operator, and the measured expectation value must be an eigenvalue of that operator.

It is clear von Neumann's construction is intended to preserve conservation. However, this argument opens the question of the possibility that measurement can change a state vector -- do work on a system or exert force on a system -- without any more effort than passively observing that system. This opens the arena to a host of problems, such as what qualifies as an observer, [20] and whether the act of observation could satisfy Lorentz covariance. [21]

Heisenberg's Argument

Heisenberg's book, [11] which Schiff [6] cited for the bubble chamber track argument, contained critiques of the corpuscular theory and of the wave theory, a presentation of statistical theory, as well as discussions of experiments that demonstrate quantum character, and finishes with a description of analytical technique. Bubble chamber tracks figure in the introduction and in his chapter on experimental results. He stands in agreement with Bohr concerning wave/particle complementarity at the end of his chapter on statistical theory. He describes measurable operators as being imbued with dynamical meaning only in the context of being measurable – exactly as noted in the above observation of the treatment of the uncertainty principle. This goes a bit further than the operationalism or logical positivism that considered that physical quantities should be defined or definable in terms of experimental procedures as much as possible as a way of removing circularities that emerge when faced with defining undefined terms.

Heisenberg's argument regarding bubble chamber tracks demonstrated that apparent examples of collapse, such as the formation of bubble-chamber tracks, could be explained using the normal dynamics embodied by Schrödinger's equations, without resorting to collapse events. As an example, he presented the formation of bubble-chamber tracks. The development described here follows Schiff.

Consider a cross-section

$$\frac{d\sigma}{d\Omega}(\vec{k}) = m^2\left(\frac{2\pi}{\hbar}\right)^4 \lim_{\varepsilon\to 0^+}\left|\left\langle \Psi_0(\vec{k})\right|T\left(E(\vec{k})+i\varepsilon\right)\left|\Psi_0(\vec{k})\right\rangle\right|^2,$$

where $\left|\Psi_0(\vec{k})\right\rangle$ is the asymptotic scattering state, and the transfer matrix $T\left(E_0(\vec{k})+i\varepsilon\right)$ satisfies $T = V + VG_0V + VG_0VG_0V + \cdots$ (a good development is presented in Taylor [22]). The leading term that includes excitations at more than one site (as in multiple vapor condensation points) is VG_0V. Now, this term may be expanded as

$$\left\langle\vec{k}\left|V_2G_0V_1\right|\vec{k}_0\right\rangle = \frac{1}{(2\pi)^3}\int d^3x_1\int d^3x_2 e^{-i\vec{k}\cdot\vec{x}_1+i\vec{k}_0\cdot\vec{x}_2}V_2(\vec{x}_1)V_1(\vec{x}_2)\left\langle\vec{x}_1\left|G_0\left(E(k_0)+i\varepsilon\right)\right|\vec{x}_2\right\rangle,$$

where

$$\left\langle\vec{x}_1\left|G_0\left(E(k_0)+i\varepsilon\right)\right|\vec{x}_2\right\rangle = \frac{-me^{ik_0|\vec{x}_2-\vec{x}_1|}}{2\pi\hbar^2\left|\vec{x}_2-\vec{x}_1\right|}.$$

For the purposes of this discussion, it is adequate to simply assume that the V's are localized so that $V_2\left(\vec{x}+\vec{R}\right) = V_1\left(\vec{x}\right) = V\left(\vec{x}\right)$, where $V\left(\vec{x}\right)\approx 0$ when $|\vec{x}| >> a$, the scattering centers are separated by a distance much larger than both the incoming particle's wavelength and the scattering center volume. Then the matrix element is

$$\left\langle\vec{k}\left|V_2G_0V_1\right|\vec{k}_0\right\rangle \approx \frac{-me^{-i\vec{k}\cdot\vec{R}+ik_0R}}{(2\pi)^4\hbar^2R}\int d^3x_1 e^{-i\left(\vec{k}-k_0\hat{R}\right)\cdot\vec{x}_1}V\left(\vec{x}_1\right)\int d^3x_2 e^{i\left(\vec{k}_0-k_0\hat{R}\right)\cdot\vec{x}_2}V\left(\vec{x}_2\right).$$

The oscillations in the integrals cancel unless $\left|\vec{k}_0-k_0\hat{R}\right|a\approx 1$ and $\left|\vec{k}-k_0\hat{R}\right|a\approx 1$, or when $\left|\hat{k}_0-\hat{R}\right|\approx 1/ka$ and $\left|\hat{k}-\hat{R}\right|\approx 1/ka$. In other words, mechanics, as embodied in Schrödinger's equation, predicts that the cross section for the production of bubble chamber tracks is insignificant if any two of the vapor molecules are not lined up with the momentum.

Necessary to the above computation is the insertion of a complete set of bases $\left|\vec{k}'\right\rangle\left\langle\vec{k}'\right|$.
This act involves a sum over all possible momenta. Some intermediate collapse in momentum space, representing a measurement of the momentum in order to determine the alignment of the scattering sites, would have changed the effective form of $\langle\vec{x}_1|G_0|\vec{x}_2\rangle$, which could, ironically, break the localization of the cross-section.

Another feature of the problem is made clear in the derivation of the scattering cross-section presented by Taylor. [22] In this case, starting from the construction of asymptotic states described by $e^{-iHt/\hbar}|\Psi\rangle \xrightarrow[t\to\pm\infty]{} e^{-iH_0t/\hbar}|\Psi_0\rangle$ where H_0 contains no interaction, and $|\Psi_0\rangle$ is an interaction-free asymptotic state vector, he builds a cross-section by integrating incoming packets over various impact parameters given a beam intensity. The inclusion of all possible intermediate states at all possible locations is also built into Feynmann's path integral formulation. [23] During the development, conservation of energy and momentum, the properties that would guarantee the consistent treatment that von Neumann's measurement postulate was designed to preserve, emerge naturally.

Conclusions

The participants in the Copenhagen school revealed some divergence. Bohr's sense was that measurement produced results leaving the system in an undetermined state. The famous arguments with Einstein at the Solvoy conference treated systems classically subject to measurement uncertainty. [24] Heisenberg considered measurement and its uncertainties to be built into the mechanics of natural systems, reflected in the very formalism from which the uncertainty principle was derived. Von Neumann argued that, in order to preserve conservation in the face of uncertainty, measurement requires its own postulate distinct from Schrödinger's equation to insist on measurements that produce consistent results, as demanded by some experimental systems, such as the Compton experiment. It is in part due to the polarization between Einstein's party and the Copenhagen party at the Solvoy conference that the notion of a single Copenhagen interpretation seems to have been established. This was further cemented by continuing debate points such as the EPR paradox. [21] The act of winning the Solvoy debate made the differences between the individuals of the Copenhagen school less visible. A second reason that the differences are not more widely presented or discussed seems to be one of style. The citation of von Neumann's measurement postulate tends to follow Dirac's monograph, [9] which, predating von Neumann's own exposition, [10] did not recite von Neumann's arguments. Rather, it presents a list of postulates upon which the development of quantum mechanics and the analysis of quantum mechanical systems is built. This has tended to exclude von Neumann's argument from common pedagogy, even though it is very short and easy to describe as a part of a lecture.

It is a curious point that the sequence of publications does not reflect the order of discourse. Von Neumann was present in Copenhagen, and had long discussions and debates with Bohr about the nature of measurement. [24] These views were communicated within the physics community orally for several years before they were finally written down, and then for the most part in monograph form rather than in the refereed literature, and not in order of priority. Of those elements that the Copenhagen group recorded, Pauli's probability interpretation is what made it into the refereed literature. Yet, their opinions are available for review, and it is clear that much of what has been offered as a "Copenhagen interpretation" actually reflects multiple and divergent opinions.

The differences in views reflected in the comparison of Heisenberg's and von Neumann's monographs emerged from a deeper question that they were trying to come to grips with. That is the correspondence between quantum state vectors $|\Psi\rangle$ and the probabilities $|\langle \vec{x}|\Psi\rangle|^2$. The depth of the problems involved are made quite clear in Feynman's development on gravitation, [5] the intractability of which explains the volume of literature that emerged in this field [3] in the face of a general reluctance to seriously consider the problem measurement, which is often characterized by the unattributable but widely quoted phrase: "shut up and calculate!" Here again, recent developments [1,2] are rendering the problem practical. Yet, it cannot be denied that a significant amount of that volume published on quantum measurement has emerged as a result of the debate reflected in the published views of Heisenberg and von Neumann as much as of the difficulty of the underlying question they were seeking to answer. These questions are no longer simply philosophical. They are now becoming experimentally accessible. The arguments that underpin our common discourse are very tractable in a classroom setting, and can give students of quantum mechanics a deeper appreciation of these questions at very little expense of effort, and it will bring to life the personalities of the research group that first struggled with the formulation of quantum mechanics, and which had first crack at these most intractable features of the formalism.

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