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The Green Zone Protection Problem

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***Abstract*—The "green zone" in combat operations refers to an area that is secured against intrusion and attacks from insurgents. During any type of military operation, the number of sensor assets that are available for detecting intrusions are limited. Thus, the size of a green zone is limited by the number of available sensors, and how those sensors are deployed. Depending on the assumptions made in modeling the coverage properties and the terrain of area being covered, the size and shape of the green zone can vary widely. The green zone protection problem is the task of determining the largest green zone area that can be defined given a limited number of sensors. The green zone protection problem is related to the problem of determining sensor coverage. In this paper, we look at the various variations of the green zone protection problem, with a range of difficulty in their solution.**

I. Introduction

The green zone in Iraq is a secured area that was the center of the coalition provisional authority and currently remains the center of international presence in Iraq. If we make the reasonable hypothesis that a similar secured area will be needed in future coalition operations in the context of most asymmetric operations. An intuitive definition of the green zone is an area needing protection that needs to be defended so that it is highly unlikely that an insurgent will be able to enter without detection. In other words, it is an area that is relatively well-protected in the context of any operation. It is the intention of this paper to generalize the concept of the green zone, and develop techniques for estimating the size of the green zone based on the number of available assets.

We define the green zone as an area which is completely secured by a combination of Intelligence, Surveillance and Reconnaissance (ISR) assets as well as defensive firepower. In the context of an asymmetric operation, we assume that the defensive firepower is sufficient to neutralize any intruder who is detected as having penetrated the green zone. Despite the superiority in firepower in asymmetric operations, any coalition force only has access to a finite number of ISR assets. Thus, it needs to determine the best way in which those assets can be used. It would be useful in asymmetric operations for the coalition commander to determine the maximum size or value of a green zone permitted by the set of available sensors that will be placed around the periphery to protect the zone from intrusion attacks. The green zone protection problem is the task of determining the optimum size of the green zone given a set of sensors. Depending on the assumptions made regarding sensor models, sensor mobility, terrain models, the impact of terrain on sensor performance, and the definition of value, different variations of the green zone definition problem can be formulated. These variations range from the simple ones to complex formulations whose solution would be NP-complete.

Securing the green zone requires the task of detecting any potential incursions into the zone, as well as taking defensive actions against any such incursion. For the purpose of this paper, we assume that detection of incursion is sufficient to take defensive action and to protect against the incursion. This assumption is likely to be true in cases of asymmetric warfare. However, there may be a lag between the detection of an incursion and the initiation of the defensive action. If we assume that the lag is zero, then the task of maximizing the green zone reduces to that of determine the maximum (or most valuable) area that can be covered by the sensors available.

Although related, the green zone definition problem is distinct from the sensor coverage planning problem, which has been extensively covered in the current research literature. The sensor coverage planning problem can be characterized as follows: Given an area to be monitored and a set of sensors with various capabilities, determine the best locations for each sensor so that the area can be effectively monitored, usually defined as completely. In contrast, the green zone definition problem tries to provide a secured area given a limited number of sensors by creating a boundary around the zone. By covering the boundary, the area will be said to be covered.

The sensor coverage planning problem has been studied in different formulations in a variety of ways. The Art Gallery Problem addresses the issue of determining the number of observers necessary to cover a space, like an art gallery with many rooms such that every point in every room is seen by at least one observer. It has found several applications in many domains such as for optimal antenna placement problems in wireless communication. It can be solved optimally in 2D and is known to be NP-hard in the 3D case [1]. Marengoni et al. [2] propose heuristics for solving the 3D case using Delaunay triangulations.

The treatment of coverage problem has been well studied and varies from theoretical analysis [3] to pragmatic usage models

[5]. While there are several variations of coverage problem examining aspects such as connectivity maintenance, mobility management, and query optimizations can be found in various papers ([6],[7],[8],[9],[11]), there has been little work on incorporating terrain considerations. Wilson, Marlin and Mackay [4] look at the real world problems of the difficulties of incorporating terrain and atmospheric environmental factors in acoustic and seismic sensors. The work by Dhillon and Chakrabarty [10] attempted to model some of the location dependence by approximating the sensor field to a Manhattan grid. Dhillon and Chakrabarty approximated the sensor field by a grid and used an asymmetric probability matrix to model the terrain, and proposed two heuristics to address the problem – both using a greedy approach with one maximizing average coverage with each sensor and the other placing sensors at the point on the grid with least amount of coverage. Brown [12] et. al. have tried to develop the impact of terrain on the coverage area of a sensor and simulated its impact on sensor coverage. Kumar et al. [13] also examine the problem of determining a belt of coverage using sensors to protect a border (e.g., U.S/Mexico border) or an enclosed area using random placement of sensors.

Since the sensor coverage planning problem provides the inverse solution to that of green zone, i.e., determining the number of sensors required to cover an area, iterative usage of the sensor coverage planning problem can be used to solve the green zone definition problem. However, such solutions may not always be the most efficient ones to use.

Defining the green zone as an area in which any intrusion will be detected will be sufficient to protect against physical incursions by an insurgent in asymmetric warfare. However, with modern day weapons, it is possible to launch grenade or mortal attacks from a remote location into the green zone. For full protection, one would need to include ISR assets which can detect such intrusions from a remote area as well, e.g. an acoustic sensor which can detect remote launch of a mortar and take steps to neutralize that attack. The approaches we have discussed in this paper can be extended to cover the case of remote sensors, but we are restricting the scope of this paper to a discussion of ISR assets that detect intruders in a physical proximity area.

In the following sections, we provide several variations of the green zone protection problem and approaches to solve them. This is followed by a section discussing the adaptation of the sensor coverage planning algorithms to address the green zone definition problem. Finally, we present our conclusions and areas for future work.

II. UNIFORM TERRAIN WITH SIMILAR SENSORS

The first variation of the green zone protection problem provides a simple formulation that is easy to solve. It assumes that covering the perimeter of the green zone is sufficient to prevent intrusions, and that any preventive actions are immediate. Furthermore for this first formulation the terrain being modeled is uniform with the implication that the coverage area of each sensor is independent of its location. The sensors are all identical. Suppose there are K sensors, and each sensor can monitor a circular area of radius s. The goal is to find the largest area in a planar surface which can be covered completely by K such sensors. The boundary must enclose an origin point (0,0) defined as the center of the green zone. We assume that the origin point contains the processing center for sensor information, and that all sensors are able to communicate with the origin point. In practice, this means that the maximum distance of each sensor from the origin is bounded by the range of its communication.

This definition problem can be seen as the task of determining the closed shape with the largest area, where the perimeter of the shape is covered completely by all the sensors together. Therefore, the green zone definition problem in this case devolves to that of finding the shape with largest area given a fixed perimeter. Such a shape in a plane is a circle.

If the zone to be covered is much larger than s, than a good approximation to the maximum size of the perimeter would be $2Ks$. (If the zone to be covered were not significantly larger, the mathematics is still easy, but the formulas look more complicated as the arc of the coverage zone and the boundary conditions would need to be included in the calculations.)

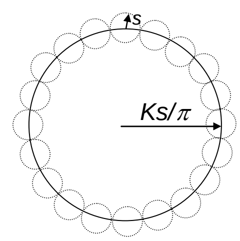

Fig. 1. The green zone for uniform terrain and uniform sensors.

Looking at Figure 1, we can conclude that the green zone in a uniform terrain given K sensors covering a radius of s each is a circle of radius Ks/π, augmented by the semi-circles of additional coverage beyond this range. The total area of the green zone is $K^2s^2/\pi + K\pi s^2/2$.

However the boundary does have a point of vulnerability. If an intruder were to traverse the boundary at any of the K points where two sensors meet and travel along a ray, there is only one point of sensing that can detect this intrusion and furthermore this point is on the extreme range of the two sensors. Thus, the solution above is valid only if the lag between detection and defensive action is zero.

There are advantages to placing sensors closer than just meeting at a point on the circumference. One advantage is that this creates an annulus of coverage rather than a minimum coverage of a single point. This can be useful for the problem mentioned above or secondarily by providing redundancy it allows some degree of non-exact placement of the sensors. The annulus provides for a finite lag to exist between detection and defensive action, such lag being the time it takes for an intruder to cross the span of the annulus.

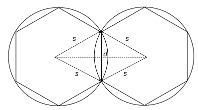

Fig. 2. Overlap among sensor coverage areas to create an annulus.

One possible way to create such an overlap is to inscribe a regular hexagon within the circle. Adjacent sensors overlap so that the adjacent inscribed hexagons are abutting as shown in Figure 2. In order to create an annulus of depth d, two adjacent sensors need to be moved closer together, creating an isosceles triangle with base length of d and equal sides being of length s. The separation between the centers of two adjacent sensors in this case equals $2\sqrt{s^2-(\frac{d}{2})^2}$. Thus, the perimeter of the circular green zone is $2K\sqrt{s^2-(\frac{d}{2})^2}$ and the maximum radius of the circular green zone is going to be:

$$\frac{K}{\pi}\sqrt{s^2-(\frac{d}{2})^2}\ .$$

In both of the cases above, it might be acceptable to have a small probability ε of undetected intrusion. If we assume that the probability of intrusion is same as the fraction of the perimeter that is not covered, then the perimeter of the green zone without any overlap would be

$$\frac{2K\sqrt{s^2-(\frac{d}{2})^2}}{(1-\varepsilon)}$$

and the corresponding green zone will be a radius of size

$$\frac{K}{(1-\varepsilon)\pi}\sqrt{s^2-(\frac{d}{2})^2}$$

For the case of no overlap, the corresponding perimeter is $2Ks/(1-\varepsilon)$ *And* the corresponding radius would be $Ks/\pi\,(1-\varepsilon)$.

III. NON-UNIFORM TERRAIN WITH SIMILAR SENSORS

In the next variation of the green zone protection problem, certain parts a region are more likely to encounter human intruders than others because of the accessibility of the terrain. Even when the sensor performance, i.e., detection range, it not affected by the terrain, the best placement of the sensors must take into account the likely routes where a security compromise can occur. To this end, the security value of any point on the ground will be proportional to its accessibility. Furthermore, one should expect the security value to decrease as the distance from the base increases.

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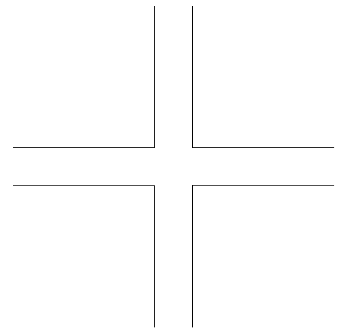

Fig 3. Terrain with different monitoring values

An example of such an environment is shown in Figure 3. The terrain consists of four roads that intersect at the origin point. It is more important to monitor the access points of the road rather than monitoring the access points that are not along the road. If there are only four sensors available, then they ought to be used for monitoring the access points rather than monitoring other locations. Assuming that the sensor coverage area can span the width of the entire road, the sensor ought to be located at the maximum range of their communication limit to maximize the green zone.

In a generalized version of the problem, we can associate a value $V(r,\theta)$ with each point (r,θ) which is the value derived from having the specific point protected. We make the pragmatic assumption that the value function V is continuous and does not increase as one moves away from the origin, i.e. the value of $\partial V/\partial r$ is non-positive. The problem of defining the green zone now becomes that of finding the area over which the surface integral of the value function is maximized with the constraint the that the total perimeter of the area is bounded. Given the constraints of K sensors, each capable of monitoring a circular area of radius s, and the central processing lying at the origin, the problem is to find a closed line $r = l\,(\theta)$ such that $\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=l(\theta)} V(r,\theta)\,dr\,d\theta$ is maximized subject to the constraint that $\int_{\theta=0}^{\theta=2\pi} r\,d\theta \le 2Ks$. In words we are looking to maximize $V(r,\theta)$ that are on closed contours less than or equal to $2Ks$.

Let us start with a circular area of perimeter $2Ks$ centered on the origin. At different points along the circle, there will be different values of V. Since the value of derivative along the radial is non-positive, one can cover more area by pushing the circle closer in towards the origin at a point where the value of V is smaller and pushing it further out at another point where the value of V is the same or larger. By iteratively repeating this process, we will converge to a shape where such an adjustment will not cause any change in the total value function covered in the area. At that point, the value of V should be equal at all the points along the curve. Thus, we can conclude that the shape which maximizes the green area will be the one which has a constant V along its perimeter.

It follows then that for any value function $V(r,\theta)$, the green zone will be defined by the equation $V(r,\theta) = c$, where c is a constant selected such that total perimeter of the shape will be less than or equal to $2Ks$.

As an example, let us consider the value function defined by

$$V(r,\theta) = \frac{1}{r(1+\varepsilon\cos(\theta))}$$

The green zone will be defined by an expression of the nature:

$$c = \frac{1}{r(1+\varepsilon\cos(\theta))}$$

If we now express the constant c as product of two other constants, a and $(1-\varepsilon^2)$, we get the expression that

$$r = \frac{a(1-\varepsilon^2)}{(1+\varepsilon\cos(\theta))},$$

which is the equation of an ellipse in polar coordinates. Thus, with the above value expression, the green zone will be an ellipse with one focus at the origin and the eccentricity as defined by the parameter ε.

If the value function V is can be expressed as the product of two independent functions of r and θ, then the determination of the green zone can be made readily. Let us assume that

$$V(r,\theta) = f(r).g(\theta),$$

then the shape of the green-zone is defined by the relationship

$$r(\theta) = f^{-1}(C/g(\theta)),$$

where C is a constant selected so that the perimeter of the shape is less than or equal to $2Ks$.

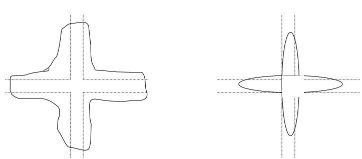

Fig 4. Solution to the Non-Uniform Terrain Green Zone

In the specific case shown in Figure 3, the shape of the green zone will be as shown in Figure 4(a), where the amount of curvature in areas that do not lie along the road will be given by the difference in value assigned to monitoring the sites along the road as compared to the value assigned to sites along the road. If we assume that the $f(r)$ is a constant, and $g(\theta)$ is given by $(1 + \cos(4\theta))/2$, giving a maximum value along the direction shown, then the shape of the green zone will be as shown in Figure 4(b).

IV. MOBILE SENSORS IN CIRCULAR MOTION

As an extension of the case of uniform terrain with similar sensors described in Section I, consider the case of mobile sensors. In the case of mobile sensors, each sensor has the ability to move about its central point to a limited extent. Due to the ability of the sensors to move about, the sensors can be spread out to cover a larger distance and define a bigger green zone.

Consider the case where all the sensors are arranged in a circular manner as discussed in Section II, and are moving around the circle at a uniform rate. By exploiting the fact that each point will be covered by each sensor at some point in time as they rotate in the circle, the size of the green zone can be extended.

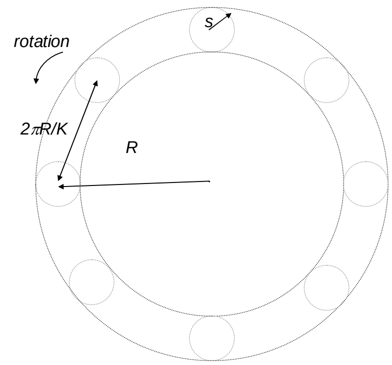

Fig. 5. Sensors in uniform rotational motion.

Suppose that the sensors are moving with a velocity of v_s along a circle of radius R. Each point is covered directly by a sensor when the sensor is within a distance of s from that point, which is $s/\pi R$ fraction of the total time in one period of rotation $T=2\pi R/v_s$. With K sensors, each point is covered for $Ks/\pi R$ fraction of time per period of rotation. If the sensors are equally spaced along the circumference of their circular path, then the maximum contiguous time period when a point is not covered by any sensors is given by $(2\pi R-Ks)/v_s$. Since any intruder has to cover a distance of 2s in that block of contiguous time to avoid detection, the intruder will need to have a velocity toward the center of at least $v_s Ks/(\pi R-Ks)$ if the intruder's angular velocity is zero.

If follows from the previous analysis that if we wanted to maintain a green zone with unit probability of detection against intruders with the maximum velocity of v_i, and we have K sensors with coverage radius of s moving along the perimeter of a circular green zone with a *sensor velocity of* v_s, then the maximum size of the green zone will be given by a circular area of radius $Ks/\pi\ (1+v_s/v_i)$.

If the velocity of the sensors is faster than the maximum possible velocity of the intruders, then the size of the green zone can be increased significantly. If the intruders are relatively slow in their movement, then the radius can be made significantly larger. On the other hand, if the intruders can move faster than the sensors, then the size of the green zone reduces to a circle with static sensors. Figure 6 shows the relative increase in the size of the green zone that can be attained as a function of the relative speeds between the intruders and the sensor.

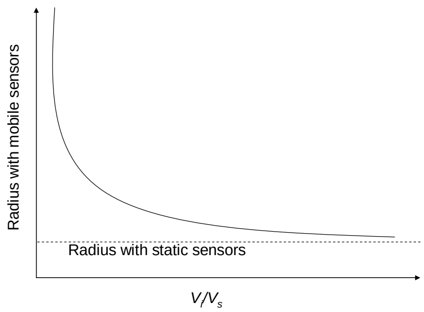

Fig. 6. Green Zone Radius

The analysis is valid for motion of sensors which are different than that of rotation around the perimeter of the green zone. As an example, the sensors may move in an osciallatory motion around the perimeter. Each sensor maintains their distance among each other, but reverse direction after traveling a fixed distance in either side. Almost any kind of motion in which the sensors move around the perimeter with a fixed distance among their positions would result in a similar characteristic for the green zone.

V. TERRAIN-SENSITIVE SENSORS

The existence of a terrain often has an impact on the ability of a sensor to monitor information. Terrain features such as mountains, buildings, and tunnels can cause obstructions and impair the ability of a sensor to cover all areas within its range. Let us consider a scenario where the position of a sensor at a specific location causes a difference in the ability of the sensor to monitor the different areas associated with the terrain.

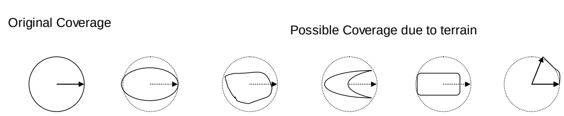

Fig. 7. Distortion of sensor range due to terrain.

Normally, if there was no distortion, a sensor coverage area would be modeled by means of a circular area of fixed radius. However, the presence of a terrain can cause distortion causing the effective sensor region to be relatively restricted and limited to a smaller area. The green zone definition problem would be to determine the maximum possible size of the green zone given a distortion function for the placement of sensors.

We do not know of any analytic method to solve the green zone problem for terrain sensitive sensors, but some heuristics can be readily defined. Since the number of sensors is bounded, one can start by placing them in a circle of radius as given by the undistorted case described in Section I. After distortion, the sensor coverage area along the circle will not be covered, and one can move the sensors inward till the distorted sensors become a connected shape.

VI. APPLYING COVERAGE TO SOLVE GREEN ZONE PROBLEM

While our formulation of the green zone problem in this paper looks at some very basic formulations, the problem of sensor coverage has been considered with many other variations ([1]-[13]) making different assumptions regarding placement, communication constraints, different modalities, competing mission requirements and varying terrain characteristics. If we can convert a solution for the sensor coverage into a solution for green zone protection, then the green zone problem can be solved for many different variations with an equally rich set of assumptions. Such an approach will provide a ready solution for those same set of assumptions.

An approach to numerically solve the green zone problem is via the use of coverage algorithms. The coverage algorithms can be described as following: Given a specific area A that needs to be covered completely but the sensors, find the number of sensors that can provide complete coverage of the area by manipulating the contour shape and making use of the bisection algorithm. Start initially with two circular areas around the origin – the first being a minimum area which can definitely be covered by *K* sensors with each having a sensor coverage area of s, and the second being a maximum area that is not likely to be covered by the K sensors. A good starting point for the minimum area will be a circle of radius s around the origin. A good starting point for the minimum area will be a circle of radius s. A good starting point for the maximum area will be a circle of radius $2Ks/\pi$, i.e. twice the radius of the simple case discussed in Section I.

At each stage of the algorithm, use the coverage problem to find the number of sensors that can be used to cover the minimum area and the maximum area. Select an area defined as halfway between the maximum and the minimum area along all radial directions as measured from the origin point. If the number of sensors required to cover this new area is less than *K*, then this becomes the minimal area for the next iteration. If the number of sensors required to cover the new area is more than *K*, then this becomes the maximal area for the next iteration.

The iteration is repeated until an area that can be covered by *K* sensors is found, or the maximal area becomes an area that is covered by less than K sensors.

This approach works well for uniform terrains which will result in a circular area solution.

VII. CONCLUSION

In this paper, we have presented several variations of the green zone protection problem, looking at approaches to determine the maximum size of a protected region with a limited number of proximity sensors. We have provided analytically tractable solutions for some simple cases, and approaches to obtain the solutions for more complex cases using simulations and numerical analysis.

Our current formulation makes the assumption that detection implies protection and there is no lag between detection and a defensive action. In future work, we intend to incorporate the concept of a finite lag between detection and defensive action and study the impact of such lag on the size of the green zone. Another future extension that we want to investigate is the relax the assumption regarding proximity sensors, and solve the green zone protection problem to protect against remote threats such as mortar attacks using non-proximity sensors.

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