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Accurate 3D Capacitance of Parallel Plates from 2D Analytical Superposition

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ABSTRACT

The two conductor strip line is a classic transmission line structure which has been extensively studied and used in endless applications for decades. Such lines with assumed zero thickness conductors have been used as a standard for calculation validations, comparisons and for many other uses. Unfortunately, the exact determination of the 3D electromagnetic E and H field of even this basic case is analytically intractable for all but the most simple cases, and hence is either approximated with 2D solutions by assuming infinite length, or by large scale numerical computations which are rather complex and time consuming. The 2D solutions assume the fields are uniform in the direction of the line length and hence neglect any fringe fields in this orthogonal direction. Simple methods to achieve the full 3D solution with high accuracy are thus highly desirable and the subject of this work. It will be shown that accurate values of the total 3D capacitance of a parallel plate capacitor having thin plates of any length, width, and separation can be determined from the superposition of the exact 2 dimensional capacitance obtained from an analytic solution using elliptic integrals, in a very simple manner. The accuracy is determined for a range of cases by comparison of the analytic values with those obtained from a 3D numerical calculation using a 64-bit work station with very large memory and processing capability. For most cases, the accuracy of the capacitance obtained by this superposition method falls nearly within the bounds of the numerical accuracy of the 3D model and the elliptical integral evaluations. This superposition method is far more accurate than would normally be expected.

Introduction

It is well known that the fringe field capacitance per unit length of a pair of parallel plates (or single plate above a ground plane) can be estimated from 2D exact calculations when the plates are very long compared to the conductor width and separation.[1] When the plates are not long, the orthogonal fringe fields in both length and width directions must be included. There is no known method for determining such 3D cases analytically. Thus these cases are typically determined by 3D numerical calculations in which the conductors are "broken" into many individual elementary cells. Each such cell is assumed to have a partial element capacitance with all the other cells within some range of influences, i.e. a limited many body problem. The capacitance of these individual cells is evaluated assuming point charges at the appropriate distances, which results is a set of linear, simultaneous equations. Additional accuracy is obtained by use of an analytical method to calculate the charge density within each cell [2]

For many typical problems, such a calculation requires a very powerful computing system and sophisticated numerical program. It will be shown that for a wide range of parallel plate parameters which require a 3D solution, the capacitance can be estimated quite accurately by the superposition of exact, analytical 2D solutions.[1] The latter can be obtained from the application of elliptic integrals

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[1] Capacitance calculations of integrated circuit, multi-conductor metal layers often make use of additions of partial capacitance between various portions of line segments. Such procedures represent a different form of field addition than that use in this paper, and typically require curve fitting to match the simple approximations to more accurate 3D calculations, e.g. [8] and [9].

to construct a universal curve applicable for all cases, as will be seen. The accuracy of the approximation has been determined for a wide range of conductor dimensions with zero thickness.

Capacitance in 3 dimensions: the problem

Consider a pair of parallel plate conductors separated by a distance, S, having a length of Wx in the x direction and width Wy in the y direction as shown in Fig 1. If a voltage is applied between these two conductors, the electric field, E, will have only a z component (no x nor y component) within an infinitesimally small xy area of length ΔW_{x1} and width ΔW_y at the exact geometric center line at which x = y = 0 as indicated in Fig. 2. Except for this small region around the center, there will be x , y and z components of E everywhere else within the space between and around the conductor plates, with the magnitude of the x and y components increasing as we move away from the center. For a fixed value of x, the E field varies with y and z approximately as indicated in Fig. 2. Similarly, in the orthogonal direction, for a fixed value of y, the field in the xz plane varies in a somewhat similar fashion with the exact pattern depending on the width and length of the conductors.

It is this spatial variation of E which gives rise to the non-uniform charge density on the conductor surface and makes the capacitance evaluation difficult for the general case.

For many applications, the length of interconnecting lines, indicated as Wx in Fig. 1 will be, or is assumed to be very much larger than the width Wy. The fringe field component, Ex, in the x direction can thus be neglected at the ends in the long direction and the 3D problem is reduced to a 2D case such as that shown in Fig. 2. This is equivalent to evaluating the capacitance per unit length of only a short length of line, ΔW_{x1}, and width Wy taken from the mid-section, as indicated in Fig. 1, and using this same capacitance per unit length for the entire length of line. This is referred to as the 2D approximation and is reasonable for long lines with Wx >> Wy There will be negligible x components of E field in this short unit length ΔW_{x1} section, and the y and z components will be similar to that shown very approximately in Fig. 2. However, if a small section such as that represented by ΔWx2 in Fig. 1, is taken nearer the edge of Wx, there will be considerable x components of the E field so the 2D solution is an approximation for any finite length. Depending on the actual dimension ratios, the capacitance determined by this approximation are sometimes not adequate because the line are not sufficiently long, or better accuracy is needed for various reasons.

Approximate 3D Calculations **(Quasi- 3D Calculations using superposition of 2D solutions)**

It is possible to estimate the capacitance of 3D geometries by the simple superposition of exact 2D analytical calculations. The accuracy varies slightly for different dimensions, as would be anticipated, but the error is typically less than inherent errors in the numerical calculations themselves, as will be seen. Only the capacitance of very thin parallel plates are included in this study.

The essential idea and fundamental assumptions are as follows.
The total capacitance of a general parallel plate capacitor is the sum of a parallel plate portion which

arises only from uniform Ez field components, plus a portion arising from only Ey components , and lastly a portion arising from only Ex components of field distortion. The approximation used here is that the C per unit length due to Ey components of field can be obtained from a 2D calculation of the Cap per ΔWx which is equivalent to using the ΔWx1 center region (x=0) in Fig. 1 where there is no Ex field components. This same C per ΔWx is then used for the entire Wx length, even where there are significant Ex components. The C portion arising from the Ex field components is done in an analogous manner, by 2D calculation of the C per ΔWy which is equivalent to that at a center region (y=0) where there is no Ey field components, then use this same C per ΔWy for the entire Wy length, even where there are significant Ey components. The inherent assumption is that near the plate corners where both Ex and Ey have significant amplitudes, they do not interfere. This is, in fact, an interesting, fundamental question, namely, if the 2D field pattern of Fig. 2 were to be drawn for an x value near the x end of the plates, how much would it change. The 3D approximation used here **assumes** that it does not change at all, and we will show that this is a good approximation for all Wx except for cases where Wx/S is less than roughly 0.5.

Another, perhaps more direct way to see this is to consider the charge density on the plates. For a true, parallel plate approximation with no fringe fields, i.e. no x nor y components of E, the charge density would appear only on the inner surfaces of the top and bottom plates, and would be constant (uniform charge) for all x and y. In an actual 3D geometry, the charge density distribution on the inner surface of the plates will be similar to that shown in Fig. 3(a), where darker shading indicates higher density. The charge density will increase as we progress from the plate center toward the edges as indicated. For a 2D approximation such as in Fig. 2, only Ey fringe fields are calculated, Ex implicitly assumed everywhere to be zero. For such a case, the charge density along the y axis will increase from the center toward the plate edge but will be a constant as a function of x, as shown in Fig. 3(b). Likewise, for a 2D approximation in the orthogonal direction, charge density along the x axis will increase from the center toward the plate edge but will be a constant as a function of y, as shown in Fig. 3(c). The 3D approximation used here assumes that the sum of charges in 2D cases of (b) plus (c) essentially equals that for the actual 3D case of (a). This cannot be an exact representation since the 2D approximation neglects some small fields at the exact corners. For instance, the E field configuration at the edges will be something like that shown in Fig. 4 -- the fields at the corners, shown in heavy, red lines, curves out around the sharp edges. However, our approximation does not have any way to include these. The magnitude of these neglected fields (and resulting charges) will typically be small, resulting in small errors in capacitance. The exact, numerical 3D calculations include these fields and will provide justification for this approximation.

Thus the essential idea is to approximate the total 3D capacitance of two parallel plates of any width, length, and separation as the summation of three components:

1. Capacitance C_{pp} of parallel plates assuming *no fringe* fields so there exists only a uniform Ez field everywhere i.e. total $C_{pp} = \varepsilon\, W_x * W_y / S$

2 Fringe Capacitance per unit length, $C_{fy} / \Delta Wx$, due only to fringe field in y direction caused by finite W_y, as in Fig. 2, assuming no Ex fringe fields, i.e. as though W_x was actually infinite; C_{fy} is

assumed to be exclusive of the parallel plate component, Cpp, and thus only due to the non-uniform E field caused by Ey components

3. Fringe Capacitance per unit length, C_{fx} /ΔWy due only to fringe field in x direction caused by finite. W_x, assuming no Ey fringe fields , i.e. as though W_y was actually infinite; C_{fx} is assumed to be exclusive of the parallel plate component, Cpp, and thus only due to the non-uniform E field caused by Ex components

To get a 3D approximation, we need all three components above. The first component is obtained directly from the well known parallel plate equation as given in (1) above. The remaining components require determination of the exact 2D capacitance for the two cases of 2 and 3 above, namely C_{fx} /ΔWx and C_{fy} / ΔWy . These are easily calculated separately by use of elliptical integrals. Note that such calculations, shown below, give the total 2D capacitance which includes the parallel plate component, Cpp, in addition to the pure fringe component. In other words, for the y-fringe field, and x-fringe field evaluation, the 2D analytical expression gives a total capacitance, Cty, and Ctx, respectively, of

$$Cty = Cfy + Cpp \qquad Ctx = Cfy + Cpp \qquad (1)$$

where subscript t refers to "total" and f to "fringe only".

Since we only want one value of C_{pp} in the final total, not two, we must subtract one Cpp from the sum of C_{ty} /ΔWx and C_{tx} /ΔWy . The final 3D approximation is obtained as follows. Since the 2D calculations are all per unit length, we must be careful to enter correct total length for each 2D value. Hence

$$\text{Total } C_{3D\ approx} = [C_{ty}/\Delta Wx\ ]^* W_x + [C_{tx}/\Delta Wy\ ]^* W_y - \varepsilon W_x {}^*W_y/S \qquad (2)$$

$$Cpp = 8.85^* Wx (Wy/S)^*10^{-3} \text{ fF} \quad \text{for W and S in microns} \qquad (3)$$

C_{ty} /ΔWx fF per unit lengh is obtained from 2D exact analytical calculation of C per unit length as in Fig 2 with plate width of Wx

C_{tx} /ΔWy fF per unit length is obtained from 2D exact analytical calculation analogous to that above but with plate width of Wx .

Both of these components can be obtained from one, universal curve which is derived and shown below.

<u>Exact, 2D Analytical Calculations using Elliptic Integrals.</u>

The exact 2D capacitance per unit length of a parallel plate configuration such as that shown in Fig 2, can be determined by using Swartz Christoffel transformation as described by Palmer[1]. This requires the evaluation of elliptic integrals. Since these functions are transcendental, the capacitance cannot be directly expressed in terms of W/S. Rather, we must assume some value for m, the modulus (argument) of the elliptic integrals. This modules is used to first calculate another argument, ϕ, for the elliptic integrals. Then W/S and the corresponding total capacitance are obtained using m and ϕ in appropriate elliptical integral expressions as given in detail below.

Assuming some value of m, the argument ϕ is obtained from

$$\sin^2\phi = \frac{K'(m) - E'(m)}{(1-m^2)*K'(m)} \quad \text{so} \quad \phi = \arcsin\left\{\frac{K(1-m)-E(1-m)}{[(1-m)*K(1-m)]}\right\}^{1/2} \qquad (4)$$

With this assumed value of m and the calculated value of ϕ , W/S is obtained from

$$\frac{W}{S} = \frac{K[1-m]*E[\phi,(1-m)] - E[1-m]*F[\phi,(1-m)]}{(E[1-m]-K[1-m])*K[m]) + K[1-m]*E[m]} \qquad (5)$$

and total capacitance per unit length from

$C_t/L = (\varepsilon_0/4\pi)$ K'/K in statfarads/cm {Palmer} (6)

By use of the conversion factor
1 Farad = 9 * 10^{11} statfarads or 1.111 * 10^{-12} Farads = 1 statfarad

the RMKS equivalent formula is

C_t/L = ε K'/ K = 8.85 * 10^{-12} ε_r K'/ K farads/meter { RMKS} (7)

This 2D capacitance can be made universal, independent of dimensions by dividing by the total parallel plate capacitance per unit length, Cpp/L = ε W/S to yield

C_t /Cpp = {K'/ K}/ (W/S) dimensionless (8)

This ratio will always be greater than 1, approaching 1 as Wy/S in Fig 2 gets very large, with Wx equal infinity. A value of 1 for this ratio simply indicates the total capacitance is that of parallel plates, i.e. no fringe fields.

Equations (4), (5), (7) and (8) are the expression and units used in this paper where

ε = 8.85 * 10^{-12} ε_r farads per meter

ε_r = relative dielectric constant = 1 for free space

K[m], K[1-m] = complete elliptic integral of 1st kind with modulus m, (1-m) respectively

K' = complementary elliptic integral of 1st kind = K[m'] = K[1-m]

E' = complementary elliptic integral of 2nd kind = E[m'] = E[1-m]

E [m], E [1-m] = complete elliptic integral of 2nd kind with modulus m, (1-m) respectively

E[ϕ, (1-m)] = incomplete integral of 2nd kind

F[ϕ, (1-m)] = incomplete integral of 1st kind

L = unit length in meter

$m' = 1-m$

Typical values for W/S of, say 1 to 11 require m to range from approximately 0.01 to 10^{-16}. Since m is the modulus in the elliptic integrals and becomes extremely small as W/S increases further, it is difficult to get 2D capacitance for very large W/S.
A universal curve for this 2D capacitance has been determined by running Mathematica elliptical integral evaluation on a large AIX 64 bit work station with large memory (up to 100 Gigabytes). This curve, shown in Fig. 5, can and has been used in conjunction with Equation (2) to obtain all Quasi 3D calculation in this paper.
There are some inconsistencies and differences in the literature on definition of modulus (arguments) used for elliptic integral evaluation --- see Appendix A for important details as well as expressions in Mathematica which were used in this paper.

Numerical (Exact) 3D Calculations

There are no known exact analytical solutions applicable to 3D configuration, hence any "exact" 3D capacitance must be obtained by large numerical calculations or approximations. In this paper, all 3D capacitance calculations were obtained using a Partial Element Equivalent Circuit (PEEC) model [3], [4]. A graphical editor was available for entering the geometries.
All models were processed on the same high performance AIX workstation as above, with very large memory (up to 100 GBy). The solution for each specific geometry dimensions was typically completed for the smallest cell (body) size possible to insure maximum accuracy. This limit is set by the memory size available to accommodate the chosen number of cells, typically 50 GBytes. To achieve this and simultaneously limit simulation run times to several hours, it was necessary to pick cell size such that the number of capacitance bodies was approximately 5 to 7K. Any significantly larger number of cells could result in run times of a few days and unexpected program crashes (e.g. – "out of memory").

Needless to say, any numerical calculation will unavoidably have some small errors, thus "exact" implies correctness to only within these limitations. Such limits are difficult to specify precisely for each case, but are estimated to be less than 5% error, in many cases even smaller.

RESULTS and Comparisons

Quasi-3D estimates of total capacitance of 3D structures were determined using the described 2D elliptical integral superposition approximation, for a family of curves in which the plate length, Wx, and separation, S, were held fixed and the plate width, Wy, was varied. The exact 3D capacitance was then determined numerically for the same cases. The value of S was held at 200 microns for all cases, and Wy was varied from 100 to 1500 microns. A family of curves was obtained for cases of Wx equal 2000, 1500, 1000, 500, and 200 microns. The total capacitance for these 3D estimates from 2D super-positions, (Q3D), and "exact" 3D numerical calculation are shown in Fig. 6(a). The quasi-3D estimates are all shown in solid lines, while the exact 3D values are shown adjacent to each, in dashed lines. It can be seen that for all cases considered, the quasi 3D estimates are very close to the actual results.

Fig 6(b) shows the curves of Fig 6(a) normalized to be universally useful. Total C is plotted vs Wy/S for various values of Wx/S comparing the exact 3D with the approximate Quasi- 3D evaluation. A magnified view of the lower part of Fig. 6(a) is shown in Fig.6(c).

Fig 7 shows the percent ERROR between the capacitance calculations using the Quasi 3D and the exact 3D value, versus W/S. Each of these curves represents the % difference between the corresponding pair of curves in Fig. 6 with % error taken as (Q3D – Exact)/ Q3D. Positive values of % error means capacitance for the Q3D approximation is larger than the 3D calculated value while negative % indicated Q3D is smaller. -- from simple arguments, it would seem that the neglect of the corner fringe fields as per Fig 4 would cause the Q3D estimates to be smaller than the exact C for small values of Wy and Wx. because the neglected part is a significant portion of the total 3D capacitance.. This is indeed the case as can be seen in Fig. 7 at small values of Wy. As Wy, and/or Wx increases, the neglected corner fringe field contribution becomes a negligible fraction of the total C and the Q3D estimates then appear to become larger than the exact C as suggested by the positive value of % Error. Furthermore, the error becomes nearly constant and is less than 3% for all cases shown. However, it becomes difficult to attach a specific rationale for both the polarity and magnitude since the % error is a combination of the errors in the 2D approximation of the 3D case, as well as numerical accuracy errors in both the evaluation of the elliptical integrals, as well as the supposedly exact 3D calculations. As a result, the main conclusion is that the Q3D approximation is quite accurate for nearly all the cases considered. A further, fundamental consequence is that the comparisons are clouded by a type of Heisenberg computational uncertainty in that both the exact analytical 2D solutions and 3D numerical solutions require numerical approximations whose accuracy cannot be tested against some absolute standard. We can keep improving the results with larger computational power and better models, but the basic problem is never eliminated.

<u>Determination of Error in using a 2D Capacitance rather than Q3D or Exact</u>

For integrated circuit interconnection lines that are thin, such as that shown in Fig.1, it is common to use the 2D capacitance per unit length, evaluated as though Wx were infinite, and multiply this value by the actual length, Wx, to obtain the total capacitance. This calculation includes only charges on the plates as depicted in Fig. 3(b), i.e. neglects the small capacitance due to charges on the ends of the line as depicted in Fig. 3(c). The error in doing this can easily be obtained by the use of Eq. (2) in combination with the universal curve of Fig. 5, which is Eq. (8). The neglected component of capacitance is C_{fx} which can be determined as follows. We will use a specific case for clarity. Assume our plates have Wy = 200 um, S = 200 um, Wx = 1000 um giving Wy/S = 1 and Wx/S = 5. Fig 5 gives total Ct/Cpp which includes the Cpp and 2D fringe component for any given W/S. For Wy/S =1, Fig. 5 gives a Ct/Cpp of 2.1 where Ct is Cty since Wy was used for the x-axis. Thus, Cty = 2.1 Cpp which represents the total capacitance of the charge configuration of Fig. 3(b) and is the sum of Cpp + Cyf so 2.1 Cpp = Cpp + Cyf
The capacitance due to finite value of Wx is obtained from Fig. 5 at Wx/S = 5 which is Ct/Cpp = 1.3 where Ct is Ctx since Wx was used for the x axis. Thus, Ctx = 1.3 Cpp = Cpp + Cfx. We only want the neglected part, namely the Cfx component (don't include Cpp twice) which is Cfx = 0.3 Cpp. The ratio of the neglected capacitance to the capacitance used for the line is 0.3 Cpp/ 2.1 Cpp = 0.143 or over 14%

We can generalize this error in neglected end capacitance as

Neglected End C error = {Ct/Cpp [@ Wx/S] -1} / {Ct/Cpp [@Wy/S]}

in fractional percent where Ct/Cpp are taken from Fig. 5 at the specified value of W/S. If we changed only Wx in the above example from 1000 to 400 um, Wx/S is now 2 and Fig. 5 gives Ctx/Cpp [@Wx/S] = 1.63 while Cty/Cpp remains at 2.1. The error in neglecting this end component is thus (1.6 – 1)/2.1 or nearly 30%.

Finite thickness plates

The evaluation of C for typical integrated circuit interconnecting lines has increasingly required the sidewall C of finite thickness conductors. Such calculations are not possible analytically and have given rise to many methods for approximation. [5] -- [9] There may well be some quasi 3D superposition methods similar to that use here, to give good estimates for such cases and are subjects for future investigations.

Conclusion

It has been shown that the 3-dimensional capacitance of thin parallel plates over a wide range of parameters can be determined quite accurately from the superposition of the two, 2-dimensional analytical components in the orthogonal directions. These 2D components are obtained from elliptic integrals and can be expressed in a universal curve. The 3D capacitance obtained by such methods have been compared with accurate values obtained using advanced numerical calculations on a large AIX workstations with 100 GBytes of main memory. The error between the 2-D superposition estimates and the more exact 3D numerical calculations typically show an error of less than 3% over an unexpectedly wide range of plate length, width, and separation. This error is approximately within the range of numerical accuracy of the calculations, thus providing convincing support for the validity of this estimation method.

Appendix: Definition of elliptic functions in calculation of 2D capacitance

MATHEMATICA was used for evaluating elliptic integrals using the following program:

```
In[11]:=
m= { .1, .12, .14, .16,.18, .2, ------   etc, user entered, as desired}

phi= ArcSin[((EllipticK[1-m]-EllipticE[1-m])/((1-m)*EllipticK[1-m]))^.5]

WoverS=(EllipticK[1-m]*EllipticE[phi,(1-m)] - EllipticE[1-m]*
          EllipticF[phi, (1-m)])/(((EllipticE[1-m] -EllipticK[1-m])*
            EllipticK[m]) +EllipticK[1-m]*EllipticE[m])

Ctotal=EllipticK[1-m]/EllipticK[m]

CoverCpp=Ctotal/WoverS
Out[11]=
```

Where m are numbers supplied by user (typed-in)
WoverS = W/S calculated value corresponding to the given m value
Ctotal = total 2D capacitance per unit length
CoverCpp = Ct/Cpp = ratio of total capacitance to the parallel plate capacitance (with no fringe fields)

The arguments (modulus) of the elliptic integrals used in some common tables [10], [11] are often k and k' rather than m and m' The use of k or m is purely arbitrary and the relationship between these arguments is relatively simple, but important, namely

$$m = k^2 \quad \text{and} \quad m' = 1\text{-}m = k'^2 = 1 - k^2 .$$

Thus we have

K == EllipticK**[k²]** == EllipticK**[m]**
K' = K[k' ²] = EllipticK[m']

Similar substitutions can be made for the remaining integrals.

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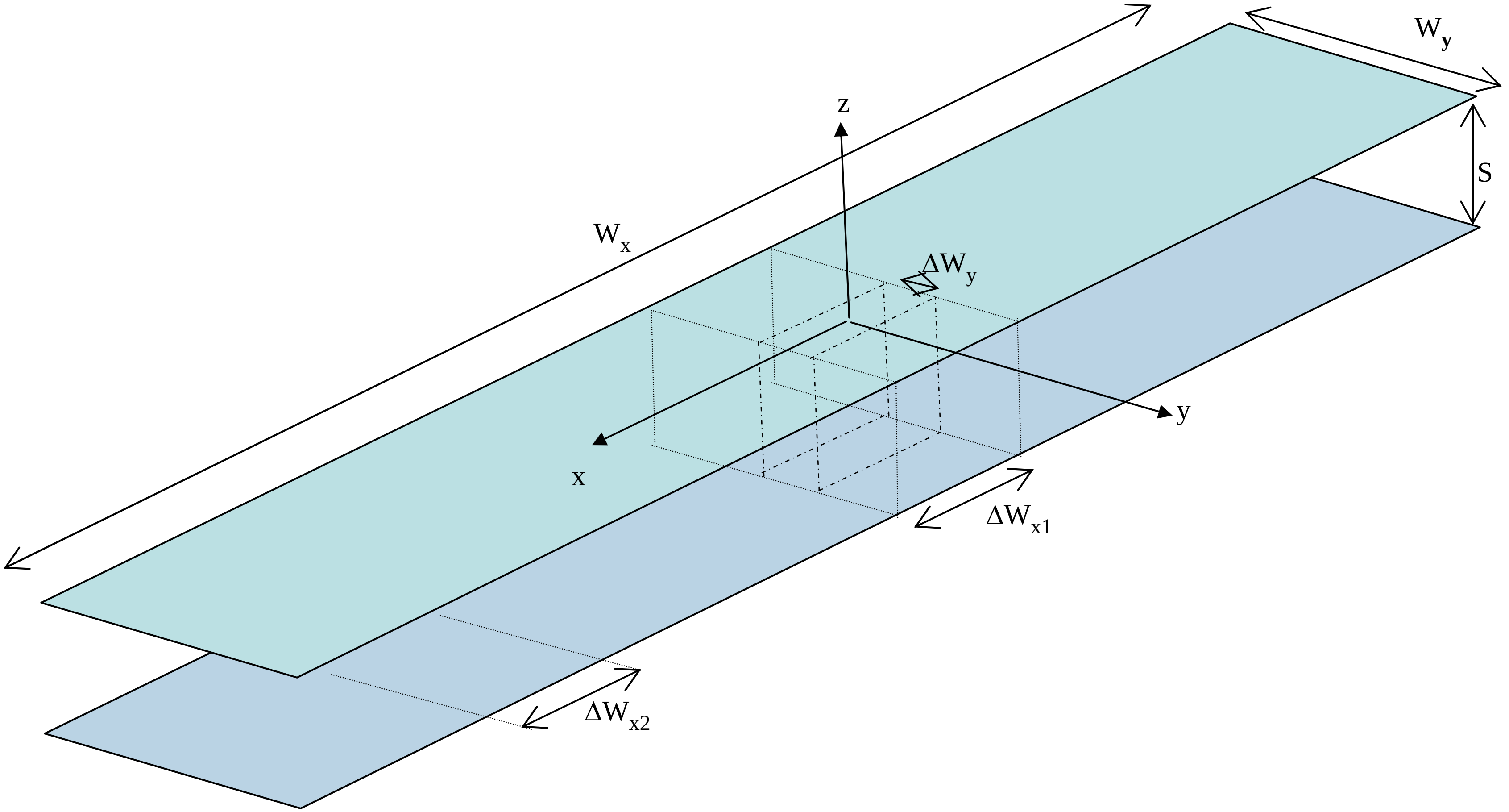


Fig. 1 Parallel plate capacitor with Wx > Wy

Fig. 2 Approximate 2D fringe field in yz direction of the small W_{x1} section from center portion of Fig. 1

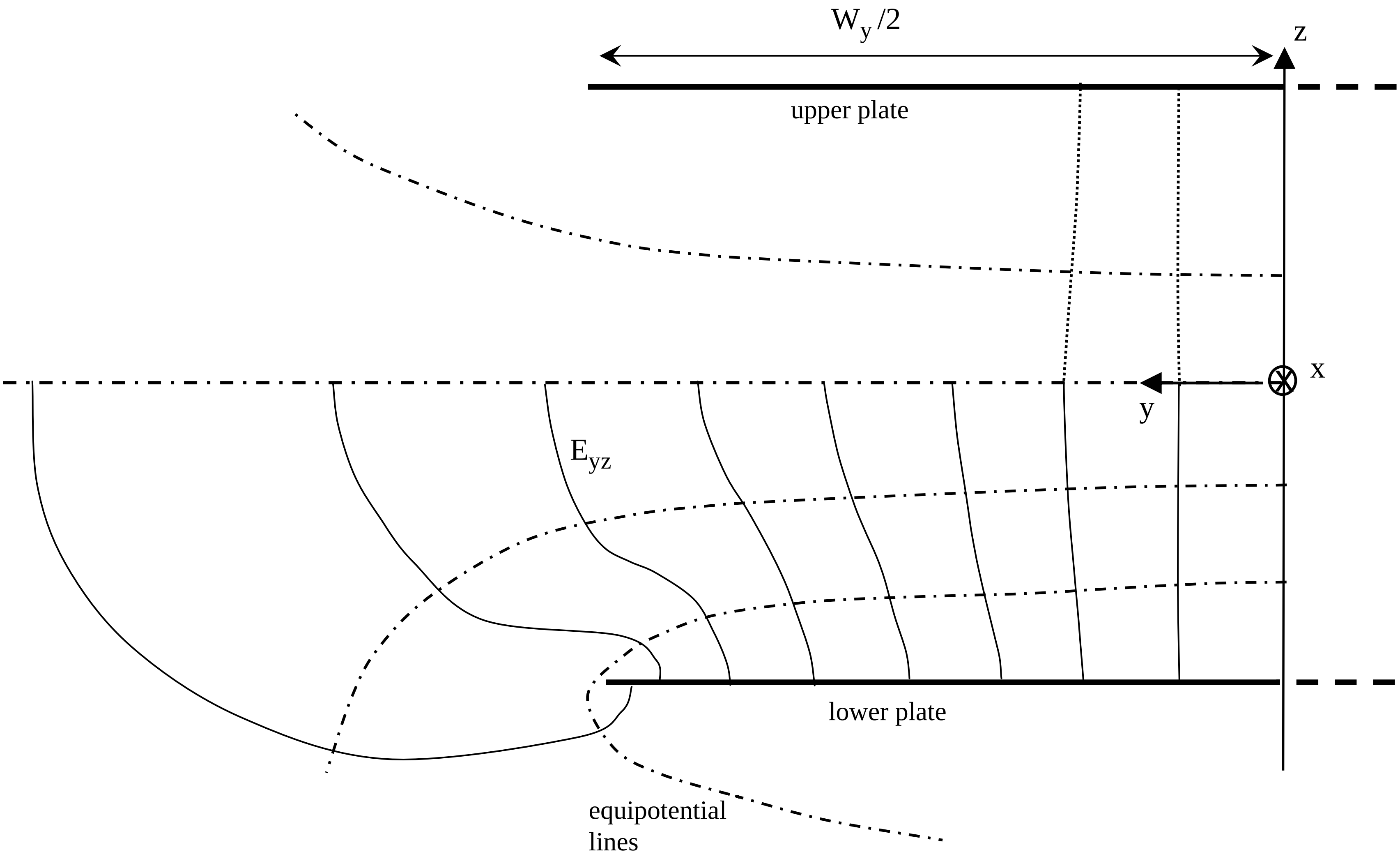
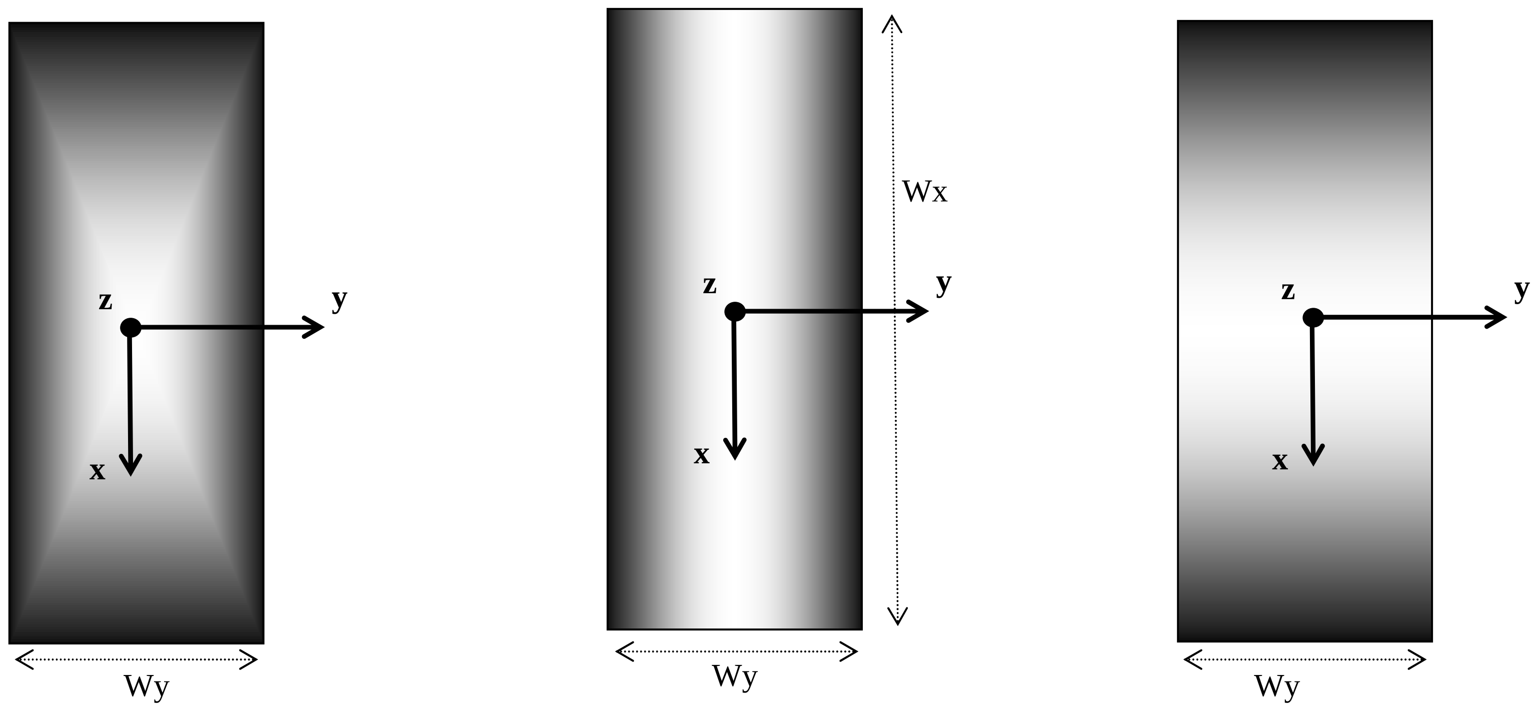


Fig. 3 Capacitor plate charge density distribution for (a) total exact 3D distribution on all 4 edges, and the assumed 2D charge density approximations on Wx edges in (b) and Wy edges in (c). Darker shading indicates larger charge density.

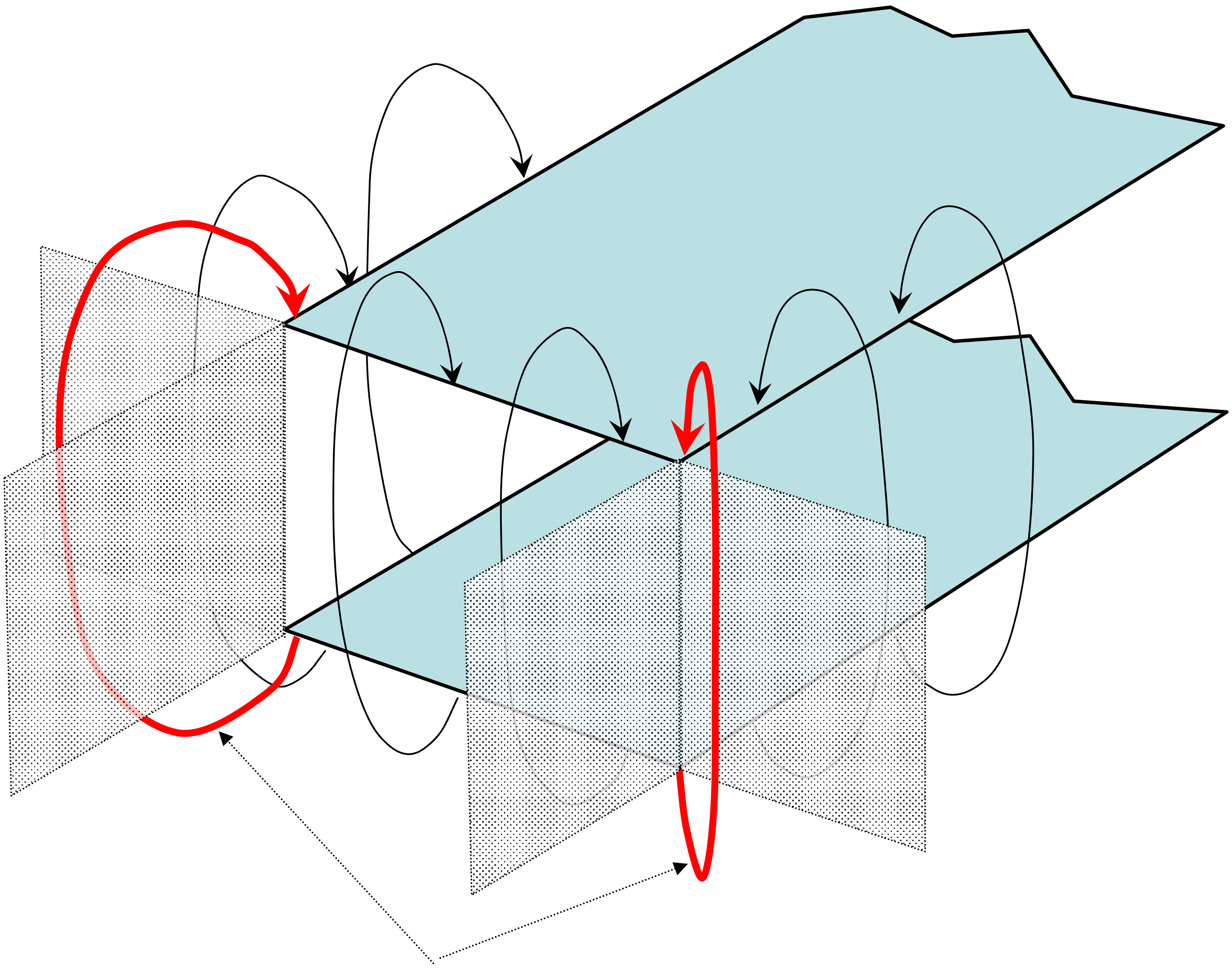


(a) 3D charge density distribution on plate with both Ex and Ey fringe fields present, as in actual case

(b) 2D charge distribution on Wx edges assuming only Ey fringe component, Ex =0 everywhere

(c) 2D charge density distribution on Wy edges sassuming only Ex fringe component, Ey=0 everywhere

Fig 4 Corner fringe fields not included in 3D fringe field, approximation



Corner fringe fields neglected in 3D approximation

Fig. 5. Universal curve for estimating fringe capacitance for 3D plates using 2D approximation as described in this paper.

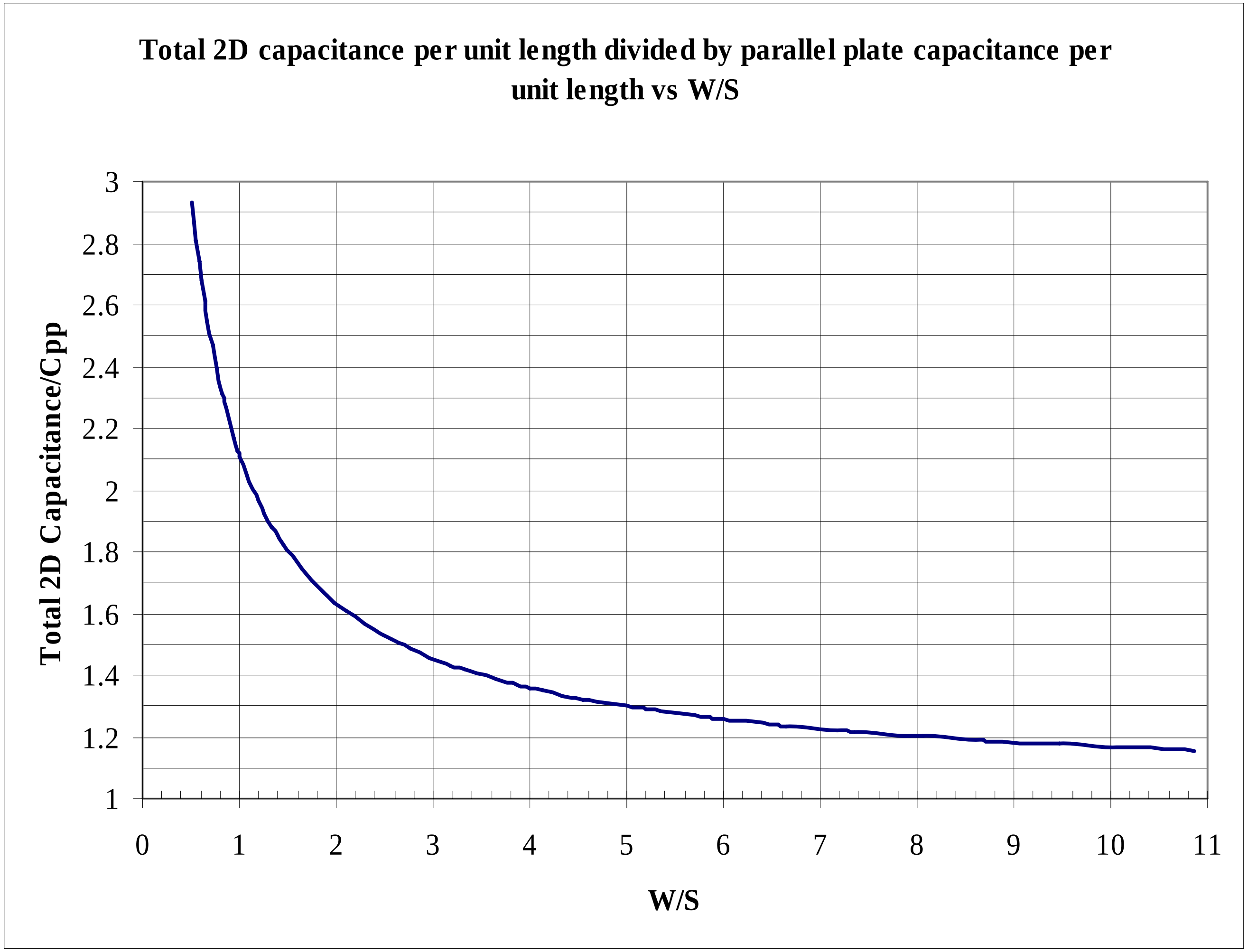


Fig. 6(a)

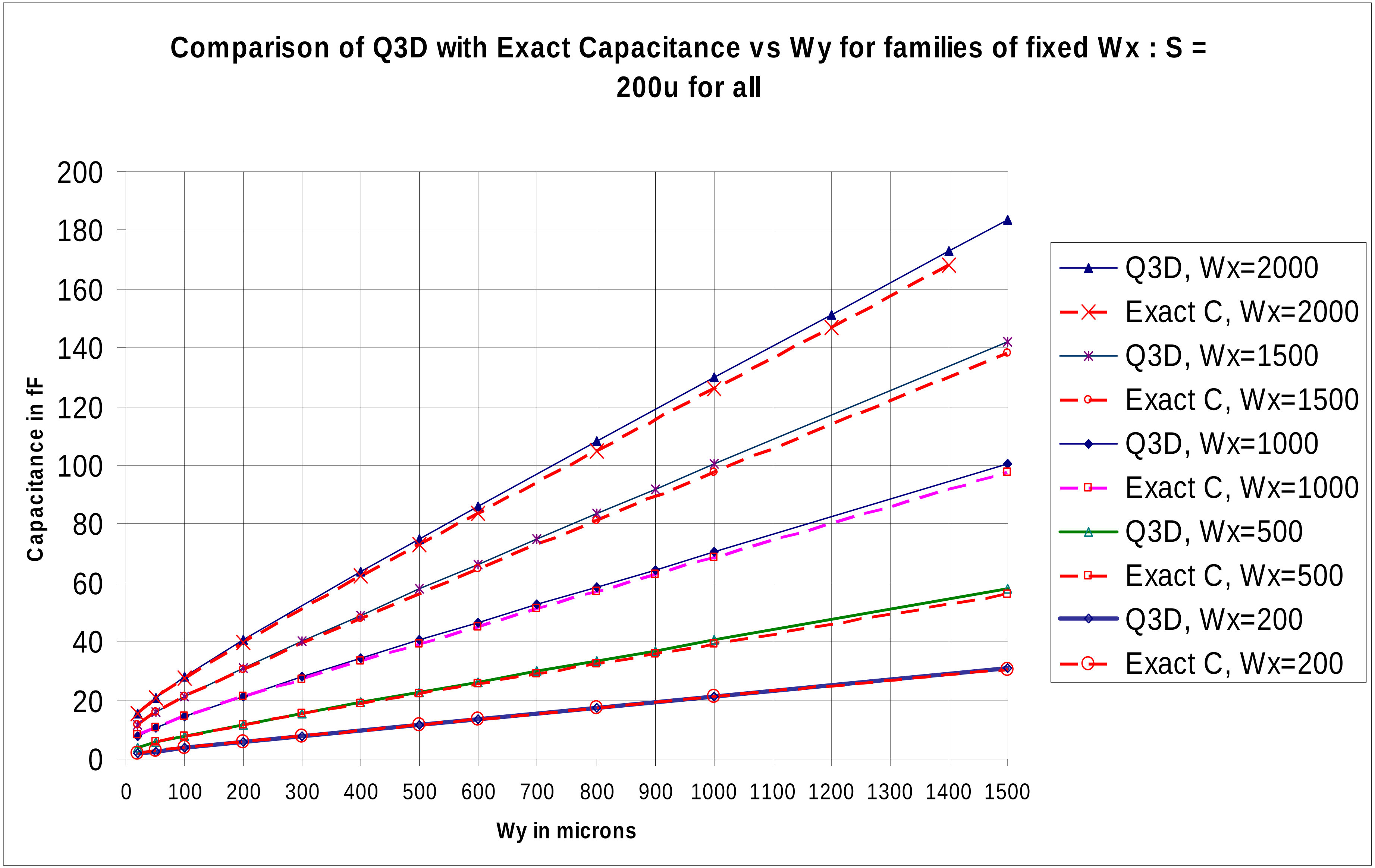

Comparison of Q3D with Exact Capacitance vs Wy for families of fixed Wx : S = 200u for all
Capacitance in fF
Wy in microns
0
20
40
60
80
100
120
140
160
180
200
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
Q3D, Wx=2000
Exact C, Wx=2000
Q3D, Wx=1500
Exact C, Wx=1500
Q3D, Wx=1000
Exact C, Wx=1000
Q3D, Wx=500
Exact C, Wx=500
Q3D, Wx=200
Exact C, Wx=200

Fig. 6(b) Normalized version of Fig 5(a) showing total C vs Wy/S for various values of Wx/S comparing exact 3D with the Quasi-3D (i.e. 2D superposition for each Wx/S

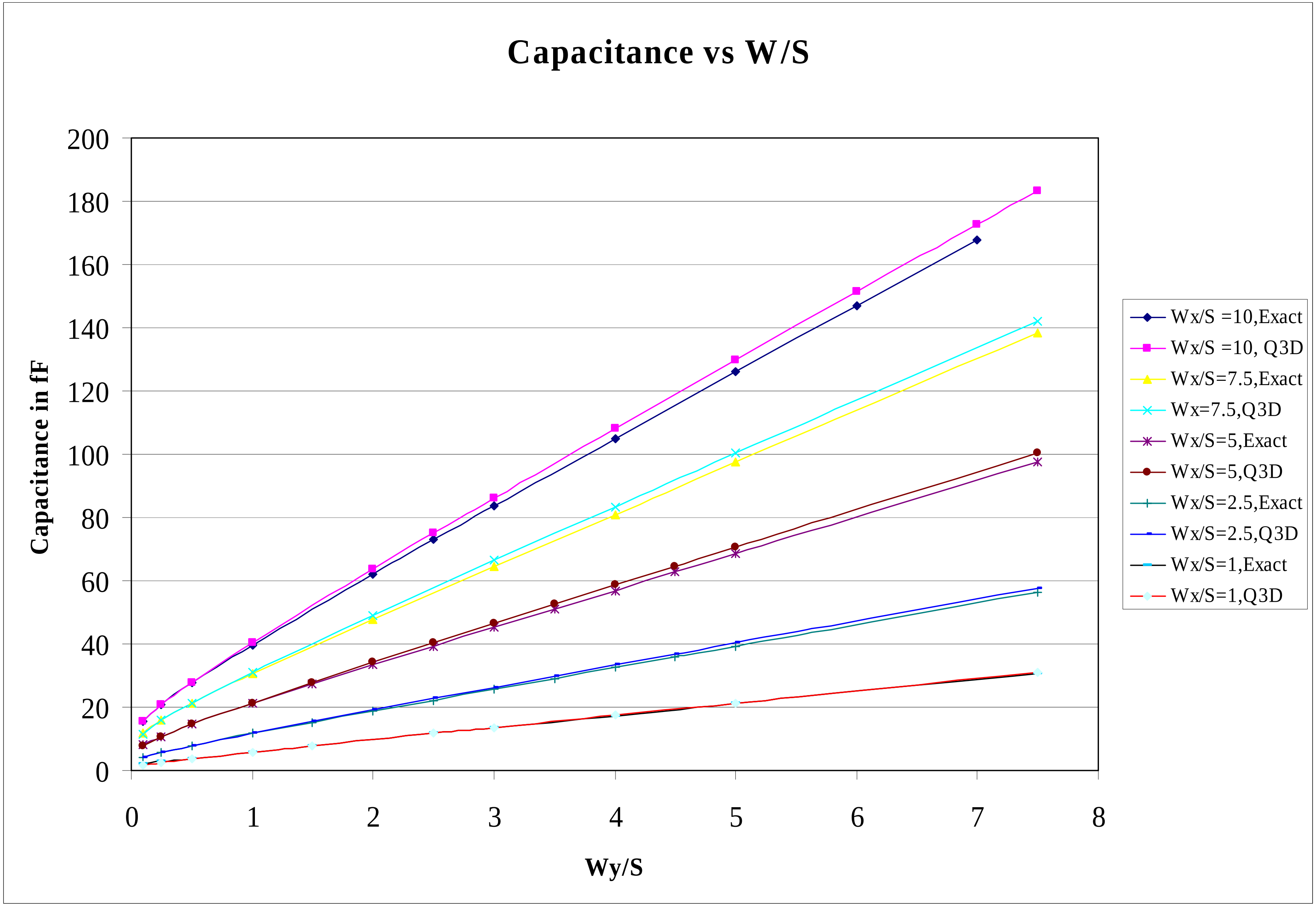


Fig. 6(c) Magnified portion of Fig 6(a) for small Wx values

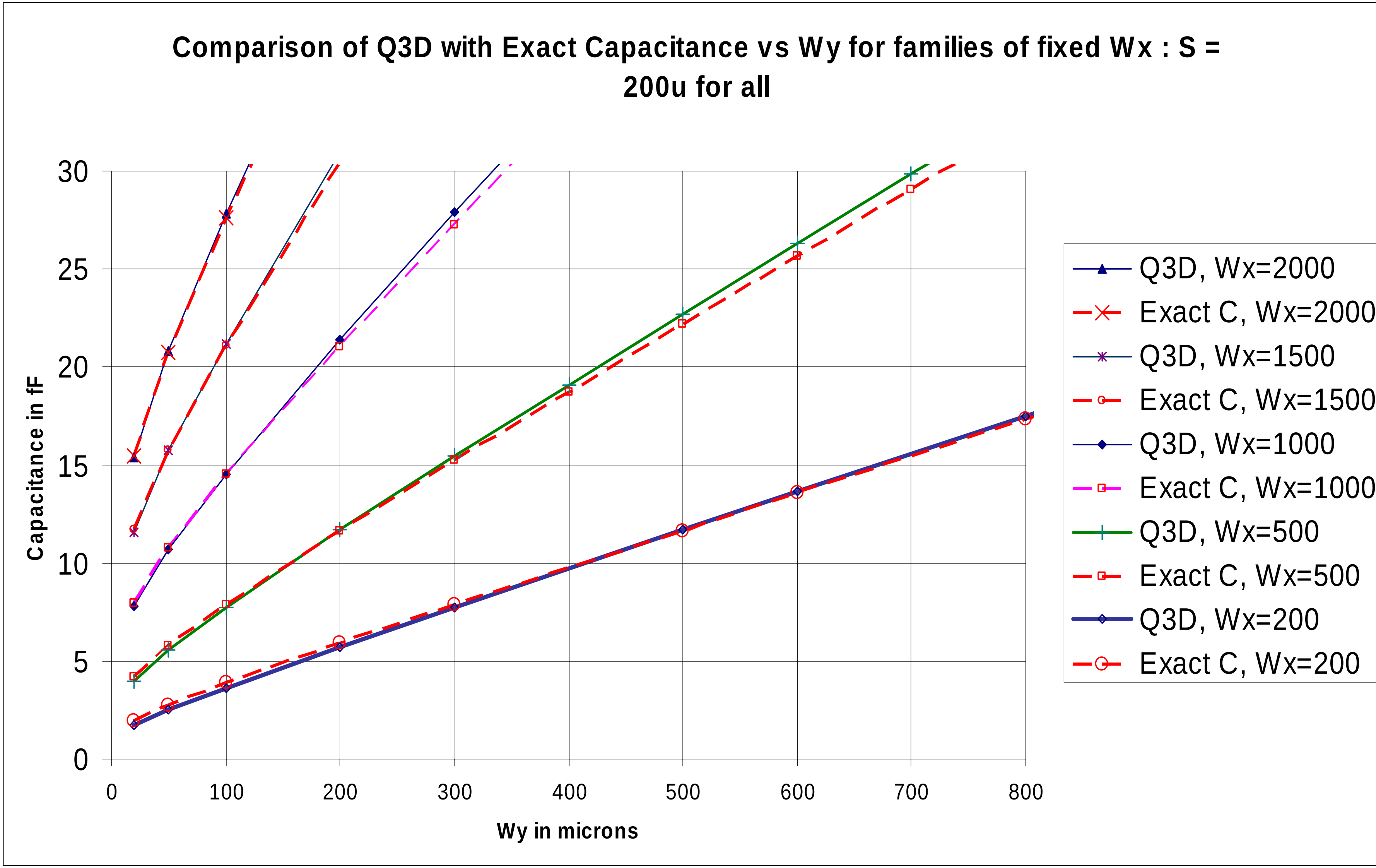


Fig. 7 Percent error between the exact 3D vs the Quasi-3D calculations of C for family of curves of Fig 5(a)