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New Substitution Policies for Assemble-to-Order Systems with Exogenous Inventory Control

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1 Introduction

Several companies struggle with the problem of synchronizing supply and demand due to the dynamic nature of supply/demand processes. Product substitution is a common method that is used by companies such as IBM, Hewlett-Packard and Dell to match customer demand with available supply by steering customers towards product configurations that can be supplied easily and profitably. For example, consider the web portal shown in Figure 1 for a distributor of computers (details of the client are withheld for confidentiality reasons). In this real-life example, customers contact

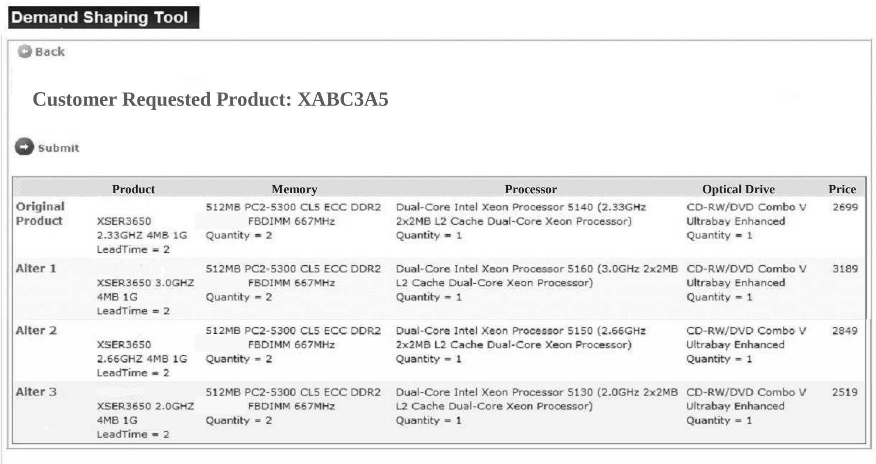

Figure 1: Example of a demand shaping portal

the sales representatives of a company in order to purchase computers. The sales representative uses the portal to identify product alternatives that should be offered based on their availability and profitability and suggests those to the customer in order to shape demand. Such an approach exploits the flexibility of customer demand to help improve the day-to-day supply chain operations of a company. In this paper, we propose policies for identifying viable product alternatives that will maximize a company's profit by mitigating the effects of supply-demand imbalances.

We study an Assemble-to-Order system with exogenous inventory control over a horizon. The bill-of-materials of the system that we study is characterized by *components* and *products*. A product is offered to a customer by assembling its components after the customer arrives. For each product, the demand over the horizon is random. We do not make any assumptions on the arrival process except that no two customers arrive at the same epoch during the horizon. It is known a priori which products are substitutable - each product is associated with a set of alternative products that can be used as substitutes when a customer demand occurs for that product. While this assumption may be restrictive, we note that several customer choice models satisfy this property (for example, the Lancaster choice model). Revenue is generated based on the products that are sold. Holding costs are incurred for component inventory that is left over at the end of the horizon. Shortage penalties are incurred on unsatisfied customer demands. The problem is to determine which alternative products to offer, if any, at every epoch when a customer arrives in order to maximize the expected profits over the horizon.

Most of the literature on assemble-to-order systems, with or without substitution, is focused on identification of optimal inventory control policies (see, for example, Hale et al. (2009) and Hsu et al. (2006)). Iravani et al. (2003) develop a quasi-birth-and-death process to obtain the performance measures of an assemble-to-order system where a customer order consists of a mix of items that are required to complete the sale and items that are optional. In contrast to the literature, our work is focused on development of policies that determine alternative products to offer with exogenous inventory control. Ervolina et al. (2009) study a similar problem, but using deterministic models.

In this paper, we propose a class of policies for Assemble-To-Order systems, which we refer to as *rationing* policies. For an Assemble-To-Order system with m products, this policy class is characterized by m^2 parameters, $\theta_{jk} \geq 0$ such that $\sum_{j=1}^{m} \theta_{jk} = 1$ for every k. According to this policy, when a customer of class k arrives, product j is offered with probability θ_{jk}. Since the optimal policy within the class of Rationing policies is difficult to characterize, we develop two optimization models that are upper and lower bounds on the optimal profit of the Assemble-To-Order system. The lower bound is formulated as a non-linear program, which we show to be concave. The optimal solutions of these two programs are used as parameters for the rationing policies. The bounds are also useful to benchmark the performance of the heuristics. Our numerical results indicate that the rationing policies are complementary over the range of problem parameters, indicating that we can gain significant benefits in profit by using the better of the two policies. The rationing policy developed based on the lower bound solution performs better than that developed based on the upper bound solution when the available component supply increases, revenue differences across products are low to moderate, supply imbalances towards high end components are high and when the demand variability is low to moderate and vice versa. The rationing policies also provide good performance compared to the upper bound and also provide significant improvements over the situation where we do not use product substitution.

The remainder of the paper is structured as follows. In section 2, we provide the notation and problem setting. In section 3, we present the models for the lower bound and upper bound along with some structural results. We also provide a discussion on efficiently identifying the parameters of the Rationing policies for Assemble-To-Order systems in Section 3. In section 4, we numerically investigate the performance of our policies and present our conclusions.

2 Notation and Problem Setting

Consider a periodically reviewed assemble-to-order backordering system with n components and m products. We use $1, 2, ..., n$ to refer to the components and $1, 2, ..., m$ to refer to the products. We use i to denote a generic component and and j and k to denote a generic product. Let the matrix, $\mathbf{v}$ be the bill-of-materials, i.e., $\forall\ i,j,\ v_{ij}=1$ if component i is used to produce product j and $v_{ij}=0$ otherwise.

We now discuss the costs incurred and the revenues generated for our system. Let r_j be the revenue obtained by selling product j. Let h_i and b_j be the holding and backordering cost parameters for component i and product j respectively. The reader may note that there are no holding costs associated with product j as no products are stocked. We will use H_j to denote the virtual holding cost of product j, i.e., $H_j=\sum_{i=1}^{n} h_i \cdot v_{ij}$.

As stated before, all inventory control is exogenous during the horizon. The initial inventory for component i is y_i. There are no replenishment epochs during the horizon. Let A_j be the set of products that may be used as substitutes for product j. We refer to a customer whose first preference is product j as a class j customer. Let D_j be the demand of product j, i.e., the number of the class j customers. We use d_j to denote a sample path of demands for the j^{th} product. Let $\phi(.)$ be the joint probability distribution function of $D_1, D_2, .., D_m$. We *do not* make assumptions on the correlation of the demands of different products.

We use ATO to denote the Assemble-To-Order system and $\mathcal{C}^*(ATO)$ to denote the optimal profit of running ATO. When a customer of class j arrives, the manager of ATO decides what product to offer. We define a policy θ to be a decision rule that determines the product offered at every epoch that a customer arrives, given the state of the system and customer class.

3 Rationing Policies

In this section, we develop two policies within the class of Rationing policies. For deriving the first policy, we construct an Assemble-To-Stock system where the products are assembled (and thus, the product quantities decided) at the beginning of the horizon before demand is realized. We denote this system as ATL. For deriving the second policy, we construct an Assemble-To-Order system where the allocation of components to products is decided after all the demand is realized. we denote this system as ATU.

We first present the ATL system. We formulate the optimal substitution rule for the ATL system as a non-linear optimization program with linear constraints. In order to formulate the program, we introduce some additional notation. Let x_{jk} be the number of units of product j that are reserved for product k demand. Let π_{jk} be the fraction of product k demand satisfied by product j. Thus, the problem of determining the optimal substitution rule for ATL may be written as

$$\begin{aligned} Z_l^* = \max_{\mathbf{x},\pi} Z = \quad E \quad & [\sum_{j=1}^{m} r_j \sum_{k=1}^{m} min\{x_{jk}, \pi_{jk} \cdot D_k\} - \sum_{k=1}^{m} b_k \sum_{j=1}^{m} (\pi_{jk} \cdot D_k - x_{jk})^+ \\ & - \sum_{i=1}^{n} h_i \cdot (y_i - \sum_{j=i}^{m} \sum_{k=1}^{m} v_{ij} x_{jk}) - \sum_{j=1}^{m} H_j \sum_{k=1}^{m} (\pi_{jk} \cdot D_k - x_{jk})^-] \qquad (1) \end{aligned}$$

subject to

$$\sum_{j=i}^{m}\sum_{k=1}^{m} v_{ij}x_{jk} \leq y_i \tag{2}$$

$$\sum_{k=1}^{m} \pi_{jk} = 1 \tag{3}$$

$$\pi_{jk} = x_{jk} = 0 \ \forall \ k \notin A_j \tag{4}$$

$$\pi_{jk}, x_{jk} \geq 0. \tag{5}$$

The objective function given in 1 models the expected profit of the system and has four components: the revenue from selling products, the penalty due to backorders and two penalties due to excess components (one at the component level and the other at the product level). Constraints 2 capture the restrictions due to available supply of components, constraints 3 capture the demand allocation requirement, constraints 4 capture restrictions on substitutability of products and constraints 5 capture the non-negativity requirements.

Theorem 1. *The optimal profit of the assemble-to-order system, $\mathcal{C}^*(ATO)$ is bounded below by the optimal profit of* ATL*, i.e.,*

$$\mathcal{C}^*(ATO) \geq Z_l^*.$$

The proof for Theorem 1 is based on standard sample path arguments and has been omitted.

Lemma 1. *The objective function of the optimal profit of the* ATL *system, Z is jointly concave in* $\mathbf{x}$ *and* π*.*

The proof for Lemma 1 is omitted due to space restrictions. As stated earlier, the constraints of the formulation of *ATL* are linear in the decision variables, $\mathbf{x}$ and π. Further, since the objective function is jointly concave in $\mathbf{x}$ and π, the lower bound in Theorem 1 can be efficiently evaluated. We also derive some parametric monotonicity properties of Z_l^*. These results provide insight on the structure of the *ATL* system. Discussion of Lemma 2 is omitted due to space constraints.

Lemma 2. *Z_l^* is supermodular in*
(i) $\mathbf{r}, \mathbf{H}, \mathbf{b}$ *and* $\mathbf{x}$*,*
(ii) $\mathbf{r}, \mathbf{H}$ *and* π*.*

We next present the formulation of the *ATU* system. Let w_{jk} be the number of units of product j that are used to satisfy product k demand. Given the demands for each product, $d_1, d_2, ..., d_m$, the optimal substitution rule of the *ATU* system may be formulated as a linear program.

$$Z_u^* = max_{\mathbf{c},\mathbf{z}} \sum_{j=1}^{m} r_j \cdot \sum_{k=1}^{m} w_{jk} - \sum_{k=1}^{m} b_k \cdot (d_k - \sum_{j=1}^{m} w_{jk}) - \sum_{i=1}^{n} h_i \cdot (y_i - \sum_{j=1}^{m} v_{ij} \sum_{k=1}^{m} \cdot w_{jk}) \tag{6}$$

subject to

$$\begin{aligned}
\sum_{j=1}^{m} w_{jk} &\leq d_k && (7)\\
\sum_{j=1}^{m} v_{ij} \sum_{k=1}^{m} \cdot w_{jk} &\leq y_i && (8)\\
w_{jk} &= 0 \; \forall \; k \notin A_j && (9)\\
w_{jk} &\geq 0. && (10)
\end{aligned}$$

The objective function given in 6 maximizes the profit and comprises three components: revenue generated by selling products, backorder penalties for not satisfying demand excess component inventory holding costs. Constraints 7 capture demand allocation, constraints 8 capture supply restrictions, constraints 9 capture restrictions on substitutions and constraints 10 capture non-negativity constraints.

Theorem 2. [1] *The optimal profit of the assemble-to-order system, $\mathcal{C}^*(ATO)$ is bounded above by the optimal profit of* ATU, *i.e.,*

$$\mathcal{C}^*(ATO) \leq \int Z_u^* \phi(\mathbf{x}) dx.$$

The proof for Theorem 2 has been omitted. Based on the optimal solutions of ATL and ATU, we suggest two heuristic policies, i.e., $\theta_{jk} = \pi_{jk}$ (RL) and $\theta_{jk} = \frac{w_{jk}}{\sum_{k=1}^{m} w_{jk}}$ (RU) respectively.

4 Numerical Results

In this section, we evaluate the performance of the RL and RU rationing policies. We also provide the benefit of using these policies over the case of no substitution (NS) for the set of problem instances that we consider. The upper bound on $\mathcal{C}^*(\mathcal{ATO})$ developed enables us to provide an upper bound on the optimality gap of the policies. We also characterize the range of problem parameters under which one rationing policy outperforms the other. We consider a 3-product, 4-component system shown in Figure 2 along with the prices, holding costs and shortage penalties.

We test our policies over 162 problem instances by varying the following: *Price Skew*, *Demand coefficient of variation (C.V.)*, *Component Stock Fractile* and *Supply Skew*. The parameter values are provided in Table 1.

Table 1: Problem Parameters for the Numerical Study

Price Skew ($\frac{r_3-r_1}{r_1}$)	C.V.	Component Stock Fractile	Supply Skew ($\frac{y_4-y_1}{y_1}$)
60%,80%, 100%	30%, 40%, $\cdots$, 80%	75%, 85%, 95%	30%, 40%, 50%

We observed that both RU and RL are within 14% of the upper bound on average (RL has a worst-case gap of 18% and RU has a worst case gap of 16%), indicating good performance since the

[1] This theorem is stated for continuous demand variables. An analogous result may be stated for discrete demands.

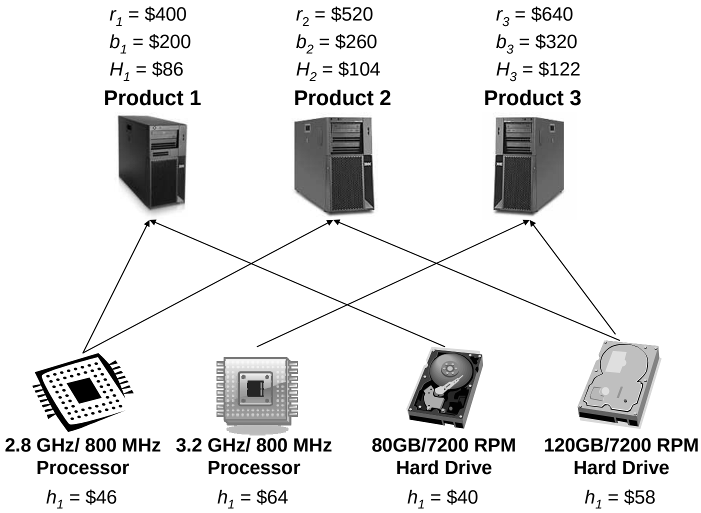

Figure 2: Problem setting for computational experiments

upper bounds are based on allocation after demands are realized. We also observed that RL and RU provide significant improvements on the NS policy - both provide average profit improvement of 10%, RL provides maximum improvement of 24% and RU provides maximum profit improvement of 26%. However, as we will see below the policies are complementary over the range of problem parameters.

In order to identify the situations under which the RL is better than RU and vice versa, we compared them in terms of the expected profit across the various factors we experimented with. Figure 4 shows the results. We observed that RL performs better than RU when the available component supply increases, revenue differences across products are low to moderate, supply imbalances towards high end components are high and when the demand variability is low to moderate and vice versa. This indicates that RL and RU are complementary over the range of problem parameters, indicating that we can gain significant benefits in profit by using the better of the two policies.

5 References

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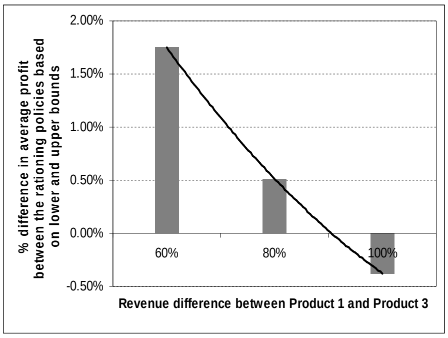
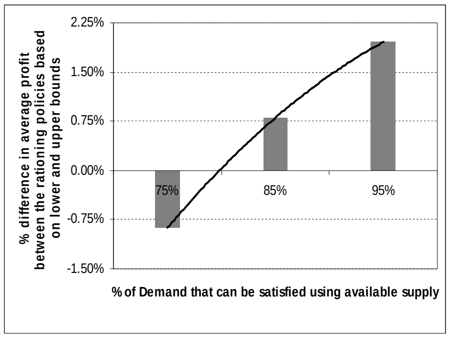
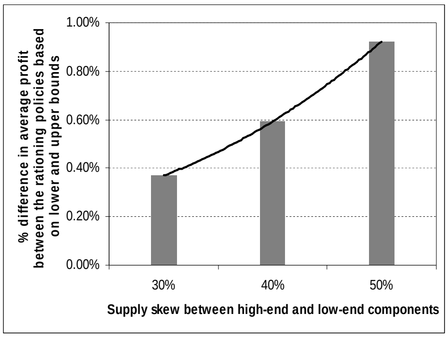


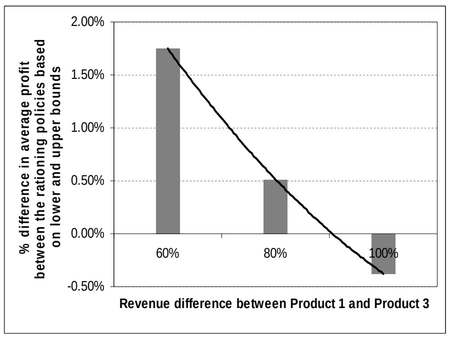
(a) Impact of revenue difference



(b) Impact of available supply



(c) Impact of supply skew between high-end and low-end components



(d) Impact of coefficient of variation of demand

Figure 3: % difference in average profit between RL and RU: (RL-RU)/RL