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Decentralized Inventory Management Under Price Protection in the High Technology Industry

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Abstract

In high-technology industries, where technological obsolescence is high, price protection has become a standard element of contracts between manufacturers and distributors. Price protection was designed to counteract the high technological obsolescence and provide distributors with incentives to stock sufficient inventory. However, empirical evidence suggests that price protection often leads to over-stocking at the distributors. Motivated by a need to decrease inventory held at the distributors of a large computer manufacturer, we model and analyze price-protection contracts as implemented in practice: full protection for a limited time, L. Formulating such price-protection contracts requires tracking the historical orders of distributors over the period of time that goods are price protected. The critical element of our analysis is a myopic reformulation that makes it easier to analyze the problem and interpret the results, while preserving the dependency of the price protection cost on L. We characterize the behavior of the optimal inventory targets under a distributor managed inventory (DMI) policy as a function of the economic parameters and the length of the price protection period. We also identify conditions that lead to over-stocking under the current system (DMI). Alternative policies that decrease incentive misalignment are then considered, including a vendor managed inventory policy (VMI) and risk sharing. We show that the incentives for "channel stuffing" that usually accompany VMI are decreased or eliminated in a price-protection environment. In a numerical study, we observe that VMI provides superior performance compared to DMI in conditions that routinely characterize the high-technology industry, i.e., high technological obsolescence, higher impact of product shortages on the manufacturer than the distributor, low inventory carrying costs (other than obsolescence costs), and low to moderate profit margins. VMI also performs well when profit margins increase over time, which often characterizes new product introductions. While risk sharing may be difficult to implement, combining VMI with risk sharing leads to significant improvements.

1 Introduction

Price protection is a common business practice, especially in the high-technology industries. It is intended to counteract double marginalization, which is particularly significant in environments with high technological obsolescence. Distributors who purchase goods from Original Equipment Manufacturers (OEMs) perceive price protection as a fair and necessary mechanism through which manufacturers decrease the effects of brutal price erosion on the operations of distributors.

To illustrate how a price-protection contract works, consider a simple example. A distributor, with no initial inventory, places an order for 200 units at $800 per unit. The order is received instantaneously and after a demand for 70 units is satisfied, the distributor is left with 130 units at the end of the period. Soon after (at the beginning of the next period) the wholesale price drops to $700 per unit. The price protection credit given by the manufacturer to the distributor is the product of the unsold inventory and the price decrease, 130*($800-$700) = $13,000. Price protection expenses have become quite significant for OEMs and there is a lot of focus in the industry on reducing these expenses. Unlike previous literature that considers simplified forms of price-protection contracts, this paper analyzes the dominating form of price-protection contracts as they are exercised in the industry, and identifies ways to reduce the total supply chain cost.

During the last 15 years, the standards of price-protection contracts went through a number of changes and eventually stabilized at the end of 1990s (Zarley, 1994a, 1994b, 1997a, 1997b; Pereira 1999; Moltzen and Campbell, 2000). The early practice of price protection (mid 1980s-mid 1990s) was an unlimited price protection policy where distributors were reimbursed for wholesale price reductions on all unsold inventory. Later, this was followed by attempts to shorten the price-protection period combined with additional incentives for retailers. In 1994, Apple and Hewlett-Packard (HP) attempted to change the terms by limiting the length of time for which price protection would be offered. However, they quickly reversed their decisions due to strong opposition from their resellers (Zarley, 1994a, 1994b). Later, in 1997, IBM decreased the price protection period to 30 days for their second tier resellers (Zarley, 1997a) and combined this new rule with an incentive of a wholesale price discount of 2.5% for those resellers who accept 15 days of price protection (Zarley, 1997b). A similar modification (with deeper discounts) was attempted at HP for a small subset of products (Pereira, 1998). Later, both policies were rescinded. In June 1999, HP changed the price protection terms from unlimited time to 60 days (Pereira, 1999) and, in April 2000, IBM started offering 45 days of price protection without any extra discount (Moltzen and Campbell, 2000). These actions at the end of the 1990s marked the eventual stabilization of the price protection terms.

In the computer industry, which is the primary motivation for this paper, price protection has become a standard element of supply contracts where distributors are eligible for *full price protection* for a *limited time* (say L weeks). That is, the manufacturer credits the distributor for any price reductions on unsold goods purchased up to L weeks before the price change. It is easy to see that having a contract with $L = \infty$ (i.e., full price protection for unlimited time) is the same as a buy-back contract. However, when L is finite, the contract differs significantly from buy-back.[1]

Clearly, by offering price protection, a significant portion of risk originally faced by the distributor due to decreasing costs and prices is now absorbed by the manufacturer. While the contract is designed to provide the distributor with an incentive to stock more inventory, in practice it often does so in an excessive manner. Many manufacturers, especially in the high-technology industry, feel that offering a price-protection contract under a traditional Distributor Managed Inventory (DMI) policy leads to excessive stocking levels at distributors and unfairly exposes manufacturers to significant price protection expenses. Consider the same example as earlier, but assume that the drop of \$100 in the unit wholesale price paid by the distributor is driven by an equivalent drop of \$100 in the manufacturing cost (from \$600 to \$500) incurred by the manufacturer. Due to price protection credits, the effective wholesale price is only \$700 (and the profit margin is only \$100) on the 130 units unsold at the end of the period, but the manufacturer could have saved \$100 per unit in manufacturing costs had some of the 130 units been manufactured later. Excessive inventories in the presence of high technological obsolescence are clearly very costly and it is well-documented that many business failures in the computer industry are due to obsolete inventory (Simchi-Levi et al., 2000; Sengupta, 2004). In this paper, we show that price protection may indeed induce excessive ordering by distributors. We then consider mechanisms that decrease the total supply chain cost based on changing the ordering rights to a Vendor Managed Inventory (VMI) policy and also by considering risk sharing. We finally present a numerical study that is designed to mimic the nature of the economic forces in the high-technology industry under price-protection contracts.

Contributions of the Paper

The model described in this paper is motivated by a large computer manufacturer who is considering the launch of a VMI system in order to improve its distribution channel operations and reduce price protection expenses. We start with a centralized model in order to provide a benchmark. Then, we follow with decentralized versions of the model, with and without price protection. The main contributions of the paper are as follows:

- We present a realistic model of price protection contracts, full price protection for limited time, that is commonly used in the high-technology industry. Capturing the dynamic nature of price protection in a decentralized system requires tracking the vector of historical orders due to the dependency of the price-protection cost on the length of price protection, L. This leads to an explosion of the state space of the dynamic programming formulation. This is

[1] This is because the price protected quantity in any period would be a function of the ordering decisions in the previous periods rather than a function of only the inventory on-hand (see Section 4.1).

possibly one of the reasons why previous literature never analyzed the true price protection problem but instead considered simplified buy-back type models.[2] The critical element of our analysis is a reformulation that incorporates the dependency of the price protection cost on L into the one-period cost. Myopic analysis of the reformulated problem allows us to develop transparent analyses and intuitive interpretations.

- For the decentralized model, we characterize the optimal inventory policy under DMI. We then develop conditions under which DMI leads to over-stocking at distributors, causing significant price protection expenses for the manufacturer and negative externalities for the whole supply chain.
- Under DMI, we show that an appropriately chosen length of price protection indeed coordinates the channel in many cases. In practice, however, any adjustments (at least, any decreases) to the length of the price protection period are not easily implementable, because the expectations related to the length of the price protection period are imposed by the market and supply contracts are difficult to change. Furthermore, the length of price protection that coordinates the channel would differ by product and by distributor which is unacceptable from a practical point of view.
- To correct the externalities of price protection, the first alternative that we consider is the use of a VMI policy. While the popularity of VMI in practice has significantly increased over the last two decades, companies realize that use of VMI requires fine-tuned implementations that impose additional constraints on supply chain operations such as maximum inventory levels, see Fry et al. (2001). We show that a price-protection environment is much more suitable for VMI contracts, because the need for additional constraints such as maximum inventory levels is decreased or eliminated. Also, we highlight that the Sarbanes-Oxley Act enacted in 2002 makes VMI easier to implement in many environments.
- The second alternative we analyze is risk sharing. While we recognize that risk sharing arrangements are difficult to implement in practice, we evaluate their potential benefits. We show that risk sharing can lead to supply chain coordination in some scenarios.
- Given a market-imposed length of price-protection period, we evaluate the performance of the decentralized models under demand and cost/price uncertainty. Our numerical analysis of VMI shows that such contracts tend to perform better than DMI in settings that characterize the high-technology industry - high price erosion (technological obsolescence), small to moderate profit margins, higher impact of product shortages on the manufacturer, and low inventory carrying costs (other than obsolescence costs). Opposite economic conditions tend to favor DMI. We also find that risk sharing can lead to significant improvements to both VMI and DMI.

[2] Another reason is simply that in the beginning of 1990s price protection was offered for an unlimited time.

The rest of the paper is organized as follows. In Section 2 we review the relevant literature. In Section 3 we define and analyze the centralized model and the DMI model without price protection. The decentralized models under price protection are introduced and analyzed in Section 4, where we reformulate the dynamic program with price-protection credits, as well as describe correcting mechanisms. Section 5 describes the design of the numerical study and lessons learned. We conclude in Section 6.

2 Relevant Literature

The model we consider can be labeled as a non-stationary decentralized inventory model with price protection. The relevant literature includes non-stationary inventory research, decentralized supply chain models, VMI contracts, revenue sharing, and price protection analysis.

Among the very first non-stationary models studied in OM literature are Karlin (1960), Iglehart and Karlin (1962), and Veinott (1963). Veinott (1966) summarizes early developments in inventory theory. Many inventory papers consider additional constraints such as multiple service classes (Veinott, 1965), limited capacity (Kapuscinski and Tayur, 1998; Aviv and Federgruen 1997), or some form of Bayesian updating (Azury, 1985; Lovejoy, 1993). In most of these papers, only demand is non-stationary while some allow for other non-stationary elements. A unified framework is described in Zipkin (2000). All of the above papers look at centralized cost structures and, consequently, none of them consider a cost function appropriate for price protection.

The supply chain literature has been growing rapidly (Tayur, Ganeshan and Magazine, 1998; Graves and de Kok, 2003). However, due to the difficulty of analysis, most decentralized models in this literature focus on single-period models or, at most, two-period models to capture the time dynamics. Some exceptions include Parker and Kapuscinski (2004), Rudi, Kapur and Pyke (2001), and Fry, Liu and Raturi (2005). To the best of our knowledge, there are no papers dealing with decentralized models in a non-stationary cost environment. Multi-period interactions with non-stationary costs are, however, at the heart of the price-protection problems.

Since we compare the traditional DMI models where a distributor determines the order quantities with VMI models where a vendor (manufacturer) determines these, we describe the VMI literature in more detail. Despite many business references to VMI, few research papers deal specifically with VMI, both in the area of empirical research as well as analytical models for VMI. Most of these papers contain some form of comparison to DMI even though these comparisons may not be their focus.[3]

The benefits of VMI depend on its definition and the corresponding model. Bernstein and Federgruen (1999) consider a model of VMI when demand has a constant rate, the vendor decides the

[3] DMI is sometimes referred to as Retailer Managed inventory, or RMI, where a retailer or distributor manages its own inventory while ordering from vendor.

quantity to replenish, while the retailer makes pricing decisions. The model reflects a consignment (shelf renting) arrangement, where the supplier pays for all holding costs, including inventory at the retailer. Cohen Kulp (2002) interprets VMI as simply information sharing, while most models give vendor ordering rights. Clark and Hammond (1997) and Cachon and Fisher (1997) compare the benefits of information sharing versus information sharing combined with VMI. Both of them indicate that most of the benefits of VMI could be attributed to and realized by information sharing without implementing the VMI process. Many models establish reasons why VMI might provide some benefits. These include Cheung and Lee (2002), Cetinkaya and Lee (2000), Aviv and Federgruen (1998), Kraiselburd et al. (2004), Mishra and Raghunathan (2004), Kim (2008), and Bertazzi et al. (2005). Nagarajan and Rajagopalan (2008) study a multi-period (or continuous time) model with fixed ordering and evaluate the effect of holding cost subsidies – these may be viewed as a special case of revenue sharing and thus related to our risk-sharing policy. They consider only Pareto-improving contracts and show that these subsidies can improve the performance of VMI contracts (the consideration of Pareto-improving contracts eliminates the possibility of stuffing the channel that most VMI implementations face - see Section 4.2 for a discussion on channel stuffing). A paper related to our study is Fry et al. (2001), which describes a VMI model with minimum and maximum bounds and shows that it is Pareto improving if the coefficients of the contract are set according to reasonable economic rules. They describe the need for imposing an upper bound on inventory, an issue that is important in our paper as well. All of the papers above consider a non-coordinated system and typically show that VMI allows, in some specific business situations, to eliminate a portion of the gap with the first-best (centralized) solution. This is similar in spirit to what we describe, but the problem is different. None of these papers consider any element of price protection, which is the focus of our paper. (While the authors focus on the effect of the manufacturer's effort, one might interpret the objective function in Kraiselburd et al. (2004) as related to infinite price protection.)

In our extensions we also consider risk sharing. Both risk and revenue sharing have been considered in multiple papers in economics and operations management, see Cachon (2003) for a recent review. In the context of VMI contracts, revenue sharing has been considered by Gerchak and Wang (2004), who study assembly operations (single-period model), Wang et al. (2004), who consider the effect of price elasticity in a consignment model, as well as Nagarajan and Rajagoplan (2008), described above. None of these papers, however, model the effect of price protection.

Price-protection contracts have been very recently studied in the OM literature by drawing similarities to buy-back contracts. Lee et al. (2000) examine the use of partial price protection (only a fraction of price drop is refunded) as a lever to coordinate the channel between a supplier and a retailer for a single-period problem and a two-period problem. While partial price protection was found to enable channel coordination in the single-period problem, it was insufficient for the two-period problem. For two periods, Taylor (2001) finds that adding a price protection element to situations where the retailer can return products mid-life and end-of-life can help coordinate the channel and this may benefit both the supplier and the retailer. In addition to returns and price protection, Lu et al. (2007) analyze a rebate policy where the supplier gives partial credits to the retailers on quantities sold after a price drop. They provide methods to determine what policies lead to Pareto improvements and also characterize the effectiveness of returns, rebates, and price protection policies under single-buying and two-buying opportunity models.

The difference between these papers and our method is that we capture the time dynamics of price protection programs as they are practiced in the industry - full price protection of unsold inventory for a limited number of periods (say, six weeks), which does not reduce itself to buy-back. Because of the difficulty in capturing the multi-period interactions, the few papers that analyze price protection contracts use single period buy-back models. This paper is the only one, to the best of our knowledge that includes the dependency of price protection costs on the length of price protection.

3 Base Models

We first describe the problem and the notation. Then we follow with the analysis of the centralized model and the decentralized model without price protection. In the next section, we describe and analyze models with price-protection contracts.

Consider a distributor (D) who buys products from a manufacturer (M) at a wholesale price and sells it to the end-customer at a retail price over T periods. At the end of each period $(t-1)$, the production cost, c_t, and wholesale price, w_t, for period t become known and the distributor places a replenishment order of size a_t which is produced by the manufacturer. In period t, the order for a_t is delivered to the distributor, demand, D_t, is revealed, and all costs are evaluated. Then the cycle repeats (production cost and wholesale price for period $t+1$ are revealed and the next order is placed). The production cost, wholesale price, and retail price decrease over time. The decrease is stochastic and becomes known at the end of the previous period. The order quantity a_t and the corresponding production (both made in period $t-1$ for delivery in period t) are made with full knowledge of period t costs and prices, and the distribution of costs and prices in future periods. The distributor is charged the actual wholesale price at the time when the order is delivered.

All production occurs in response to a distributor's order and, therefore, the manufacturer holds no inventory. The distributor, on the other hand, faces uncertain demand and attempts to satisfy demand from on-hand inventory. Any excess inventory is carried over to future periods. In the case of a shortage, the requested products are produced and expedited to the distributor. When goods are expedited, the distributor incurs the wholesale price for purchasing the product and some additional cost penalties - a cost for expediting orders, a cost pertaining to the loss of customer goodwill, and possibly, costs due to price concessions offered to the customers to compensate for the time delay in delivering the product. The manufacturer is also penalized for distributor shortages and incurs a cost due to loss of customer goodwill in addition to the regular cost of producing the product and possibly the cost of accelerated (expedited) production. This is a close representation of actual practice, where the manufacturer is able to produce additional products and expedite them at an additional cost if the required components are available and the distributor is able to complete the sale.

Both the manufacturer and the distributor are risk neutral and discount their cash flows at the same rate. The state of the system is observable and all information is fully transparent to the manufacturer and distributor.

We make the following assumption:

Assumption 1 *All variables are exogenous, except ordering quantities.*

The following notation is used:

Exogenous Variables

- T - time horizon, indexed $t = 1, ..., T$
- D_t - demand in period t, with cdf $\Phi_t(.)$ and pdf $\phi_t(.)$
- $D[t,k]$ - convolution of demands D_t, ..., D_k, $k \geq t$, with cdf $\Phi_{[t,k]}(.)$. $\Phi_{[t,t]}(.) = \Phi_t(.)$
- L - length of price-protection period
- c_t - production cost in period t
- w_t - wholesale price in period t
- p_t - retail price in period t
- g_t^z - shortage penalty to $z = M, D$, including all costs and shortage penalties such as expediting cost, loss of goodwill cost and any additional production costs, $g_t = g_t^M + g_t^D$
- h_t - inventory carrying cost for holding inventory at distributor's site in period t
- β - discount factor

Endogenous Variables

- x_t - starting inventory
- a_t - quantity ordered in period t
- e_t - quantity expedited in period t
- y_t - inventory on hand, after order arrives, $y_t = x_t + a_t$, but before expediting
- $\bar{a}_t$ - cumulative quantity shipped until (and including) period t, $\bar{a}_t = \bar{a}_{t-1} + a_t + e_t$

3.1 Centralized Model

The centralized problem is easy to formulate and solve. We briefly outline it, as it will serve as a benchmark for the decentralized solutions. Due to the presence of expediting, the amount of goods sold in any given period is equal to the demand in that period. Thus, it is sufficient to use a cost-based formulation and omit the gross profit $(p_t - c_t) * D_t$. The cost minimization formulation for the centralized system is given by:[4]

$$\begin{aligned} V_t^{SC}(x_t) &= \min_{y_t \geq x_t} [W(y_t) + \beta E_{D_t} V_{t+1}^{SC}((y_t - D_t)^+)] \\ \text{where } W^{SC}(y_t) &= E\{g_t(D^t - y_t)^+ + (c_t - \beta c_{t+1} + h_t)(y_t - D_t)^+\} \end{aligned}$$

and $V_0^{SC}(x_{T+1}) = 0$. All of the above expectations are with respect to future demands and costs. Clearly, this is a generalization of non-stationary inventory problem described in Karlin (1961). The added elements are the expediting costs and uncertain costs dynamics for the future periods. Due to Assumption 1 we have

Theorem 1 *(a) The optimal policy is a non-stationary base-stock policy. (b) Assuming that inventory can be salvaged in the next period at cost c_{t+1}, the myopic base-stock level is*

$$y_t^{C*} = \Phi_t^{-1}\{\frac{g_t}{g_t + (c_t - \beta E c_{t+1}) + h_t}\} \qquad (1)$$

Proof is in the appendix. ■

Sufficient Condition *For the myopic policy to be optimal, it is sufficient that demand is stationary and the production costs, c_t for $t = 1, ..., T$, are convex decreasing when all other parameters are constant.*

For the myopic policy to be optimal, the myopic inventory level y_t^{C*} should be reachable in every period t. Note that $c_{t-1} - c_t \geq c_t - c_{t+1}$ and c_t is non-increasing implies that $c_{t-1} - \beta c_t \geq c_t - \beta c_{t+1}$ for any $\beta \in [0, 1]$, which further implies non-decreasing myopic ratios. From Veinott (1965), myopic levels are, therefore, optimal.

Having described the centralized model, we now analyze a decentralized model (DMI) where the distributor makes the ordering decisions in the absence of price protection.

3.2 Decentralized Model: DMI without Price Protection

In a DMI system, the distributor is an active player who makes all ordering decisions, and the manufacturer is delivering the quantities ordered by the distributor. Because no decisions are made by the manufacturer, the problem can be expressed as a minimization of the distributor's cost. Since there is no price protection in this preliminary setting, the structure of the distributor's cost mirrors that of the supply chain, except that the wholesale price w plays the role of the

[4]The term $c_t x_t$ is omitted in the initial period and, in all other periods, reassigned to the previous period. By explicitly expressing $c_t x_t$ as $c_t E(y_{t-1} - d_{t-1})^+$, the correction, which is usually a part of the myopic formulation, is directly embedded in the function $W(.)$.

production cost c and the cost of shortage is g_t^D. We modify the one-period cost function similar to the centralized case and assign credit for the remaining inventory to the current period.

$$\begin{aligned} V_t^D(x_t) &= \min_{y_t \geq x_t} [W^D(y_t) + \beta E_{D_t} V_{t+1}^D((y_t - D_t)^+)] \\ \text{where } W^D(y_t) &= E\{g_t^D(D^t - y_t)^+ + (w_t - \beta w_{t+1} + h_t)(y_t - D_t)^+\} \end{aligned}$$

and $V_0^D(x_{T+1}) = 0$.

Theorem 2 *Without price protection:*

(a) The optimal policy is a non-stationary base-stock policy.

(b) Assuming that inventory can be salvaged in the next period at the purchase cost, w_{t+1}, the myopic policy has the optimal up-to level

$$y_t^{D*} = \Phi^{-1}\{\frac{g_t^D}{g_t^D + (w_t - \beta E w_{t+1}) + h_t}\}.$$

(c) If the decrease in the expected wholesale price is at least equal the decrease in the expected cost, $w_t - \beta E w_{t+1} \geq c_t - \beta E c_{t+1}$, the distributor's myopic quantity is (weakly) smaller than the supply-chain myopic quantity, $y_t^{D*} \leq y_t^{C*}$.

Proof: Parts (a) and (b) follow exactly as for the centralized problem, see Theorem 1. For part (c) note that the critical ratio for the supply chain is guaranteed to be strictly larger than the critical ratio for the distributor, if $(w_t - \beta E w_{t+1}) - (c_t - \beta E c_{t+1})$ is non-negative, since $g_t^M \geq 0$. ■

Theorem 2 illustrates the well-known effect of double marginalization in a decentralized environment. While set in a dynamic framework with decreasing prices, it reflects the main distortion of overage and underage costs, which leads to insufficient incentives for the distributor to order supply-chain optimal quantities and is the primary source of coordination friction.

The condition in part (c) of Theorem 2 intuitively holds in the long term as it is unlikely that, in the long term, absolute margins are increasing when prices are decreasing. To illustrate the point, consider no discounting ($\beta = 1$) and assume cost $c_1 = \$800$ and wholesale price $w_1 = \$1000$ for a profit margin of 25%. When costs drop to $c_2 = \$600$, with a constant margin the wholesale price would become \$800 while with a more intuitive constant mark-up it would become \$750. One can expect that wholesale prices above \$800 would be very unlikely, which corresponds to condition (c) in the theorem above.[5]

4 Decentralized Models Under Price Protection

In this section, we explore two types of decentralized models under price protection: first, a traditional DMI model where the distributor makes the stocking decisions; second a VMI model in

[5] The sufficient condition may not hold, while the myopic policy may still be optimal. E.g., this may be the case during production ramp-up which we describe in our numerical study.

which the manufacturer performs these functions. We also analyze the impact of using a risk sharing agreement under both decentralized models.

4.1 DMI Under Price Protection

Part (c) of Theorem 2 above shows that the myopic up-to levels from the distributor's perspective are likely to be smaller than the myopic up-to levels for the centralized system. Price protection, in general, helps to eliminate or reduce the risk of price erosion for the distributor. Today, the dominating form of price protection practiced in the industry is full price protection for a limited time (Pereira, 1999; Moltzen and Campbell, 2000). It is governed by L, the length of the price-protection period. In any time period, when the wholesale price is decreased, the distributor is credited for the smaller of the inventory on hand and the amount purchased in the last L periods, multiplied by the decrease in the wholesale price.

Formally, the price protection credit is $(w_{t-1} - w_t) * \min\{x_t, \bar{a}_{t-1} - \bar{a}_{t-L-1}\}$ and, thus, the distributor's expected cost under a price-protection contract with price-protection length L is:

$$\begin{aligned}
\bar{V}_t^{D,PP}(x_t, \bar{a}_{t-1}, \ldots, \bar{a}_{t-L-1}) &= \min_{y_t \geq x_t} [\bar{W}_t^{D,PP}(y_t, \bar{a}_{t-1}, \bar{a}_{t-L-1}) \\
&\quad + \beta E_{D_t} \bar{V}_{t+1}^{D,PP}((y_t - D_t)^+, \bar{a}_t, \ldots, \bar{a}_{t-L})], \\
\text{where } \bar{W}_t^{D,PP}(y_t, \bar{a}_{t-1}, \bar{a}_{t-L-1}) &= W_t^D(y_t) - (w_{t-1} - w_t) * \min\{x_t, \bar{a}_{t-1} - \bar{a}_{t-L-1}\}\}
\end{aligned}$$

and $\bar{V}_0^{D,PP}(x_{T+1}, \ldots) = 0$.

From the above formulation it is clear that including price protection requires the history of orders placed in the last L periods, increasing the state space of the dynamic program. The following two properties are pivotal for the rest of the analysis. They enable us to reformulate the problem, by including expected price-protection cost in future within the current-period cost and to express the cost as a function of the target inventory level, y_t, without losing the dependence on the length of the price-protection period. Then, we discuss a more subtle relationship with the typically used myopic formulations.

Critical Property 1 *The price protected quantity,* $\min\{x_t, \bar{a}_{t-1} - \bar{a}_{t-L-1}\}$*, due to a price action in period* t*, can be re-written as*

$$\sum_{k=1}^{L} \min\{(y_{t-k} - D[t-k, t-1])^+, a_{t-k}\}$$

Proof: The property above is easy to justify in intuitive terms. Any unit purchased in period $t-k$, for $k = 1, \ldots, L$, is a part of a_{t-k} and, is price protected, if it is not sold by period t. Because the underlying idea of price protection is first-in first-out, this specific unit will be sold only when the cumulative demand since the unit was purchased exceeds the inventory level, y_{t-k}, to which inventory was raised in period $t-k$. Clearly it is sufficient to show that

$$\begin{aligned}\min \quad & \{(y_{t-k} - D[t-k, t-1])^+, \bar{a}_{t-k} - \bar{a}_{t-L-1}\} = \qquad (2)\\ & \min\{(y_{t-k} - D[t-k, t-1])^+, a_{t-k}\} + \min\{(y_{t-k-1} - D[t-k-1, t-1])^+, \bar{a}_{t-k-1} - \bar{a}_{t-L-1}\}\end{aligned}$$

and apply it iteratively. From the definition of $\bar{a}_{t-k}$, the left-hand side of (2) can be expressed as

$$\min\{((y_{t-k}-D[t-k,t-1]))^+, a_{t-k}+e_{t-k}\}+\min\{(y_{t-k}-D[t-k,t-1]-a_{t-k}-e_{t-k})^+, \bar{a}_{t-k-1}-\bar{a}_{t-L-1}\}.$$

If $y_{t-k} > D_{t-k}$, we have $e_{t-k} = 0$ and the first term matches the first term of the right-hand side of (2). For the second term, since $e_{t-k} = 0$ and $y_{t-k} - a_{t-k} = x_{t-k} = (y_{t-k-1} - D_{t-k-1})^+$, we have

$$\begin{aligned}(y_{t-k} - D[t-k, t-1] - a_{t-k} - e_{t-k})^+ &= ((y_{t-k-1} - D_{t-k-1})^+ - D[t-k, t-1])^+\\ &= (y_{t-k-1} - D[t-k-1, t-1])^+\end{aligned}$$

which completes this case.
If $y_{t-k} \leq D_{t-k}$, we have $((y_{t-k} - D[t-k, t-1])^+ = 0$. Also, $(y_{t-k-1} - D_{t-k-1}) \leq x_{t-k} \leq y_{t-k}$. Thus, $y_{t-k-1} - D[t-k-1, t-1] \leq y_{t-k} - D[t-k, t-1]$, implying that all terms in (2) are 0, which completes the proof. ■

Using Property 1, we can write the price protection cost due to a price action in period t as $(w_{t-1} - w_t)\sum_{k=1}^{L} \min\{(y_{t-k} - D[t-k, t-1])^+, a_{t-k}\}$. By grouping all components with purchase quantity a_t in period t and incorporating the appropriate discounts, the future price protection credit for an order placed in period t, $G(x_t, y_t)$, can be expressed as:

$$G(x_t, y_t) = \sum_{k=1}^{L} \beta^k (w_{t+k-1} - w_{t+k}) \min\{(y_t - D[t, t+k-1])^+, y_t - x_t\}$$

since $a_t = y_t - x_t$.

Critical Property 2 *The dynamic programming formulation for DMI under price protection is equivalent to:*

$$V_t^{D,PP}(x_t) = \min_{y_t \geq x_t} \; [W^{D,PP}(y_t) + \beta E_{D_t} V_{t+1}^D((y_t - D_t)^+)].$$

where

$$\begin{aligned}W^{D,PP}(y_t) &= E\{g_t^D(D^t - y_t)^+ + (w_t(1-\beta) + h_t)(y_t - D_t)^+\\ &\quad +\beta^{L+1}(Ew_{t+L} - Ew_{t+L+1})(y_t - D[t, t+L])^+\}\end{aligned}$$

Proof: Based on Property 1, grouping the terms with the same purchase quantity a_t and incorporating appropriate discounting, we get

$$V_t^{D,PP}(x_t) = \min_{y_t \geq x_t} \quad [W^D(y_t) - G(x_t, y_t) + \beta E_{D_t} V_{t+1}^D((y_t - D_t)^+)]$$

In order to transform the price-protection credit, $G(x_t, y_t)$, note that:

$$
\begin{aligned}
E(\min\{(y-D)^+, y-x\}) &= \int_0^x (y-x)dF(D) + \int_x^y (y-D)dF(D) \\
&= \int_0^y (y-D)dF(D) - \int_0^x (x-D)dF(D) \\
&= E(y-D)^+ - E(x-D)^+ \qquad (3)
\end{aligned}
$$

Thus, using $x_t = (y_{t-1} - d_{t-1})^+$, we have

$$E\min\{(y_t - D[t, t+k-1])^+, (y_t - x_t)\} = [E(y_t - D[t, t+k-1])^+ - E(y_{t-1} - D[t-1, t+k-1])^+]$$

Denoting $u_t^k = \beta^k(Ew_{t+k-1} - Ew_{t+k})$, we have $E[G(x_t, y_t)] = E[\bar{G}_t^0(y_t) - \bar{G}_t^1(y_{t-1})]$ where

$$\bar{G}_t^0(y_t) = \sum_{k=1}^{L} u_t^k E(y_t - D[t, t+k-1])^+ \text{ and } \bar{G}_t^1(y_{t-1}) = \sum_{k=1}^{L} u_t^k E(y_{t-1} - D[t-1, t+k-1])^+$$

for all t. Notice that the difference between $\bar{G}_t^0(y_t)$ and $\bar{G}_t^1(y_{t-1})$ is the inclusion of D_{t-1} in the demand convolution in $\bar{G}_t^1(y_{t-1})$. In the dynamic program formulation, terms containing y_{t-1} may be assigned to the preceding period $t-1$ (after multiplying them by the discount factor β). Thus, in period t, we have

$$
\begin{aligned}
E[-\bar{G}_t^0(y_t) + \beta\bar{G}_{t+1}^1(y_t)] &= -\sum_{k=1}^{L} u_t^k E(y_t - D[t, t+k-1])^+ + \beta\sum_{k=1}^{L} u_{t+1}^k E(y_t - D[t, t+k])^+ \\
&= -u_t^1 E(y_t - d_t)^+ + u_t^{L+1} E(y_t - D[t, t+L])^+
\end{aligned}
$$

resulting in the one-period cost function:

$$
\begin{aligned}
W^{D,PP}(y_t) &= W^D(y_t) - \bar{G}_t^0(y_t) + \beta\bar{G}_{t+1}^1(y_t) \\
&= E\{g_t^D(D^t - y_t)^+ + (w_t - \beta w_{t+1} + h_t)(y_t - D_t)^+ \\
&\quad -\beta(w_t - w_{t+1})(y_t - D_t)^+ + \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1})(y_t - D[t, t+L])^+\} \\
&= E\{g_t^D(D^t - y_t)^+ + (w_t(1-\beta) + h_t)(y_t - D_t)^+ \\
&\quad +\beta^{L+1}(Ew_{t+L} - Ew_{t+L+1})(y_t - D[t, t+L])^+\}
\end{aligned}
$$

and dynamic program formulation:

$$V_t^{D,PP}(x_t) = \min_{y_t \ge x_t}[W^{D,PP}(y_t) + \beta E_{D_t} V_{t+1}^D((y_t - d_t)^+)] \qquad \blacksquare$$

We will refer to the quantity $W^{D,PP}$ as the myopic function. Property 2 highlights that our myopic formulation is different from the traditional myopic function. In a typical myopic formulation, full credit is given to the remaining inventory (which becomes the starting inventory in the next period). In a case when purchasing costs are nonstationary or the cost is discounted, it is standard to normalize costs to 0, by adding the change of expected cost to the holding cost. These corrections award the decision maker with an appropriate credit for the inventory at the beginning of the next period, since this inventory does not have to be purchased. With price protection, there are additional consequences of a purchase, reflected in our derived one-period cost, which contains two components for cost adjustments. The first one, $w_t(1-\beta)$, reflects the loss in value of excess inventory at the end of period t from cash discounting since the price protection credit is paid only in cash in the period of a price reduction. The second term is a charge of $\beta^{L+1}(Ew_{t+L} - Ew_{t+L+1})$, taken in period $t+L+1$ on the inventory remaining after period $t+L$. This term is similar to the salvage cost of excess inventory in traditional formulations, but has a time delay of L periods due to the presence of price protection.

The properties above allow us to express the distributor's policy in terms of the target inventory levels, y_t. In the next theorem we describe the optimal policy. Recall that, without price protection, we expect distributors to under-stock. A sufficient length of the price-protection period, L, brings the up-to level above the supply-chain optimal level, as shown in point (c) below.

Theorem 3 *(a) The optimal policy is base-stock level.*
(b) If $w_t - w_{t+1} \geq w_{t+1} - w_{t+2}$, for all t, the myopic base-stock level, $y_t^{D,PP*}(L)$ is a non-decreasing function of price-protection period L.
(c) If

$$\frac{w_t(1-\beta)+h_t}{(c_t-\beta Ec_{t+1})+h_t} \quad < \quad \frac{g_t^D}{g_t} \tag{4}$$

then there exists a price protection length, L, such that $y_t^{C*} \leq y_t^{D,PP*}(L)$.

Proof: (a) The one-period profit function, $W^{D,PP}(y_t)$, is clearly convex. By induction, assuming V_{t+1} is convex, since expectation and minimization preserve convexity, we have $EV_{t+1}((y-D)^+)$ is convex (recall that expediting is equivalent to the backlogging case with appropriately redefined costs and, therefore, preserves convexity). From Property 2, the objective function is expressed in terms of the target inventory y_t and is independent of the starting inventory x_t, thus the optimal policy is base-stock.

(b) The derivative of the distributor's one-period profit function is given by

$$\begin{aligned} W^{D,PP'}(y_t) &= -g_t^D + (g_t^D + w_t(1-\beta)+h_t) * \Phi_t(y_t) \\ &\quad + \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) * \Phi_{[t,t+L]}(y_t) \end{aligned}$$

If $w_t - w_{t+1} \geq w_{t+1} - w_{t+2}$ for all t, notice that the absolute value of the component of the derivative of the profit function $W^{D,PP}(y_t)$ that depends on L is non-increasing with L. Consequently, the minimizing point does not decrease with L.

(c) To show that there exists an L such that $y_t^{C*} \leq y_t^{D,PP*}(L)$ it is sufficient to show that there exists an L such that the derivative of the profit function $W^{D,PP'}(y_t)$ is non-positive at y_t^{C*}.

$$\begin{aligned} W^{D,PP'}(y_t^{C*}) &= -g_t^D + (g_t^D + w_t(1-\beta)+h_t) * \Phi_t(y_t^{C*}) \\ &\quad + \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) * \Phi_{[t,t+L]}(y_t^{C*}) \end{aligned}$$

Using (1), we have,

$$W^{D,PP'}(y_t^{C*}) \quad = \quad -g_t^D + \alpha g_t + \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) * \Phi_{[t,t+L]}(y_t^{C*}),$$

where
$$\alpha = \frac{g_t^D + w_t(1-\beta) + h_t}{g_t + (c_t - \beta E c_{t+1}) + h_t}$$

For $W^{D,PP'}(y_t^{C*}) \leq 0$ to hold, we need to find an L such that

$$\beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) * \Phi_t^L(y_t^{C*}) \quad \leq \quad g_t^D - \alpha g_t \tag{5}$$

For $\beta \in [0,1]$, the left-hand side of (5) converges to 0, so the existence of such an L is equivalent to the right-hand side of (5) being positive

$$\begin{aligned} \frac{g_t^D + w_t(1-\beta) + h_t}{g_t + (c_t - \beta E c_{t+1}) + h_t} &< \frac{g_t^D}{g_t}, \text{ or} \\ \frac{w_t(1-\beta) + h_t}{(c_t - \beta E c_{t+1}) + h_t} &< \frac{g_t^D}{g_t} \end{aligned}$$

which is the assumed condition. ■

While Theorem 3(b) guarantees that the up-to levels are monotonic in L, the monotonicity does not always hold. The example below illustrates a scenario with non-monotonic up-to levels.

Example

Figure 1 shows the optimal order up-to levels in period 1, as a function of the length of price protection. The value of the optimal base-stock initially decreases, then increases, and then remains constant. Given the price reduction from \$30 to \$20 in period 4, when price protection is for three

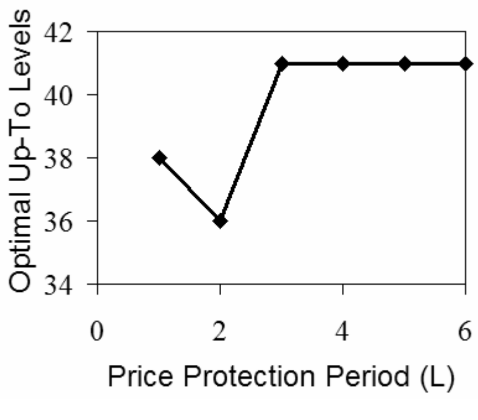


Figure 1: Effect of length of price protection on the optimal stocking levels. Demand is stationary and discrete U[0,49]. $w_1 = w_2 = w_3 = 30$ and $w_4 = w_5 = \ldots = 20$. $g_t^D = 3$ and $h_t = 0.5$ for all t.

periods or longer ($L \geq 3$), the distributor is not concerned about the higher price in periods 1, 2, and 3 and orders up to the appropriate myopic level for $w_4 = 20$ of 41 units. With two periods of price protection, the distributor will significantly overpay for any inventory not sold in the first three periods, resulting in a lower up-to level of 36 units.

The scenario with one period of price protection, $L = 1$, results in interesting dynamics. We observe a higher up-to level in period 1 for $L = 1$ than for $L = 2$. When $L = 1$, the optimal up-to level for period 2, $y_2^{L=1}$, is 27 units, which is much smaller than optimal up-to level for period 1, $y_1^{L=1} = 38$ due to a higher risk of price reduction in period 4. When $L = 2$, the optimal up-to level for period 2, $y_2^{L=2}$ is 41 units, which is higher than the optimal up-to level for period 1, $y_1^{L=2} = 36$ as price protection covers any price reductions on orders placed in period 2. In both cases, every unit of inventory left-over from period 1 faces the same risk from the price drop in period 4. However, if the inventory left over from period 1 is greater than 27 when $L = 1$, it would help reduce the underage cost in period 2 as the actual up-to level would be higher than the optimal one. However, such an externality does not exist when $L = 2$ since $y_1^{L=2} < y_2^{L=2}$. This explains why $y_1^{L=2}$ is less than $y_1^{L=1}$.

Thus, when the anticipated price decrease takes place just outside of the price protection period, it is beneficial for the distributor to under-order. This is commonly observed in practice where distributors with advance information on (or strong expectations about) price reductions delay placing orders until the price decreases are covered by price protection. This results in a "dip" in the order size L periods before the price reduction becomes effective, followed by a noticeable increase.

If we consider the price protection period L as a continuous variable, then part (c) of Theorem 3 states that for DMI: if condition (4) is satisfied, we can find an L that coordinates the chain. While theoretically possible, in practice, it is difficult to adjust the length of the price protection period for the reasons described earlier.

In the following section, therefore, we concentrate on some of the mechanisms practiced or considered as an addition or a refinement to price-protection contracts.

4.2 VMI Under Price Protection

While the previous section considered the traditional case where the distributor determines the order quantities, here we consider a manufacturer who controls the up-to levels at the distributor and acts to minimize its individual cost. In this case, the distributor becomes a passive player who serves demand from the stock (if available) and passes the demand information to the manufacturer. As before, no inventory is held at the manufacturer's site. The manufacturer earns a profit margin $(w_t - c_t)$ on all ordered products and incurs a penalty for orders that cannot be served from on-hand inventory.

Channel Stuffing and the Sarbanes-Oxley Act

It is easy to see that, *in the absence of price protection*, the manufacturer has multiple reasons to sell huge (infinite) volume of products to the distributor: the manufacturer's revenue increases with the quantity placed with the distributor, the products are sold at a higher price in the current period than in a future period, and the revenue realized in the current period is not discounted. Consequently, the total amount that the manufacturer wants to sell to the distributor in the current period may be equal to infinity, or at least is excessive. We label such behavior as **channel stuffing**. The problems related to full control of inventory by the vendor are well recognized in practice. Multiple sources report that channel stuffing took place in the past where firms attempted to book sales in an earlier accounting period in order to meet previously announced financial targets (e.g., Osterland 2006; Khattab 2007). There are few papers that formally model managerial incentives to stuff the channel (see Lai et al., 2008, and references therein). This can be interpreted as more aggressive discounting of future cash flows (smaller β) and VMI makes such behavior easier as the manufacturer is in control of inventory. However, given the range of benefits that VMI provides, the distributor and the manufacturer typically agree on some form of a physical or a financial constraint such as an upper bound on the inventory level to control such undesirable behavior, see Fry et al. (2001). We show that, in the case when the manufacturer gives a price protection credit to the distributor, the incentive to stuff the channel decreases and stuffing is not necessarily optimal. Nevertheless, in some cases, it remains a possibility.

In practice, the recent Sarbanes-Oxley (SOX) regulations impose financial penalties on firms that drive excessive inventory into their distribution channel (Greenwood 2004). Due to these penalties, firms are paying extra attention to managing inventory by following well-defined rules that limit inventory levels (or inventory increases) at their distributors (Jones 2003). Pharmaceutical Commerce (2006) describes "inventory management agreements ... which sought to limit the amount of inventory kept at the wholesaler." While there seems to be no common standard, most control mechanisms that were introduced prevent sudden increases of channel inventories. Thus, in addition to the economic reason described in Fry et al. (2001), SOX provides an additional reason to impose the upper bound on inventory levels.

Analysis and Insights

Using a similar modeling of the price protection component as in Critical Property 2, we have:

$$\begin{aligned} V_t^{M,PP}(x_t) &= -(c_t - w_t)x_t - \sum_{k=1}^{L} \beta^k E(w_{t+k} - w_{t+k-1})(x_t - D[t, t+k-1])^+ \\ &\quad + \min_{y_t \geq x_t} [W^{M,PP}(y_t) + \beta E_{D_t} V_{t+1}^{M,PP}((y_t - d_t)^+)] \end{aligned}$$

where the one-period profit function is given by:

$$W_t^{M,PP}(y_t) = g_t^M (D^t - y_t)^+ + (c_t - w_t)(y_t - D_t)^+$$

$$+\sum_{k=1}^{L}\beta^k E(w_{t+k}-w_{t+k-1})(y_t-D[t,t+k-1])^+ \quad (6)$$

(We omitted the constant $(c_t-w_t)D_t$ and used $y_t+(D^t-y_t)^+=D_t+(y^t-D_t)^+$.)

Definition 1 *The manufacturer has an incentive to* **stuff the channel***, if there exists a period t, such that* $\lim_{y_t\to\infty}V_t^{M,PP}(x_t,y_t)=-\infty$.

It is easy to obtain necessary and sufficient conditions for stuffing the channel.

Property 3 *The manufacturer will stuff the channel if and only if there exists t such that $c_t < E(1-\beta)*(w_t+\beta w_{t+1}+\ldots+\beta^{L-1}w_{t+L-1})+\beta^L w_{t+L}$.*

Proof: Clearly, from (6), $c_t<(1-\beta)*(w_t+\beta w_{t+1}+\ldots+\beta^{L-1}w_{t+L-1})+\beta^L Ew_{t+L}$ is equivalent to $W_t^{M,PP}(y_t)\to-\infty$, which is the necessary and sufficient condition for some period $\bar{t}\geq t$ in order for $V_{\bar{t}}^{M,PP}\to-\infty$. ■

While Property 3 establishes necessary and sufficient conditions to stuff the channel, the period in which stuffing takes place, could be the one with the largest unit benefit,[6] i.e., largest negative discounted value: $c_t-(1-\beta)*(w_t+\beta w_{t+1}+\ldots+\beta^{L-1}w_{t+L-1})-\beta^L Ew_{t+L}$. Note, however, that $c_t-(1-\beta)*(w_t+\beta w_{t+1}+\ldots+\beta^{L-1}w_{t+L-1})-\beta^L Ew_{t+L}<\beta[c_{t+1}-(1-\beta)*(w_{t+1}+\ldots+\beta^{L-1}w_{t+L})-\beta^L Ew_{t+L+1}]$ is equivalent to $c_t-\beta c_{t+1}-(1-\beta)w_t-\beta^{L+1}(w_{t+L}-\beta w_{t+L+1})<0$, which points to "best" periods for stuffing the channel (the highest t for which inequality still holds).

Note from Property 3 that under many reasonable combinations of parameters (not extremely small discount factor and not extremely high margins), price protection eliminates any incentives to stuff the channel. This is intuitive, as the benefit (to the manufacturer) of realizing revenue earlier due to higher wholesale prices is significantly reduced or eliminated by having to reimburse any reductions in the wholesale price during the next L periods, while the benefit of producing at a lower cost is given up.

However, the manufacturer may still have an incentive to "stuff" the channel under VMI in some situations. Following practices described in Fry et al. (2001), we impose an upper bound on the inventory at the distributor in such situations. The new clear trend in the industry after SOX was passed is to curb excessive inventories, which makes this assumption fit reality very well.

Using a similar rearrangement of the price protection component as in Property 2, we have:

$$V_t^{M,PP}(x_t)=\min_{y_t\geq x_t}\quad [W^{M,PP}(y_t)+\beta E_{D_t}V_{t+1}^{M,PP}((y_t-d_t)^+)].$$

where the one-period profit function is given by:

$$\begin{aligned}W_t^{M,PP}(y_t) &= g_t^M(D^t-y_t)^+ + [(c_t-\beta c_{t+1})-w_t(1-\beta)](y_t-D_t)^+\\ &\quad -\beta^{L+1}(Ew_{t+L}-Ew_{t+L+1})(y_t-D[t,t+L])^+\end{aligned}$$

[6]If, for example, only a large but finite storage was available.

Differentiating the one-period cost function wrt y_t, we have

$$W_t^{'M,PP}(y_t) = -g_t^M + [g_t^M + (c_t - \beta E c_{t+1}) - w_t(1-\beta)] * \Phi_t(y_t) \\ -\beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) * \Phi_{[t,t+L]}(y_t) \tag{7}$$

Define positive constants

$$\delta = \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) \text{ and}$$

$$\gamma = \frac{[g_t^M + (c_t - \beta E c_{t+1}) - w_t(1-\beta)]}{\beta^{L+1}(Ew_{t+L} - Ew_{t+L+1})}.$$

Property 4 *If the pdf of demand D_t,$\phi_t(z)$, is IFR for all t, then $W_t^{'M,PP}(y_t)$ is unimodal.*

Proof:

$$W_t^{'M,PP}(y_t) = -g_t^M + \delta[\gamma\Phi_t(y_t) - \Phi_{[t,t+L]}(y_t)]$$

Unimodality of $W_t^{'M,PP}(y_t)$ is equivalent to unimodality of

$$\gamma\Phi_t(y_t) - \Phi_t^L(y_t) = \int_0^{y_t} \phi_t(z)[\gamma - \Phi_{[t+1,t+L]}(y_t - z)]dz$$

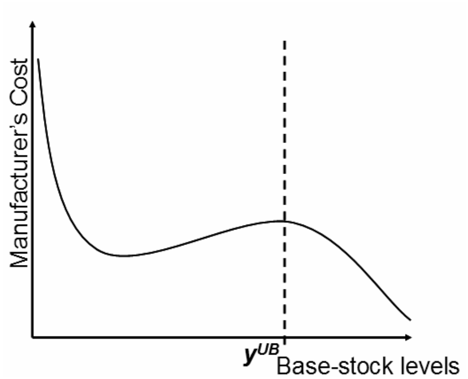
Notice that $\Phi_{[t+1,t+L]}(y_t - z)$ is a non-increasing function of z. Thus, $[\gamma - \Phi_{[t+1,t+L]}(y_t - z)]$ is unimodal. This gives us that $[\gamma\Phi_t(y_t) - \Phi_{[t,t+L]}(y_t)]$ is unimodal in z since $\phi_t(z)$ is IFR (Rosling, 2002). ■

Since the first derivative of the one-period cost function is unimodal (Property 4), the function has at most two zero points, giving it a shape as shown in Figure 2(a) when stuffing is a possibility in our model. As mentioned earlier, we impose an upper bound on the inventory level that would control this behavior. In our numerical evaluations, we do not penalize for stocking inventory above the upper bound, but instead remove the benefit of extra shipments by the manufacturer, see the upper bound y_t^{UB} in Figure 2(b). The bound can be created in many ways (see Fry et al. 2001). We chose to use the zero points of $W_t^{'M,PP}(y_t)$ which can be easily evaluated.

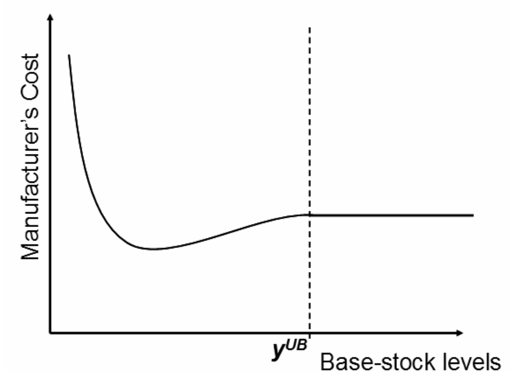
We now compare the optimal up-to levels under VMI with price protection to the centralized model. This theorem builds on Theorem 3.

Theorem 4 *For VMI, under the assumptions of Theorem 3, part (c), and assuming that the demand distribution is IFR, there exists a price-protection length, L, such that $y_t^{C*} \geq y_t^{M,PP*}(L)$.*

Proof: We first observe that $W_t^{SC}(y_t) = W_t^{D,PP}(y_t) + W_t^{M,PP}(y_t)$ and thus $W_t^{SC'}(y_t) = W_t^{'D,PP}(y_t) + W_t^{'M,PP}(y_t)$. If (4) is satisfied, we have from part (c) of Theorem 3 that there exists an L such that $y_t^{D,PP*} > y_t^{C*}$. For such an L, we have $W_t^{'D,PP}(y_t^{C*}) < 0$ and thus, $W_t^{'M,PP}(y_t^{C*}) > 0$. Given that $W_t^{'M,PP}(y_t)$ is unimodal (Property 4) and the modified cost based on Figure 2(b), we have $y_t^{C*} > y_t^{M,PP*}$. ■



(a) Example of manufacturer's cost with incentive to stuff the channel.



(b) Bound on manufacturer's cost to prevent stuffing (in numerical study, we are indifferent for levels above the bound, y^{UB}).

Figure 2: Manufacturer's cost function under VMI

The results above show that price protection can be an effective inventory control in either the DMI (Theorem 3) or VMI setting (Theorem 4) if implemented with an appropriate length of the price protection period L. While this is a pleasing theoretical result, there is a high level of resistance towards any adjustment of price-protection terms in the industry. As manufacturers tend to match each other's price-protection terms and conditions, changing the length of the price protection period may be impossible from a practical point of view.

We next consider a risk-sharing mechanism that involves the manufacturer paying a portion of expediting cost to the distributor.

4.3 Risk Sharing

Risk sharing (or alternatively revenue sharing) agreements are difficult to implement in most real situations. Given, however, their theoretical appeal, we analyze an example that uses the spirit of risk sharing applied to the specific business case of price protection.

In the presence of price protection, over-stocking at a distributor under DMI may be observed, and it corresponds to under-stocking under VMI, $y_t^{M,PP*} < y_t^{C*} < y_t^{D,PP*}$, as indicated by Theorems 3 and 4. A natural correction might be to change the economic tradeoffs faced by the decision maker by using a risk sharing mechanism. One of the candidates is an adjustment in backlog penalties.[7] Specifically, we consider the manufacturer providing the distributor with a subsidy on the expediting cost (recall that the expediting cost is paid by the distributor when shortages occur). This shifts more shortage risk to the manufacturer and moves the order up-to levels for both DMI

[7] Adjustments in wholesale prices would be effective, but they can dramatically influence the revenue streams and are more difficult to implement.

and VMI in the desired direction: increasing the up-to levels under VMI, while decreasing the up-to levels under DMI. The effect of this risk sharing mechanism can be evaluated for the myopic levels. Let $g_t^D = \bar{g}_t^D + b_t$, where b_t is the expediting cost per unit paid by the distributor on shortages. Let

$$\begin{aligned} \Omega_t &= (1-\alpha)(g_t) - g_t^M - \beta^{L+1}(Ew_{t+L} - Ew_{t+L+1}) * \Phi_{[t,t+L]}(y_t^{C*}) \\ \text{and } \omega_t &= \frac{\Omega_t}{b_t * (1 - \Phi_t(y_t^{C*}))} \end{aligned}$$

We have $\Omega_t > 0$, as we assumed $y_t^{M,PP*} < y_t^{C*}$. ω_t is the desirable portion of the expediting cost that the manufacturer should pay.

Theorem 5 *Assume that for L-period price protection, $y_t^{M,PP*} < y_t^{C*} < y_t^{D,PP*}$. If $\omega_t \leq 1$, then coordination can be achieved under DMI and VMI through a risk sharing mechanism where the manufacturer pays $\omega_t b_t$ of the expediting cost per unit.*

Proof: The updated cost functions for the manufacturer under VMI and distributor under DMI are

$$W^{M,PP,Sub}(y_t) = W^{M,PP}(y_t) + \omega_t b_t (D_t - y_t)^+$$

$$W^{D,PP,Sub}(y_t) = W^{D,PP}(y_t) - \omega_t b_t (D_t - y_t)^+$$

It is easy to verify that $W_t^{M,PP,Sub'}(y_t^{C*}) = W_t^{D,PP,Sub'}(y_t^{C*}) = 0$, which implies that the channel is coordinated. ∎

In practice, the myopic levels need not be optimal. Also, the decision about the level of risk sharing (splitting of expediting costs) will not be made by a central decision maker,[8] but will either be negotiated or decided by one of the parties. We consider the extreme case when one of the parties makes the decision. Since, in general, some division of decision rights is practiced, we assume that the decision on the amount of risk sharing is made by the party *not* in control of the ordering decision (this provides that party with some control over their cost). Specifically, under DMI, where the distributor makes the stocking decision, we assume that the manufacturer decides on the amount of risk sharing (subsidy on expediting cost). Note that, even though the cost is paid by the manufacturer, under DMI the manufacturer may have an incentive to pay a portion of the expediting costs so that inventory is reduced and price protection expenses go down (see Section 4.1). In the case of VMI, the distributor would be given the right to decide the subsidies on expediting costs. The subsidy will provide the distributor with extra cash, but also will raise the stocking levels at the distributor and increase its holding costs. It is not clear by how much (if at all), such distributed decision rights may help to improve the ordering decisions.

In the next section, we numerically evaluate the effect of switching from DMI to VMI and also evaluate the effect of adding the above risk sharing mechanism.

[8] In a central decision maker setting, risk sharing leads to theoretically pleasing results, but is not practical.

5 Computational Experiments

Our computational study is based on settings of a large computer manufacturing company. The company's supply chain is currently operating under DMI with six weeks of price protection, $L = 6$, for the distributor. There is a three-week lead time for order replenishment which includes the time from order placement to delivery. The production costs and sale prices decline between 0.5%-1.5% a week on average, reflecting the presence of high technological obsolescence in the business. It is widely known and reluctantly acknowledged that the distributors are over-stocking products due to the presence of price protection, resulting in significant price protection expenses for the manufacturer and reduced net profits. The manufacturer, in response, decided to explore the possibility of moving to a VMI system, where stocking levels at the distributor are decided by the manufacturer, possibly with negotiated up front lower and upper bounds on the distributor's inventory. The ownership of product is transferred to the distributor upon order delivery.

In order to clearly differentiate between the effects of each individual lever, we evaluate the two policies, DMI and VMI. The conceptual set up is similar to that in the real setting. The numerical values of some parameters have been changed to preserve confidentiality, but the nature and the scale of critical economic tradeoffs is fully preserved. In our numerical experiments, one period corresponds to 3 weeks, which is the replenishment lead time for the orders. Thus, we use a price protection length of 2 periods to reflect the 6 week price protection period. We assume that there is a single product for which the demand follows a Triangular distribution, similar to the assumption made by Birge (1995), Chen (2007), Dasci and Laporte (2005), Papastavrou et al. (1996), and many more.[9] We assume that cost and price drops are stochastic and model them as a Markov chain in which each state consists of a production cost and a wholesale price, as described below. We assume that the manufacturer's shortage penalty, g_t^M, includes the cost due to loss of customer goodwill and that there are no other shortage penalties, i.e, $g_t^M = \bar{g}_t^M$. Since our parameters are designed to reflect realistic values, we set up a base case as the most realistic scenario and then vary one parameter at a time.

Table 1 lists the parameters we use for the base case. We assume that the demand per period has a mean of 500 units and a coefficient of variation of 25%. Initial production cost is \$80, wholesale price is \$100, and they evolve according to a Markov chain indexed by a single parameter. The system may stay in the same state with a probability of 0.3, move to the next one (one step forward) with a probability of 0.4, or move 2 steps forward with a probability of 0.3. Both the production cost and the wholesale price decrease by 3%, when moving from state to state, preserving constant margins in the base scenario (we relax this assumption later). Inventory carrying cost per unit per period is \$.5, which is equivalent to the average observed interest rate of 12% per year on c_t. Since

[9] Also, Wikipedia, 30 October 2008, says that triangular distribution "is often used in business decision making, particularly in simulation."

Parameter	Base Value
D_t	Triangular with mean = 500 units and c.v. = 0.25 for all t%
c_0	\$80
w_0	\$100
Cost/price drops between states	3% per period for c and w for all t
Cost/price transition probabilities	$p_{t,t} = 0.3, p_{t,t+1} = 0.4, p_{t,t+2} = 0.3$
h_t	\$0.5 for all t
b_t	\$10 for all t
g_t^M	\$33.5 for all t
$\bar{g}_t^D$	\$16.5 for all t

Table 1: Experimental Design for Base Scenario

the loss of goodwill costs are often difficult to estimate, we used the critical ratio to set the sum of loss of goodwill and expediting costs to result in a 95% service level for the centralized model. The expediting cost is set to 10% of the wholesale price in period 1 to reflect realistic numbers. The distributor often stocks products from multiple manufacturers and, thus, the cost due to loss of goodwill from product shortages for the distributor does not to exceed that of the manufacturer. The loss of goodwill costs are chosen such that the ratio $(\frac{\bar{g}_M}{\bar{g}_D})$ is equal to 2. We experiment with three different policies:

Centralized System: as described in Section 3.1. Price protection and subsidies do not play any part here.

DMI: as described in Section 4.1.

VMI: as described in Section 4.2.

To evaluate these policies, we run a 10-period stochastic dynamic program. The total cost is the sum of costs for the manufacturer and the distributor. To find the "best" subsidy, we discretize the percentage of subsidy in increments of 5%. We use the centralized system as the basis for evaluating the other policies in terms of the expected total cost and show the percentage by which the expected total costs of the different policies are above the expected total cost for the centralized system. Note that any cost savings are additions to the net profit, e.g., using a gross profit margin of 25%, these results imply that the relative effect on profit is approximately four times bigger. The specifics of various scenarios we experimented with are discussed in the following sub-sections.

5.1 Effect of Rate of Price Decrease

To explain the effect of different rates of price erosion, Figure 3 shows the percent cost increase of DMI and VMI over the centralized system as a function of the average price drop per period. We vary the price drops between 1% and 5%, between successive states, which translates to expected price drops of 0.3%-1.7% per week, given that each period in the model corresponds to 3 weeks. Such a range covers the realistic rates of price changes. Production costs and wholesale prices are assumed to drop at the same rate, maintaining the 25% gross profit margin for the manufacturer. All other parameters are set to their base values.

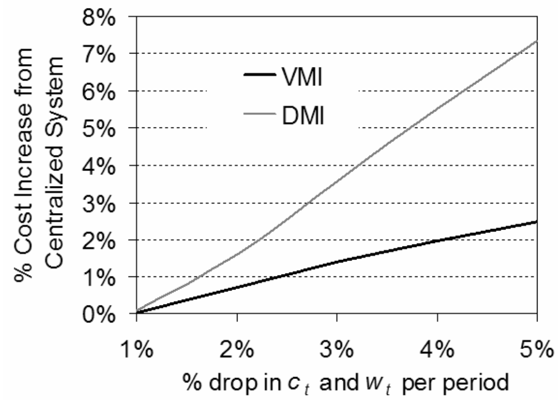


Figure 3: Performance of VMI and DMI under different rates of price drops.

Under DMI, we expect the order up-to levels to decrease with steeper price drops as the financial risk due to early procurement/production increases, but price protection should increase these levels. Indeed over-stocking takes place under DMI and, observe in Figure 3, that the gap with the centralized system increases. The VMI ordering, on the other hand, is more sensitive to the price changes. Since most of the cost incurred by the manufacturer is due to unnecessarily high production costs that could be avoided, as expected, VMI results in under-stocking. The total expected costs increase with steeper price reductions, but due to the consistency of forces between VMI and the supply chain, VMI performs significantly better than DMI (with a lower expected total cost) and the difference increases with a higher rate of price drops.

We also evaluated scenarios with delayed price reductions: the production cost decreases in one period while wholesale price remains constant, and the wholesale price decreases in the next period while the production cost remains constant. Virtually the same behavior was observed.

5.2 Effect of Penalties on Inventory Shortage and Excess

We now examine the effect of various changes in the structure of penalties.

Unit cost of expediting: In order to isolate the effect of expediting cost, we kept the sum of the expediting cost and the loss of goodwill costs constant, and maintained the same ratio between the manufacturer's and distributor's loss of goodwill costs as the base case. Thus, we measure the relative contribution of expediting cost to the total penalty.

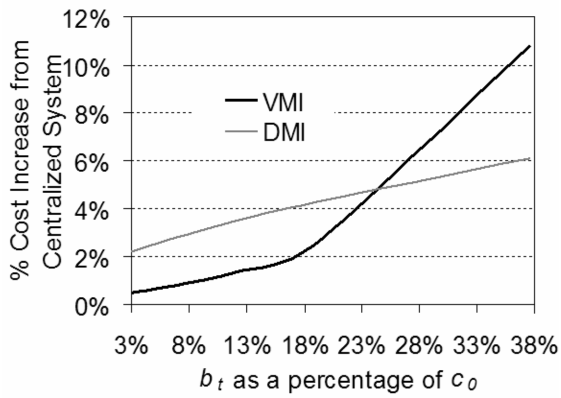


Figure 4: Performance of VMI and DMI under different unit expediting costs

Increasing the expediting costs, while keeping the total shortage cost per unit constant, results in an increase in the tendency to under-stock under VMI and over-stock under DMI and, generally, a poor performance compared to the centralized system, see Figure 4. VMI outperforms DMI when the cost of expediting is less than 25% of the product cost (expediting costs are usually around 10% of product cost in practice). However, when the expediting costs are high, VMI performs worse than DMI, understandably as the manufacturer is then bearing only a very small portion of the total shortage cost.

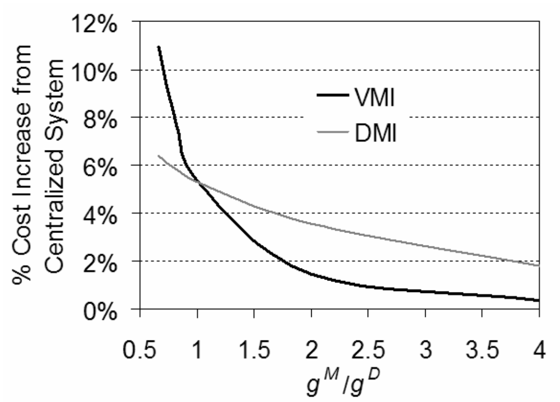


Figure 5: Performance of VMI and DMI under different ratios of goodwill costs.

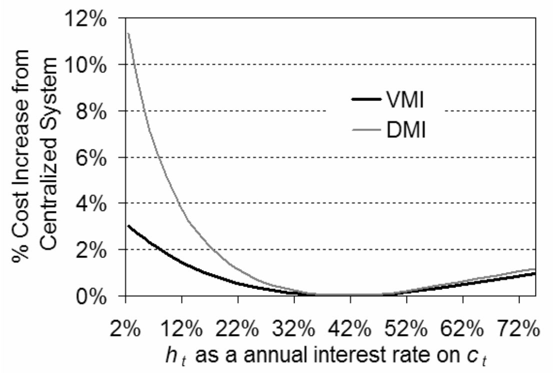


Figure 6: Performance of VMI and DMI under different holding costs at the distributor.

Ratio of goodwill: Figure 5 evaluates the effect of the ratio of goodwill of $\frac{\bar{g}_M}{\bar{g}_D}$ (a ratio of 2 being the base case). When the ratio is high, the effect of shortages shifts from the distributor to the manufacturer and, not surprisingly, VMI performs better in terms of the expected total cost than DMI.

Distributor's holding cost: For very low holding costs, the distributor is penalized neither by inventory costs nor by decreasing prices (due to price protection) and, thus, VMI can significantly help. With higher holding cost, the inefficiencies decrease (Figure 6) and then start diverging away.

Overall, VMI proved to be the more robust compared to DMI across the broad range of parameters we have tested. Specifically, VMI clearly outperforms DMI when the price erosion is high, the shortages penalize the manufacturer more heavily, and when the inventory carrying costs (other than obsolescence) are low - conditions typical in the high technology industry. On the other hand, stable prices, very high expediting costs, low shortage penalty for the manufacturer, or high inventory holding costs reduce the benefits of VMI and can even lead to VMI under-performing DMI.

5.3 Effect of Reducing the Length of Price Protection

In this section, we explore the effects of reducing the length of price protection. In the actual problem that motivated this study, the length of the price protection period is 6 weeks, or $L = 2$. This section evaluates the performance of the policies in a hypothetical case, if it was possible to decrease the length of price protection period to $L = 1$.

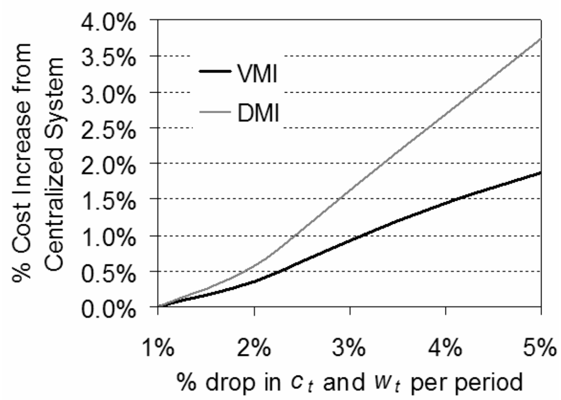
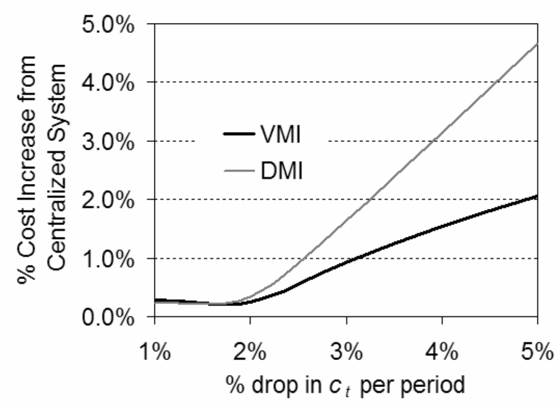


Figure 7: Performance of VMI and DMI under different rates of price drops when $L = 1$.

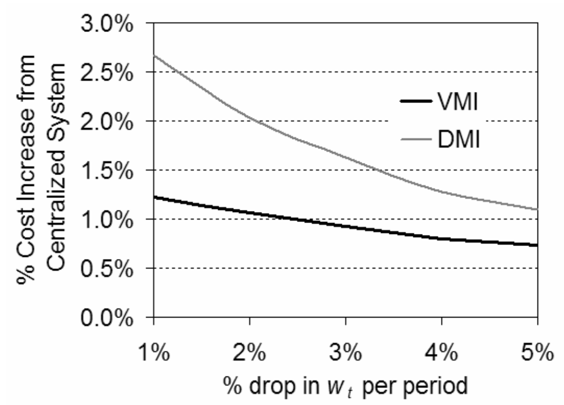
Figure 7 shows the performance of the policies under different rates of price drops. The VMI system still outperforms DMI, although the policies perform closer to the centralized system compared to $L = 2$ (Figure 3). This is consistent with our theoretical results that indicate that by adjusting the length of price protection (if it were possible in practice), one could achieve, or at least move towards, supply chain coordination.

Similarly for a range of profit margins (the margins are identical in all periods within an experiment) we found that VMI continues to outperform DMI, although the difference between VMI and DMI is reduced for higher margin. With increase in profit margin both VMI and DMI get closer to the centralized system, but the effect is fairly slow.

Figures 8(a) and (b) evaluate a drop in cost c that is different from the drop in wholesale cost w. In Figure 8(a) we fix the drop in c_t between successive states to 3% and we vary the



(a) The drop in w_t between successive states is set to 3%. We vary the drop in c_t between successive states.



(b) The drop in c_t between successive states is set to 3%. We vary the drop in w_t between successive states.

Figure 8: Performance of VMI and DMI when production cost and wholesale price decrease at different rates. Length of price protection period $L = 1$

drop in w_t between successive states. In Figure 8(b), we fix the drop in w_t between successive states to 3% and vary the drop in c_t. Thus, the margins change across the periods within a given experiment. Margins usually increase during the product launch phase, when the wholesale prices drop at a slower pace compared to production costs and decrease during the withdrawal phase of a product, when the wholesale prices drop faster compared to the production costs. Figures 8(a) and (b) indicate that the superiority of VMI over DMI increases when the margins are increasing over time. Using a VMI policy during product introductions has other advantages such as risk pooling (outside the scope of our model) that allow manufacturers to more effectively deal with high demand uncertainty.

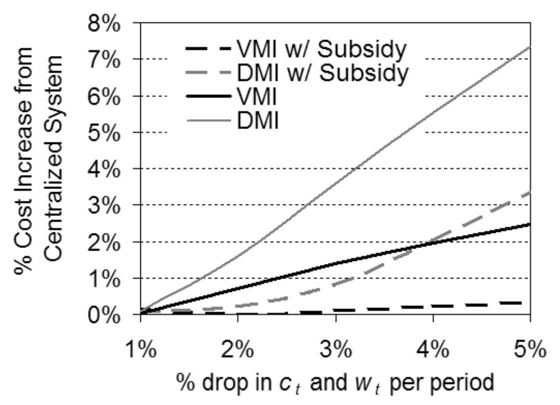
5.4 Effect of Risk Sharing

In this section, we explore the benefits of risk sharing. We allow for risk sharing through subsidies on the expediting costs, as explained in Section 4.3. Let us denote:

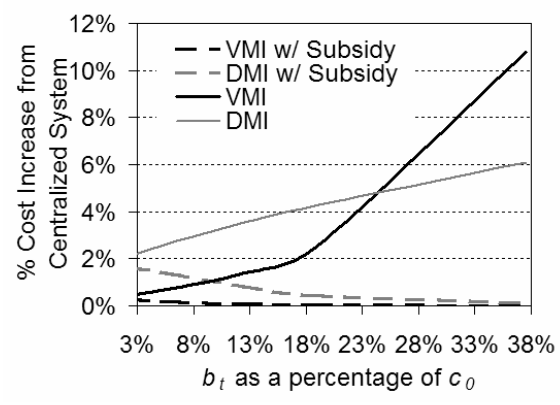
DMI with Subsidy: the manufacturer chooses the subsidy (given to the distributor) on expediting costs, to minimize his total cost.

VMI with Subsidy: the distributor chooses the subsidy on expediting costs (received from the manufacturer) to minimize his total cost.

By offering the distributor a subsidy on expediting costs, the penalty on shortages is increasing for the manufacturer and decreasing for the distributor. This counteracts the over-stocking under



(a) Performance of VMI and DMI under different rates of price drops in the presence of risk sharing



(b) Performance of VMI and DMI under different unit expediting costs in the presence of risk sharing

Figure 9: Effect of using risk sharing on the performance of VMI and DMI

DMI and under-stocking under VMI. Figure 9(a) shows that VMI with subsidy performs very close to the centralized system across all rates of price drops. While DMI is also improved when subsidies are offered, the costs are higher compared to VMI with subsidy.

Since, in our setting, the size of the subsidy is limited by cost of expediting, we look at the performance of the systems as a function of the expediting costs in Figure 9(b). While VMI with subsidy dominates DMI with subsidy, both policies perform relatively close to centralized system. As stated in the previous section, the expediting cost is a natural upper bound on the amount of risk sharing. When increased, it allows for more appropriate risk sharing choices.[10] VMI itself is not effective for very high costs of expediting (distributor incurs these costs and supplier does not attempt to sufficiently limit them.)

6 Conclusions

In this paper, we study a two-echelon non-stationary decentralized inventory model with price protection. Price protection is designed to protect distributors against price erosion, which is observed especially in industries with high technological obsolescence.

To the best of our knowledge, this paper is the only one that models price protection in a natural multi-period setting, reflecting the actual time limit for price protection, which is a critical element of currently used price-protection contracts. The pivotal element of the analysis is a reformulation of the problem that allows us to express the multi-period price-protection credit in an expected-

[10]While it is theoretically possible to implement subsidies higher than the expediting costs, the manufacturer paying more than the cost of expediting might lead to gaming behavior by the distributor.

cost form and incorporate it into a modified myopic problem. The myopic reformulations allow us to describe the ordering patterns of the decentralized policies as a function of the length of the price-protection period and of technological obsolescence. We show that the total cost is convex under DMI, which results in the optimal policy being a non-stationary base-stock policy.

We use the reformulated problem to illustrate the performance of decentralized policies in standard settings characterizing the high-technology industry. While the lack of price protection might lead to under-stocking, we found that distributors tend to over-stock under DMI with price protection, leading to excessive costs for the manufacturer driven by unnecessarily early, and therefore expensive, production. This is consistent with what we have observed in practice.

The length of the price protection period is shown to be a potentially effective method to coordinate the supply chain. However, its use in practice is extremely limited as different products and different suppliers would require a different length of price protection and because price protection terms are usually difficult to change. We therefore analyzed other contractual remedies, including VMI and risk sharing.

Our analysis of VMI benefits from the same type of myopic reformulation as used in the analysis of DMI. The use of VMI, however, is not without drawbacks. Specifically, all VMI contracts have the general tendency to "stuff" the channel. This tendency exists in price-protection environments as well, but is less acute (in some situations, setting upper bounds on inventory is still necessary). Our analysis of VMI shows that it is likely to result in under-stocking in the presence of price protection. Subsidies on expediting costs can coordinate the supply chain under both VMI and DMI. When choosing subsidies, to make them practical, we assign the decision right to the supply chain entity not in control of the ordering process (rather than assuming the presence of a central decision maker who would set them).

We numerically evaluate the performance of the policies using parameters that capture the nature and the scale of critical economic tradeoffs in the computer industry. Using the centralized system as the benchmark, we observed that VMI performs closer to the centralized system than DMI in terms of total supply chain costs for a broad range of parameters. Also, the use of VMI helps to reduce price protection expenses to the manufacturer. VMI is especially beneficial when technological obsolescence is high, shortages have a higher impact on the manufacturer, inventory carrying costs (other than technological obsolescence) are low and profit margins are low to moderate. Such situations typically characterize the computer industry. This along with the benefits of risk-pooling and better manufacturing planning makes VMI a suitable mechanism for inventory control in the computer industry. We also found that risk sharing (in the form of subsidies on expediting costs) helps both DMI and VMI, with the combination of VMI with risk sharing performing closer to the centralized system than DMI in terms of the total supply chain costs.

In situations where there is a higher impact of shortages on the distributor than the manufacturer, or where the inventory carrying costs (other than obsolescence costs) are high, or where obsolescence is low and prices are stable VMI was no better or worse than DMI. Moreover, VMI can lead to "channel stuffing" (although mitigated by price protection), especially in situations when the prices are stable.

Appendix

Proof of Theorem 1 The problem is equivalent to the case of backlogging. Cost of shortage could be re-assigned to the beginning of the following period (with the appropriate state and cost-function redefinition, with the starting inventory in the following period $y_t - d_t$). The penalties are the same, but the production decision is shifted to the next period. Clearly, it is optimal to immediately satisfy the backlog. Thus, the penalty for shortage would be g_t and the penalty for over-ordering would be $c_t - \beta E c_{t+1} + h_t$. For the equivalent backlogging case, standard proof shows that the optimal policy is a non-stationary base-stock policy. The derivative of the current-period value function w.r.t. y_t is:

$$W_t^{SC'}(y_t) = -g_t + [g_t + h_t + (c_t - \beta E c_{t+1})]\Phi_t(y_t)$$

and the myopic up-to level can then be obtained as (1) by setting $W_t^{SC'}(y_t)$ to zero. (1) is solution to Newsvendor problem with cost of underage $c_u = g_t$ and the cost of overage, c_o, composed of holding inventory h_t as well as decrease in value due to cost decreases and discounting, $c_t - \beta c_{t+1}$. ■

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