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## Dummy Fill Density Analysis with Coupling Constraints

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# Dummy Fill Density Analysis with Coupling Constraints

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## Abstract

In modern VLSI manufacturing processes, dummy fills are widely used to adjust local metal density in order to improve layout uniformity and yield optimization. However, the introduction of a large amount of dummy features also affects wire electrical properties. In this paper, we propose the first Coupling constrained Dummy Fill (CDF) analysis algorithm which identifies feasible locations for dummy fills such that the fill induced coupling capacitance can be bounded within the given coupling threshold of each wire segment. The algorithm also makes efforts to maximize ground dummy fills, which are more robust and predictable. The output of the algorithm can be treated as the upper bound for dummy fill insertion, and it can be easily adopted in density models to guide dummy fill insertion without disturbing the existing design.

**Category:** B.7.2 [Integrated Circuits]: Design Aids - Placement and routing; J.6 [Computer Applications]: Computer-Aided Engineering - Computer-aided design

**Terms:** Algorithms, Design

**Keywords:** dummy fills, coupling, CMP

## 1. Introduction

In modern VLSI manufacturing processes, foundries usually require an effective metal density to be satisfied in order to achieve layout uniformity and yield optimization. Dummy fills are widely used to increase the density of sparse regions. In general, layout density control consists of two phases: density analysis and fill synthesis. Density analysis is to decide the available positions for dummy fills. Fill synthesis computes the amount of dummy features for each density window [1].

Most previous works [1, 2, 3, 4, 8, 9] focus on fill synthesis. And none of these works consider the dummy fill coupling impacts on neighboring wires. [5] formulated a performance impact limited fill approach. But their model assumes that the dummy feature width  $w$  is much smaller than the spacing between two signal wires  $d$ , i.e.,  $w \ll d$ . Based on this assumption, they derived the capacitance formulas that are only related to the distance of two wires, and totally ignore the spacing between dummy fills and wires. However, the dummy fill induced coupling capacitance is tightly related to the distance between dummy fills and wires. Furthermore, in the current technology,  $w \ll d$  does not hold at all! For example, in one foundry, the width of a floating metal fill shape on a metal layer can be four and a half times of the minimum spacing of that metal level. Figure 1 shows the dummy fill impacts on wire capacitance. The simulation is based on the test structure in Figure 1 (a) in 65nm technology node. One signal wire is placed between two power rails. The dummy features, which are floating metal squares, are inserted around the signal wire. When the spacing between dummy

fills and the signal wire changes, the induced wire capacitance is also different, varying from 3% to 10%. The large coupling impacts from dummy fills definitely cannot be ignored.

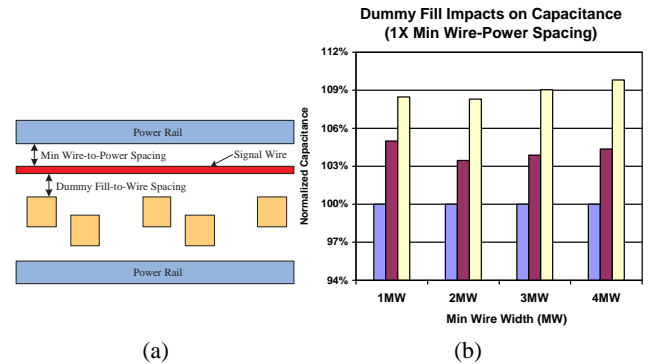


Figure 1: (a) Test structure: one signal wire is placed between two power rails. Dummy fills are inserted around the signal wire. (b) Dummy fill introduced coupling capacitance on wires with different widths. *No fill* is the case without dummy fills. *Min/Max* is the minimum/maximum capacitance increase that is introduced by dummy fills due to the different spacing between dummy fills and the signal wire. *MW* refers to the minimum wire width.

In this paper, we address the dummy fill density analysis problem, and propose the first Coupling constrained Dummy Fill (CDF) algorithm which identifies feasible locations for dummy fills such that the fill induced coupling capacitance can be bounded within the given coupling threshold of each wire segment. The output of our algorithm can be easily adopted in density models for fill synthesis. For example, the CMP density calculation in [1, 2, 3, 4, 8, 9] require that the upper bound of dummy fills for each window must be given. Based on a CDF solution, it is very easy to derive the maximum amount of dummy fills in each window. Furthermore, our algorithm outlines the dummy fill regions such that no coupling violations will be introduced as long as dummy fills are inserted in those regions. This allows different dummy fill patterns to be applied.

The dummy fills can either be connected to ground or be left floating. Both types are supported and adopted [11] in the industry. Overall, the coupling capacitance of floating dummy fills is less than that of ground fills. Thus we are expected to get even larger capacitance increase from ground fills. On the other hand, ground fills are more robust and predictable. They can be used for noise shielding, and avoid fill induced uncertain floating voltage. Therefore, ground dummy fills are still favored over floating dummy fills. Furthermore, ground fills can help to reduce the IR-drop of the power

distribution network [7]. In our algorithm, we also make efforts to maximize locations for ground fills.

For the rest of the paper, we first give the formal formulation of the CDF problem in Section 2. In Section 3, we present the CDF algorithm to identify feasible ground/floating dummy fill positions with capacitive coupling constraints. The algorithm first locates dummy fill regions using ILP formulations. Then it calculates the maximum ground dummy fills that can be inserted. The exact ground/floating fill locations are determined by a dynamic programming based algorithm that maximizes the total amount of floating dummy fills as well. The experimental results are presented in Section 4, and Section 5 concludes the paper.

## 2. Problem Formulation

The dummy fills are metal tiles, and they are to be inserted in the empty space among signal wires. Since ground dummy fills need connection to power grids on upper/lower layers, their width and pitch are usually larger than those of the floating dummy fills. Assume the half width of a ground fill is  $w_g$  and the spacing between two ground fills is  $s_g$ . Let  $w_f$  and  $s_f$  be the half width and spacing of floating dummy fills, respectively. The spacing between a ground dummy fill and a floating dummy fill is also  $s_g$ .

Given a routing solution  $R$  on layer  $L$ , there are  $N$  signal wire segments  $S_1, \dots, S_N$ . For each wire segment  $S_i$ , it has a coupling threshold  $C_i$ . The dummy fills are inserted in the empty space with the following constraints:

1. The spacing between a dummy feature and a signal wire must be larger than the min-spacing requirement  $s$ .
2. The spacing between two dummy features (ground or floating) should satisfy the given spacing requirements.
3. If the total capacitive coupling on  $S_i$  after inserting dummy fills is  $c_i$ , then  $c_i \leq C_i$ .

Our target is to identify as many as possible feasible positions for ground dummy fills. At the same time, maximize floating fill insertion as well.

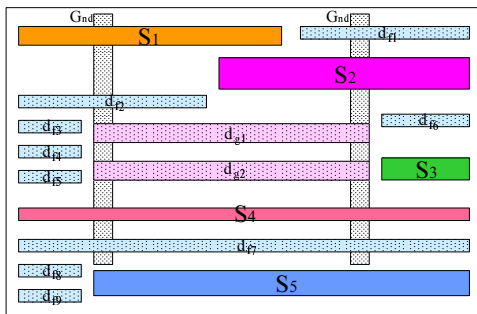


Figure 2: A dummy fill solution for a CDF problem.

Figure 2 shows an example. In this example, there are 5 wire segments  $S_1, S_2, S_3, S_4$ , and  $S_5$ .  $d_{g1}$  and  $d_{g2}$  are ground fills, while  $d_{fi}$  ( $i = 1, \dots, 9$ ) are floating fills. Since  $S_1$  has a tight coupling bound, dummy fills have to be placed far from  $S_1$ . The power rail pair is on the lower layer. For  $d_{g1}$  and  $d_{g2}$ , they can be connected to  $G_{nd}$ . Therefore, they are inserted as ground fills. Although  $d_{f7}$

is long enough to cross the power rail pair, the allowable spacing between  $S_4$  and  $S_5$  is not enough to hold a ground fill. Still a floating fill is inserted.

## 3. CDF Algorithm

The whole flow of the CDF algorithm can be summarized as follows.

**Algorithm CDF**

1. Build the coupling capacitance lookup table;
2. Slice the layout into slots;
3. Identify dummy fill regions for each slot;
4. Determine ground/floating fill locations;

The output is the feasible locations for ground/floating dummy fills satisfying the coupling constraints. It can be treated as the upper bound for fill synthesis with the guarantee that no coupling violations will be introduced as long as dummy fills are placed on these locations. Without loss of generality, we assume that all wire segments are horizontal, and dummy fills are horizontal metal tiles as well.

### 3.1 Coupling Capacitance Lookup Table

In general, each segment has the coupling effect on all other segments. However, the coupling capacitance decreases dramatically if the segment is out of the neighborhood of the other segment [10, 12]. Therefore, we only consider the capacitive coupling between two neighboring parallel wires and suppose the neighborhood distance is  $D$ . Then the capacitive coupling between two segments can be expressed in the following formula:

$$c = \begin{cases} \alpha \cdot \frac{L}{d^\beta} & d \leq D \\ 0 & d > D \end{cases}$$

where  $\alpha$  is the coupling parameter,  $\beta$  is an experimentally estimated constant [10],  $L$  is the coupling length, and  $d$  is the distance between two segments.

Although the behavior of floating dummy fills is less predictable, the above formula can totally bound their coupling impacts. Therefore, we apply this model for all coupling calculations.

To speed up execution, we build up a lookup table. For each entry, it records the distance of two wires and the corresponding unit length coupling capacitance. Then the coupling between two wires can be calculated in constant time.

### 3.2 Slot Partition

Since the coupling capacitance is closely related to the coupling length between two wires, we first slice the whole layout vertically according to  $x$  coordinates of the end points of each wire segment. As shown in Figure 3, the layout is partitioned into 5 slots. For each slot, the wire distribution may be different, and it directly affects dummy fill insertion. Therefore, our dummy fill insertion strategy is to insert dummies slot by slot. This slicing step can be easily achieved by sorting  $x$  coordinates of all end points, and the running time is bounded by  $O(N \log N)$ .

Once the layout is partitioned into slots, the capacitive coupling threshold of each wire segment is redistributed according to its length in each slot, i.e., if the length of wire segment  $S_i$  is  $L_i$ , and the slot length is  $L'_i$ , then the capacitive coupling threshold of  $S_i$  in the given slot is  $C_i * (L'_i/L_i)$ . Without confusion, we still use  $C_i$  to

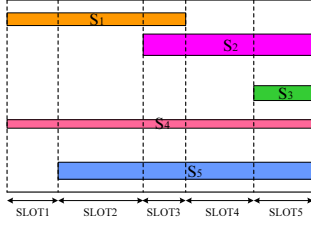


Figure 3: The illustration of layout slicing.

represent the coupling threshold of  $S_i$  in the given slot.

For some positions, we know clearly whether dummy fills can be inserted or not. Therefore, we can further partition the slot into smaller pieces, and solve the dummy fill insertion problem for each slot piece independently. Since the dummy fill insertion in each slot piece has no impact on other slot pieces, the quality of the final solution is guaranteed.

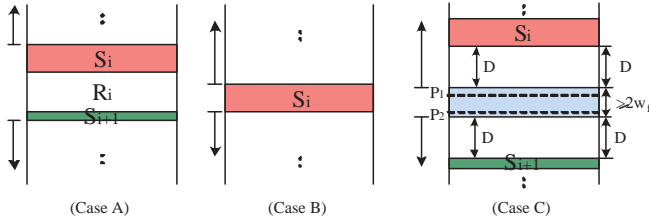


Figure 4: Three cases for slot partition

There are three cases as shown in Figure 4.

**Case A:** There is no feasible position between two wires  $S_i$  and  $S_{i+1}$ , i.e., the spacing between  $S_i$  and  $S_{i+1}$  is less than  $2 \cdot w_f + 2 \cdot s$ . Then no dummy fills can be inserted between  $S_i$  and  $S_{i+1}$ . Therefore, the dummy fill insertion above  $S_i$  has no impacts on the fill solution below  $S_{i+1}$ . We can safely divide the slot into two parts: a slot piece above  $S_i$  and a slot piece below  $S_{i+1}$ .

**Case B:** If there is no coupling constraint on  $S_i$ , then dummy fills can be inserted around  $S_i$  as long as the min-spacing requirement is satisfied. Therefore, we can apply the dummy fill insertion algorithm for the slot piece above  $S_i$  and the slot piece below  $S_i$  separately.

**Case C:** If the spacing between  $S_i$  and  $S_{i+1}$  is larger than  $2 \cdot D + 2 \cdot w_f$ , then at least one dummy fill can be inserted. As shown in Figure 4 (Case C), the middle cyan region can hold at least one dummy fill. The spacing between  $P_1$  and  $S_i$  is  $D$ , and the spacing between  $P_2$  and  $S_{i+1}$  is also  $D$ . If  $|P_1 - P_2| \geq w_f$ , the piece between  $P_1$  and  $P_2$  is safe for dummy insertion, and the dummy fill insertion above  $P_1$  is independent of the fill insertion below  $P_2$ . Therefore, we only need to process two slot pieces: the piece above  $P_1$  and the piece below  $P_2$ .

Combining these three cases, we can cut a slot into smaller pieces, and identify dummy fill regions for each slot piece separately. For high density region, Case A helps to slice a slot into smaller pieces; for low density region, Case C can be applied. These

three cases help to control the number of wires in each slot piece within a reasonable range. This greatly reduces the problem size and speeds up the execution. For convenience, we also refer a slot piece as a slot since the dummy fill algorithm is applied on each slot piece independently.

### 3.3 Dummy Fill Region Identification

In this section, we identify dummy fill regions for the given slot subject to the coupling capacitance constraints. We first partition the slot into FillRegions which covers all possible positions for the dummy insertion. Then an optimal ILP based algorithm is proposed to maximize effective dummy fill regions assuming that each FillRegion has at least one dummy fill assigned. Next, we extend the algorithm to allow no fills in some FillRegions.

#### 3.3.1 FillRegion

Suppose there are  $m$  wire segments  $S_1, \dots, S_m$  in the given slot. The slot width is  $L$ . For each wire segment  $S_i$ , let  $y_i$  be the  $y$  coordinate of the center line,  $w_i$  be the half width of the segment, and  $C_i$  be the coupling threshold. Also define  $d_i = \sqrt[3]{\alpha L / C_i}$ , which specifies the minimum spacing between the dummy fills and  $S_i$  respect to the given coupling threshold.

For convenience, we let the region between two wire segments  $S_i$  and  $S_{i+1}$  ( $i = 1, \dots, m-1$ ) be  $R_i$ . The region above  $S_1$  is  $R_0$ , and  $R_m$  is the region below  $S_m$ . For each region  $R_i$  ( $i = 1, \dots, m-1$ ), its **FillRegion** is defined as  $[V_i, U_i] = [y_{i+1} + w_{i+1} + w_f + \max\{d_{i+1}, s\}, y_i - w_i - w_f - \max\{d_i, s\}]$  since the spacing between a dummy fill and a wire segment must satisfy both coupling constraint and min-spacing constraint. So the FillRegion size is  $r_i = U_i - V_i + 1$ . This bound covers all feasible locations for dummy fills, but not all positions are valid to insert dummies due to the coupling constraints. Similar rules apply for  $R_0$  and  $R_m$ . For convenience, we also refer  $R_i$  to the FillRegion of  $R_i$ .

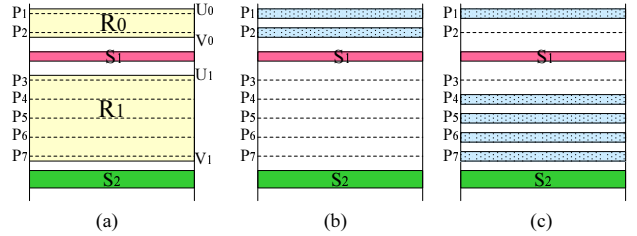


Figure 5: (a) The slot piece includes two wire segments  $S_1$  and  $S_2$ . In  $R_0$ , two dummy fills can be inserted as denoted by  $P_1$  and  $P_2$ , and  $R_1$  can hold five dummy fills. (b) When two dummy features are inserted in  $R_0$ , no fills can be placed in  $R_1$  since the coupling between  $P_2$  and  $S_1$  has reached the coupling threshold of  $S_1$ . (c) Only  $P_1$  is used for dummy insertion in  $R_0$ . 4 dummy fills can be inserted in  $R_1$ .

If we arbitrarily assign dummy fills into the FillRegions, the amount of inserted fills may be limited. Figure 5 gives an example. In Figure 5 (a), there are two signal wire segments  $S_1$  and  $S_2$ . In  $R_0$ , two dummy fills can be inserted as denoted by  $P_1$  and  $P_2$ .  $R_1$  can hold five dummy features. If two dummy features are inserted in  $R_0$  as shown in Figure 5 (b), they consume all the coupling budget of  $S_1$ , and no dummies can be inserted within  $R_1$ . On the other hand, if only one dummy is inserted in  $R_0$  as shown in Figure 5

(c), four dummy fills can be placed in  $R_1$ . Therefore, three more dummies can be inserted than the solution in Figure 5 (b).

### 3.3.2 Optimal Dummy Fill Region Generation

In this section, we present an optimal algorithm to maximize the total effective regions for dummy fill insertion. The algorithm identifies dummy fill regions from FillRegions such that no capacitive coupling violations are introduced if dummy fills are inserted in those regions. At the same time, the total dummy fill regions are maximized. The algorithm is based on the ILP formulation. We assume that at least one dummy feature is to be inserted in each FillRegion. In the next section, we will extend our ILP formulation to allow no-fills in some FillRegions.

For each wire segment  $S_i$  ( $i = 1, \dots, m$ ), two sets of variables  $X_i$  and  $Y_i$  are created:  $X_i = \{x_{i1}, \dots, x_{ir_i}\}$  and  $Y_i = \{y_{i1}, y_{i2}, \dots, y_{ir_i}\}$ . The values of  $x_{ij}$  and  $y_{ik}$  are either 0 or 1. To facilitate expression, two auxiliary variables  $T_i$  and  $B_i$  are used for each  $S_i$ . The following is the ILP formulation to identify maximum dummy fill regions.

$$\min \{T_1 + \sum_{i=1}^{m-1} (B_i + T_{i+1}) + B_m\}$$

subject to:

$$x_{i1} + \dots + x_{ir_i-1} = 1; \quad i \in [1, m] \quad (1)$$

$$y_{i1} + \dots + y_{ir_i} = 1; \quad i \in [1, m] \quad (2)$$

$$T_i = 1 \cdot x_{i1} + 2 \cdot x_{i2} + \dots + r_{i-1} \cdot x_{ir_{i-1}} - 1; \quad i \in [1, m] \quad (3)$$

$$B_i = 1 \cdot y_{i1} + 2 \cdot y_{i2} + \dots + r_i \cdot y_{ir_i} - 1; \quad i \in [1, m] \quad (4)$$

$$B_{i-1} + T_i < r_{i-1}; \quad i \in [2, m] \quad (5)$$

$$\sum_{j=1}^{r_i-1} p_{ij} \cdot x_{ij} + \sum_{k=1}^{r_i} q_{ik} \cdot y_{ik} \leq C_i + c_i^{i-1} + c_i^{i+1}; \quad i \in [1, m] \quad (6)$$

$$x_{ij} = 0, 1; \quad i \in [1, m]; \quad j \in [1, r_{i-1}];$$

$$y_{ik} = 0, 1; \quad i \in [1, m]; \quad k \in [1, r_i];$$

$x_{ij}$  refers to the  $j^{\text{th}}$  position above  $S_i$ , and  $y_{ik}$  represents the  $k^{\text{th}}$  position below  $S_i$ . If  $x_{ij} = 1$ , it means that the  $j^{\text{th}}$  position and its above positions are selected. Therefore, only one value of  $x_{ij}$  can be 1. Equation (1) expresses this constraint. Similarly, if  $y_{ik} = 1$ , it means the  $k^{\text{th}}$  position and positions below it are selected. Still only one value of  $y_{ik}$  can be 1, and Equation (2) is used to bound the constraint.

$T_i$  means how many positions from the bottom of  $R_{i-1}$  are not used, and  $B_{i-1}$  means how many positions from the top of  $R_{i-1}$  are not used. Therefore, the total number of unused positions should be less than the total positions in the region  $R_{i-1}$ . This constraint is expressed by Equation (5).

Finally, Equation (6) describes the coupling constraint on each wire segment. The capacitive coupling between any two given positions can be easily derived from the lookup table. Let  $c_i^{i-1}$  ( $i = 2, \dots, m$ ) be the coupling capacitance between  $S_i$  and  $S_{i-1}$ . For convenience, set  $c_1^0 = 0$ . Similarly,  $c_i^{i+1}$  ( $i = 1, \dots, m-1$ ) is the capacitive coupling between  $S_i$  and  $S_{i+1}$ , and  $c_m^{m+1} = 0$ . Also let  $p_{ij}$  be the coupling capacitance between  $x_{ij}$  and  $S_i$ , and  $q_{ik}$  is the coupling capacitance between  $y_{ik}$  and  $S_i$ .  $\sum_{j=1}^{r_i-1} p_{ij} \cdot x_{ij}$  is the coupling capacitance between  $S_i$  and the dummy fill right above it;  $\sum_{k=1}^{r_i} q_{ik} \cdot y_{ik}$  is the coupling capacitance between  $S_i$  and the dummy fill right below it. Due to the dummy fill insertion, the capacitive coupling between  $(S_i, S_{i-1})$  and  $(S_i, S_{i+1})$  is blocked, and the total coupling on  $S_i$  is adjusted accordingly.

Obviously, our target is to maximize dummy fill regions, i.e., to minimize the number of unused positions, which is expressed as the

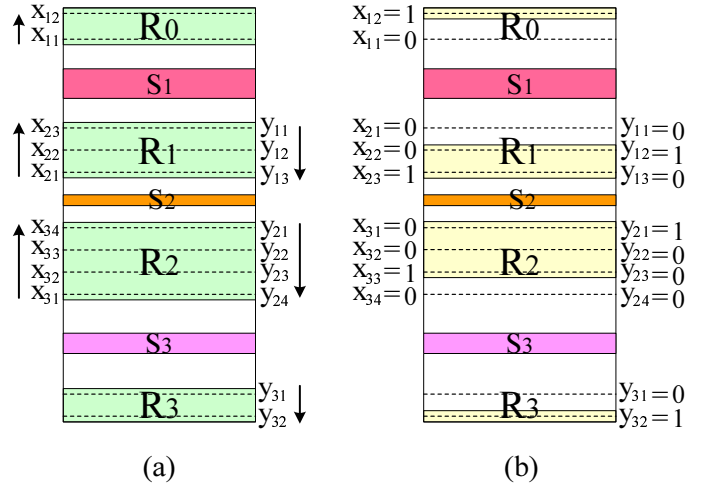


Figure 6: (a) The slot includes three wire segments  $S_1$ ,  $S_2$ , and  $S_3$ . For each position in the FillRegion, it is associated with variables  $x_{ij}$  and/or  $y_{ik}$ . (b) A dummy fill solution from an ILP solution with  $x_{12} = x_{23} = x_{33} = 1$  and  $y_{12} = y_{21} = y_{32} = 1$ .

sum of  $T_i$  and  $B_i$ .

Figure 6 shows an example with three wire segments  $S_1$ ,  $S_2$  and  $S_3$ . The green areas are the FillRegions. For each position in the FillRegions, it is associated with  $x_{ij}$  and/or  $y_{ik}$  accordingly as shown in Figure 6 (a). Figure 6 (b) gives the dummy fill region solution when  $x_{12} = x_{23} = x_{33} = 1$  and  $y_{12} = y_{21} = y_{32} = 1$ .

### 3.3.3 General Dummy Fill Region Generation

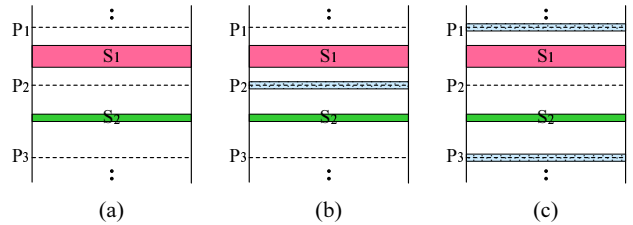


Figure 7: (a)  $P_1$ ,  $P_2$  and  $P_3$  are feasible positions to insert dummies. (b) When a dummy is inserted on  $P_2$ , the introduced coupling capacitance on both  $S_1$  and  $S_2$  reaches the coupling thresholds of  $S_1$  and  $S_2$ . No dummies can be inserted on  $P_1$  and  $P_3$ . (c) Two dummies can be inserted on  $P_1$  and  $P_3$  when  $P_2$  is left empty.

In the above section, we assume that each FillRegion has at least one dummy inserted. However, in order to maintain coupling constraints and maximize the amount of inserted dummies, not all FillRegions can get one dummy assigned. Figure 7 shows an example. In Figure 7 (a), there is one feasible position  $P_2$  between  $S_1$  and  $S_2$ . If one dummy is inserted on  $P_2$  as shown in Figure 7 (b), it consumes the coupling budget of both  $S_1$  and  $S_2$ , and no fills can be placed on  $P_1$  and  $P_3$ . On the other hand, two dummies can be placed on  $P_1$  and  $P_3$  when  $P_2$  is not used as illustrated in Figure 7 (c). Therefore, we can insert one more dummy in Figure 7 (c).

On the other hand, even density distribution is strongly preferred for yield improvement and design overhead reduction. If there is a



FillRegion between two wires, it means that the spacing between two wires is large and it is desirable to insert dummies for density uniformity. Therefore, we introduce a no-fill weight  $\lambda$ . If a FillRegion has no fills inserted, we add a penalty in the objective function to discourage the large empty space between two wires. This helps to balance the two requests of maintaining coupling constraints and maximizing dummy fills.

Based on the ILP formulation in the previous section, we add two more variables  $x_{i0}$  and  $y_{i0}$  for each  $S_i$ . When  $x_{i0} = 1$ , it means no fills are inserted in the region  $R_{i-1}$ . When  $y_{i0} = 1$ , it means no fills are inserted in  $R_i$ . Then we extend the previous ILP formulation for the general dummy fill region generation.

$$\min \{ [T_1 + \sum_{i=1}^{m-1} (B_i + T_{i+1}) + B_m] + \lambda \cdot (y_{m0} + \sum_{i=1}^m x_{i0}) \}$$

subject to:

$$x_{i0} + x_{i1} + \dots + x_{ir_{i-1}} = 1; \quad i \in [1, m] \quad (1)$$

$$y_{i0} + y_{i1} + \dots + y_{ir_i} = 1; \quad i \in [1, m] \quad (2)$$

$$T_i = 1 \cdot x_{i1} + 2 \cdot x_{i2} + \dots + r_{i-1} \cdot x_{ir_{i-1}} - 1; \quad i \in [1, m] \quad (3)$$

$$B_i = 1 \cdot y_{i1} + 2 \cdot y_{i2} + \dots + r_i \cdot y_{ir_i} - 1; \quad i \in [1, m] \quad (4)$$

$$B_{i-1} + T_i < r_{i-1}; \quad i \in [2, m] \quad (5)$$

$$\sum_{j=1}^{r_i-1} p_{ij} \cdot x_{ij} + \sum_{k=1}^{r_i} q_{ik} \cdot y_{ik} \leq (1 - x_{i0}) \cdot c_i^{i-1} + (1 - y_{i0}) \cdot c_i^{i+1} + C_i; \quad i \in [1, m] \quad (6)$$

$$y_{(i-1)0} + x_{i1} + \dots + x_{ir_{i-1}} = 1; \quad i \in [2, m] \quad (7)$$

$$x_{(i+1)0} + y_{i1} + \dots + y_{ir_i} = 1; \quad i \in [1, m-1] \quad (8)$$

$$x_{ij} = 0, 1; \quad i \in [1, m]; \quad j \in [0, r_{i-1}]$$

$$y_{ik} = 0, 1; \quad i \in [1, m]; \quad k \in [0, r_i]$$

Compared to the ILP formulation in the previous section,  $\lambda \cdot (y_{m0} + \sum_{i=1}^m x_{i0})$  are added in the objective function to penalize the FillRegions without dummy fills. In Equations (1) and (2),  $x_{i0}$  and  $y_{i0}$  are added, respectively, so that both cases (with-fills and without-fills) are covered. Equations (3), (4) and (5) are not changed. Equation (6) is adjusted to include the situation such that no fills are inserted in the FillRegions above/below  $S_i$ . Meanwhile, if the FillRegion  $R_i$  has no fills, both  $x_{(i+1)0}$  and  $y_{i0}$  should be 1. Therefore, two more equations (7) and (8) are used to force  $x_{(i+1)0}$  and  $y_{i0}$  to be 1 at the same time. This ILP formulation accurately describes the coupling constraints and can produce dummy fill region solutions that balance both the maximum fills and the density uniformity.

### 3.4 Dummy Fill Insertion

Once we identify the dummy fill regions, we need specify the exact dummy fill locations. Furthermore, for each dummy fill region, they can be used for either floating fills or ground fills. Since ground fills are favored over floating fills for their reliability and predictability, we should insert as many ground fills as possible. In this section, we first calculate the maximum amount of the ground fills that can be inserted in the dummy fill regions. Then we propose a dynamic programming based algorithm to decide the positions for both ground fills and floating fills. The algorithm also maximizes the insertion of floating dummy fills.

#### 3.4.1 Ground Dummy Fill Estimation

Ground dummies usually have a larger width and spacing, and they must connect to power rails. Figure 8 (a) shows an example with

4 slots. The regions  $F_i$  ( $i = 1, \dots, 9$ ) are the dummy fill regions obtained by our dummy fill region generation algorithms. SLOT2, SLOT3 and SLOT4 cover a pair of  $Gnds$ . As illustrated in Figure 8 (b), two ground fills  $d_{g1}$  and  $d_{g2}$  are inserted going through  $F_3$ ,  $F_5$  and  $F_8$ . Although  $d_{f5}$  can reach both two  $Gnds$ , the width of  $F_4$ ,  $F_6$  and  $F_9$  is not enough to hold one ground fill. For the rest of the dummy fill areas, they are used for floating dummy fills ( $d_{f1}, \dots, d_{f8}$ ).

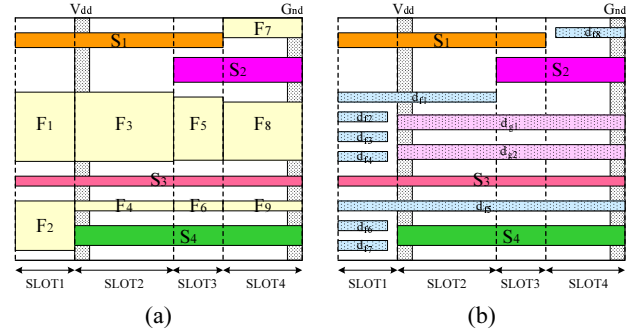


Figure 8: (a)  $F_1, \dots, F_9$  are dummy fill regions obtained from the ILP dummy fill region generation algorithm. (b) Dummy fill solution with floating fills and ground fills.

For each position in the region outlined by a pair of power rails, if it is fully covered by dummy fill regions, the position is feasible for ground fill insertion. As shown in Figure 9 (a), there are six dummy fill regions within two  $Gnds$ . Two regions  $G_1$  and  $G_2$  can be used for ground fills.

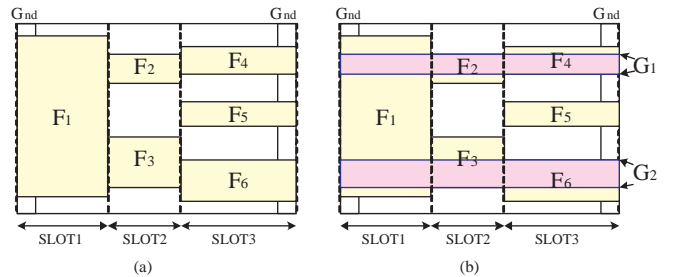


Figure 9: (a) There are 6 dummy fill regions between two  $Gnds$ . (b)  $G_1$  and  $G_2$  can be used for ground fill insertion.

For each ground fill region, suppose its height is  $H_g$  ( $H_g \geq 2 \cdot w_g$ ). Then the maximum number of ground fills in this region is  $\lceil (H_g - 2 \cdot w_g) / (2 \cdot w_g + s_g) \rceil + 1$ .

Although the number of ground fills can be determined very easily, the exact locations of ground fills have large impacts on floating fills. Figure 10 shows an example. In Figure 10 (a), the region outlined by blue lines can insert two ground fills. If the ground fills are placed as Figure 10 (b), no floating fills can be inserted in  $F_1$ . On the other hand, if the two ground fills are placed as Figure 10 (c),  $d_{f1}$  can be inserted.

This tells us that the ground fill locations have large impacts on floating fill assignment. Therefore, we propose the following dynamic programming based algorithm to determine dummy fill

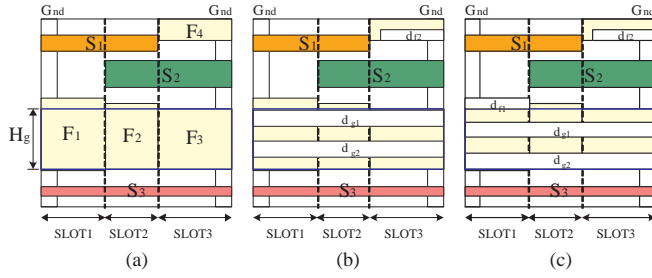


Figure 10: (a) Two ground fills can be inserted in the region outlined by blue lines. (b) No floating fills can be inserted in  $F_1$ . (c) A floating fill  $d_{f1}$  can be placed in  $F_1$ .

positions so that the number of floating fills can be maximized as well.

### 3.4.2 Dummy Fill Position Calculation

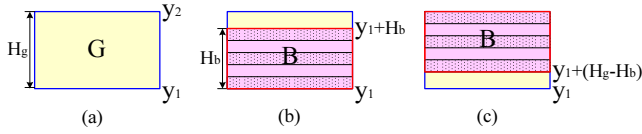


Figure 11: (a)  $G$  is a ground fill region with a height  $H_g$ . (b)  $B$  represents the area occupied by ground fills. (c) The bottom edge of region  $B$  in  $G$ .

If the ground fills are packed tightly, i.e., the ground fills are placed with minimum ground fill spacing  $s_g$ , then more room is left for floating fills. Therefore, we group the ground fills together, and treat them as a whole. In Figure 11 (a),  $G$  represents a ground fill region with a height  $H_g$ .  $y_1$  and  $y_2$  are the  $y$  coordinates of the bottom and upper edges of  $G$ , respectively. Suppose the number of ground fills inserted in  $G$  is  $N_g$ , and let  $B$  represent the total area occupied by ground fills. The height of  $B$  is  $H_b = (N_g - 1) \cdot (2 \cdot w_g + s_g) + 2 \cdot w_g$ . Therefore, the  $y$  coordinate of the bottom edge of  $B$  varies within the range  $T = [y_1, y_1 + (H_g - H_b)]$  as illustrated in Figure 11 (b) and (c). Obviously,  $H_g - H_b < 2 * w_g + s_g$ . If the position of  $B$  is fixed, then we can calculate how many floating fills can be inserted. This motivates us for the following dynamic programming based dummy fill insertion algorithm.

**FP (Fill Position) Problem** Between a pair of  $Gnds$ , there are  $n$  dummy fill regions  $F_1, \dots, F_n$ , and  $m$  ground fills regions  $G_1, \dots, G_m$ . Assume the ground fills occupy a region  $B_i$  in  $G_i$ . Suppose the height of  $G_i$  is  $H_{gi}$ , and its  $y$  coordinate of the bottom edge is  $y_{gi}$ . The height of  $B_i$  is  $H_{bi}$ , and its  $y$  coordinate of the bottom edge is  $y_{bi}$ . The target is to determine the values of  $y_{bi}$  so as to maximize the number of floating dummy fills.

Without loss of generality, suppose the  $y$  coordinate of the whole region is  $[0, Y]$ . Also let  $T_i = [y_{gi}, y_{gi} + (H_{gi} - H_{bi})]$ , which specifies all the possible positions of the bottom edge of  $B_i$ .

Our approach is to scan the ground fill regions from bottom to top (i.e., from  $G_1$  to  $G_m$ ). For each position in  $T_i$ , we calculate the maximum floating fills that can be inserted below it, and record the values in a two-dimension array  $Rec$ . We also use another two-dimension array  $Pre$  for tracing purpose. After processing  $G_m$ , we

can get the maximum number of floating fills. Once the positions of ground fills are settled, the floating fill positions can be determined easily.

#### Algorithm Fill\_Position

1. Initialize  $Rec$  and  $Pre$ ;
2. For each position  $p$  in  $T_1$
3.  $Rec[1][p] =$  number of floating fills within  $[0, p - s_g]$ ;
- 4.
5. For  $i = 2$  to  $m$
6. For each position  $p$  in  $T_i$
7. For each position  $q$  in  $T_{i-1}$
8.  $fills =$  number of floating fills within  $[q + H_{b(i-1)} + s_g, p - s_g]$ ;
9. if ( $fills + Rec[i-1][q] > Rec[i][p]$ )
10. then  $Rec[i][p] = fills + Rec[i-1][q]$ ;
11.  $Pre[i][p] = q$ ;
- 12.
13.  $maxfills = 0$ ;
14. For each position  $p$  in  $T_m$
15.  $fills =$  number of floating fills within  $[p + H_{bi} + s_g, Y]$ ;
16. if ( $maxfills < fills + Rec[m][p]$ )
17. then  $maxfills = fills + Rec[m][p]$ ;
18.  $maxpre = p$ ;

In the FP algorithm, we need to calculate the number of floating fills in the given range  $[Q, P]$ . So we need to check each  $F_i$ . If  $F_i$  have no intersection with  $[Q, P]$ , then no floating fills will be inserted in  $[Q, P]$  from  $F_i$ . Otherwise, we have four cases as shown in Figure 12. Suppose the  $y$  coordinates of the bottom and upper edges of  $F_i$  are  $y_{1i}$  and  $y_{2i}$ , respectively.

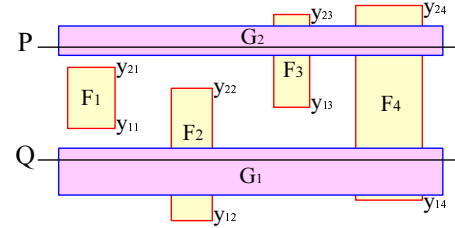


Figure 12: Four cases for floating dummy fill calculation

- Case 1: The whole dummy fill region  $F_i$  is within  $[Q, P]$ , i.e.,  $Q \leq y_{1i}$  and  $y_{2i} \leq P$ . If  $y_{2i} - y_{1i} \geq 2 \cdot w_f$ , then the total number of floating fills in  $F_i$  is  $\lceil (y_{2i} - y_{1i} - 2 \cdot w_f) / (2 \cdot w_f + s_f) \rceil + 1$ . Otherwise, no fills can be inserted. One example is  $F_1$  in Figure 12. For  $F_1$ ,  $Q < y_{11}$  and  $y_{21} < P$ , and the floating fill insertion in  $F_1$  is totally determined by  $y_{11}$  and  $y_{21}$ .
- Case 2: The dummy fill region  $F_i$  is partially within  $[Q, P]$  such that  $y_{1i} \leq Q$  and  $Q \leq y_{2i} \leq P$ . If  $y_{2i} - Q \geq 2 \cdot w_f$ , then the total number of floating fills in  $F_i$  is  $\lceil (y_{2i} - Q - 2 \cdot w_f) / (2 \cdot w_f + s_f) \rceil + 1$ . Otherwise, no fills can be inserted.  $F_2$  in Figure 12 gives an example.
- Case 3: Similar to Case 2. The dummy fill region  $F_i$  is partially within  $[Q, P]$  with  $Q \leq y_{1i} \leq P$  and  $P \leq y_{2i}$ . If  $P - y_{1i} \geq 2 \cdot w_f$ , then the total number of floating fills in  $F_i$  is  $\lceil (P - y_{1i} - 2 \cdot w_f) / (2 \cdot w_f + s_f) \rceil + 1$ . Otherwise, no fills can be inserted.  $F_3$  in Figure 12 shows an example.

Table 1: Test Results

File	Layout Area( $\mu\text{m}^2$ )	Wire Segs	Wire Cons.	Orginal MaxDens	CPU Time	Violations		Max-Density			Ave-Density		
						NoCap	CDF	NoCap	CDF	Diff	NoCap	CDF	Diff
Test1	483.8x441.4	1405	1079	12.95%	1s	563	0	49.74%	49.74%	0.00%	44.81%	39.85%	4.96%
Test2	4654.0x4660.4	6370	4437	23.36%	2s	2209	0	50.37%	50.01%	0.36%	48.41%	47.46%	0.95%
Test3	438.2x437.4	8670	5694	11.73%	2s	4528	0	49.18%	49.14%	0.04%	42.99%	37.63%	5.36%
Test4	4660.4x4682.8	17594	12241	23.06%	9s	6033	0	50.14%	50.13%	0.01%	47.99%	46.27%	1.72%
Test5	1478.0x1497.2	29936	21953	24.25%	10s	13988	0	50.37%	49.68%	0.69%	45.87%	37.03%	8.84%
Test6	7548.8x7467.2	44078	32378	24.61%	40s	6257	0	51.49%	50.26%	1.23%	48.10%	45.99%	2.11%
Test7	1479.5x1497.0	53082	31569	19.89%	12s	22355	0	50.09%	50.09%	0.00%	46.73%	44.89%	1.84%
Test8	4683.8x4683.4	58169	42889	23.86%	108s	25229	0	50.47%	49.74%	0.67%	47.94%	40.95%	6.99%
Test9	7477.2x7550.4	119490	84971	24.72%	209s	30529	0	53.36%	51.16%	2.20%	49.11%	44.53%	4.58%

Case 4: The dummy fill region  $F_i$  fully covers  $[Q, P]$ , i.e.,  $y_{1i} \leq Q$  and  $P \leq y_{2i}$ . If  $P - Q \geq 2 \cdot w_f$ , then the total number of floating fills in  $F_i$  is  $\lceil (P - Q - 2 \cdot w_f) / (2 \cdot w_f + s_f) \rceil + 1$ . Otherwise, no fills can be inserted.  $F_4$  in Figure 12 is an example.

For each  $F_i$ , the number of floating fills can be calculated in constant time. In the FP algorithm,  $T_i < 2 \cdot w_g + s_g$ . Therefore, the total running time of FP algorithm can be bounded by  $O(m \cdot n \cdot T^2)$ , where  $T = 2 \cdot w_g + s_g$ .

## 4. Experimental Results

Our algorithm was implemented in C on a linux workstation (2.3GHz). We use GLPK (GNU Linear Programming Kit) as our ILP solver. The test cases are derived from the industry designs and the coupling thresholds are generated randomly. Set the dummy fill spacing be the same as as the fill width, i.e.,  $2 \cdot w_f = s_f$  and  $2 \cdot w_g = s_g$ . The density window size is set as  $24\mu\text{m}$ . Table 1 shows the results of the nine test cases. “Wire Segs” is the total number of wire segments in each test case. “Wire Cons.” is the number of wires with coupling constraints. “Original MaxDens” is the maximum density of all the density windows before fill insertion. For all the test cases, the CDF algorithm can find feasible positions for dummy fills within a short time. We compared our results with the filling results without considering coupling constraints. For “NoCap”, dummy fills are inserted in the empty space as long as they satisfy the spacing rules. Using our algorithm, the maximum density of the final postfill layouts is around 50% for all the test cases. Compared with NoCap, the final maximum density and average density are pretty close, which means that our algorithm can identify feasible fill locations very effeciently. A small density gap (both maximum density and average density) between CDF and NoCap exists. This is caused by wire patterns and coupling constraints. However, as shown in NoCap, when the coupling constraints are not considered during dummy fill insertions, the dummy fill induced coupling may make the total coupling on wires exceed the given threshold. As shown in Table 1, “Violations” is the number of wires whose total capacitive coupling is larger than the given threshold. For all the test cases, if the dummy fills are inserted regardless the coupling constraints, a large number of wires have coupling violations.

## 5. Conclusion

In this paper, we present the first dummy fill density analysis algorithm which takes coupling impacts into consideration. The algorithm also makes efforts to maximize ground dummy fills, which

are more robust and predictable. The output of the algorithm can be treated as the upper limit of dummy fill insertion, and it can be easily adopted in density models to guide dummy fill insertion without disturbing the existing design. Furthermore, our algorithm outlines the dummy fill regions such that no coupling violations will be introduced if dummy fills are inserted in those regions. This allows different dummy fill patterns to be applied.

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