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On the Analysis of Finite MANETs

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***Abstract*—Because of their importance in military and other applications, Mobile Ad-Hoc Wireless Networks or MANETs have attracted significant attention in the research community. However, almost all of the literature has focused on analyzing MANETs in an asymptotic case with a very large number of nodes under varying levels of node density and distribution. While the asymptotic analysis is extremely valuable, practical usage of MANETs requires us to be able to analyze networks of finite size. In this paper, we present an approach to analyze MANETs with a fixed number of nodes which can be used in many practical applications related to MANETs.**

I. Introduction

Mobile ad-hoc wireless networks are an important area of study with many applications in the military and civil domains. These networks can be used for a variety of applications, e.g. creating a communication channel between several vehicles on the move in a military convoy or operation, managing connectivity among a group of unmanned aerial vehicles, creating ad-hoc networks based on buses or other vehicles moving on roads (e.g., DieselNet [1]), collecting information from a bunch of sensors distributed in a geographic area, monitoring animals using RFID or other sensor tags (e.g., ZebraNet [2] and TurtleNet [3]), and a host of other applications

Because of their importance several attempts have been made to analyze and characterize the properties of wireless sensor networks. Several properties and results regarding the operation and properties of MANETs in the asymptotic case are known. By the asymptotic case, we mean a situation where there are a large number of mobile nodes which are distributed according to some density function and their movements are governed by a probabilistic random mobility model.

While the value of such analysis cannot be denied, asymptotic networks which move according to some known mobility models are unlikely to be realized in practice. All practical instance of a MANET will have a finite number of nodes that may be too small for asymptotics to apply; these nodes would often be moving in some type of mobility model dictated by the needs of their mission or operation. A convoy of trucks may form a mobile network moving along the roadways in the region, while a UAV would be programmed to fly on a predetermined path over a sensor field to collect information from the different sensors in the field. Clearly, we need to develop techniques and methodologies that can analyze the properties of the dynamic network created by such motion.

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The analysis of finite networks is hampered by the fact that such networks do not enjoy general ergodic properties that facilitate asymptotic analyses. Thus, particular attention must be paid to realization dependencies. We consider wireless networks, and as such the interactions between the `links' or `edges' cannot be ignored. In particular, notions such as supportable rate or outage capacity of a link depends not only upon the transmitter (e.g., transmit power), the receiver (e.g., sophistication of the signal processing), but also upon the characteristics of the link (terrain dependent aspects such as fading and shadowing, but also interference due to traffic on nearby links). Finally, mobility induced effects (e.g., Doppler) cannot be ignored.

If the network elements were not mobile, the analysis of the properties of the network is relatively straight-forward. One can analyze the graph defined by the nodes and edges of the network to obtain several properties such as the bandwidth and latency between two nodes in the network, the points of maximum vulnerability as defined by the minimum cut, the diameter of the node, etc. It would be highly desirable to obtain the same properties in a network where the elements were mobile and dynamic.

We show in this paper that it is possible to represent any MANET which is stable, finite and defined motion paths into an equivalent graph with less complex motions which would have the same average properties as the properties of the graph generated by the time-varying MANET. In some cases, and for some graph properties, we can create a static graph that can be used to analyze the dynamic MANET and have the same properties as that of dynamic MANET on the average. Using this approach, we can analyze the properties of many types of mobile networks, and we present examples of some such analyzable networks subsequently in the paper.

II. Problem Formulation

We consider the analysis of the network connectivity property among a set $N = \{n_1, \ldots n_k\}$ representing k mobile nodes. Each of the nodes has a special vector property defined as its position, where the position of node n_i is given by the vector $\vec{p}_i(t)$. At any instant of time, the velocity of node n_i is defined as

$$\vec{v}_i(t) = \frac{\partial}{\partial t}\vec{p}_i(t)$$

The set of all velocities $\{v_1,\dots v_k\}$ is represented as **V**.

Definition 1. A dynamic MANET is a 3-tuple { N, P_0, **V**}, where

- N represents the set of nodes $\{n_1,\dots n_k\}$
- P_0 represents the initial position of the nodes at time t = 0.
- **V** represents the set of velocity vectors of the different nodes $\{v_1,\dots v_k\}$.

An edge in dynamic MANET connects two vertices, and is associated with a set of time-varying scalar properties. Each property is a time-varying mapping from the set NxN to the set of real numbers. Examples of properties associated with the edge include the capacity of the edge, the delay of the edge, or the packet loss-probability of the edge. Each of these properties of the edges would have an instantaneous value. For simplicity, we are only considering nodes which have identical properties (e.g. transmission power, receiver fidelity, etc.) and transmit with a fixed power under uniform channel conditions.

Definition 2. An edge-property of a dynamic MANET is an operation mapping the position of two nodes in the MANET to a real number.

In other words, for any two nodes i and j, an edge property $ep_{i,j} = f(\vec{p}_i, \vec{p}_j)$. Since the positions are functions of time, the edge properties are a function of time as well.

Definition 3. A network property np of a dynamic MANET with k nodes is an operation mapping the k^2 edge properties to a real number.

Since the edge properties are time-varying, the network properties will be time-varying as well.

Examples of a network property (in addition to edge properties) include the diameter (maximum latency between any two points in the MANET), total capacity (sum of all edge capacity in the MANET), etc. in the dynamic MANET. Any combination of edge properties, e.g. shortest latency between a pair of nodes will also be a network property. It follows from the definition that any individual edge property is also a network property. Note that all of these properties are time-varying, so at any time the property has an instantaneous value.

If the network were not mobile, the network properties would be determined by means of graph-theoretic algorithms on the graph representing the nodes and edges of the network. We would reduce the problem of the determining MANET properties to that of determining the properties on an equivalent graph with a simpler set of motions. In order to do so, we would demonstrate a set of congruence relationships between MANETs that have different velocity vectors but consist of the same set of nodes.

In order to demonstrate the congruence, we use the following definitions.

Definition 4. An isotropic edge property of a MANET is an edge property which only depends on the properties of the nodes it connects and the distance between the two nodes of the edge.

In other words, an isotropic property $ep_{i,j} = f(|\vec{p}_i - \vec{p}_j|)$. Some examples of isotropic edge properties would be the propagation delay on an edge and the loss rate of an edge if the edge is in a homogenous medium.

Definition 5. An isotropic network property is a network property which is independent of any non-isotropic edge properties of the MANET.

In other words, an isotropic network property is obtained by a combination of one or more isotropic edge properties of the node.

Definition 6. A dynamic MANET M1 = { N, P0, V } is defined to be congruent to a dynamic MANET M2 = { N', P'0, V', } with respect to a network property np iff N = N' and np(M1,t) = np(M2,t) for all value of time t.

In other words, two MANETs are congruent for a network property if the two have the same set of nodes, and they have the same instantaneous value of the network property at all times t under consideration.

For the analysis of the MANETS, we will also introduce the notion of the center of a MANET, which is a hypothetical position defined by taking a average of the position of the different elements of the MANETs. More formally,

Definition 7. Given a k-dimensional weight metric W = { $w_1\dots w_k$} such that $\sum_{i=1}^{i=k} w_i = 1$ and all weights are positive, the weighted center of a MANET with k nodes is a hypothetical node with the position vector defined by $\sum_{i=1}^{i=k} w_i\vec{p}_i$ and the velocity vector $\sum_{i=1}^{i=k} w_i\vec{v}_i$.

The weighted center of MANET will be useful in showing the congruence properties discussed later in the paper.

III. CONGRUENCE UNDER TRANSLATION

Given any type of velocity of the different members of a MANET, we first establish the congruence of isotropic properties under the notion of translation.

Let us consider a graph with k nodes, and define a homogenous velocity vector $\boldsymbol{C} = \{c, \dots c\}$, i.e. a velocity vector where all k nodes are moving with a (not necessarily constant) velocity *c*. Given two velocity vectors, they can be added and subtracted using the normal rules for vector addition and subtraction, i.e. if **V** = $\{v_1, \ldots v_n\}$ and **V'** = $\{v'_1, \ldots v'_n\}$ then **V-V'** = $\{v_1 - v'_1, \ldots v_n - v'_n\}$, etc.

Theorem 1. If ***C*** is a homogenous velocity vector then the dynamic MANET $M1=\{N, P_0, \boldsymbol{V}\}$ is congruent to the dynamic MANET $M2=\{N, P_0, \boldsymbol{V}\text{-}\boldsymbol{C}\}$ with respect to any isotropic edge property.

Proof: Consider any isotropic edge property of the edge between nodes n_i, and n_j where $n_i \in N$ and $n_j \in N$. Due to the definition of the isotropic edge property, it suffices to show the MANET will be congruent with respect to the edge property if the distance between the corresponding nodes in M1 and M2 is the same. Since the two nodes started out from the same initial position vectors (P_0 is common between both MANETs), the position of n_i at time *t* in *M1* equals $p_{0,i} + \int_0^t v_i dt$ where p_{0i} is the initial starting position of n_i (i.e. the i^{th} entry in P_0) and v_i is the i^{th} entry in ***V***. Similarly, the position of n_j at time *t* in *M1* equals $p_{0,j} + \int_0^t v_j dt$.

In *M2*, the corresponding positions are $p_{0,i} + \int_0^t (v_i - c)dt$ and $p_{0,j} + \int_0^t (v_j - c)dt$ respectively. If we take the difference in position vectors of the two nodes in *M2*, the common term $\int_0^t cdt$ cancel out and we see that it is equal to the difference in the position vectors of the two nodes in M1. The congruence then follows from the definition of the isotropic edge property.

Some interesting corollaries can be derived from Theorem 1.

Corollary 1.1: If ***C*** is a homogenous velocity vector then the dynamic MANET $M1=\{N, P_0, \boldsymbol{V}\}$ is congruent to the dynamic MANET $M2=\{N, P_0, \boldsymbol{V}\text{-}\boldsymbol{C}\}$ with respect to any isotropic network property..

Proof: All edge properties that the network property is dependent on are isotropic by definition, and the two MANETs will be congruent with respect to all those edge properties, it follows that it will be congruent with respect to the network property.

Corollary 1.2 For any isotropic edge property, a MANET is congruent with respect to that edge property to another MANET in which an arbitrarily selected node is stationary.

Proof: For any arbitrary node n_i, replace the homogenous velocity vector in Theorem 1 with the velocity vector where all nodes are moving with the velocity of v_i. Then, the resulting MANET has the node n_i as stationary and is congruent to the original MANET.

Corollary 1.3 For any isotropic edge property, a MANET is congruent with respect to that edge property to another MANET in which the weighted center of the MANET is stationary.

Proof: replace the homogenous velocity vector in Theorem 1 with the velocity vector of the weighted center of the MANET.

Corollary 1.4 For any isotropic network property, a MANET is congruent with respect to that network property to another MANET in which the weighted center of the MANET is stationary.

Proof: Combine corollary 1.3 with the fact that isotropic network property is composed as function of isotropic edge properties.

IV. CONGRUENCE UNDER ROTATION

Let us consider a graph with *k* nodes, and define a rotation velocity vector ***R*** = $\{\vec{r}_i \ldots \vec{r}_k\}$ with respect to an origin $\vec{\mathrm{p}}_0$ where each of the rotation velocity vectors satisfies the following conditions at all times:

$\vec{r}_i.(\vec{p}_0 - \vec{p}_i) = 0$, where . is the vector dot product.

$$|\,\vec{r}_i\,|\,/\,|\,\vec{r}_j\,| = |\,(\vec{p}_0 - \vec{p}_i)\,|\,/\,|\,(\vec{p}_0 - \vec{p}_j)\,|$$

The rotation vector describes the type of motion which would be created when the entire MANET is viewed as being on a virtual fixed plane rotating around the origin with some rotational speed. The rotational speed need not be a constant during the time of the rotation.

Theorem 2. If ***R*** is a rotation velocity vector then the dynamic MANET $M1=\{N, P_0, \boldsymbol{V}\}$ is congruent to the dynamic MANET $M2=\{N, P_0, \boldsymbol{V}\text{-}\boldsymbol{R}\}$ with respect to any isotropic edge property.

Proof: Consider any isotropic edge property of the edge between nodes n_i, and n_j where $n_i \in N$ and $n_j \in N$. Due to the definition of the isotropic edge property, it suffices to show the MANET will be congruent with respect to the edge property if the distance between the corresponding nodes in M1 and M2 is the same. Since the two nodes started out from the same initial position vectors (P_0 is common between both MANETs), the position of n_i at time *t* in *M1* equals $p_{0,i} + \int_0^t v_i dt$ where p_{0i} is the initial starting position of n_i (i.e. the i^{th} entry in P_0) and v_i is the i^{th} entry in ***V***. Similarly, the position of n_j at time *t* in *M1* equals $p_{0,j} + \int_0^t v_j dt$.

In *M2*, the corresponding positions are $p_{0,i} + \int_0^t (v_i - r_i)dt$ and $p_{0,j} + \int_0^t (v_j - r_j)dt$ respectively. If we take the difference in position vectors of the two nodes in *M2*, the difference in position would be $\int_0^t (r_i - r_j)dt$. However, that is the difference in positions obtained among two points each of whom is rotating at a uniform speed around the origin point $\vec{p}_0$, which would always be zero. A more formal proof can also be defined based on the definition of the rotation vector above.

The following corollaries follow from Theorem 2.

Corollary 2.1 For any isotropic edge property, a MANET is congruent with respect to that edge property to another MANET in which nodes have no net rotational component around the weighted center of the MANET.

Proof: Choose the weighted center as the origin point and remove the sum of rotational components of other nodes around the origin.

Corollary 2.2 For any isotropic network property, a MANET is congruent with respect to that network property to another MANET in which the weighted center of the MANET is stationary, and at least one node of the MANET has no rotation component.

Proof: Combine corollary 1.3 with Theorem 2, and remove the rotation component for any one node.

Corollary 2.3 For any isotropic network property, a MANET where the movement of each node is independent of the others is congruent with respect to that network property to another MANET in which the nodes movement is restricted to be radial movement towards or away from a fixed location.

Proof: Combine corollary 2.2 with the fact that with independent motions of particles, a zero rotation component on the average is only possible if the rotation component of each of the independent nodes is zero. This implies that the independent motion of nodes will be restricted to moving towards or away from the weighted center of mass. [1]

The congruence under translation and rotation implies that we can consider any finite set of MANETs as a system which does not have a net rotational or net translational component in its motion. Thus, the finite MANET can be considered as comprising of nodes whose motion consists primarily of movement towards or away from a weighted center, and a rotation vector that is dependent.

[1] Note that the statement is not true for dependent motions, e.g. if one node were moving at the exact opposite of another node, or one node were moving twice as fast as another node, and a third one rotating thrice as fast in opposite direction, then it is possible to have a net rotation of zero with non-zero rotation components of individual nodes.

An appropriately selected weight function can thus be used to determine a good weighted center, and to analyze the characteristics of a MANET using an appropriately constructed graph.

V. CONGRUENCE TO OSCILLATORY MOTION

In this section, we look at some of the properties of the finite MANETs which have no rotation or translation component, i.e. each node is oscillating towards a weighted center. As discussed earlier, for any network property, any arbitrary moving MANET would have a MANET that has these characteristics and is congruent with respect to that property with the former.

If the nodes are constrained to only move towards or away from a weighted center of the network, then their motion along this direction needs to be constrained. In order for the node to not fall into the center, or to fly away to an infinite distance away from the center, the oscillations of each node needs to be bounded within a minimum and a maximum bound at any time. Furthermore, since the center is static, any node must not have a net motion away or towards the center when averaged over a sufficiently large period of time. In other words, the net motion of any node would be to oscillate around an average distance from the weighted center, such that the time-average value of displacement around the distance is zero.

Let us define a motion vector **V** = $\{v_1,\ldots v_n\}$ as an oscillatory vector with respect to an origin $\vec{p}_0$ where the position of each individual node has both a finite minimum and a finite maximum bound on the value of $|\vec{p}_0 - \vec{p}_i|$.

We can then prove the following theorem.

Theorem 3. For any isotropic edge property and any stable dynamic MANET *M1*={ *N*, P_0, ***V***} where the motions of individual nodes are independent, one can find a dynamic MANET *M2*={ *N*, P_0, ***V'***} with respect to any isotropic edge property such that M2 is congruent to M1 with respect to that edge property, and ***V'*** is an oscillatory vector with respect to the weighted center of M2.

Proof: We know from corollaries 1.4 and 2.3 that M1 can be shown to be congruent to a dynamic MANET where the weighted center is stationary and has no rotation component in the motion of the individual nodes. Using mathematical induction, we can show that an oscillatory motion with or without bounds is the only feasible motion possible in that case. Furthermore, if the bounds on the movement of a node do not exist, that node will eventually need to be at an infinite distance from the origin point, which would contradict the fact that the MANET is stable.

If a MANET has individual motions that are not independent, then one can remove the translation components, and end up with a congruent MANET in which at least one node is stationary, and the other nodes have motion vectors which have a rotation component and an oscillatory component.

For any isotropic edge property, the variation of the edge property as a function of time can be determined if the dependency of the edge property on the distance between nodes, and the dependency of the distance between the nodes and the node are known. Once the determination of that function is know, then the evaluation of the characteristics of the edge property as a function of time, and determining its metrics such as mean and variances are straight-forward.

VI. From Edge Properties to Network Properties

In this section, we examine how we can move from isotropic edge properties to determining the isotropic network properties of a MANET. The network property of a path would be a function of a combination of edge properties along that path in the network. Similarly, the network property of the overall network is a combination of the edge properties of all the edge property in the network.

Corollary 2.3 and Theorem 3, while valid, provide only limited simplification in the analysis of finite MANETs. In general, even with a stationary weighted center, the relative motion of two nodes are fairly complex. However, Theorem 1 and 2 take together provide the insight that the motion of the nodes around the weighted center would contain of two primary components – a rotation component and an oscillatory component. If $\vec{r}_i$ is the vector from the weighted center to the node, and $\vec{t}_i$ is the perpendicular to that vector, then the motion of any node around the weighted center can be expressed as the sum of two vectors ($\omega_i \vec{r}_i + o_i \vec{t}_i$), where the mean of both ω and o is zero. If follows that the relative motion of two nodes is also the difference in the oscillation and rotation vectors of the two nodes – which would also have a mean value of zero.

While a mean value of zero can be obtained from the motion of many different types, the most common type of motion that results in a mean of zero is a periodic relative motion. With periodic motion, we can confine our analysis to a finite period [0..T] where T is the period of the motion. In many other cases, we are also able to confine our studies to a finite period.

If we plot any instantaneous property of an edge between the time-period of [0..T], we would get a time-varying curve representing its path. As an example, if we consider two UAVs, one rotating around a central point with a rotational speed of 1 rotation every 10 minutes at a distance of 1 km from the weighted center and the other rotating at a distance of 2 Km from the weighted center at 2 rotations every 10 minutes, we may get a distance-time plot as shown in Figure 1(a). If we consider the instantaneous capacity, we may get a distance-time plot as shown in Figure 1(b), whereas if we consider the instantaneous propagation delay between two nodes, we may get a chart like Figure 1(c).

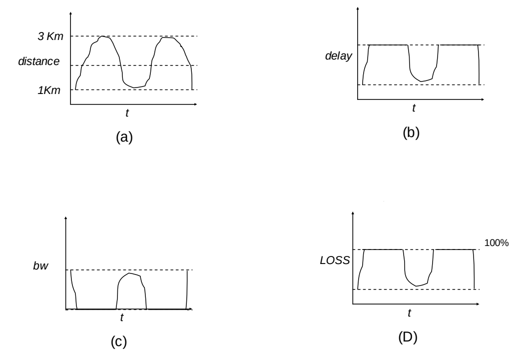

Figure 1

In general, the network properties of a finite MANET would be a composition of the time-dependent edge properties of the MANET. An example situation is illustrated in Figure 2. Any edge property can be represented by a time-varying function between the nodes. Figure 2 represents the time varying edge properties of the nodes in a hypothetical graph.

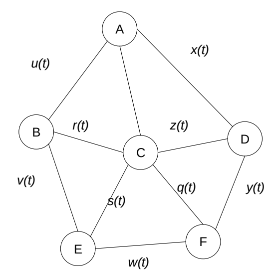

Figure 2

Consider a path in the network shown in Figure 2 which follows the path A-B-E-F. and consider that an isotropic edge property varies according to the functions shown on the edges of the graph above. Different properties of the path can be obtained by performing different types of operations on the functions represented by the paths, e.g.

- If the edge property represents the instantaneous delay on the link, then the delay experienced by a packet sent at time t on path ABEF would be u(t) + v(t+u(t)) + w(t + v(t)+u(t)).
- If the edge property represents the instantaneous capacity of the link AB, then the capacity of the path ABEF would be min (u(t), v(t)) , w(t)
- If the edge property represents the loss rate on the edge, then (assuming the loss rates are small enough to ignore the variations), the loss rate on the path would be u(t) + v(t) + w(t).

Using the fact that E{f(t)+u(t)} = E(f(t)+E(u)), and that the expected value of a function would not change by adding a constant offset, one can determine that the expected value of the link delays along the path would be E(u)+E(v)+E(w). Similarly, the average loss rate of the path can be determined by means of adding the averages of the path lengths. Similarly, the probability that a path is connected can be determined by the probability that each of the edges along the path is connected. However, a property like minimum capacity can not be determined that easily.

For the properties which are determined by addition or multiplication operations of time-varying properties of the path, the expected value of the combined property of the path can be determined by the average property of each of the edges (assuming independent edge properties in the multiplicative case). For such properties, we can convert a MANET into a static equivalent graph which can be analyzed using static graph analysis methods to provide an estimate of the average property of the network. Figure 3 demonstrates the average representation of some of the time varying properties shown in Figure 2, and would be a static equivalent graph for the MANET for the purpose of determining the average values of isotropic network properties.

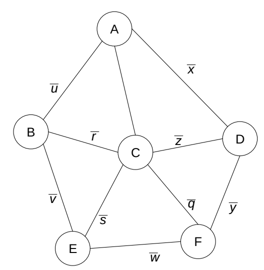

Figure 3

While not all of the network properties can be estimated in this manner, we can analyze quite a few properties (e.g. network latency, network bandwidth, error rates, connectivity) etc. using this approach of a static graph. In other cases, we would have to resort to analyzing the time-varying properties of an equivalent oscillatory graph.

VII. Examples of Analysis

In this section, we provide some example scenarios of finite MANETs that we can examine using the properties and equivalence that we have described above. We consider three examples, and show how one can analyze the characteristics of those nodes using the results above. To simplify the results of our examples, we have purposely selected the motion vectors to be relatively simple. However, the results can be applied to more realistic motion vectors.

A. River Sensors

A set of 5 buoys with sensors are placed in the river, and they flow downstream from location A to location B as shown in Figure 4. The amount of power used for transmitting information is proportional to the distance between two nodes. The river current is at the speed of 5 mph. A boat sails in the river to collect the information from one of the sensors. Which of the sensors should act as the repository of data collection for the sensors in order to minimize the bandwidth used by the entire sensor system?

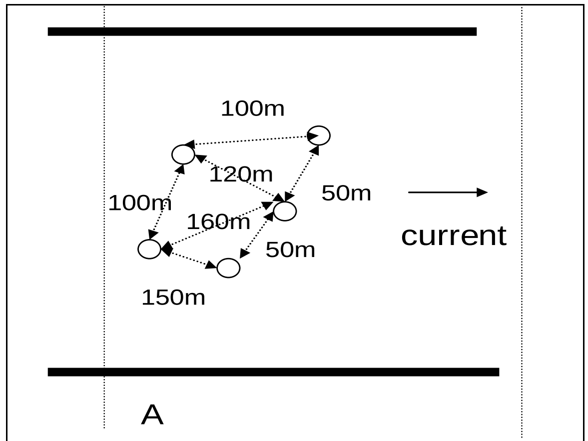

Figure 4

Answer: Since all of the sensors are floating downstream at the same velocity, we can consider them to be stationary with respect to each other. Thus, we have the task of deciding which of the 5 nodes should act as the center for collecting information so as to provide the collection boat with all of the information required. This would be the node which when selected as the root node, would result in a spanning tree of minimum weight – when all the edges are directed towards the root node. Using a stationary graph analysis algorithm, the proper sensor can be determined in a straight-forward manner.

B. Surveillance UAVs

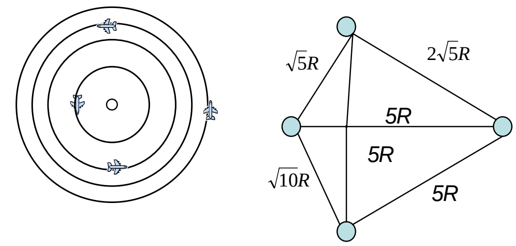

Figure 5

In order to detect suspicious vehicles, four UAVs are circulating over the headquarters of the US/UK headquarters in Bagdad. The UAVs can sustain a bandwidth b with each other which is given by the formula $b = Be^{-d/D}$, where d is the distance between the two nodes, and B and D are constants. [2]Assuming that the UAVs are rotating at 1 rotation every 10 minutes, at the distance of R apart from each other, and phased equally along a circle, what is the bandwidth possible for communication between each pair of UAVs.

Answer: Since the UAVs are rotating at the same rotational velocity, they can be considered to be stationary with respect to each other. The distance between the different nodes and the center is as shown in the right hand side of the diagram. Given

[2] This modeling is consistent with various models of power and bandwidth fading, e.g. one presented in [4].

the angle between the center and the locations of the UAVs (90o each), and the distances from the center, the distance between the UAVs can be calculated using trigonometric relationships. That distance is shown in the right hand image of Figure 2. Knowing the distance and the formula mapping the bandwidth to distance, the feasible bandwidth between each pair of UAVs calculated.

If the UAVs were rotating not at the same rate, but at different rates, then we can use Theorem 2 to make one of the UAVs stationary in a congruent MANET. Then, one can compute the variation of the distance of the other UAV from the stationary one, and then calculate the net bandwidth available as a function of time with the motion of the other UAV. As an example, let us consider the bandwidth between the UAV that is closest to the center, and the one next to it. The first UAV is rotating at 1 revolutions per 10 minutes while the second one is rotating at 2 revolutions per 10 minutes. We can consider this system as congruent to a system in which the first UAV is stationary and the second UAV is rotating at the speed of 1 revolution per 10 minute. In this revolution, the distance between the UAVs will vary in a sinusoidal manner between R and 3R, resulting in a correspondingly fluctuating bandwidth. The average bandwidth between the two nodes can then be computed from the provided expressions.

C. Military Convoy

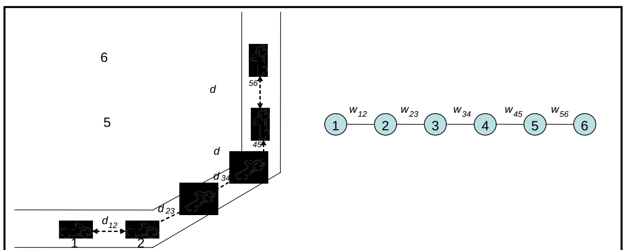

Figure 6

In this scenario, each vehicle sets up a communication channel with its immediate neighbor, and the communication between two vehicles that are not right in front of or behind each other takes place through multi-hop connection. We index the vehicles from the head of the group to the tail with *1, 2, …, 6*, and denote the distance between adjacent two adjacent vehicles by d_{12}, d_{23}, …, d_{56}. These inter-vehicle distances can vary over time, and we assume these distances follow stationary distributions with the respective density functions $f_{12}(d)$, $f_{23}(d)$, …, $f_{56}(d)$ with maximum and minimum distances d_{max} and d_{max}. Given the achievable bandwidth, *B(d)*, as a arbitrary decreasing function of distance *d* between two nodes communicating directly over the wireless channel, and the minimum bandwidth, C_{min}, required for two nodes to be able to successfully communicate with each other, what is the probability of this network of vehicles being connected? Also what is the achievable bandwidth between any pair of vehicles, assuming there is no interference between different wireless links thanks to, e.g., multi-channel allocation?

Answer: Since the vehicles are moving back-to-back along the same path and the network property (i.e., end-to-end connectivity) we are concerned with in this scenario is dependent only on the distance between vehicles, they can be simply regarded moving along a 1-dimensional line, where there is clearly no rotational component in their mobility. If we take any arbitrary vehicle as the center of the network, then the original mobile network of the vehicles is congruent to a network in which the vehicle selected as the center is stationary and all others oscillate around that vehicle. Now since the minimum bandwidth is C_{min}, in order for two vehicles to be able to communicate with each other (i.e., to be connected), they must be within distance $d^* = B^{-1}(C_{min})$. Therefore, assuming $d_{min} < d^* < d_{max}$, the probability, $p_{i,j}$, that two adjacent vehicles *i* and *j* are connected is $p_{i,j} = \int_{d\min}^{d^*} f_{ij}(x)dx$. Also the effective average bandwidth, $C_{i,j}$, between two adjacent vehicles *i* and *j* is $C_{i,j} = \int_{d\min}^{d^*} C(x) f_{ij}(x)dx$ (If $d^* < d_{min}$, $p_{i,j} = 0$ and $C_{i,j} = 0$, and if $d^* > d_{min}$, $p_{i,j} = 1$ and $C_{i,j} = \int_{d\min}^{d_{\max}} C(x) f_{ij}(x)dx$).

Hence, our answers can be found by looking at the static graph on the right-hand image of Figure 3, with the edge weights replaced by $p_{i,j}$ (for connectivity) or $C_{i,j}$ (for bandwidth). More specifically, the connectivity probability $P_c(i,j)$, and the effective average bandwidth, $B_e(i,j)$, between arbitrary pair of vehicles *i* and *j* (*i* < *j*), are $P_c(i,j) = 1 - \prod_{k=i}^{j-1}(1 - p_{k,k+1})$, and $B_e(i,j) = \min_{k=i,\dots,j-1} C_{k,k+1}$.

D. Reconnaissance Vehicles

In this scenario, four reconnaissance vehicles leave a central location to travel a distance of D along the four compass directions at equal speeds and then return back to the base. The effective bandwidth between any two pair of vehicles is B when the distance between the vehicles is less than $\sqrt{2}D$ and zero otherwise. What is the effective average bandwidth between the vehicles that are proceeding directly across from each other?

Answer: While each of the trucks has not reached the distance of $\sqrt{2}D$ away from the center, they are each in direct communication with each other, and thus have the bandwidth of B between themselves. However, when the vehicles are between the distance of $\sqrt{2}D$ and 2D away from the origin, they can only communicate via one of the vehicles moving perpendicular to them. Since each of the reconnaissance vehicles moving perpendicular to each other are only going to be at a maximum distance of $\sqrt{2}D$, such a pair of vehicles is always connected. Thus, this set of 4 vehicles switches between the two graphs (a) and (b) which are shown in Figure 8.

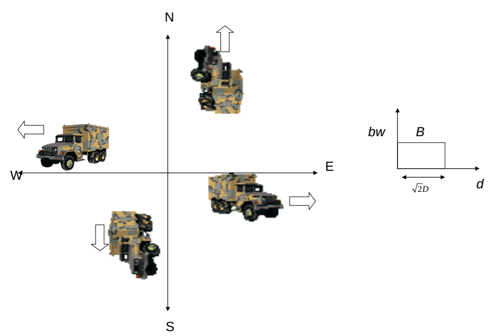

Figure 7

When the vehicles are connected before they have reached the distance of $\sqrt{2}D$, any pair of vehicles can communicate with the full bandwidth B in configuration (a). When the vehicles have reached configuration (b), each of the vehicles moving perpendicular to each other has the bandwidth of B, but do not have the ability to communicate with the one directly across. Using symmetry arguments, we can show that the bandwidth possible between any two vehicles in this case would be 2B/3 where the system is in configuration (b).

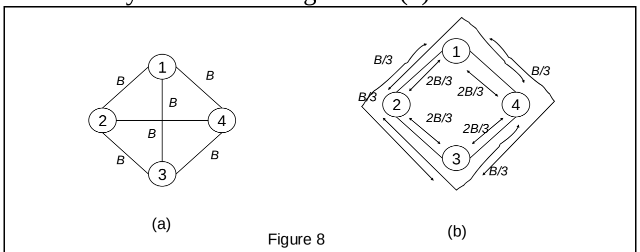

Figure 8

The ratio of time in which the system is in stage (a) to when it is in stage (b) is $\sqrt{2}/(2-\sqrt{2})$. Taking the weighted average of the time when the system is in the two different stages, one can show that the average bandwidth between a pair of vehicles directly across from each other would be $B\frac{\sqrt{2}+1}{3\sqrt{2}}$.

Although the examples provided above can be viewed as toy examples illustrating the analysis of finite MANETs, it should be apparent that they can be extended to analyze more complex motion vectors and a larger number of nodes.

In a more pragmatic case, when the number of nodes in a MANET is finite, one can develop a software package that tracks the instantaneous velocity vectors of the nodes, and then builds a quasi-static model of the static network using the velocity vectors at any given instance. Such a system can then be used to answer questions regarding which node in the MANET is best connected, which is likely to lose connectivity in the near future, and which one needs to use more than one path to maintain a given bandwidth need.

VIII. CONCLUSIONS

In this paper, we have presented a method to analyze wireless MANETs of finite size by converting them into congruent MANETs with a simpler type of motion vector. The method is applicable to isotropic properties – which are independent of the positions of individual nodes in the network. The method has been shown to be useful in the context of some example scenarios, and can be used to analyze the average values of some properties of finite sized MANETs.

In future work, we would like to develop a scheme to understand and analyze non-isotropic properties of the network, as well as develop the concept of analyzing graphs with time-varying edge properties that are not readily convertible to a static equivalent graph. We would like to combine our results with work on graph algorithms that handle node additions and deletions [5] to address a larger set of analysis problems related to dynamic mobile networks.

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