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Energy-Based Source Tracking and Motion Pattern Recognition Using Acoustic Sensor Networks

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Abstract

Acoustic sensor networks can be used for localization of an acoustic-energy emitting source. While maximum-likelihood (ML) methods are widely used for estimating the pattern of motion, more advanced machine learning schemes should be employed for improving the accuracy of localization. In this paper, we develop a learning Bayesian tracking algorithm that is capable of reconstructing the target transition model using passive wireless acoustic sensors. The adaptive scheme is intended to track targets that exhibit a complex motion pattern that cannot be adequately modeled prior to the implementation of the filtering algorithm. The derivation of the algorithm begins by modeling the likelihood assuming perfect knowledge of the sensors locations. Since this assumption is inadequate when the number of sensors is large, it is further relaxed by resorting to a probabilistic representation of the underlined locations. A Markov random field (MRF) model facilitates the implementation of an optimization method for estimating the unknown sensor locations. The convergence of this method is proven using the Kullback-Leibler divergence measure. Modeling the source path as a stochastic process yields a Bayesian localization filter. The filtering algorithm is rendered adaptive by incorporating a novel motion pattern recognition procedure based on the Baum-Welch (BW) algorithm, implemented when the target dynamics is inadequately modeled or completely unknown. Simulations show that the the tracking accuracy of the new adaptive algorithm outperforms the conventional ML scheme.

Index Terms

Sensor Network, Maximum Likelihood, Baum-Welch Algorithm, Expectation Maximization, Acoustic Energy.

I. Introduction

Wireless sensor networks (WSN) are distributed systems composed of static and/or dynamic devices that are capable of monitoring the environment and transmitting the gathered data using transceivers. WSN can be used for a variety of applications, such as wide-area surveillance, underwater acoustics, oil-field monitoring, geophysics and seismic monitoring [1], [2].

The design and analysis of WSN have two aspects: The micro aspect, that deals with the architecture of a single sensor, including the transceiver, battery, sensing device, microprocessor and logic; and the macro aspect, that deals with the the data collection and fusion processes, aimed at transforming a number of inter-communicating, cooperative sensors into a single virtual sensor. The current work focuses on the latter aspect.

The gathering of data using WSN can be performed using a variety of sensors. These include optical sensors, temperature and pressure sensors, and acoustic sensors. Since sensor networks are characterized by limited battery power, limited communication bandwidth and limited processing power, efficient cooperative signal processing methods must be developed. Acoustic sensors (microphones) are simple devices that constitute a potentially promising method for creating a virtual sensor that consumes very little power while requiring minimal communication bandwidth [3]. However, designing an accurate source localization algorithm using acoustic sensors is challenging, because of the limited sensing range of each microphone and the noisy nature of the source signature. In addition, sensor locations are often uncertain to some degree; thus, trilateration or time-difference-of-arrival methods, used for relative sensor localization, must be upgraded to account for this uncertainty.

WSN were employed for source localization in [3]–[5] using maximum likelihood (ML)-based optimization schemes. These methods, however, are intended for localizing a stationary source and may perform poorly in the case of moving targets owing to inefficient modeling of time correlations induced by the motion.

Bayesian target tracking and detection methods based on WSN observations were proposed in [6]–[10]. With no exception, all the algorithms presented by these works belong to the class of sequential Monte Carlo methods also known as particle filters (PFs). In particular, decentralized computation schemes were derived for improving the estimation performance and for reducing the computational burden. Particle filtering methods are well-known for their performance, flexibility and ease of implementation in a wide range of applications. These Bayesian methods solve for the optimal probability density function (pdf) of inherently nonlinear, non-Gaussian systems, which in turn makes them appealing in the WSN context. Thus, an energy-based source localization PF was derived in [6]. This algorithm assumes a simple linear transition model of the moving target while an approximation procedure is used to obtain the origin source energy. The decentralized particle filtering algorithms proposed in [7], [9], [10] exploit the sensor network's architecture for reducing computational load. These PFs tend to spread the associated computations over the network, thereby allowing parallel processing. In addition, much effort is devoted for reducing the computational load over sensor cliques [9].

In another work [11], a multi-target tracking scheme is derived based on a hidden Markov model (HMM) representation of the moving source dynamics. Following this, Viterbi decoding is employed for searching the most likely target states.

In this paper, we develop an adaptive Bayesian localization filter that uses passive wireless acoustic sensor readings for tracking a moving energy source of which the transition model is inaccurate or completely unknown. The derivation of the algorithm begins by modeling the likelihood pdf, which forms the core of the conventional ML methods, and proceeds with the development of the adaptive filtering method.

The energy source attenuation model is used for constructing an approximate likelihood function relating the noisy sensors' readings and the source location. Firstly, the derivation of the likelihood pdf assumes perfect knowledge of the sensors locations. Since this assumption is inadequate when the number of sensors is large, it is relaxed by resorting to a probabilistic representation of the locations. Following this approach, it is assumed that every sensor communicates with its nearest neighbors, providing them with a relative distance information. We subsequently use a Markov random field (MRF) to model the network's communication topology, which in turn facilitates the derivation of an expectation maximization (EM) scheme for estimating the unknown sensor locations. We prove the convergence of this optimization method using the Kullback-Leibler [12] divergence measure.

The second part of this work is devoted to the derivation of an adaptive Bayesian filtering algorithm that is capable of tracking a moving source of which the Markovian transition model is inaccurate or completely unknown. Thus, a motion recognition procedure is incorporated into the plain localization filter, thereby providing it with an online adaptive tracking capability. The resulting scheme yields the energy source location as well as an estimated transition probability kernel.

The source motion pattern is estimated using the Baum-Welch (BW) algorithm [13]. The BW procedure is an iterative batch algorithm aimed at estimating the parameters of a HMM. In particular, it is aimed at yielding the transition, emission and initial probabilities associated with the HMM and the corresponding observed data. Using the BW procedure, the estimated quantities are computed in an iterative manner using an analytic recipe (assuming Gaussian emission probabilities). This algorithm turns out to be very efficient when the HMM is specified by a large number of parameters (i.e., many transition states), thereby allowing adequate representation of complex learning models.

The main contributions of this work are therefore

1) Development of a learning Bayesian tracking algorithm capable of reconstructing the target unknown transition model. As opposed to other Bayesian algorithms, the method developed herein assumes no knowledge of the Markovian transition model describing the energy source movement. This adaptive scheme is intended to track targets that exhibit a complex motion pattern that cannot be adequately modeled prior to the implementation of the filtering algorithm. Motion learning is facilitated by the using a novel pattern recognition algorithm based on the BW mechanism. To the best of the authors' knowledge, this is the first application of the BW learning scheme for acoustic pattern recognition in the sensor networks field. It is shown in this work that the BW-based learning outperforms the conventional ML schemes.
2) Design of an iterative optimization procedure for localizing sensors with unknown initial locations over the entire network. The derived method is essentially an EM procedure aimed at maximizing a network auxiliary function. The convergence of the proposed method is proved using the Kullback-Leibler divergence measure.

This paper is organized as follows. The next section derives a likelihood pdf based on the energy source attenuation model. As part of this, a likelihood model that accounts for uncertainties in sensor locations is obtained. An optimization method for localizing sensors in large networks is then described. The Bayesian localization filter and its grid-based implementation are given in Section III following by a detailed derivation of the BW procedure and its implementation for motion estimation. Section V presents a numerical study in which the adaptive filtering algorithm is compared against the conventional ML scheme. Conclusions are given in the last section.

II. MAXIMUM-LIKELIHOOD-BASED SOURCE LOCALIZATION

In this section we present a maximum likelihood approach for solving the source localization problem. We obtain a likelihood expression relating the measurements and the energy source position while assuming that the sensor locations are known. This assumption is relaxed based on two approaches that utilize the network structure for communicating distance information between nodes.

Throughout this work, the following notational conventions are used. Random variables and their realizations are denoted by lower- and upper-case letters, respectively. If the random variable is denoted by a Greek letter, its corresponding realization is denoted by a different Greek letter.

A. Observation Model

Let $\{\theta^i\}_{i=1}^M$, $\theta^i \in \mathbb{R}^2$ be a coordinate set of M acoustic sensors comprising a network, $\mathcal{T} = \{1, \ldots, M\}$. Let also $X_k \in \mathbb{R}^2$ be the location of some acoustic source (energy source) at time k. The energy emitted at the source decays approximately proportionally to the square of the distance from the ith sensor's. This allows formulating a basic measurement equation that relates the ith sensor's reading with the source energy, S_k, that is [4],

$$y_k^i = g_i \frac{S_k}{\| \theta^i - X_k \|^\alpha} + n^i \tag{1}$$

where y_k^i, n^i and $g_i \in \mathbb{R}$ denote the ith sensor's reading, measurement noise random variable, and a proportionality constant, respectively. The value of the energy decay factor, α, is roughly 2.08 [4]. Furthermore, it is assumed that n^i, $i = 1, \ldots, M$ are statistically independent with some known probability density function (pdf) $n^i \sim p_{n^i}$.

The basic measurement model in Eq. (1) assumes the knowledge of the source energy, S_k. This assumption is relaxed by combining two distinct sensor readings, y_k^i and y_k^j, to yield

$$y_k^i = \frac{g_i}{g_j} \left(\frac{\| \theta^j - X_k \|}{\| \theta^i - X_k \|} \right)^\alpha y_k^j + v^{i,j} \tag{2}$$

where

$$v^{i,j} = n^i - \frac{g_i}{g_j} \left(\frac{\| \theta^j - X_k \|}{\| \theta^i - X_k \|} \right)^\alpha n^j \tag{3}$$

is the effective measurement noise. Notice that the measurement model thus derived renders the sensor readings statistically dependent. Moreover, this dependency is strongly affected by the distance between the sensor locations.

In what follows we derive two approximate likelihood expressions based on the model in Eq. (2) assuming either known or uncertain sensor locations. The obtained relations facilitate the application of an ML-based localization algorithm.

B. Approximate Likelihood Derivation

The model in Eq. (2) implies

$$p_{y_k^i \mid X_k, y_k^j}(Y_k^i \mid X_k, Y_k^j) = p_{v^{i,j}} \left(Y_k^i - \frac{g_i}{g_j} \left(\frac{\| \theta^j - X_k \|}{\| \theta^i - X_k \|} \right)^\alpha Y_k^j \right) \tag{4}$$

where the sensors locations act as parameters in Eq. (4). At this point it is assumed that any statistical dependency manifested by the effective measurement noise random variables $\{v^{i,j} \mid i \neq j\}$ is negligible. That in turn allows deriving a simplified expression for the likelihood of the source location given the complete measurement set.

Denote by $\mathscr{Z}_k = \{y_k^i\}_{i=1}^M$ and $Z_k = \{Y_k^i\}_{i=1}^M$ the set of all sensors measurements and its realization at time k, respectively. Assuming that the virtual edges $\{\{Y_k^i, Y_k^j\} \mid i, j \in \mathcal{T}\}$ form a tree with root $l \in \mathcal{T}$, the likelihood of X_k given Z_k is

$$p_{\mathscr{Z}_k|X_k}(Z_k \mid X_k) = p_{y_k^l|X_k}(Y_k^l \mid X_k) \prod_{i,j\in\mathcal{T}} p_{y_k^i|X_k, y_k^j}(Y_k^i \mid X_k, Y_k^j) \tag{5}$$

When M is sufficiently large, the contribution of the root term $p_{y_k^l|X_k}$ in Eq. (5) is negligible. Following this, Eqs. (5) and (4) yield

$$p_{\mathscr{Z}_k|X_k}(Z_k \mid X_k) \approx \prod_{i,j\in\mathcal{T}} p_{v^{i,j}} \left(Y_k^i - \frac{g_i}{g_j} \left(\frac{\| \theta^j - X_k \|}{\| \theta^i - X_k \|} \right)^\alpha Y_k^j \right) \tag{6}$$

where the pdf of $v^{i,j}$ is obtained using the convolution operator, as

$$p_{v^{i,j}}(V^{i,j}) = \int_{-\infty}^{+\infty} p_{n^i} \left(V^{i,j} + \frac{g_i}{g_j} \left(\frac{\| \theta^j - X_k \|}{\| \theta^i - X_k \|} \right)^\alpha N^j \right) p_{n^j} \left(\frac{g_j}{g_i} \left(\frac{\| \theta^i - X_k \|}{\| \theta^j - X_k \|} \right)^\alpha N^j \right) \mathrm{d}N^j \tag{7}$$

C. Uncertainty in Sensors Locations

The derivation of the preceding likelihood pdf assumes perfect knowledge of the sensors locations $\{\theta^i\}_{i=1}^M$. This underlying assumption is inadequate when the number of sensors is prominently large. In such cases, resorting to a probabilistic representation of the locations provides a promising solution. Following this approach, every sensor communicates with its nearest neighbors, providing them with relative distance information. It can be verified that within a sensor network consisting of at least 3 sensors whose locations are known, the ambiguity in determining the locations of the rest of the members can be resolved (see Fig. 1 and [14]).

Denoting by $d^{i,j}$ the relative distance between two neighboring sensors yields

$$\| \vartheta^i - \vartheta^j \| = d^{i,j} + w^{i,j} \tag{8}$$

where $\{\vartheta^i\}_{i=1}^M$ and $w^{i,j}$ denote the sensors locations random vectors (as opposed to the deterministic locations $\{\theta^i\}_{i=1}^M$) and the relative measurement noise, respectively. Eq. (8) allows formulating the pdf $p_{\vartheta^i|\vartheta^j}$ in terms of the (known) pdf of $w^{i,j}$, that is

$$p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j) = p_{w^{i,j}} \left(\| \theta^i - \theta^j \| - d^{i,j} \right) \tag{9}$$

Further assuming that $\{w^{i,j} \mid i \neq j\}$ are statistically independent allows writing the joint pdf of all sensor locations as a pairwise Markovian network $\{i, j\} \in \mathcal{T}$

$$p_{\vartheta^M, \vartheta^{M-1} \dots, \vartheta^1}(\theta^M, \theta^{M-1}, \dots, \theta^1) = \prod_{i,j\in\mathcal{T}} p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j) \tag{10}$$

where $p_{\vartheta^i|\vartheta^j}$ act as the clique potentials. As mentioned above, the knowledge of $\{\theta^j\}_{j=1}^3$ provides sufficient information for determining the rest of the sensors locations in the network. Considering the uncertainty introduced by the relative measurement noise, it can be deduced that in this case having at least 3 perfectly known locations renders the joint pdf in Eq. (10) sharply peaked [1].

[1]Assuming sufficiently low intensity noise $w^{i,j}$.

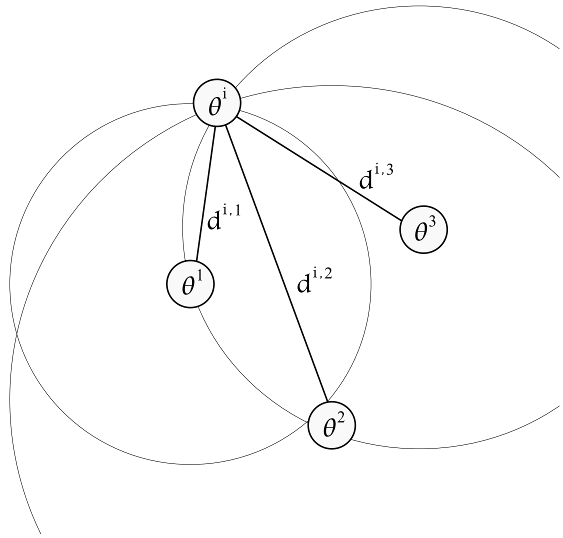


Fig. 1. The 4th sensor location θ^i is determined based on the locations of the other 3 sensors θ^1, θ^2 and θ^3.

The likelihood pdf in Eq. (5) can now be modified to account for sensor locations uncertainty. Thus,

$$\begin{aligned} p_{\mathscr{Z}_k|X_k}(Z_k \mid X_k) = \\ \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p_{\mathscr{Z}_k,\vartheta^M,\vartheta^{M-1},\ldots,\vartheta^4|X_k}(Z_k, \theta^M, \theta^{M-1}, \ldots, \theta^4 \mid X_k)\mathrm{d}\theta^M \mathrm{d}\theta^{M-1} \cdots \mathrm{d}\theta^4 \\ \approx \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \prod_{i,j\in\mathcal{T}} p_{y_k^i|X_k,y_k^j,\vartheta^i,\vartheta^j}(Y_k^i \mid X_k, Y_k^j, \theta^i, \theta^j) \times \\ \prod_{i,j\in\mathcal{T}} p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j)\mathrm{d}\theta^M \mathrm{d}\theta^{M-1} \cdots \mathrm{d}\theta^4 \\ \approx \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \prod_{i,j\in\mathcal{T}} p_{v^{i,j}} \left(Y_k^i - \frac{g_i}{g_j} \left(\frac{\| \theta^j - X_k \|}{\| \theta^i - X_k \|} \right)^\alpha Y_k^j \right) \times \\ \prod_{i,j\in\mathcal{T}} p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j)\mathrm{d}\theta^M \mathrm{d}\theta^{M-1} \cdots \mathrm{d}\theta^4 \end{aligned} \tag{11}$$

where the known locations $\{\theta^j\}_{j=1}^3$ act as parameters of $p_{\mathscr{Z}_k|X_k}(Z_k \mid X_k)$.

The likelihood expression in Eq. (11) relies on the potentials $p_{\vartheta^i|\vartheta^j}$, which form the network's communication topology. This formulation implicitly assumes that each node transmits its estimated location computed based on the position of its neighbors. In this setting, the transmitted signals carry the nodes' location (and possibly other classification parameters), so that the signals traveling time is proportional to the relative distance. The network's root consists of the fixed nodes of which the locations are perfectly known.

The above method suffers from the following shortcomings. Firstly, the likelihood in Eq. (11) may become computationally excessive as the number of sensors increases owing to the marginalization of θ^i over the entire network. Secondly, the energy consumption of individual nodes and thereby of the overall network may be intensive due to transmission of multiple identification messages consisting of the nodes' location and labeling information.

In light of the above, we suggest an alternative approach based on a rather different communication topology for estimating the unknown sensor locations.

D. Expectation Maximization for the Sensor Network

In this section we derive an efficient EM optimization procedure for localizing sensors within $\mathcal{T}$. The technique described herein assumes a unique structure of the sensor network, which alleviates the previously-mentioned drawbacks.

Network's Communication Topology: Suppose that the sensor network $\mathcal{T}$ consist of three types of nodes: 1) Passive (or unlabeled) nodes. These nodes do not transmit any information (i.e., location and labeling) but rather reflect the transmitted signals from other nodes. It is assumed that the location of the passive nodes are perfectly known. 2) Observed nodes. These nodes have their transmitted signals reflected by the passive nodes. In other words, the observed nodes acquire relative distance measurements with respect to the unlabeled sources (i.e., the passive nodes). 3) Unobserved nodes. These nodes do not share a direct communication link with the passive nodes.

Now, let $\theta = \{\theta^i\}_{i \in \mathcal{T}}$ denote the collection of all observed and unobserved node locations. Let also $\{m \in \mathcal{T}\}$ be the subset of observed nodes of which a likelihood function $p_{y^m|\vartheta^m,w}$ is specified, where y^m and w denote the measurement acquired by m and its passive node (unknown) labeling parameter, respectively. In this context, w is a nuisance variable used to associate measurements with their origin (passive) nodes. We further assume that each measurement y^m is a set of relative distances computed based on reflected signals from L distinct passive nodes (see Fig. 2). The labeling parameters $\{w_1, \ldots, w_L\} \in w$ associated with y^m satisfy $q_w(W) = \prod_{t=1}^{L} q_{w_t}(W_t)$, where $q_{w_t}(W_t)$ denotes the class probability of w_t, that is, $q_{w_i}(j)$ represents the probability of the ith signal to be reflected by the jth passive node (see Fig. 2). In order to illustrate this construction, consider a case in which the relative distance measured by m with respect to the ith passive node satisfy

$$d_i^m = \| \theta^m - \theta_i^p \| + r^m \tag{12}$$

where θ_i^p denotes the ith passive node (known) location and $r^m \sim p_{r^m}$ is the measurement noise. Further assuming there are exactly n_p passive nodes, yields

$$p_{y^m|\vartheta^m,w}(Y^m \mid \theta^m, W) = \prod_{j=1}^{L} \sum_{i=1}^{n_p} p_{r^m} \left(d_i^m - \| \theta^m - \theta_i^p \|\right) \delta(W_j - i) \tag{13}$$

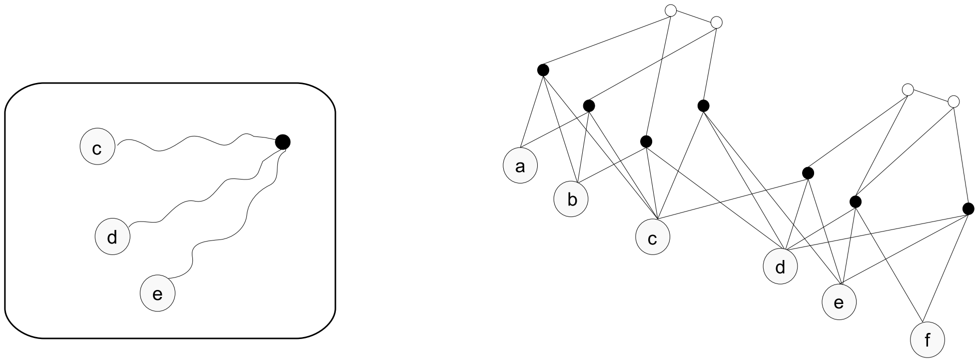
where $\delta(\cdot)$ denotes the Dirac delta measure.

Following the above and assuming, as before, that the cliques are restricted to pair of nodes that are directly linked (see Eq. (9)), the network $\mathcal{T}$ can be represented via a pairwise Markov random field (MRF) model

$$p_{\vartheta|y,w}(\theta \mid Y, W) = c(Y, W)^{-1} \prod_{i,j \in \mathcal{T}} p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j) \prod_{m \in \mathcal{T}} p_{y^m|\vartheta^m,w}(Y^m \mid \theta^m, W) \tag{14}$$

where $y = \{y^m\}_{m \in \mathcal{T}}$ denotes the entire measurement set and $p_{\vartheta^i|\vartheta^j}$, $\{i, j \in \mathcal{T}\}$ are the clique potentials.

The network MRF model in Eq. (14) facilitates the derivation of an iterative optimization scheme for maximizing $p_{\vartheta|y}(\theta \mid Y)$. This method is detailed below.



(a) A packet of $L = 3$ reflected signals from the passive nodes c, d and e forms the unlabeled measurement set y^m.

(b) Communication topology

Fig. 2. An example of sensor network communication topology. Showing the passive nodes a, b, c, d, e and f, the observed nodes (bold circles) and the unobserved nodes (empty circles). In this network each measurement y^m is constructed based on $L = 3$ reflected signals.

Network Auxiliary Function: Let us define an auxiliary function over the network $\mathcal{T}$

$$F(q_w, \theta, Y) = \int q_w(W) \log \frac{\prod_{i,j \in \mathcal{T}} p_{\vartheta^i | \vartheta^j}(\theta^i \mid \theta^j) \prod_{m \in \mathcal{T}} p_{y^m | \vartheta^m, w}(Y^m \mid \theta^m, W)}{q_w(W)} \mathrm{d}W \tag{15}$$

Proposition 1: Using the above, the following iteration for $\mathcal{T}$:

$$q_w^{n+1} = \arg\max_{q_w} F(q_w^n, \theta_n, Y) \tag{16a}$$

$$\theta_{n+1} = \arg\max_{\theta} F(q_w^{n+1}, \theta_n, Y) \tag{16b}$$

constitutes an EM recursion that converges to a local maximum of $p_{\vartheta|y}(\theta \mid Y)$.

Proof: Defining $w/w_k = \{w_i \mid w_i \in w, \quad i \neq k\}$ yields

$$F(q_w, \theta, Y) = \int q_{w/w_k}(W/W_k) q_{w_k}(W_k) \log \left[\frac{p_{\vartheta|y,w}(\theta \mid Y, W)}{q_{w/w_k}(W/W_k) q_{w_k}(W_k)} \right] \mathrm{d}W_k \mathrm{d}(W/W_k)$$

$$= \int q_{w/w_k}(W/W_k) q_{w_k}(W_k) \log \left[\frac{p_{\vartheta|y,w}(\theta \mid Y, W)}{q_{w_k}(W_k)} \right] \mathrm{d}W_k \mathrm{d}(W/W_k)$$

$$- \int q_{w/w_k}(W/W_k) \log q_{w/w_k}(W/W_k) \mathrm{d}(W/W_k)$$

$$= \int q_{w/w_k}(W/W_k) q_{w_k}(W_k) \log \left[\sum_{i,j \in \mathcal{T}} \log p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j) + \sum_{m \in \mathcal{T}} \log p_{y^m|\vartheta^m,w}(Y^m \mid \theta^m, W) \right.$$

$$\left. - \log q_{w_k}(W_k) \right] \mathrm{d}W_k \mathrm{d}(W/W_k) - \int q_{w|w_k}(W/W_k) \log q_{w/w_k}(W/W_k) \mathrm{d}(W/W_k)$$

$$= \sum_{i,j \in \mathcal{T}} \log p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j)$$

$$+ \int q_{w_k}(W_k) \left[\sum_{m \in \mathcal{T}} \int q_{w/w_k}(W/W_k) \log p_{y^m|\vartheta^m,w}(Y^m \mid \theta^m, W) \mathrm{d}(W/W_k) \right] \mathrm{d}W_k$$

$$- \int q_{w_k}(W_k) \log q_{w_k}(W_k) \mathrm{d}W_k - \int q_{w/w_k}(W/W_K) \log q_{w/w_k}(W/W_K) \mathrm{d}(W/W_k)$$

$$= \sum_{i,j \in \mathcal{T}} \log p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j) + \int q_{w_k}(W_k) \log \frac{\bar{p}(Y \mid \theta, W_k)}{q_{w_k}(W_k)} \mathrm{d}W_k$$

$$- \int q_{w/w_k}(W/W_k) \log q_{w/w_k}(W/W_k) \mathrm{d}(W/W_k) \quad (17)$$

where

$$\log \bar{p}(Y \mid \theta, W_k) = \sum_{m \in \mathcal{T}} \int q_{w/w_k}(W/W_k) \log p_{y^m|\vartheta^m,w}(Y^m \mid \theta^m, W) \mathrm{d}(W/W_k) \quad (18)$$

Now, letting

$$g(W_k \mid Y, \theta) = b(Y, \theta)^{-1} \bar{p}(Y \mid \theta, W_k) \quad (19)$$

where

$$b(Y, \theta) = \int \bar{p}(Y \mid \theta, W_k) \mathrm{d}W_k \quad (20)$$

yields

$$F(q_w, \theta, Y) = \sum_{i,j \in \mathcal{T}} \log p_{\vartheta^i|\vartheta^j}(\theta^i \mid \theta^j) - KL\left(q_{w_k}(W_k) \parallel g(W_k \mid Y, \theta)\right)$$

$$+ \log b(Y, \theta) - \int q_{w/w_k}(W/W_k) \log q_{w/w_k}(W/W_k) \mathrm{d}(W/W_k) \quad (21)$$

where KL denotes the Kullback-Leibler distance.[2] From the above it easily follows that

$$q_{w_k}^{n+1}(W_k) = g(W_k \mid Y, \theta) = \frac{\bar{p}(Y \mid \theta, W_k)}{\int \bar{p}(Y \mid \theta, W_k) \mathrm{d}W_k} \tag{22}$$

which constitutes an E-step (see Eq. (18)). Notice that the optimal class probabilities $q_{w_k}^{n+1}$ are exclusively evaluated over the observed nodes $\{m \in \mathcal{T}\}$. The new estimate $q_w^{n+1} = \prod_{t=1}^{L} q_{w_t}^{n+1}$ is then used in the M-step for obtaining θ_{n+1} which follows from Eq. (16b). ■

The EM recursion in Proposition 1 can be used to obtain estimates of θ over the entire network based on the knowledge of the passive nodes fixed locations and of the data y.

E. Maximum Likelihood Estimate

The energy source location can be now obtained as the ML estimate based on either approximate likelihood pdfs derived in Eqs. (6) (assuming the sensor locations are perfectly known or estimated using Proposition 1) and (11). At every time step the ML estimate is given as

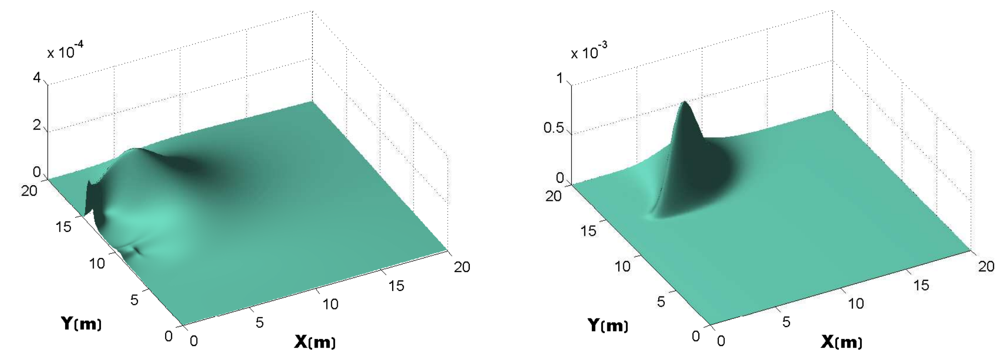
$$\hat{X}_k^{ML} = \arg\max_X p_{\mathscr{Z}_k \mid X_k}(Z_k \mid X) \tag{23}$$

The likelihood map of $p_{\mathscr{Z}_k \mid X_k}$ for various sensor networks is illustrated in Fig. 3 over a 20 m^2 region. The role of the amount of nodes used in the network is perfectly demonstrated as the likelihood pdf becomes sharply peaked owing to an increasing number of sensors.

In practice, the ML estimate can be obtained using either parametric or non-parametric optimization methods. An extensive survey of parametric methods for solving Eq. (23) is given in [4]. In the simulation section of this work the likelihood $p_{\mathscr{Z}_k \mid x_k}$ is computed over a discrete grid for obtaining $\hat{X}_k^{ML}$.

[2]The Kullback-Leibler distance [12] is a most frequently used information-theoretic "distance" measure. If p_0, p_1 are two probability densities, the Kullback- Leibler distance is defined to be

$$KL(p_1 \parallel p_0) = \int p_1(x) \log \frac{p_1(x)}{p_0(x)} \mathrm{d}x$$



(a) 10 Sensors

(b) 20 Sensors

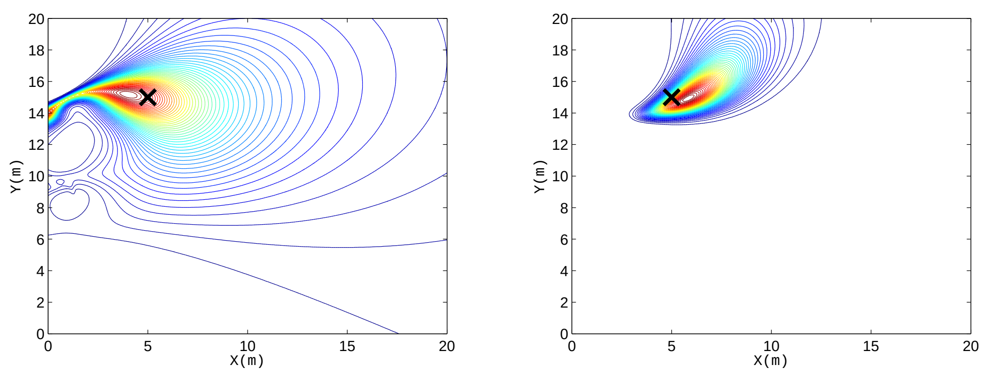


Fig. 3. Likelihood pdfs and their corresponding level curves of 10 and 20 sensor networks. The actual target location is cross marked in the level curves figures.

III. Bayesian Tracking Using the Sensor Network

The ML localization method performs poorly when applied for tracking a dynamic energy source. This is manifested by a rather noisy behavior of the algorithm owing to inefficient modeling of time correlations. In that case, improved tracking performance is achieved by implementing a filtering scheme that assumes a probabilistic transition model of the target motion.

Considering the energy source path to be a stochastic process leads to a Bayesian formulation of the localization filter. Let $\{x_k\}_{k=0}^{\infty}$ be a random process representing the energy source trajectory with some initial pdf $p_{x_0}(X_0)$. Assuming that the motion is characterized by a Markovian transition kernel $p_{x_k|x_{k-1}}(X_k \mid X_{k-1})$ facilitates the implementation of the conventional Bayesian filtering recursion, that is [15], [16]

$$p_{x_k|\mathscr{Z}^{k-1}}(X_k \mid Z^{k-1}) = \int_{-\infty}^{+\infty} p_{x_k|x_{k-1}}(X_k \mid X_{k-1}) p_{x_{k-1}|\mathscr{Z}^{k-1}}(X_{k-1} \mid Z^{k-1}) \mathrm{d}X_{k-1} \tag{24a}$$

$$p_{x_k|\mathscr{Z}^{k}}(X_k \mid Z^{k}) = \frac{p_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) p_{x_k|\mathscr{Z}^{k-1}}(X_k \mid Z^{k-1})}{\int_{-\infty}^{+\infty} p_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) p_{x_k|\mathscr{Z}^{k-1}}(X_k \mid Z^{k-1}) \mathrm{d}X_k} \tag{24b}$$

where $\mathscr{Z}^k = \{\mathscr{Z}_1, \ldots, \mathscr{Z}_k\}$ and $Z^k = \{Z_1, \ldots, Z_k\}$ denote the measurements time history and its realization up to time k, respectively.

A. Exact Grid-Based Filter

When the energy source location can take a finite number of states it is convenient to implement a grid-based filter for numerically solving Eqs. (24) [17]. In such cases, the obtained solution is an exact one. In other cases, this approach can be implemented for approximating the Bayesian solution of continuous state space models.

Having a discrete sample space, the pdfs constituting the Bayesian recursion in Eqs. (24) are replaced by probability mass functions (pmf) [18]. Thus, an equivalent discrete version of this recursion is given by

$$f_{x_k|\mathscr{Z}^{k-1}}(X_k \mid Z^{k-1}) = \sum_{X_{k-1} \in \mathbb{X}_{k-1}} f_{x_k|x_{k-1}}(X_k \mid X_{k-1}) f_{x_{k-1}|\mathscr{Z}^{k-1}}(X_{k-1} \mid Z^{k-1}) \tag{25a}$$

$$f_{x_k|\mathscr{Z}^{k}}(X_k \mid Z^{k}) = \frac{f_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) f_{x_k|\mathscr{Z}^{k-1}}(X_k \mid Z^{k-1})}{\sum_{X_k \in \mathbb{X}_k} f_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) f_{x_k|\mathscr{Z}^{k-1}}(X_k \mid Z^{k-1})} \tag{25b}$$

where the pmf is defined as $f_{a|b}(A \mid B) = \Pr(a = A \mid b = B)$ and $\mathbb{X}_{k-1}$ denotes the set of all possible outcomes of x_{k-1}. Recalling that

$$f_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) = p_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) \mathrm{d}\mu \tag{26}$$

for some σ-finite measure μ, the pmf $f_{\mathscr{Z}_k|x_k}(Z_k \mid X_k)$ can be replaced by the conventional likelihood pdf $p_{\mathscr{Z}_k|x_k}(Z_k \mid X_k)$ (which was previously derived) in Eq. (25b). The minimum mean square error (MMSE) estimate of x_k is then obtained as

$$\hat{X}_k^{MMSE} = \sum_{X_k \in \mathbb{X}_{k-1}} X_k f_{x_k|\mathscr{Z}^{k}}(X_k \mid Z^{k}) \tag{27}$$

IV. Motion Pattern Recognition

This part of the work is concerned with the implementation of the Bayesian tracking filter of the preceding section in cases where the energy source motion model is inaccurate or completely unknown. In such situations the sensor network can be further utilized for reconstructing the motion pattern using the Baum-Welch algorithm [13].

The motion pattern recognition procedure derived herein is incorporated into the Bayesian filter, thus providing it with an online adaptive tracking capability. The resulting scheme yields the energy source location as well as an estimated transition probability kernel $\hat{f}_{x_k|x_{k-1}}(X_k \mid X_{k-1})$.

A. Transition Model

For simplicity and mathematical tractability, the following assumptions are made: 1) the energy source motion model can be described via a homogeneous Markov chain, and 2) the finite set of possible movements is independent of the location. The former assumption allows the proposed learning algorithm to infer the motion pattern by processing observations gained over a time interval whereas the latter allows significant reduction of the search space (i.e. of possible movements).

Let Ω be the set of N_s possible movements of the energy source independently of its location. This set is constructed out of subsets of an optional movement label (name) and a specific associated action. In this work, the following set which describes $N_s = 9$ possible movements is considered

$$\Omega = \left\{ \begin{array}{ccc} \{\text{'Left-Up'}, [-1,+1]^T\}, & \{\text{'Up'}, [0,+1]^T\}, & \{\text{'Right-Up'}, [+1,+1]^T\}, \\ \{\text{'Left'}, [-1,0]^T\}, & \{\text{'Stay'}, [0,0]^T\}, & \{\text{'Right'}, [+1,0]^T\}, \\ \{\text{'Left-Down'}, [-1,-1]^T\}, & \{\text{'Down'}, [0,-1]^T\}, & \{\text{'Right-Down'}, [+1,-1]^T\} \end{array} \right\} \tag{28}$$

The actions associated with each movement are described by a vector whose elements are the number of steps along each axis. Because the movement is independent of the location, the transition kernel in this case is given as

$$f_{x_k|x_{k-1}}(X_k \mid X_{k-1}) = \Pr(u_k = X_k - X_{k-1}) \tag{29}$$

where the possible outcomes of the random variable u_k are the actions contained in the set Ω. The transition model associated with Ω is further illustrated in Fig. 4.

B. Likelihood Derivation

The derivation of the likelihood associated with each action u_k yields two expressions for two different purposes. The first likelihood expression takes into account the immediate past of the measurements which in turn renders the motion learning algorithm independent of the Bayesian filter outcome (i.e., the posterior pmf). This allows the motion reconstruction procedure to be executed prior to the application of the tracking filter and therefore it is most suitable for initialization purposes. The second likelihood expression, however, takes into account the measurements time history and is adequate for online motion estimation.

Recognizing that given U_k

$$f_{\mathscr{Z}_k|x_k}(Z_k \mid X_k) = f_{\mathscr{Z}_k|x_k}(Z_k \mid X_{k-1} + U_k) = f_{\mathscr{Z}_k|x_{k-1},u_k}(Z_k \mid X_{k-1}, U_k) \tag{30}$$

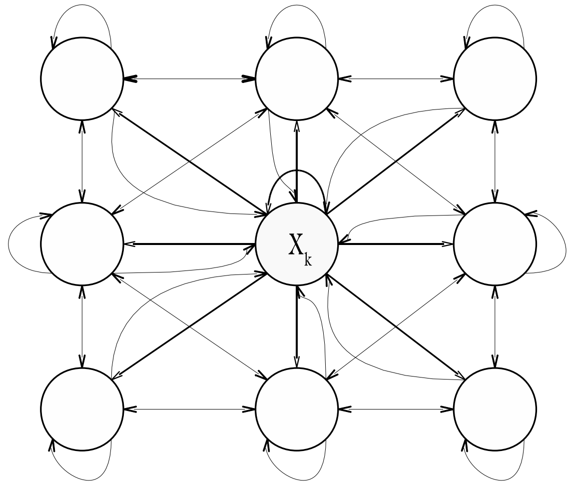


Fig. 4. The Markovian motion model. Showing the 9 possible actions (bold arrows) at location X_k. The empty circles represent all possible locations at time step $k+1$.

the first likelihood expression is obtained by merging two consecutive measurements $\mathscr{Z}_k$ and $\mathscr{Z}_{k-1}$ to yield

$$
\begin{aligned}
f_{\mathscr{Z}_k,\mathscr{Z}_{k-1}|u_k}(Z_k, Z_{k-1} \mid U_k) &= \sum_{X_{k-1}\in\mathbb{X}_{k-1}} f_{\mathscr{Z}_k,\mathscr{Z}_{k-1},x_{k-1}|u_k}(Z_k, Z_{k-1}, X_{k-1} \mid U_k) \\
&= \sum_{X_{k-1}\in\mathbb{X}_{k-1}} f_{\mathscr{Z}_k|x_{k-1},u_k}(Z_k \mid X_{k-1}, U_k) f_{\mathscr{Z}_{k-1}|x_{k-1}}(Z_{k-1} \mid X_{k-1}) f_{x_{k-1}}(X_{k-1}) \\
&= \sum_{X_{k-1}\in\mathbb{X}_{k-1}} f_{\mathscr{Z}_k|x_k}(Z_k \mid X_{k-1}+U_k) f_{\mathscr{Z}_{k-1}|x_{k-1}}(Z_{k-1} \mid X_{k-1}) f_{x_{k-1}}(X_{k-1}) \quad (31)
\end{aligned}
$$

where the prior $f_{x_{k-1}}(X_{k-1})$ is assumed to be a uniform pmf (since the transition kernel is unknown).

The expression of the second likelihood function is derived as follows

$$
\begin{aligned}
f_{\mathscr{Z}^k|u_k}(Z^k \mid U_k) &= f_{\mathscr{Z}_k,\mathscr{Z}_{k-1}|u_k,\mathscr{Z}^{k-2}}(Z_k, Z_{k-1} \mid U_k, Z^{k-2}) f_{\mathscr{Z}^{k-2}}(Z^{k-2}) \\
&\propto f_{\mathscr{Z}_k,\mathscr{Z}_{k-1}|u_k,\mathscr{Z}^{k-2}}(Z_k, Z_{k-1} \mid U_k, Z^{k-2}) \\
&= \sum_{X_{k-1}\in\mathbb{X}_{k-1}} f_{\mathscr{Z}_k,\mathscr{Z}_{k-1},x_{k-1}|u_k,\mathscr{Z}^{k-2}}(Z_k, Z_{k-1}, X_{k-1} \mid U_k, Z^{k-2}) \\
&= \sum_{X_{k-1}\in\mathbb{X}_{k-1}} f_{\mathscr{Z}_k|x_{k-1},u_k}(Z_k \mid X_{k-1}, U_k) f_{\mathscr{Z}_{k-1}|x_{k-1}}(Z_{k-1} \mid X_{k-1}) f_{x_{k-1}|\mathscr{Z}^{k-2}}(X_{k-1} \mid Z^{k-2}) \\
&= \sum_{X_{k-1}\in\mathbb{X}_{k-1}} f_{\mathscr{Z}_k|x_k}(Z_k \mid X_{k-1} + U_k) f_{\mathscr{Z}_{k-1}|x_{k-1}}(Z_{k-1} \mid X_{k-1}) f_{x_{k-1}|\mathscr{Z}^{k-2}}(X_{k-1} \mid Z^{k-2})
\end{aligned} \tag{32}
$$

Observing Eqs. (31) and (32), it can be recognized that the prior pmf term in Eq. (31) is replaced by the propagated pmf $f_{x_{k-1}|\mathscr{Z}^{k-2}}(X_{k-1} \mid Z^{k-2})$ computed by the Bayesian filter.

C. Motion Pattern Reconstruction Using the Baum-Welch Algorithm

The transition kernel in Eq. (29) is estimated using the BW algorithm. The BW is an iterative batch algorithm aimed at estimating the parameters of a HMM. In particular, it is aimed at yielding the transition, emission and initial probabilities associated with the HMM and the corresponding observed data. Using the BW procedure, the estimated quantities are computed in an iterative manner using an analytic recipe (assuming Gaussian emission probabilities). This algorithm turns out to be extremely efficient when the HMM is specified by a large number of parameters (i.e., consisting of many transition states), thereby allowing adequate representation of complex learning models.

In this work it is assumed that the sensors noise pdfs are perfectly known, therefore the BW algorithm is applied for the estimation of $f_{x_k|x_{k-1}}(X_k \mid X_{k-1})$ exclusively. The derivation proceeds as follows. Let $\mathscr{G}_k = \{\mathscr{Z}_k, \mathscr{Z}_{k-1}\}$ and $G_k = \{Z_k, Z_{k-1}\}$. Let also $\{\mathscr{G}_l\}_{l=1}^{N_m}$ and $\{G_l\}_{l=1}^{N_m}$ be a measurement batch and its realization, respectively. The transition between two actions u_{k-1} and u_k is described by the stochastic matrix

$$
A = \{a_{i,j}\} \in \mathbb{R}^{N_s \times N_s}, \quad a_{i,j} = \Pr(u_k = \Omega(j) \mid u_{k-1} = \Omega(i)) \tag{33}
$$

where $\Omega(i)$ denotes the ith action in the set of all possible movements Ω. The first-order Markovian transition model, $f_{x_k|x_{k-1}}(X_k \mid X_{k-1})$, implies that the actions $\{u_i\}_{i=1}^{k}$ performed at every time step are all independent. Therefore,

$$
\Pr(u_k = \Omega(j) \mid u_{k-1} = \Omega(i)) = \Pr(u_k = \Omega(j)) \tag{34}
$$

Eqs. (33) and (34) imply

$$
\Pr(u_k = \Omega(j)) = \frac{1}{N_s} \sum_{i=1}^{N_s} a_{i,j} \tag{35}
$$

From Eqs. (29) and (35) it follows that

$$
f_{x_k|x_{k-1}}(X_k \mid X_{k-1}) = \frac{1}{N_s} \sum_{j=1}^{N_s} \sum_{i=1}^{N_s} a_{i,j} \delta(\Omega(j), X_k - X_{k-1}) \tag{36}
$$

where $\delta(i, j)$ denotes the Kronecker delta. Eq. (36) implies that the stochastic matrix A completely specifies the transition kernel of the moving source. In what follows, a single BW iteration, aimed at estimating the elements of A, is described.

The Baum-Welch Iteration: The so-called forward and backward probabilities, denoted by $\beta_i^F(l)$ and $\beta_i^B(l)$, are defined for every $i = 1, \ldots, N_s$ and $l = 1, \ldots, N_m$, as

$$\beta_i^F(l) = \Pr(\mathscr{G}_1 = G_1, \ldots, \mathscr{G}_l = G_l, u_l = \Omega(i)) \tag{37a}$$

$$\beta_i^B(l) = \Pr(\mathscr{G}_{l+1} = G_{l+1}, \ldots, \mathscr{G}_{N_m} = G_{N_m} \mid u_l = \Omega(i)) \tag{37b}$$

These probabilities are computed recursively assuming the knowledge of $\hat{A}$, the previous-time estimate of the transition matrix. Thus,

$$\beta_i^F(l+1) = \mathscr{L}(G_{l+1}, \Omega(i)) \sum_{j=1}^{N_s} \hat{a}_{j,i} \beta_j^F(l) \tag{38a}$$

$$\beta_i^B(l) = \sum_{j=1}^{N_s} \hat{a}_{i,j} \mathscr{L}(G_{l+1}, \Omega(j)) \beta_j^B(l+1) \tag{38b}$$

where

$$\beta_i^F(1) = \frac{1}{N_s} \sum_{j=1}^{N_s} \hat{a}_{j,i} \mathscr{L}(G_1, \Omega(i)), \quad \beta_i^B(N_m) = 1 \tag{39}$$

and $\mathscr{L}(G_l, \Omega(i))$ is either one of the likelihoods in Eqs. (31) and (32) [3]. Now, it can be verified that

$$\gamma_i(l) = \Pr(u_l = \Omega(i) \mid \{\mathscr{G}_t = G_t\}_{t=1}^{N_m}) = \frac{\beta_i^F(l) \beta_i^B(l)}{\sum_{j=1}^{N_s} \beta_j^F(l) \beta_j^B(l)} \tag{40a}$$

$$\begin{aligned} \xi_{i,j}(l) &= \Pr(u_l = \Omega(i), u_{l+1} = \Omega(j) \mid \{\mathscr{G}_t = G_t\}_{t=1}^{N_m}) \\ &= \frac{\beta_i^F(l) \beta_j^B(l+1) \hat{a}_{i,j} \mathscr{L}(G_{l+1}, \Omega(j))}{\sum_{r=1}^{N_s} \sum_{t=1}^{N_s} \beta_r^F(l) \beta_t^B(l+1) \hat{a}_{r,t} \mathscr{L}(G_{l+1}, \Omega(t))} \end{aligned} \tag{40b}$$

Recalling the definition of $a_{i,j}$, the following re-estimation formula is obtained using Eqs. (40)

$$\hat{a}_{i,j} = \frac{\sum_{l=1}^{N_m - 1} \xi_{i,j}(l)}{\sum_{l=1}^{N_m - 1} \gamma_i(l)} \tag{41}$$

This estimate is then used in the next time step carried out following Eqs. (38) - (41).

Following the algorithm's convergence, Eqs. (36) and (41) are used to yield the energy source estimated transition kernel, that is

$$\begin{aligned} \hat{f}_{x_k \mid x_{k-1}}(X_k \mid X_{k-1}) &= \frac{1}{N_s} \sum_{j=1}^{N_s} \sum_{i=1}^{N_s} \hat{a}_{i,j} \delta(\Omega(j), X_k - X_{k-1}) \\ &= \frac{1}{N_s} \sum_{j=1}^{N_s} \sum_{i=1}^{N_s} \frac{\sum_{l=1}^{N_m - 1} \xi_{i,j}(l)}{\sum_{l=1}^{N_m - 1} \gamma_i(l)} \delta(\Omega(j), X_k - X_{k-1}) \end{aligned} \tag{42}$$

[3]For instance, if the likelihood in Eq. (31) is used then $\mathscr{L}(G_l, \Omega(i)) = f_{\mathscr{Z}_l, \mathscr{Z}_{l-1} \mid u_l}(Z_l, Z_{l-1} \mid \Omega(i))$.

Practical Implementation: The BW algorithm is a special case of the EM algorithm. As such its convergence is sensitive to initial conditions. For that reason it is suggested that the estimated quantities obtained using this algorithm will be averaged upon several runs. Following this approach, the stochastic matrix $\hat{A}$ used for reconstructing the transition kernel in Eq. (42) is replaced by $\hat{A}^* = \frac{1}{N_B}\sum_{i=1}^{N_B} \hat{A}(i)$, where $\hat{A}(i)$ denotes the ith outcome of the BW algorithm applied to a single measurement batch realization out of as many as N_B such realizations.

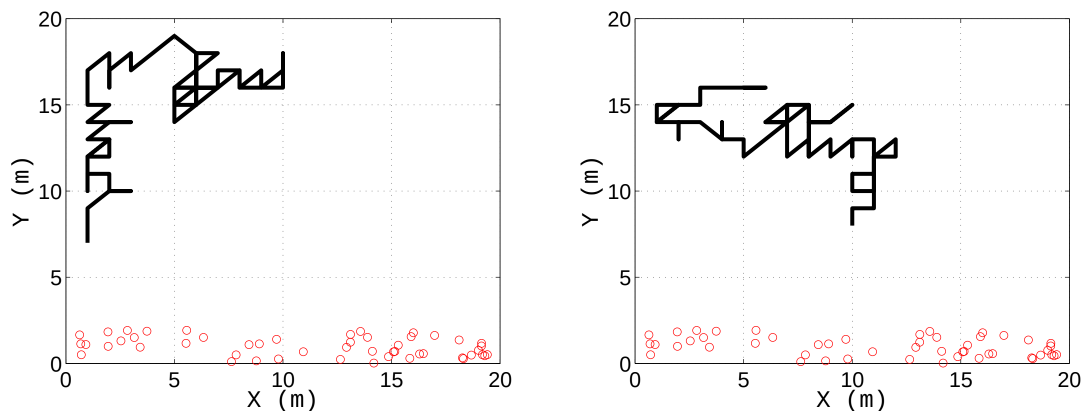
V. Numerical Study

The tracking algorithms derived in the preceding sections are tested numerically using a network consisting of 50 acoustic sensors. The sensors are spread uniformly in a 20×2 meters region. The noise associated with each sensor is modeled by a zero-mean Gaussian random variable with standard deviation of 10^{-3}.

The energy source movement is modeled by a Markov chain bounded in a 20×20 meters region. The probabilities associated with the various actions/movements are set arbitrarily as

$$\Pr(\text{Action}) = \left\{ \begin{array}{ll} \text{Action = 'Up-Left'}, & 0.05 \\ \text{Action = 'Up'}, & 0.1 \\ \text{Action = 'Up-Right'}, & 0 \\ \text{Action = 'Left'}, & 0.11 \\ \text{Action = 'Stay'}, & 0.40 \\ \text{Action = 'Right'}, & 0.20 \\ \text{Action = 'Down-Left'}, & 0 \\ \text{Action = 'Down'}, & 0.09 \\ \text{Action = 'Down-Right'}, & 0.05 \end{array} \right\} \tag{43}$$

The stochastic motion of the energy source is exemplified by two distinct path samples illustrated in Fig. 5.



(a) Sample 1 (b) Sample 2

Fig. 5. Two samples of the energy source path (consisting of 50 steps). Circles represent sensor locations.

The Bayesian grid-based tracking filter sample space consists of $20 \times 20 = 400$ possible states. That is, each 1 m^2 rectangle in the total region is mapped as a possible location of the energy source. In all runs, the initial location of the energy source is sampled from a uniform distribution over the total region. Correspondingly, the prior pmf of the tracking filter is set as $p_{x_0}(X_0) = 1/400$.

The motion recognition algorithm is used for providing the tracking filter with the estimate of the transition kernel. The likelihood function used by the BW algorithm is the one in Eq. (31). The BW algorithm itself is implemented for $N_B = 100$ measurement batches each consisting of $N_m = 50$ measurements. A single BW iteration is used for each measurement batch. Following the argumentation in Section IV-C, the estimated transition kernel is computed using the matrix $\hat{A}^*$.

The performance of the motion recognition algorithm is demonstrated in Fig. 6. In this figure, both true and estimated transition kernels are represented by a probabilistic decision wheel. It can be seen that the BW algorithm manages to capture the source motion as the largest estimation error is approximately 4% (in both the 'Right' and 'Down' actions).

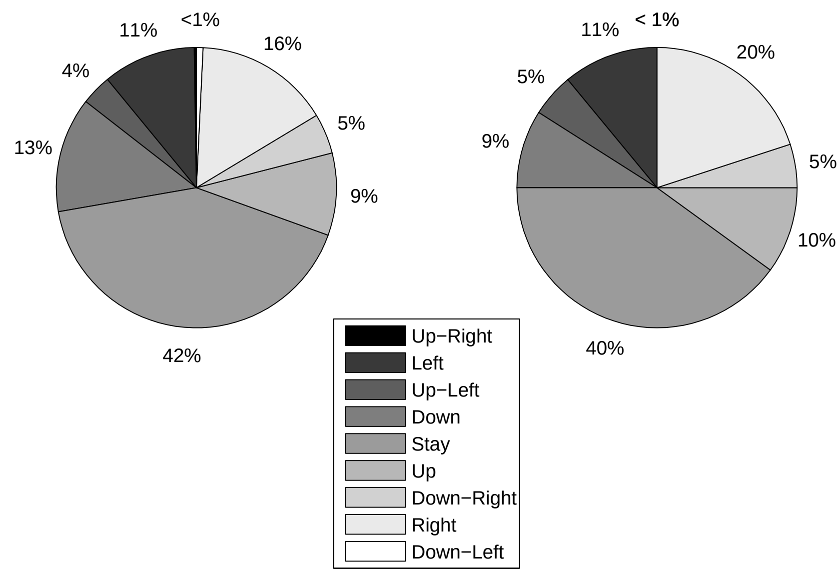


Fig. 6. True (right) and estimated (left) energy source decision wheels (transition probabilities). Estimated transition kernel constructed based on $N_B = 100$ batches, each consisting of $N_m = 50$ measurements.

Figure 7 depicts the cumulative distribution function of the mean norm estimation error of both the Bayesian tracking filter (which is fed with the transition kernel estimate obtained by the motion recognition algorithm) and the ML estimator derived in Sections III-A and II, respectively. The performance measure used here (i.e., the mean norm estimation error) is defined as

$$e = \frac{1}{T} \sum_{k=1}^{T} \| \, x_k - \hat{x}_k \, \| \tag{44}$$

where $\hat{x}_k$ is either the MMSE or ML estimates. The distribution is constructed based on 500 Monte Carlo runs, each consisting of $T = 50$ time steps. The advantage of using the Bayesian filter over the ML estimator is clearly manifested in this figure. It can be recognized that

$$\Pr(e^{ML} < 2.3 \text{ m }) = 0.9, \quad \Pr(e^{MMSE} < 1.5 \text{ m }) = 0.9 \tag{45}$$

which reads as follows: The mean norm position estimation error of the ML estimator is less than 2.3 meters in 90% of the runs whereas the same measure is less than 1.5 meters when using the Bayesian filter.

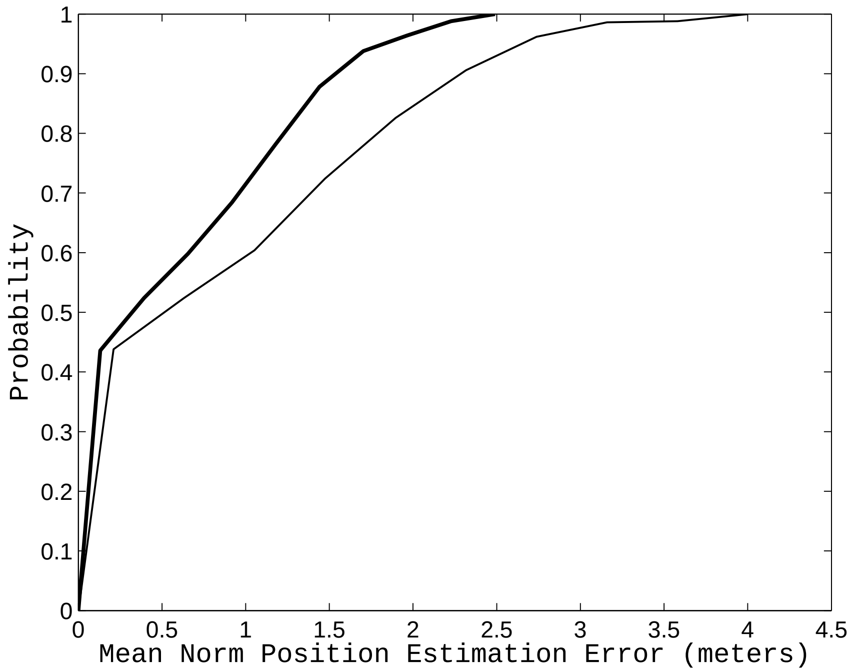


Fig. 7. Mean norm position estimation error cumulative distribution function of the Bayesian grid filter equipped with the motion pattern recognition algorithm (thick line) and of the ML estimation scheme (thin line). 500 Monte Carlo runs.

VI. Conclusions

An efficient method for tracking and motion pattern recognition of an acoustic source using sensor networks was developed. The new algorithm is a Bayesian filter equipped with a novel motion pattern recognition scheme based on the Baum-Welch (BW) mechanism. The BW procedure uses the sensor readings to reconstruct the unknown Markovian transition kernel, which is then fed into the localization filter. The new filtering algorithm is implemented using an exact grid-based scheme. The results of a Monte Carlo simulation are shown in which the adaptive algorithm outperforms the conventional ML method.

In addition, an iterative optimization scheme was derived for estimating unknown sensor locations in prominently large networks. The suggested method is essentially an expectation maximization recursion aimed at maximizing a network auxiliary function, which is composed based on the underlying communication topology. The convergence of this method was proven using the Kullback-Leibler divergence measure.

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