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Analyzing Finite MANETs by Transformations to Equivalent Static Graphs

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Analyzing Finite MANETs by Transformations to Equivalent Static Graphs

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Abstract—**Because of their importance in military and other applications, Mobile Ad-Hoc Wireless Networks or MANETs have attracted significant attention in the research community. However, almost virtually all of the literature has focused on analyzing MANETs in an asymptotic case with a very large number of nodes under varying levels of node density and distribution. While the asymptotic analysis is extremely valuable, practical usage of MANETs requires us to be able to analyze networks of finite size. In this paper, we present an approach to analyze MANETs with a fixed number of nodes which can be used in many practical applications related to MANETs. Our approach is based on simplifying the motion paths of a MANETs by applying a set of transformations, and decomposing the motion paths into a generalized Fourier series transformation of simpler periodic motions.**

I. Introduction

Mobile ad-hoc wireless networks are an important area of study with many applications in the military and civil domains. These networks can be used for a variety of applications, e.g. creating a communication channel between several vehicles on the move in a military convoy or operation, managing connectivity among a group of unmanned aerial vehicles, creating ad-hoc networks based on buses or other vehicles moving on roads (e.g., DieselNet [1]), collecting information from a bunch of sensors distributed in a geographic area, monitoring animals using RFID or other sensor tags (e.g., ZebraNet [2] and TurtleNet [3]), and a host of other applications

Because of their importance several attempts have been made to analyze and characterize the properties of wireless mobile ad hoc networks. Several properties and results regarding the operation and properties of MANETs in the asymptotic case are known [4] [5] [6]. By the asymptotic case, we mean a situation where there are a large number of mobile nodes which are distributed according to some density function and their movements are governed by a probabilistic random mobility model.

While such analysis and asymptotic results are valuable in providing insights about network behavior, asymptotic networks which move according to some known mobility models are unlikely to be realized in practice. All practical instance of a MANET will have a finite number of nodes that may be too small for asymptotics to apply; these nodes would often be moving according to some type of mobility model dictated by the needs of their mission or operation. A convoy of trucks may form a mobile network moving along the roadways in the region, while a UAV would be programmed to fly on a predetermined path over a sensor field to collect information from the different sensors in the field. Clearly, we need to develop techniques and methodologies that can analyze the properties of the dynamic network created by such motion.

The analysis of finite networks is hampered by the fact that such networks do not enjoy general ergodic properties that facilitate asymptotic analyses. Thus, particular attention must be paid to realization dependencies. We consider wireless networks, and as such the interactions between the `links' or `edges' cannot be ignored. In particular, notions such as supportable rate or outage capacity of a link depends not only upon the transmitter (e.g., transmit power), the receiver (e.g., sophistication of the signal processing), but also upon the characteristics of the link (terrain dependent aspects such as fading and shadowing, but also interference due to traffic on nearby links). Finally, mobility induced effects (e.g., Doppler) cannot be ignored.

If the network elements were not mobile, the analysis of the properties of the network is relatively straight-forward. One can analyze the graph defined by the nodes and edges of the network to obtain several properties such as the bandwidth and latency between two nodes in the network, the points of maximum vulnerability as defined by the minimum cut, the diameter of the network, etc. It would be highly desirable to obtain the same properties in a network where the elements were mobile and dynamic. There is a body of work on the Delay Tolerant Network (DTN) that develops routing protocols in intermittently connected networks based on predicted mobility pattern of mobile nodes [7] [8], but the literature lacks the effort to analyze fundamental characteristics of mobile networks of finite size. Even in the static case, the literature on the analysis of finite sized networks is scanty; see, e.g., [9].

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We show in this paper that it is possible to represent any MANET which is stable, finite and has defined motion paths into an equivalent graph with less complex motions paths in a manner such that the resulting graph would have the same average properties as the properties of the original graph. We discuss two types of transformations which result in the simplification of the motion paths – the first type may be viewed as rigid body transformations while the other type may be viewed as elastic transformations. By using a series of transformations, we show that a dynamically varying network may be approximated by a static network which would have network properties similar to that of the original graph.

The structure of this paper is as follows: In section 2, we provide a formalization of the problem of analyzing finite networks. In Section 3, we discuss the results of applying a rigid body transformations to a finite MANET, followed by a scheme for applying elastic transformations on the finite MANETs. Finally, we show how we can combine these transformations to simplify the motion of a finite-sized MANET, and areas for future work.

II. PROBLEM FORMULATION

We consider the analysis of the network connectivity property among a set $N = \{n_1, \ldots n_k\}$ representing k mobile nodes. Each of the nodes has a special vector property defined as its position, where the position of node n_i is given by the vector $\vec{p}_i(t)$. At any instant of time, the velocity of node n_i is defined as

$$\vec{v}_i(t) = \frac{\partial}{\partial t}\vec{p}_i(t)$$

The set of all velocities $\{v_1, \ldots v_k\}$ is represented as **V**. For the sake of notational simplicity, we often drop the explicit dependence on time; e.g., V rather than V(t).

Definition 1. A dynamic MANET is a 3-tuple { N, P_0, **V**}, where

- N represents the set of nodes $\{n_1, \ldots n_k\}$
- P_0 represents the initial position of the nodes at time t = 0.
- **V** represents the set of velocity vectors of the different nodes $\{v_1, \ldots v_k\}$.

An edge in dynamic MANET connects two vertices, and is associated with a set of time-varying scalar properties. Each property is a time-varying mapping from the set NxN to the set of real numbers. Examples of properties associated with the edge include the capacity of the edge, the delay of the edge, or the packet loss-probability of the edge. Each of these properties of the edges would have an instantaneous value. For simplicity, we are only considering nodes which have identical properties (e.g. transmission power, receiver fidelity, etc.) and transmit with a fixed power under uniform channel conditions.

Definition 2. An edge-property of a dynamic MANET is an operation mapping the position of two nodes in the MANET to a real number.

In other words, for any two nodes i and j, an edge property $ep_{i,j} = f(\vec{p}_i, \vec{p}_j)$. Since the positions are functions of time, the edge properties are a function of time as well.

Definition 3. A network property np of a dynamic MANET with k nodes is an operation mapping the k^2 edge properties to a real number.

As a convenience, we will use the convention *np(M)* to refer to the network property np of a MANET M. Since the edge properties are time-varying, the network properties will be time-varying as well.

Examples of a network property (in addition to edge properties) include the diameter (maximum latency between any two points in the MANET), total capacity (sum of all edge capacities in the MANET), etc. in the dynamic MANET. Any combination of edge properties, e.g. shortest latency between a pair of nodes will also be a network property. It follows from the definition that any individual edge property is also a network property. Note that all of these properties are time-varying, so at any time the property has an instantaneous value.

If the network were not mobile, the network properties would be determined by means of graph-theoretic algorithms on the graph representing the nodes and edges of the network. We would reduce the problem of the determining MANET properties to that of determining the properties on an equivalent graph with a simpler set of motions. In order to do so, we would demonstrate a set of congruence relationships between MANETs that have different velocity vectors but consist of the same set of nodes.

In order to demonstrate the congruence, we use the following definitions.

Definition 4. An isotropic edge property of a MANET is an edge property which only depends on the properties of the nodes it connects and the distance between the two nodes of the edge.

In other words, an isotropic property $ep_{i,j} = f(|\vec{p}_i - \vec{p}_j|)$. Some examples of isotropic edge properties would be the propagation delay on an edge and the loss rate of an edge if the edge is in a homogenous medium.

Definition 5. An isotropic network property is a network property which is independent of any non-isotropic edge properties of the MANET.

In other words, an isotropic network property is obtained by a combination of one or more isotropic edge properties of the node.

Definition 6. A dynamic MANET M1 = { N, P0, V } is defined to be congruent to a dynamic MANET M2 = { N', P'0, V', } with respect to a network property np iff N = N' and np(M1) = np(M2,) at all times.

In other words, two MANETs are congruent for a network property if the two have the same set of nodes, and they have the same instantaneous value of the network property at all times *t* under consideration. If the graphs representing two MANETS are isomorphic at any instance, then their network properties should be the same at that instance. If the graphs representing two MANETs are always isomorphic, then they will be congruent for any network property. However, just because two MANETs are congruent for a network property does not imply that their graphs are isomorphic. Thus isomorphism of the graphs of two MANETs is sufficient for congruence under a network property, but not necessary.

For the analysis of the MANETS, we will also introduce the notion of the center of a MANET, which is a hypothetical position defined by taking an average of the position of the different elements of the MANETs. More formally,

Definition 7. Given a *k*-dimensional weight metric $W = \{ w_1 \ldots w_k\}$ such that $\sum_{i=1}^{i=k} w_i = 1$ and all weights are positive, the weighted center of a MANET with *k* nodes is a hypothetical node with the position vector defined by $\sum_{i=1}^{i=k} w_i \vec{p}_i$ and the velocity vector $\sum_{i=1}^{i=k} w_i \vec{v}_i$.

The weighted center of MANET will be useful in showing the congruence properties discussed later in the paper.

III. Rigid Body Transformations

The transformations we perform to simplify the motion paths of a MANET can be provided an informal treatment by creating the imaginary body which is obtained by connecting each node in the MANET to the other nodes by a wire. Under some types of motion of the MANET nodes, there would be no deformation in any of the wires connecting the different nodes in the MANET. We can define transformation of motion paths of the different nodes of the MANET which cause no deformation, and show that the resulting MANET will be congruent to the original MANET. Since the imaginary body created above would remain unchanged during these transformations, we refer to these transformations as rigid body transformations.

Two types of rigid-body transformations are the notion of translation and rotation. In this section, we establish the congruence of isotropic properties under the notion of translation and rotation. In the next section, we examine the issue of congruence under different types of non-rigid body transformations.

Let us consider a graph with *k* nodes, and define a homogenous velocity vector $\boldsymbol{C} = \{c, \ldots c\}$, i.e. a velocity vector where all k nodes are moving with a (not necessarily constant) velocity *c*. Given two velocity vectors, they can be added and subtracted using the normal rules for vector addition and subtraction, i.e. if $\mathbf{V} = \{v_1, \ldots v_n\}$ and $\mathbf{V'} = \{v'_1, \ldots v'_n\}$ then $\mathbf{V\text{-}V'} = \{v_1 - v'_1, \ldots v_n - v'_n\}$, etc.

Theorem 1. If $\boldsymbol{C}$ is a homogenous velocity vector then the dynamic MANET $M1=\{N, P_0, \boldsymbol{V}\}$ is congruent to the dynamic MANET $M2=\{N, P_0, \boldsymbol{V\text{-}C}\}$ with respect to any isotropic edge property.

Proof: Consider any isotropic edge property of the edge between nodes n_i, and n_j where $n_i \in N$ and $n_j \in N$. Due to the definition of the isotropic edge property, it suffices to show the MANET will be congruent with respect to the edge property if the distance between the corresponding nodes in M1 and M2 is the same. Since the two nodes started out from the same initial position vectors (P_0 is common between both MANETs), the position of n_i at time *t* in *M1* equals $p_{0,i} + \int_0^t v_i dt$ where p_{0i} is the initial starting position of n_i (i.e. the i^{th} entry in P_0) and v_i is the i^{th} entry in $\boldsymbol{V}$. Similarly, the position of n_j at time *t* in *M1* equals $p_{0,j} + \int_0^t v_j dt$.

In *M2*, the corresponding positions are $p_{0,i} + \int_0^t (v_i - c)dt$ and $p_{0,j} + \int_0^t (v_j - c)dt$ respectively. If we take the difference in position vectors of the two nodes in *M2*, the common term $\int_0^t cdt$ cancel out and we see that it is equal to the difference in the position vectors of the two nodes in M1. The congruence then follows from the definition of the isotropic edge property.

Some interesting corollaries can be derived from Theorem 1.

Corollary 1.1: If $\boldsymbol{C}$ is a homogenous velocity vector then the dynamic MANET $M1=\{N, P_0, \boldsymbol{V}\}$ is congruent to the dynamic MANET $M2=\{N, P_0, \boldsymbol{V\text{-}C}\}$ with respect to any isotropic network property..

Proof: All edge properties that the network property is dependent on are isotropic by definition, and the two MANETs will be congruent with respect to all those edge properties, it follows that it will be congruent with respect to the network property.

Corollary 1.2 For any isotropic edge property, a MANET is congruent with respect to that edge property to another MANET in which an arbitrarily selected node is stationary.

Proof: For any arbitrary node n_i, replace the homogenous velocity vector in Theorem 1 with the velocity vector where all nodes are moving with the velocity of v_i. Then, the resulting MANET has the node n_i as stationary and is congruent to the original MANET.

Corollary 1.3 For any isotropic edge property, a MANET is congruent with respect to that edge property to another MANET in which the weighted center of the MANET is stationary.

Proof: replace the homogenous velocity vector in Theorem 1 with the velocity vector of the weighted center of the MANET.

Corollary 1.4 For any isotropic network property, a MANET is congruent with respect to that network property to another MANET in which the weighted center of the MANET is stationary.

Proof: Combine corollary 1.3 with the fact that isotropic network property is composed as function of isotropic edge properties.

Let us now demonstrate the congruence of MANETs under the notion of rotation of a rigid body.

Let us consider a graph with k nodes, and define a rotation velocity vector $\boldsymbol{R} = \{\vec{r}_i ... \vec{r}_k\}$ with respect to an origin $\vec{\mathrm{p}}_0$ where each of the rotation velocity vectors satisfies the following conditions at all times:

$$\vec{r}_i . (\vec{p}_0 - \vec{p}_i) = 0$$, where . is the vector dot product.

$$|\vec{r}_i| / |\vec{r}_j| = |(\vec{p}_0 - \vec{p}_i)| / |(\vec{p}_0 - \vec{p}_j)|$$

The rotation vector describes the type of motion which would be created when the entire MANET is viewed as being on a virtual fixed plane rotating around the origin with some rotational speed. The rotational speed need not be a constant during the time of the rotation.

Theorem 2. If $\boldsymbol{R}$ is a rotation velocity vector then the dynamic MANET $M1 = \{N, P_0, \boldsymbol{V}\}$ is congruent to the dynamic MANET $M2 = \{N, P_0, \boldsymbol{V}\text{-}\boldsymbol{R}\}$ with respect to any isotropic edge property.

Proof: Consider any isotropic edge property of the edge between nodes n_i, and n_j where $n_i \in N$ and $n_j \in N$. Due to the definition of the isotropic edge property, it suffices to show the MANET will be congruent with respect to the edge property if the distance between the corresponding nodes in M1 and M2 is the same. Since the two nodes started out from the same initial position vectors (P_0 is common between both MANETs), the position of n_i at time t in $M1$ equals $p_{0,i} + \int_0^t v_i dt$ where p_{0i} is the initial starting position of n_i (i.e. the i^{th} entry in P_0) and v_i is the i^{th} entry in $\boldsymbol{V}$. Similarly, the position of n_j at time t in $M1$ equals $p_{0,j} + \int_0^t v_j dt$.

In $M2$, the corresponding positions are $p_{0,i} + \int_0^t (v_i - r_i) dt$ and $p_{0,j} + \int_0^t (v_j - r_j) dt$ respectively. If we take the difference in position vectors of the two nodes in $M2$, the difference in position would be $\int_0^t (r_i - r_j) dt$. However, that is the difference in positions obtained among two points each of whom is rotating at a uniform speed around the origin point $\vec{\mathrm{p}}_0$, which would always be zero. A more formal proof can also be defined based on the definition of the rotation vector above.

The following corollaries follow from Theorem 2.

Corollary 2.1 For any isotropic edge property, a MANET is congruent with respect to that edge property to another MANET in which nodes have no net rotational component around the weighted center of the MANET.

Proof: Choose the weighted center as the origin point and remove the sum of rotational components of other nodes around the origin.

Corollary 2.2 For any isotropic network property, a MANET is congruent with respect to that network property to another MANET in which the weighted center of the MANET is stationary, and at least one node of the MANET has no rotation component.

Proof: Combine corollary 1.3 with Theorem 2, and remove the rotation component for any one node.

Corollary 2.3 For any isotropic network property, a MANET where the movement of each node is independent of the others is congruent with respect to that network property to another MANET in which the nodes movement is restricted to be radial movement towards or away from a fixed location.

Proof: Combine corollary 2.2 with the fact that with independent motions of particles, a zero rotation component on the average is only possible if the rotation component of each of the independent nodes is zero. This implies that the independent motion of nodes will be restricted to moving towards or away from the weighted center of mass. [1]

The congruence under translation and rotation implies that we can consider any finite set of MANETs as a system which does not have a net rotational or net translational component in its motion. Thus, the finite MANET can be considered as comprising of nodes whose motion consists primarily of

[1] Note that the statement is not true for dependent motions, e.g. if one node were moving at the exact opposite of another node, or one node were moving twice as fast as another node, and a third one rotating thrice as fast in opposite direction, then it is possible to have a net rotation of zero with non-zero rotation components of individual nodes.

movement towards or away from a weighted center, and a rotation vector that is dependent.

An appropriately selected weight function can thus be used to determine a good weighted center, and to analyze the characteristics of a MANET using an appropriately constructed graph.

IV. Analyzing Elastic body transformations

The concept of rigid body translation and rotations can only simplify the motion paths of MANETs to a limited extent. A more comprehensive simplification of their motion paths may be obtained by means of transformations that allow the body formed by the different nodes in the MANET to twist and adapt its shape.

In order to provide an intuitive understanding of the notion behind elastic transformations, let us imagine a planar MANET which is drawn on an elastic membrane. Under the rigid body transformations, the elastic membrane was rotated or moved about. Under the elastic transformations, let us imagine an individual pulling on the elastic sheet so that it doubles in size along its two axes and then pushing it back into the original size at regular periodic intervals. As the membrane is stretched in this manner, the distance among the different nodes in the MANET varies in a sinusoidal fashion.

In this section, we first describe how to analyze a MANET which is subject to sinusoidal elastic transformations, and then using a combination of such sinusoidal elastic transformations to analyze MANETs with arbitrary motion paths.

A. MANET Analysis under elastic oscillations

Consider any finite MANET which starts from some original position of the various nodes, and then is subjected to a sinusoidal elastic transformation. One specific property of the stretching all dimensions of elastic members equally along all dimensions is that each edge stretches by an equal amount at a regular rate. As a result, all nodes in the MANET have the same transformation (lengthening or shortening) that are synchronized in time. As a result of the changes in the path links, all isotropic edge properties of the MANET will vary and expand in the same manner.

With any such sinusoidal motion, that the mean value of an edge property will not be affected. However, the variance (second momentum) and higher momentum measure of the variations of the edge property over time would be different.

Let us consider a MANET in an initial position where the length of a specific edge initially is l_0. Under a sinusoidal elastic transformation of amplitude A, and frequency ω, the length at any given time t will be $l_0+A\sin(\omega t)$.

Any isotropic edge or network property of the MANET would vary periodically in accordance with the amplitude and period of the sinusoidal elastic motion. Since the size of the edge at any instant is known, the mean and other higher moments of the edge (and corresponding network properties) can be calculated in a relatively straight-forward manner.

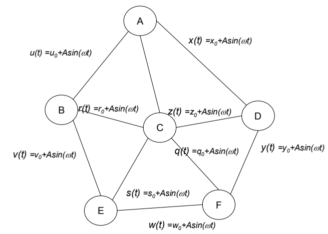

Figure 1

As an example, consider the MANET which starts in an initial position with the nodes and edges as shown in Figure 1. As it goes through the sinusoidal expansion and contraction, the length of the different edges change and correspondingly the edge properties and associated network properties of the graph would change. As an example, Figure 2 shows the variation of the length of edge AD and the corresponding variation in the edge properties of bandwidth, delay and loss rate on that edge. Note that the bandwidth is caped below by 0 and loss rate above by 100%.

The mean and other moments of any edge or network properties that are not subject to a threshold (e.g. delay in this example) can be calculated analytically based on their periodic variation. For properties where a threshold is involved, analytic forms would tend to be more complex, but can be computed numerically in a straight-forward manner.

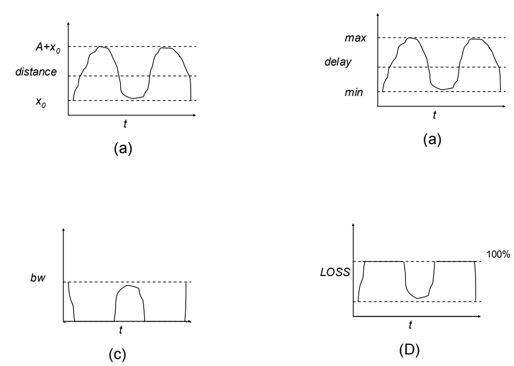

Figure 2

B. Analyzing Arbitrary MANET motion paths

Most MANETs do not tend to have motion that can be nicely modeled by oscillatory functions. However, for a variety of complex motions, one can use the results from the analysis of oscillatory motion to get a reasonable estimation of the properties of the network.

Consider the oscillatory stretching movements of a graph drawn on an elastic membrane, but being done at different frequencies of ω, 2ω,3ω,4ω, we can see that these oscillatory motions can be viewed as making a Fourier series in 2-dimensions, and most types of random stretching of the elastic membrane can be expressed as a linear combination of the various oscillatory motions as a 2-dimensional Fourier expansion characterizing the actual oscillation [13]. The result is applicable to 3-dimensional oscillatory motion as well.

One can thereby approximate the arbitrary oscillatory movement of a MANET whose center is stationary by means of its Fourier expansion in the various oscillatory components, and then calculate the impact on the edge property by accumulating the impact from the accumulation of the oscillatory movements at the various base frequencies that make up the Fourier expansion of the same.

On a more formal note:

Let us define a base oscillatory motion vector $\mathbf{E}(\omega, A, \vec{p}_0) = \{v_1, \ldots v_n\}$ as an oscillatory vector with respect to an origin $\vec{p}_0$ and amplitude (maximum change in position) **A**, where the position of the i^{th} node in the MANET is given by the expression $\vec{p}_i(t) - \vec{p}_0 = [\vec{p}_i(0) - \vec{p}_0]\sin(\omega t)$, which is akin to having the MANET on an elastic surface being stretched by a maximum of unit dimension back and forth. Consider the basic motion vectors described by the set of motion vectors $\mathbf{E}(\omega, A, \vec{p}_0), \mathbf{E}(2\omega, A, \vec{p}_0), \mathbf{E}(3\omega, A, \vec{p}_0) \ldots$

We can express the motion paths of any MANET in terms of

$$\boldsymbol{V} = \boldsymbol{R} + \alpha_1 \mathbf{E}(\omega, A, \vec{p}_0) + \alpha_2 \mathbf{E}(2\omega, A\ \vec{p}_0) + \alpha_3 \mathbf{E}(3\omega, A, \vec{p}_0) + \ldots$$

Where α_i are the constants marking the coefficients in the Fourier expansion of the motion of the MANET, and ***R*** is a rigid body motion, i.e. comprising of a translation and rotational motion vector only.

For analyzing the properties of the MANET, we can select the amplitude A to be a distance at which the effective bandwidth between any two nodes in the network would be zero, and we can choose the original point for the elastic body transformations to be the weighted center of the MANET. In that case, we simply need to understand what happens to the network and edge properties of the network in the presence of the different elastic body transformations. Thus, the properties of the MANET can be reduced to that of a congruent MANET with a motion defined by

$$\boldsymbol{V'} = \alpha_1 \mathbf{E}(\omega) + \alpha_2 \mathbf{E}(2\omega) + \alpha_3 \mathbf{E}(3\omega) + \ldots$$

The above can be viewed as the Fourier expansion of the motion characterizing the MANET after the rigid body transformations are accounted for. For many types of MANET motion paths, only the first few series in the Fourier expansion need to be considered for a reasonably approximate solution. For such motion paths, the analysis method provided in the preceding subsection can be used to estimate the impact of each types of motion component, and then the net result combined to obtain the final characteristics of a finite MANET, providing the characteristic of the MANET under the oscillatory elastic properties are analyzable.

The Fourier expansion may not simplify all types of motion paths for MANETs, but given its applicability in many other disciplines, we anticipate that many various types of motion paths can be analyzed in this manner.

V. Application Examples of Analysis

In this section, we provide some example scenarios of finite MANETs that we can examine using the properties and equivalence that we have described above. We consider some examples, and show how one can analyze the characteristics of those nodes using the results provided above.

We want to point out that the motion vectors in the examples shown below are relatively simple – intended primarily so that we can show that the results are applicable and the final analysis presented in a simple manner. However, using numerical approaches, the decomposition provided in the earlier sections can be done for the more complex motion that we encounter in real MANETs..

A. Military Squadron

A squadron of 5 airplanes is flying from New York to California in the formation shown in Figure 3. The airplanes maintain a common speed maintaining the distances shown between them, and are able to communicate with each other if they are within 500 meters of each other and transmit at a fixed bandwidth of 512 Kbps. What is the best spanning tree structure that should be used for communication among the airplanes?

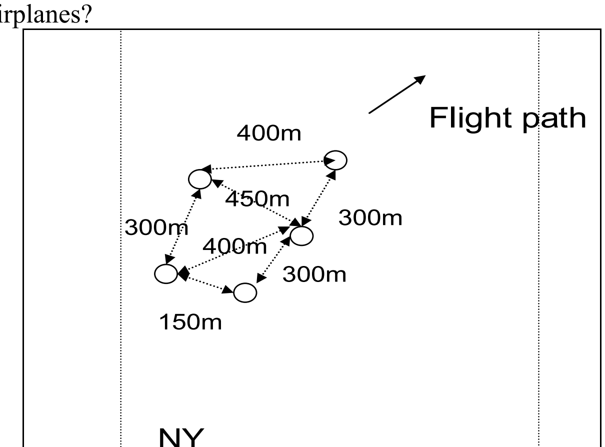

Figure 3

Answer: Let us define the direction of the x-axis along the flight path taken by the planes. Each of the 5 planes has a velocity vector of $\boldsymbol{V} = \{v_0, v_0, v_0, v_0, v_0\}$, where v_0 is the time-varying common velocity of the planes. As per Theorem 1, we can remove this homogenous velocity vector, and the resulting MANET will be congruent to the original MANET.

The resulting MANET is a static graph with no motion, and the nodes with the edge-distances as shown in Figure 3. It is straight-forward to determine the best minimum spanning tree on a static graph.

In a real scenario, the planes would have stochastic variations in their relative positions which would also need to be accounted for. Example C below provides such a scenario.

B. Surveillance UAVs

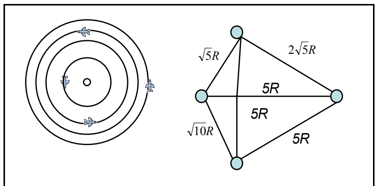

Figure 4

In order to detect suspicious vehicles, four UAVs are circulating over the headquarters of the US/UK headquarters in Baghdad. The UAVs can sustain a bandwidth b with each other which is given by the formula b = $Be^{-d/D}$, where d is the distance between the two nodes, and B and D are constants. [2]Assuming that the UAVs are rotating at 1 rotation every 10 minutes, at the distance of R apart from each other, and phased equally along a circle, what is the bandwidth possible for communication between each pair of UAVs.

Answer: Since the UAVs are rotating at the same rotational velocity, Theorem 2 tells us that they are congruent to a MANET where they can be considered to be stationary with respect to each other. The distance between the different nodes and the center is as shown in the right hand side of the diagram. Given the angle between the center and the locations of the UAVs (90o each), and the distances from the center, the distance between the UAVs can be calculated using trigonometric relationships. That distance is shown in the right hand image of Figure 4. Knowing the distance and the formula mapping the bandwidth to distance, the feasible bandwidth between each pair of UAVs calculated.

C. Military Convoy

In this scenario, each vehicle sets up a communication channel with its immediate neighbor, and the communication between two vehicles that are not right in front of or behind each other takes place through multi-hop connection. We index the vehicles from the head of the group to the tail with *1, 2, …,*and *6*, and denote the distance between adjacent two adjacent vehicles by d_{12}, d_{23}, …, and d_{56}. These inter-vehicle distances can vary over time, and we assume these distances follow stationary distributions with the respective density functions $f_{12}(d)$, $f_{23}(d)$, …, $f_{56}(d)$ with maximum and minimum distances d_{max} and d_{max}. Given the achievable bandwidth, *B(d)*, as an arbitrary decreasing function of distance *d* between two nodes communicating directly over the wireless channel, and the minimum bandwidth, C_{min}, required for two nodes to be able to successfully communicate with each other, what is the probability of this network of vehicles being connected? Also what is the achievable bandwidth between any pair of vehicles, assuming there is no interference between different wireless links thanks to, e.g., multi-channel allocation?

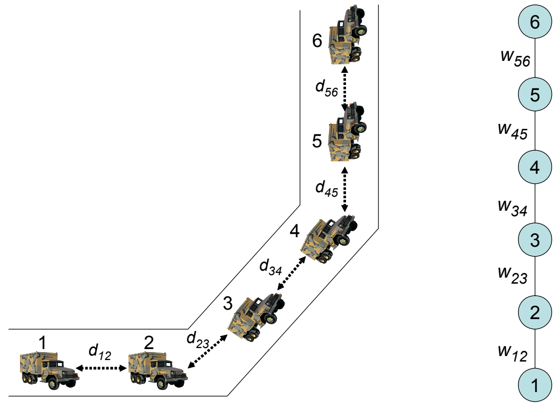

Figure 6

Answer: Since the vehicles are moving back-to-back along the same path and the network property (i.e., end-to-end connectivity) we are concerned with in this scenario is dependent only on the distance between vehicles, they can be simply regarded moving along a 1-dimensional line, where there is clearly no rotational component in their mobility. Let us select vehicle 3, and subtract the motion of vehicle 3 from each of the other vehicle's motion. According to Theorem 1, the original mobile network of the vehicles is congruent to a network in which vehicle 3 is stationary and all others oscillate around it. Now since the minimum bandwidth is C_{min}, in order for two vehicles to be able to communicate with each other (i.e., to be connected), they must be within distance $d^* = B^{-1}(C_{min})$. Therefore, assuming $d_{min} < d^* < d_{max}$, the probability, $p_{i,j}$, that two adjacent vehicles *i* and *j* are connected is $p_{i,j} = \int_{d\min}^{d^*} f_{ij}(x)dx$. Also the effective average bandwidth, $C_{i,j}$, between two adjacent vehicles *i* and *j* is $C_{i,j} = \int_{d\min}^{d^*} C(x) f_{ij}(x)dx$ (If $d^* < d_{min}$, $p_{i,j} = 0$ and $C_{i,j} = 0$, and if $d^* > d_{min}$, $p_{i,j} = 1$ and $C_{i,j} = \int_{d\min}^{d_{\max}} C(x) f_{ij}(x)dx$). Hence, our answers can be found by looking at the static graph on the right-hand image of Figure 5, with the edge weights replaced by $p_{i,j}$ (for connectivity) or $C_{i,j}$ (for bandwidth). More specifically, the connectivity probability $P_c(i,j)$, and the effective average bandwidth, $B_e(i,j)$, between arbitrary pair of vehicles *i* and *j* $(i < j)$, are $P_c(i,j) = 1 - \prod_{k=i}^{j-1}(1 - p_{k,k+1})$, and $B_e(i,j) = \min_{k=i,\dots,j-1} C_{k,k+1}$.

D. Reconnaissance Vehicles

[2] This modeling is consistent with various models of power and bandwidth fading, e.g. one presented in [10].

In this scenario, four reconnaissance vehicles leave a central location to travel a distance of D along the four compass directions at equal speeds and then return back to the base. The reconnaissance vehicles keep on doing this at the constant speed returning to the base once every hour. The effective bandwidth between any two pair of vehicles is $Be^{-d/D}$. The vehicles communicate in the pattern shown by the dotted arrows. What is the mean and variance of the communication bandwidth possible along any one link?

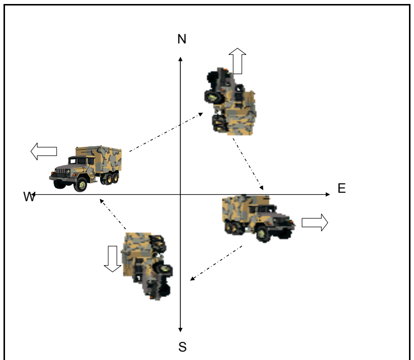

Figure 6

Answer: The graph modeling the communication network between the vehicles is shown in Figure 7. As the vehicles move back and forth, they can be viewed as performing an oscillatory motion with a period of 1 hour. Thus, applying the principles of Section 2.A, we can model the distance of any single link as varying between the limits of 0 and $\sqrt{2}D$ regularly over a time-period of T, and the bandwidth varying according to the formula provided above. The mean and standard deviation can each be computed to be $B(e+1)/2e$.

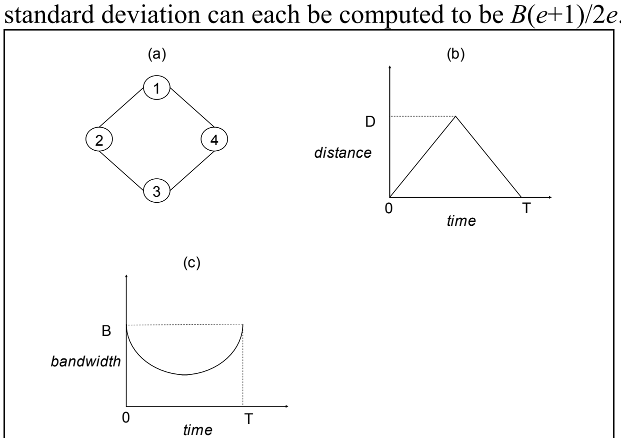

Figure 7

Although the examples provided above can be viewed as toy examples illustrating the analysis of finite MANETs, it should be apparent that they can be extended to analyze more complex motion vectors and a larger number of nodes.

It is possible to develop a software package that tracks the instantaneous velocity vectors of the nodes, and then builds a quasi-static model of the static network using the velocity vectors at any given instance. Such a system can then be used to answer questions regarding which node in the MANET is best connected, which is likely to lose connectivity in the near future, and which one needs to use more than one path to maintain a given bandwidth need.

VI. CONCLUSIONS

In this paper, we have presented a method to analyze wireless MANETs of finite size by converting them into congruent MANETs with a simpler type of motion vector. The method is applicable to isotropic properties – which are independent of the positions of individual nodes in the network. The method has been shown to be useful in the context of some example scenarios, and can be used to analyze the average values of some properties of finite sized MANETs.

In future work, we would like to develop a scheme to understand and analyze non-isotropic properties of the network, as well as develop the concept of analyzing graphs with time-varying edge properties that are not readily convertible to a static equivalent graph. We would like to combine our results with work on graph algorithms that handle node additions and deletions [11] to address a larger set of analysis problems related to dynamic mobile networks. We would also develop a software package that can analyze the properties of arbitrary finite MANETs.

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