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# IBM Research Report

## Inventory Management under Price Protection

**Karthik Sourirajan, Roman Kapuscinski*, Markus Ettl, Pu Huang**
IBM Research Division
Thomas J. Watson Research Center
P.O. Box 218
Yorktown Heights, NY 10598

*Ross School of Business
University of Michigan
Ann Arbor, MI 48109

# Inventory Management under Price Protection[1]

Karthik Sourirajan[a] • Roman Kapuscinski[b] • Markus Ettl[a] • Pu Huang[a]

[a] *IBM T.J. Watson Research Center, Yorktown Heights, New York 10598, USA*

[b] *Ross School of Business, University of Michigan, Ann Arbor, MI 48109, USA*

*ksourira@us.ibm.com* • *kapuscin@bus.umich.edu* • *msettl@us.ibm.com* • *puhuang@us.ibm.com*

## 1 Introduction

Price protection is a common business practice intended to counteract the effects of high technological obsolescence. Distributors perceive price protection as a fair and necessary mechanism through which many Original Equipment Manufacturers (OEMs) decrease the effects of brutal price erosion on operations of distributors. The following example illustrates how price protection works. Assume that a distributor, with no initial inventory, places an order for 100 units at \$800/unit. The order is received instantaneously. After a demand for 70 units is satisfied, the distributor is left with 30 units at the end of the period. At the beginning of the next period, the wholesale price drops to \$700/unit. The price protection credit given by OEM to the distributor is the product of the unsold inventory and the price decrease, 30*(\$800-\$700) = \$3,000.

Price protection contracts are designed to provide the distributor with an incentive to stock more, but many OEMs feel that the market-imposed length of price protection leads to excessive stocking at distributors and unfairly exposes the OEM to significant price protection expenses. While price protection contracts have been studied in literature in a single-period setting using parallels to buy-back contracts ([3], [4]), this paper is the first to model price protection in a more realistic multi-period setting. We are able to analyze temporal dependencies by reformulating the problem and expressing the multi-period price-protection credit as a part of the myopic problem.

In order to provide a benchmark and explain why price protection leads to negative externalities on the whole supply chain, we start with a centralized model followed by decentralized versions of the model, both with and without price protection. Given market-imposed length of price-protection periods, we evaluate the performance of the decentralized models under demand and cost/price uncertainty. Our myopic reformulation leads to intuitive interpretations. Based on structural properties, we demonstrate that a distributor managed inventory policy (DMI) could potentially lead to overstocking at distributors, causing significant price protection expenses for the manufacturer, while the use of a Vendor Managed Inventory policy (VMI) could lead to understocking at distributors. For each of the two settings, we then describe the effects of subsidies that help to reduce the externalities of price protection observed in the decentralized models and are acceptable from practitioners' point of view. Computational experiments in Section 3 suggest

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that VMI typically performs better than DMI. VMI policies adapt well to a wide range of realistic economic parameters that characterize today's high-technology market. Also, VMI is a more natural candidate in an environment where prices decrease rapidly (this is where price protection is observed) rather than in a stationary environment.

# 2 Problem Description

Consider a distributor (D) who buys a product from a manufacturer (M, OEM) over $T$ time periods (indexed by $t$) at a wholesale price ($w_t$) and sells it to the end-customer at a retail price ($p_t$). At the end of each period, the distributor places an order which is produced by the manufacturer and delivered to the distributor at the beginning of the next period. The production cost ($c_t$), wholesale price, and retail price decrease over time. The decrease is stochastic and becomes known at the beginning of each period. All decisions in a given period are made with a full knowledge of current-period prices and the distribution of changes in future periods. The distributor is charged the actual wholesale price at the time when the order is placed.

The production of goods is triggered by orders placed by the distributor. The manufacturer does not hold any inventory. The distributor is facing uncertain demand ($D_t$ with cdf $\Phi_t(.)$ and pdf $\phi_t(.)$ independent of cost and price) and attempts to satisfy demand from stock. Any excess inventory at the distributor is held to the next period at a unit cost $h_t$. In the case of product shortages, the required products are produced and expedited to the distributor. When goods are expedited, the distributor incurs two types of costs in addition to the wholesale price for purchasing the product, i.e., the cost of expediting orders ($b_t$), and a cost due to a loss of customer goodwill to the distributor ($g_t^D$). The manufacturer is also penalized for distributor's shortage and incurs a cost due to loss of customer goodwill ($g_t^M$) in addition to the regular cost of producing the product. Both manufacturer and distributor are risk neutral and both discount their cash flows at the same rate $\beta$. Let $D[t,k]$ be the convolution of demands $D_t$, ..., $D_k$, $k \geq t$, with cdf $\Phi_t^{k-t}(.)$. We use $Ez$ denote the expected value of $z$. The endogenous variables are the starting inventory ($x_t$), the quantity ordered in period $t$ ($a_t$), the quantity expedited in period $t$ ($e_t$), the inventory on hand after the order arrives but before expediting ($y_t = x_t + a_t$), and the cumulative quantity shipped until (and including) period $t$, $\bar{a}_t = \bar{a}_{t-1} + a_t + e_t$.

## 2.1 Centralized Channel

The centralized problem will serve as a benchmark for decentralized solutions. The cost minimization formulation for the centralized system is given by:[2]

$$V_t^{SC}(x_t) = \min_{y_t \geq x_t} [W(y_t) + \beta E_{D_t} V_{t+1}^{SC}((y_t - D_t)^+)]$$

[2]The term $c_t x_t$ is omitted in the initial period and, in all other periods, reassigned to the previous period. By explicitly expressing $c_t x_t$ as $c_t E(y_{t-1} - d_{t-1})^+$, the correction, which is usually a part of the myopic formulation, here is directly embedded in function $W$.

$$\text{where } W(y_t) \;=\; E\{(b_t+g_t)(D^t-y_t)^+ + (c_t-\beta c_{t+1}+h_t)(y_t-D_t)^+\}$$

and $V_0^{SC}(x_{T+1}) = 0$. Clearly, this is a generalization of the non-stationary inventory problem described in [2]. The added elements are the expediting costs and uncertain costs dynamics for the future periods. Given our assumptions about the exogenous variables, we have

**Theorem 1** *(a) The optimal policy is a non-stationary base stock policy. (b) Assuming that inventory can be salvaged next period at the cost $c_{t+1}$, myopic up-to level is $y_t^{C*} = \Phi_t^{-1}\{\frac{(b_t+g_t)}{(b_t+g_t)+(c_t-\beta Ec_{t+1})+h_t}\}$*

## 2.2 Distributor Managed Inventory (DMI)

We start with the case of no price protection. We show that the optimal policy is a non-stationary base-stock policy and, if $w_t - \beta Ew_{t+1} \geq c_t - \beta Ec_{t+1}$, the distributor's myopic optimal quantity is (weakly) smaller than the supply-chain myopic optimal quantity, $y_t^{D*} \leq y_t^{C*}$.

In the computer industry (the primary motivation for this problem) price protection has become a standard type of contract where distributors are eligible for *full price protection* for a *limited time* (say $L$ weeks). This price protection credit can be expressed as $(w_{t-1}-w_t)*\min\{x_t, \bar{a}_{t-1}-\bar{a}_{t-L-1}\}$.

**Property 1** *The price protected quantity, $\min\{x_t, \bar{a}_{t-1}-\bar{a}_{t-L-1}\}$, can be re-written as*

$$\sum_{k=1}^{L} \min\{(y_{t-k} - D[t-k, t-1])^+, a_{t-k}\}$$

*resulting in the following dynamic programming formulation:*

$$V_t^{D,PP}(x_t) = \min_{y_t \geq x_t} \; [W^{D,PP}(y_t) + \beta E_{D_t} V_{t+1}^{D}((y_t - D_t)^+)].$$

*where $W^{D,PP}(y_t) = E\{(b_t + g_t^D)(D^t - y_t)^+ + (w_t - \beta w_{t+1} + h_t)(y_t - D_t)^+\} - \beta(w_t - Ew_{t+1})(y_t - d_t)^+ + \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1})(y_t - D[t, t+L])^+$.*

This formulation allows us to express distributor's policy in terms of target levels. In the next theorem we justify that the optimal policy has the desired structure.

**Theorem 2** *(a) The optimal policy is base-stock level. (b) If $w_t - w_{t+1} \geq w_{t+1} - w_{t+2}$, for all $t$, the myopic base-stock level, $y_t^{D,PP*}(L)$ is a non-decreasing function of price-protection period $L$. (c) If*

$$\frac{w_t(1-\beta)+h_t}{(c_t-\beta Ec_{t+1})+h_t} \;<\; \frac{b_t+g_t^D}{b_t+g_t} \tag{1}$$

*then there exists a price protection length, $L$, such that $y_t^{C*} \leq y_t^{D,PP*}(L)$.*

If we consider price protection period $L$ as a continuous variable, then part (c) of Theorem 2 states that if (1) is satisfied, there exists an $L$ that coordinates the chain. Since in practice it is difficult to adjust the length of the price protection period, in the following sections we concentrate on the mechanisms practiced or considered as an addition or refinement to price-protection contracts.

### 2.3 Vendor (Manufacturer) Managed Inventory

Assume that the manufacturer controls the up-to levels at the distributor, holds no inventory at its site, and acts to minimize its individual cost. Price protection policy in place between the manufacturer and the distributor is the same as in Section 2.2. We then have,

$$V_t^{M,PP}(x_t) = \max_{y_t \geq x_t}[W^{M,PP}(y_t) + \beta E_{D_t} V_{t+1}^{M,PP}((y_t - d_t)^+)]$$

where one-period profit function is given by:

$$\begin{aligned} W_t^{M,PP}(y_t) &= g_t^M(D^t - y_t)^+ + (c_t - w_t)(y_t - D_t)^+ \\ &\quad - \sum_{k=1}^{L} \beta^k E(w_{t+k} - w_{t+k-1})(y_t - D[t, t+k])^+ \end{aligned}$$

It is easy to show that the manufacturer has an incentive to sell huge (infinite) volume of products to the distributor *in the absence of price protection* since there is no cost to the manufacturer for having excess inventory at the distributor, but there are gains from selling earlier at a higher price. We label such behavior as stuffing (see definition below). The problems related to control of inventory by the vendor are well recognized in practice and various firms impose physical or financial constraints and incentives to deal with them, see [1] for a discussion.

**Definition 1** *The manufacturer has an incentive to* **stuff the channel***, if there exists period $t$, such that* $\lim_{y_t \to \infty} V_t^{M,PP}(x_t, y_t) = -\infty$.

It is easy to obtain necessary and sufficient conditions for stuffing the channel.

**Lemma 1** *The manufacturer will stuff the channel if and only if there exists $t$ such that* $c_t < (1 - \beta) * (w_t + \beta E w_{t+1} \ldots + \beta^{L-1} E w_{t+L-1}) + \beta^L E w_{t+L}$.

By re-arranging the terms and differentiating the one-period cost function wrt $y_t$, we get

$$\begin{aligned} W_t^{\prime M,PP}(y_t) &= -g_t^M + [g_t^M + (c_t - \beta E c_{t+1}) - w_t(1 - \beta)] * \Phi_t(y_t) \\ &\quad -\beta^{L+1}(E w_{t+L} - E w_{t+L+1}) * \Phi_t^L(y_t) \end{aligned} \tag{2}$$

**Lemma 2** *If $\phi_t(z)$, the pdf of demand $D_t$, is IFR for all $t$, then $W_t^{\prime M,PP}(y_t)$ is unimodal.*

In many VMI settings an upper bound is used to limit the manufacturer's inventory decisions and disallow degenerate behavior [1]. If the manufacturer has an incentive to stuff the channel, we propose the use of upper bounds on the stocking levels. The upper bound $y^U B_t$ (Figure 1)

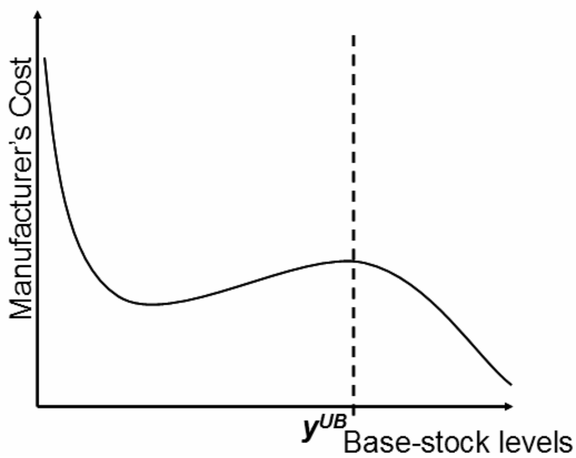


Figure 1: Example of manufacturer's cost with incentive to stuff the channel

prevents extreme and practically unacceptable ordering that could result in nonintuitive cost gains. The bounds can be easily evaluated by looking at the zero points of $W_t^{'M,PP}(y_t)$. We remove the incentive, but not penalize for being above the specific bound (by keeping cost constant). We modify the cost so that the manufacturer loses incentive to ship additional units.

**Theorem 3** *Under price protection, if there exists an $L$ such that $y_t^{D,PP*} > y_t^{C*}$, then we have $y_t^{C*} > y_t^{M,PP*}$.*

The results above show that price protection, implemented with an appropriate length of the price protection period, can be an effective form of control of the orders placed in either the DMI or VMI setting. While this is an interesting theoretical result, the length of the price protection period is often dictated by the marketplace and therefore difficult (if not impossible) to change. Our analyses with industry data show that current price protection policies result in inventory levels that are clearly in excess of the supply-chain optimal levels under DMI. We therefore propose a modification of existing contracts with subsidies of the expediting cost.

## 2.4 Correcting the Externalities of Price Protection

Due to typical over-stocking at distributor's location under DMI and under-stocking under VMI, a natural way to correct them is to either decrease the distributor's stockout penalties or increase its inventory holding cost. The first can be achieved by subsidizing the expediting cost paid by the distributor. The decision about the size of the subsidies could, in ideal circumstances, be made by a central decision maker or negotiated based on the maximum "size of the pie" to be divided. More realistically, one of the parties makes this decision.

Under DMI, the distributor makes the stocking decision, while we assume that the manufacturer decides on subsidies. Consequently, stocking levels will be reduced. Similarly, the distributor decides subsidies on expediting costs under VMI and the stocking levels increase. While such considerations makes the theoretical models intractable, they can be easily captured using computational experiments that we show in the next section.

# 3 Computational Experiments

Our computational study is based on settings of a big computer manufacturing company, that explores the possibility of moving to a VMI system. Ownership of product is transferred to the distributor upon order delivery. We evaluate the two policies DMI and VMI, both with and without subsidies on expediting costs. The conceptual set up is similar to that in real world, but the numerical values of some parameters have been modified to preserve confidentiality.

In our experiments, one period corresponds to 3 weeks, which is the replenishment lead time. We use a price protection of 2 periods to reflect the 6 week price protection period. There is a single product for which the demand follows a triangular distribution. The inventory carrying cost per unit per period is equivalent to the average observed interest rate of 12% per year on $c_t$. The critical ratio was used to set the sum of goodwill costs and expediting cost such that we get a 95% service level for the centralized model. The expediting cost was set to be 50% of the distributor's gross profit in period 1 to reflect the approximation of the actual ratio. The distributor stocks products from multiple manufacturers and thus, the cost due to loss of goodwill in case of product shortage is set to not exceed that of the manufacturer. The goodwill costs were then obtained such that the ratio ($\frac{g_M}{g_D}$) was approximately equal to 2. We consider price drops between 1%-5% per 3-week period, which translates to 0.3%-1.7% per week (such a range covers the realistic rates of price changes). Production costs and wholesale prices are assumed to drop at the same rate, maintaining the 25% gross profit margin for the manufacturer. Both the manufacturer and the distributor have complete information regarding the price drops.

We experiment with five different policies: *Centralized System*, VMI without subsidy (*VMI w/o Subsidy*), DMI without subsidy (*DMI w/o Subsidy*), VMI with subsidy (*VMI w/ Subsidy* - under this policy, distributor chooses the subsidy given by the manufacturer on expediting costs so that his total cost is optimized) and VMI without subsidy (*VMI w/o Subsidy* - under this policy, manufacturer chooses the subsidy on expediting costs so that his total cost is optimized). To evaluate all of the policies, we run a 10-period stochastic dynamic program. The total cost is the sum of costs for the manufacturer and the distributor.

With steeper price drops, we expect the order up-to levels to decrease as the cost risk due to early procurement/production increases. However, the presence of price protection makes the DMI system less sensitive. Overstocking continues under DMI at minimally changed levels and the gap with the centralized system increases with faster price erosion. The VMI system, on the other

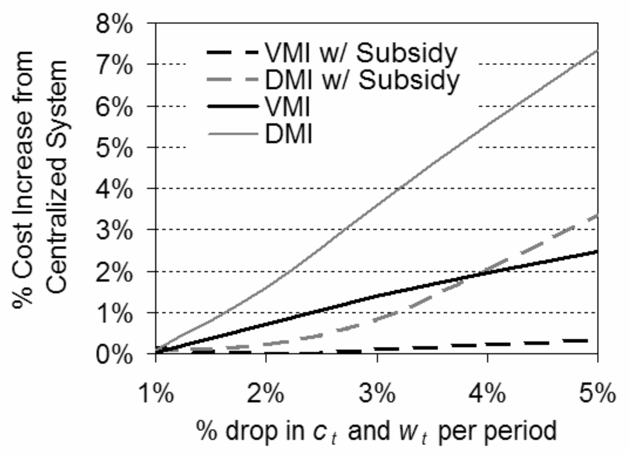


Figure 2: Performance of VMI and DMI under different rates of price drops

hand is more sensitive to price changes. Its cost risk, due to steeper price drops, comes primarily from unnecessarily high production costs that could be avoided. Combined with lower margins (compared to the centralized system), this leads to understocking under VMI. The total expected costs under VMI worsen with steeper price drops. However, due to the consistency of forces between VMI and the supply chain, we expect VMI to perform better than DMI. Indeed, we find that VMI performs significantly better than DMI (lower expected total cost) with the difference increasing with higher rate of price drops. By offering to the distributor a subsidy on expediting costs, the penalty on shortages is increasing for the manufacturer and decreasing for the distributor. This counteracts the overstocking under DMI and understocking under VMI. Figure 2 shows that *VMI w/ Subsidy* performs very close to the *Centralized System*. A similar logic applies for the DMI system with subsidies. While DMI is also improved when subsidies are offered (Figure 2), the costs are higher compared to *VMI w/ Subsidy*.

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