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Gate Capacitance of Cylindrical Nanowires with Elliptical Cross-Sections

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We investigate gate capacitance of cylindrical nanowires with elliptical cross-sections because most fabricated Si nanowires have elliptical cross sections. We derive an exact result for the capacitance of confocal elliptical capacitors and an approximate expression for the capacitance of conformal elliptical capacitors. Using numerical simulations for conformal elliptical capacitors, we show that the analytical results for the confocal and conformal elliptical capacitors are within 5 % of the numerical values for eccentricity < 0.85. We also provide correction factors to the analytical results that match the numerical conformal elliptical capacitances to within 5 % for all values of eccentricity.

For over a decade, there has been a significant amount of experimental research interest in cylindrical nanowire (NW) geometry for fabricating metal oxide semiconductor field effect transistors (MOSFETs).[1-10] This is because the NW geometry provides better gate control over the channel region of a short-channel MOSFET than any other MOSFET geometry.[11-13] An interesting consequence of NW fabrication is that the NW cross-sections are generally elliptical and not circular.[9] Furthermore, the gate oxide is uniform across the perimeter, implying that the geometry is that of conformal ellipses. Since the capacitance of a MOSFET is an important parameter that not only determines on-state current but also affects the short-channel characteristics,[11-13] theoretical work on the capacitance of NWs with confocal and conformal elliptical cross-sections is required to understand experimental data from NWs with elliptical cross-sections.

In this letter, we investigate the capacitance of cylindrical NWs with elliptical cross-sections using analytical and numerical methods. We derive an exact analytical expression for the capacitance of confocal elliptical capacitors and an approximate result for the capacitance of conformal elliptical capacitors. We use numerical simulations for conformal capacitors to show that the analytical capacitance expressions are within 5 % of the numerical capacitances for eccentricity < 0.85. We also show that the differences between the analytical and numerical results increase sharply for eccentricity > 0.85. Therefore, we provide correction factors to the analytical results that match the numerical capacitances to within 5 % for all values of eccentricity.

The elliptical NW geometry is modeled as follows. We assume that the inner and outer conductors have elliptical cross-sections. The semi-major and semi-minor axes are denoted as a_{in} and b_{in} for the inner conductor, and a_{out} and b_{out} for the outer conductor. The eccentricity of the inner conductor is given by $e_{\text{in}} = \sqrt{1 - b_{\text{in}}^2 / a_{\text{in}}^2}$ and that of the outer conductor by $e_{\text{out}} = \sqrt{1 - b_{\text{out}}^2 / a_{\text{out}}^2}$. We define the elliptical coordinates (μ,ν) as $x = k \cosh(\mu) \cos(\nu)$ and $y = k \sinh(\mu) \sin(\nu)$, where (x,y) are Cartesian coordinates and $x = \pm k$ are the location of the foci along the x-axis. One should note that surfaces with constant μ are ellipses while those with constant ν are hyperbolae.

We first present exact analytical results for the capacitance of confocal elliptical capacitors, where the inner and outer conductors have the same value of k. For confocal elliptical capacitors, of the four geometrical parameters a_{in}, b_{in}, a_{out}, and b_{out}, only three are free parameters because $k^2 = a_{\text{in}}^2 - b_{\text{in}}^2 = a_{\text{out}}^2 - b_{\text{out}}^2$. One should note that confocal ellipses are not conformal, with the exception being the special case of a circle. This is because the distance between the inner and outer conductors of a confocal capacitor, denoted by T_{OX}, is a function of angle ν and is given by $T_{\text{OX}}(\nu) = \sqrt{(a_{\text{out}} - a_{\text{in}})^2 \cos^2 \nu + (b_{\text{out}} - b_{\text{in}})^2 \sin^2 \nu}$. Given that the inner and outer conductors with constant potentials are ellipses, equipotential surfaces are ellipses while electric field lines are along the hyperbolae. Therefore, the electrostatic potential is $\phi = \phi(\mu)$ and the electric field is $\vec{E} = E_\mu \, \hat{\mu} = -(1/h) d\phi/d\mu \, \hat{\mu}$, where E_μ is the component of electric field along the μ direction, $\hat{\mu}$ is the unit vector along the μ direction, and $h = k\sqrt{\sinh^2 \mu + \sin^2 \nu}$ is the scale factor for coordinate transformation.[14] Using either elliptical symmetry or conformal mapping[15], one can evaluate ϕ and E_μ, which are given by $\phi(\mu) = \phi(\mu_{\text{in}}) + (\lambda/2\pi\varepsilon_{\text{OX}})(\mu - \mu_{\text{in}})$ and $E_\mu(\mu,\nu) = \lambda \big/ 2\pi\varepsilon_{\text{OX}} k \sqrt{\sinh^2 \mu + \sin^2 \nu}$, respectively, where μ_{in} describes the inner conductor, λ is the linear charge density, and ε_{OX} is the permittivity of the dielectric between the conductors.

Having obtained the potential and electric-field profiles, it is straightforward to calculate capacitance. The capacitance per unit length of cylindrical conductors with confocal elliptical cross-sections, denoted as C_{L1}, is given by

$$C_{\text{L1}} = \frac{2\pi\varepsilon_{\text{OX}}}{\ln\left(\dfrac{a_{\text{out}} + b_{\text{out}}}{a_{\text{in}} + b_{\text{in}}}\right)}. \qquad (1)$$

The circular cross-section limit with $a_{\text{in}} = b_{\text{in}} = R$ and $a_{\text{out}} = b_{\text{out}} = R + T_{\text{OX}}$, where R is the radius of the circle, leads to $C_L = 2\pi\varepsilon_{\text{OX}} / \ln(1 + T_{\text{OX}}/R)$, which is the correct limit for cylindrical capacitors with circular cross-sections.[16]

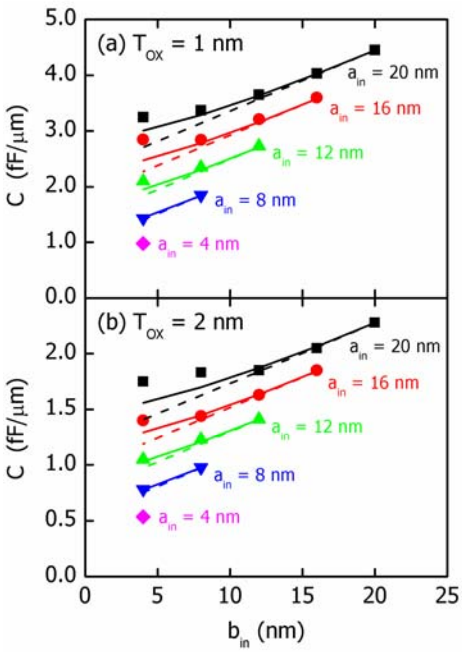

FIG. 1. Capacitance per unit length C_L of conformal elliptical conductors versus semi-minor axis of inner conductor b_{in} for different values of semi-major axis of inner conductor a_{in}, and dielectric thickness T_{OX} = 1 nm (a) and 2 nm (b). Symbols: numerical simulations, dashed lines: Eq. (1), and solid lines: Eq. (3).

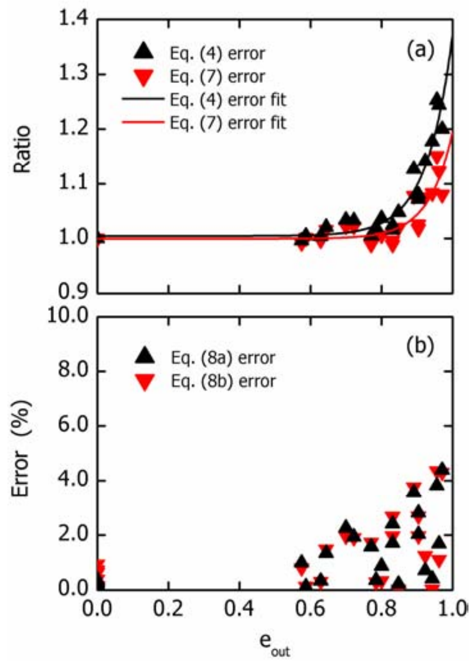

FIG. 2. (a) Ratio of numerically-calculated capacitance per unit length to analytical capacitance per unit length from Eq. (1) (up triangles) and Eq. (3) (down triangles) of conformal elliptical capacitors versus eccentricity e_{out} of the outer conductor. The symbols represent data from Fig. 1 while the lines are least-square fits to the data given by Eqs. (1) and (3). (b) Error between calculated capacitance per unit length of conformal elliptical capacitors and corrected analytical capacitance $C_{L1}f_{C1}$ and $C_{L2}f_{C2}$ versus e_{out}, where C_{L1}, C_{L2}, f_{C1}, and f_{C2} are given by Eqs. (1), (3), (4a), and (4b), respectively.

We next investigate the planar limit with $a_{in} >> b_{in}$ and $a_{in} >> T_{OX}$. In this case, $a_{out} \approx a_{in}$ and $b_{out} \approx \sqrt{2T_{OX}a_{in}}$, and therefore, Eq. (1) reduces to $C_{L1} \approx \sqrt{2}\pi\varepsilon_{OX}\sqrt{a_{in}/T_{OX}}$. For a planar geometry, the relevant quantity is capacitance per unit area, which we define as $C_A = C_L / P$, where P is the perimeter of the ellipse. The perimeter of an ellipse is $P = 4 a_{in} E(e_{in})$, where $E(e)$ is the complete elliptic integral of second kind. Since $E(e)$ has no closed form solution, we use the following well-known approximate form:[17]

$$P = \pi[3(a_{in} + b_{in}) - \sqrt{(3a_{in} + b_{in})(a_{in} + 3b_{in})}\,]. \qquad (2)$$

For $a_{in} >> b_{in}$, $P \approx \pi\sqrt{3}(\sqrt{3}-1)a_{in}$. Since T_{OX} is a function of ν for con-focal ellipses, we define an effective oxide thickness by $T_{OXE} = (2/\pi)\int_0^{\pi/2} T_{OX}(\nu)\,d\nu$. The integral in this equation is also a complete elliptic integral of the second kind, which can be approximated by the result of Ref. 17. For $a_{in} >> b_{in}$, we obtain $T_{OXE} \approx \sqrt{3}(\sqrt{3}-1)\sqrt{T_{OX}a_{in}/2}$. Therefore, in the planar limit, we obtain $C_A \approx \varepsilon_{OX} / T_{OXE}$, which is the correct value for planar capacitance upon properly defining the effective oxide thickness.

As mentioned previously, the geometry of fabricated NWs shows that the inner and outer conductors have a constant distance between them, that is, the dielectric thickness T_{OX} is constant. This implies that the inner and outer conductors in fabricated NWs are conformal ellipses. Conformal ellipses have separate foci and therefore, are not confocal. Thus, one cannot use conformal mapping or elliptical symmetry to analytically calculate the capacitance of conformal elliptical conductors. Therefore, we present a simple approximate analytical result for the capacitance of conformal elliptical conductors. The idea is to use C_L of capacitors with circular cross-section with radius R being dependent on ν and given by $R(\nu) = a_{in}\sqrt{1 - e_{in}^2 \sin^2\nu}$. That is, an approximate value of conformal elliptical capacitance, denoted by C_{L2}, is $C_{L2} = 4\varepsilon_{OX}\int_0^{\pi/2} d\nu / \ln(1 + T_{OX}/R(\nu))$. In order to obtain a closed-form solution, we expand the integrand to second order in $T_{OX}/R(\nu)$. Then C_{L2} is given by

$$C_{L2} \approx \varepsilon_{OX}\left(\frac{P}{T_{OX}} + \pi\right), \qquad (3)$$

where P is given by Eq. (2).

We now discuss the results of numerical modeling of the capacitance of conformal elliptical conductors and examine the validity of Eqs. (1) and (3) for the conformal case. We performed numerical simulations using Synopsis Sentaurus Device simulator.[18] We simulated technologically relevant conformal elliptical structures with a_{in} and b_{in} in the 4 to 20 nm range, and T_{OX} = 1 and 2 nm. For conformal elliptical conductors, we define $a_{out} = a_{in} + T_{OX}$ and $b_{out} = b_{in} + T_{OX}$. We show the numerically-computed values of C_L of conformal elliptical capacitors versus b_{in} as symbols in Fig. 1 for different values of a_{in}, and T_{OX} = 1 and 2 nm. We also show C_{L1} [Eq.(1)] and C_{L2} [Eq.(3)] for these structures in Fig. 1 as lines. One should note that even though Eq. (1) was derived for confocal elliptic conductors, one can always use it to calculate C_L of conformal elliptic conductors.

Figure 1 shows that both Eq. (1) and Eq. (3) are in good agreement with the numerical capacitance for $b_{in}/a_{in} > 0.5$ and that these equations lead to significant error for $b_{in} << a_{in}$. We quantify this error in terms of eccentricity e_{out} of the outer conductor. We plot the ratio of numerical C_L to C_L from Eqs. (1) and (3) in Fig. 2(a) versus e_{out}. We find that for the error is less than 5% for $e_{out} < 0.85$ for Eq. (1) and $e_{out} < 0.9$ for Eq. (3). For larger values of e_{out}, the error increases sharply with e_{out}. Figure 2(a) also shows that the error between the numerically-calculated capacitance and capacitance from Eqs. (1) and (3) is a monotonic function of e_{out}. Therefore, one can define a correction factor to Eqs. (1) and (3) to match the numerically-calculated capacitance for conformal elliptical conductors. We find that the following functions are fairly good approximations for correction factors to C_{L1} [Eq.(1)] and C_{L2} [Eq.(3)], as shown as lines in Fig. 2(a):

$$f_{C1} = 1 + (4 \times 10^{-7}) \exp(13.73\, e_{out}) , \tag{4a}$$

and

$$f_{C2} = 1 + (6.25 \times 10^{-8}) \exp(14.95\, e_{out}) . \tag{4b}$$

When we calculate the capacitance of conformal elliptic capacitors using the correction factors, that is, as $C_{L1} f_{C1}$ or $C_{L2} f_{C2}$, we find that the error compared to the numerically-calculated values of capacitance is less than 5% for all values of e_{out}, as shown in Fig. 2(b).

In conclusion, we have investigated capacitance of cylindrical conductors with elliptical cross-sections. We have derived an exact expression for the capacitance of confocal elliptical conductors and an approximate result for the capacitance of conformal elliptical capacitors. Using numerical simulations for conformal elliptical capacitors, we have shown that the exact confocal and approximate conformal elliptical capacitances are within 5 % of the numerical values for eccentricity < 0.85. We have also provided correction factors to the analytical results that match the numerical results to within 5 % for all values of eccentricity.

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