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Visualizing Risk

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Abstract—We describe the issues surrounding the visualization and communication of risk. Colloquially, "risk" has many meanings; we define it as a probability distribution (discrete or continuous), the entirety of which is important to be understood. While the display of uncertainty has had some measure of attention, though primarily in one dimension, the communication of risk is complicated by the difficulty for many people to accurately understand and contemplate the idea of a probability distribution to begin with, in part due to well-documented psychological biases. We review the scientific and experimental background of this subject and propose guidelines for the effective presentation of risk information, in the specific application area of transportation in a metropolitan area.

Index Terms—Risk, uncertainty, probability.

1 Introduction

While the science of transforming data into graphical displays has had a rich history over the last several decades, the presentation of risk information has a much thinner body of work. For our purposes we define "risk" as a situation where there is an underlying probability distribution which must be understood in order to fully apprehend the problem. Risk implies uncertainty, in the sense that the outcome is unknown ahead of time, but by considering the entire probability distribution we distinguish risk from the usual understanding of uncertainty which is often expressed (at least in one dimension) using various sorts of error bars or confidence regions, and in multiple dimensions using a variety of novel visualization methods (see [49, 48, 32]). In addition, uncertainty often reflects *limitations* of the data, in the sense that there is a "true" value to be expressed, but that we do not know it, while we are trying to represent a range of values which are *all* true, in the sense that there is some probability of any of them occurring. In many cases the challenge of risk visualization involves the difficulties that humans have in processing and understanding risk information as we will discuss below. But beyond this, reducing a probability distribution to measures such as mean and variance fails to capture aspects of the distribution which may be important to people; and not all people or situations may have the same utility function with respect to the distribution. For example, for a person contemplating the distribution of travel times to reach the airport using different travel modes, the *maximum*, or perhaps a 95% confidence, time to reach the airport may be of much more importance than the *average* time. Alternatively, portions of the distribution for a financial portfolio or a hurricane prediction (for example, the region of catastrophic loss) may be of more importance than other regions.

Some potential application areas are understanding the risk in undertaking a particular set of projects, which may fail or succeed to various degrees, the balancing of short-term *vs.* long-term risks of medical treatment, the evaluation of systemic risks in a food-supply chain, or the distribution of travel times via different modes of public transport in an urban area.

Note that the term "Risk Communication," as used, for example, in conference presentations of the Society for Risk Analysis, often refers more to public policy (for example the need for communication with stateholders and open and transparent decision-making) than to concrete visual representations of risk. Our interest is rather in graphical risk communication. This paper will survey the history and science of risk visualization from a variety of perspectives: the psychology of risk, a brief review of some of the scientific underpinings of graphical display of information, a discussion of the visualization of uncertainty, a survey of the results in one area in which significant experimental work has been done in risk visualization: the medical and health arena, and other topics in risk visualization. Finally we will present our recommendations for effective visual representation of risk, with a focus on continuous probability distributions.

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2 Visualization

Graphical visualizations are an invaluable tool to ease the communication of information between individuals. The most apparent advantage of producing a visualization is that, if certain requirements are met, people are able to instantaneously and effortlessly decompose it into its constituent objects, and to understand the underlying information very quickly. This ability is defined by Julesz [22] as *preattentive vision*, and it has been the focal point of a number of studies [7, 17, 4, 12].

In 1983 Jacques Bertin [4] presented a number of visual variables that he believed would lead to good quality graphics. While developed for the printed page, Bertin's visual variables are still used by researchers within the area of computer generated visualizations, whether they are aware of Bertin or not. Bertin suggested that size, value, colour, orientation and shape were the best methods of communicating information in a visual framework. However each had its advantages and disadvantages and the information you wished to communicate would determine which visual variable you would include in your graphic.

The first variable presented by Bertin was size. According to Bertin the use of size in a graphic would allow the user to visually order the data that they were trying to communicate. However the issue with using size to communicate information is that you are relying on your target audience's ability to judge the differences in size. While size can be used to present ordered data, if your data is unordered then Bertin suggested using colour to communicate it. To use colour to communicate information Bertin was referring to the colour itself rather then the saturation level; an extension of that is the use of value which refers to the brightness of a colour or the intensity of the colour. Orientation refers to the variation in the angle between marks. This variable can be used when a researcher wishes to communicate a proportions or quantities, for example a pie chart. The final visual variable that was considered by Bertin was shape.

According to Bertin, visual elements could be grouped in two different classes of visual variables, planar and retinal. Planar variables are mapped to spatial dimensions of the plane, while retinal variables are encoded using different means, like color or shape. In the same book, Bertin explicitly define two basic properties of these variables: "Length" and "Level of organization." Length is defined as the number of different perceptible states: while designing a visualization, it is crucial to map a component to a variable with at least the same number of states of the component. Failure to comply with this would result in the user being unable to distinguish between different values, effectively degrading the performances. The level of organization of a visual variable is something more complex, and represents the different

	Associative	Selective	Ordered	Quantitative
Planar	Yes	Yes	Yes	Yes
Size		Yes	Yes	Yes
Brightness		Yes	Yes	If scaled
Texture	Yes	Yes	Yes	
Color (Hue)	Yes	Yes	Limited	
Orientation	Yes	Yes		
Shape	Yes	Yes		
Motion:Velocity		Yes	Yes	If scaled
Motion:Direction		Yes		
Flicker:Frequency		Yes	Yes	If scaled
Flicker:Phase		Yes		
Disparity		Yes	Yes	

Table 1. Different levels of organization of visual variables, as proposed by Green

levels of data which it may represent. Bertin proposes four different aspects of organization for each variable [4]:

- Associative: represent the ability of the feature to visually grouping a series of elements, despite of other differences.
- Selective: permits the viewer to select a category of elements, effectively ignoring the others.
- Ordered: let the user visually guess that some features represent larger or smaller quantities that others, although it could not be always possible to quantify the difference.
- Quantitative: the highest level of organization, permits the viewer to directly extract proportions between values, without the need of consulting a legend.

Some of these aspects are more controversial than others: in [17] Green suggests that shape could be or not be selective, depending on the particular shape, and that users could be trained to use preattentive vision to select shapes that previously required cognitive effort.

In the same work, Green tries to extend the visual variables proposed by Bertin [4], to include the new and more dynamic visualization methods available now. In order to do that, motion and flicker variables are added to the ones originally developed by Bertin. In Table 1 we report the variables proposed by Green and Bertin, indicating the level of organization that each one of these can represent.

In a different work, Cleveland and McGill [7] focused on the quantitative level of organization, performing experiments to estimate which aspects are best interpreted by preattentive vision. The list resulting from their work (ordered from the most appropriate to the least) can be used to decide whenever use one variable in place of another:

1. Position along a common scale
2. Position on identical but non-aligned scales
3. Length Angle and Slope (with θ not too close to 0, $\pi/2$, or π radians)
4. Area
5. Volume, Density and Color saturation
6. Color hue

Whenever possible it is advisable to prefer the topmost aspect to encode the information, so that it can be decoded more effectively; Moreover, when defining a visualization, it is important to focus on the aspect of the data that needs to be communicated, and do it in the most explicit possible way. When presenting series of data, for instance, it is advisable to explicitly plot the difference between values if that is a key aspect; not doing so would force the user to estimate slopes or distance (length) in order to infer that information.

3 Psychology of Risk

Psychologists have accumulated a large body of research examining the ways that people understand and estimate risk. Most of this research has been conducted within the framework of "heuristics and biases" pioneered by Daniel Kahneman, Amos Tversky and their colleagues in the 1970s [43, 23]. This framework posits that, when estimating probabilities, people tend to rely on heuristics or "rules of thumb" which give reasonably good approximations under some circumstances. For example, when estimating the frequency of an event people may rely on the "availability heuristic," in which they make their estimate proportional to the ease by which an example of the target event can be brought to mind [42]. This works well when the subjective indicator (ease of recall) is correlated with the frequency of the target event, but when it is not the availability heuristic breaks down. The two may become uncorrelated for various reasons, such as exposure to mass media which tend to focus on dramatic and unusual events, such as homicide or airline accidents, and ignore more routine, less sensational events, such as common diseases or car accidents. For example, when asked to rate the probability of a variety of causes of death, people tend to rate more "newsworthy" events as more likely because they can more readily recall an example from memory [28]. These systematic deviations from estimates arrived at by our best normative theories such as the probability calculus are known as cognitive biases.

In addition to the availability heuristic, Tversky and Kahneman identified various other heuristics, of which the two most important are representativeness, and anchoring and adjustment. The representativeness heuristic is used when a person is asked to estimate the chances that an item belongs to a category. They observe that people tend to use similarity to guide their judgment. For instance, an introvert and tidy person is more likely to be associated with being a librarian than with being a salesman or farmer because the description is closer to the librarian stereotype. One shortcoming of this heuristic is the insensitivity to prior, which means ignoring (or not adjusting enough for) the base rate of librarians against farmers and salesmen in the general population, even when those rates are explicitly stated. Another reported bias associated with the representativeness heuristic is insensitivity to sample size, *i.e.*, not accounting for the fact that deviations from population statistics are more likley in small samples than in large samples. The anchoring and adjustment heuristic corresponds to the process of starting from a base value (either given or quickly estimated) and making adjustments to it. This heuristic is useful when asked to provide a numerical prediction. One limitation of this rule of thumb is that people can be very much influenced by the base value they start from and do insufficient adjustments. This is especially salient when the assessment process suggests a starting value. For instance, Tversky and Kahneman [43], report that when asked to estimate the percentage of African countries within the United Nations, people from the group that was asked first whether this number was more than 10% reported on average significantly lower estimates that those from the group who were asked directly.

Having identified heuristics and associated biases, psychologists are now attempting to devise ways to help people overcome their limitations. Various "debiasing" methods have been developed, with varying success. For example, Gigerenzer and Hoffrage have shown that some cognitive biases are diminished when statistical information is presented in terms of natural frequencies as opposed to probabilities [16]. They argue that this is because frequency formats correspond to the sequential way information is acquired in natural sampling, such as in animal foraging, which our brains have been shaped to do by natural selection. Such research suggests that new forms of data visualization could provide "corrective" representations that might help counter the documented cognitive biases. For example, Inbar [20] discusses graphical representations of probabilities in which care is taken to counter the documented biases in judgment and decision-making. In particular, he designed an experiment dealing with Allais's paradox [2], also known as the "certainty effect," in which people overweight outcomes that are certain, relative to outcomes which are merely probable [1]. While the sample size was small, they found that graphical representation of the expected value resulted in people making different choices than when presented with the (original) textual description. They speculate that the graphical presentation reduced the cognitive load in computing the probabilities. They also investigated the Ellsberg paradox, which is generally interpreted as uncertainty aversion. However, in this case the graphical representation they devised did not result in a change in participants' behavior.

Finally, there is a prolific literature related to risk perception which focuses on identifying the characteristics of risks that (i) make them be over or underestimated by the general public and (ii) make some more acceptable than others. A recent cover story of Time magazine explores Americans' faulty risk perceptions [25]. [38, 39] provide a review of the work that has been carried out in that domain in the past decades. In particular, researchers have observed that two aggregate factors can predict the perceived risk of diverse hazards. The first one labeled *dread* captures characteristics such as whether the hazard can be catastrophic, involve fatalities, or cannot be controlled. The second factor captures whether the risk is *known* and is based on whether the hazard is observable, its effect immediate or delayed, and whether it is known to science. Some of the findings are especially relevant to the visual communication of risk. For instance, there is strong evidence that the way of presenting the information, e.g., choice of unit or presenting mortality rates versus survival rate, influences people's perceptions of and reactions to risk. Also, a very recent study [30] explores how people are influenced by political borders in their perception of disasters (such as earthquakes or radioactive accidents) which do not respect borders. They report in particular that whether to present a map with light or dark borders had a significant effect on the perceived risk from an environmental hazard (radioactive contamination). People presented with light borders provided higher estimates of risk than those provided with a map with dark border.

4 Uncertainty Visualization

In [21] Johnson and Sanderson make a strong case that the visualization community needs to do a better job of taking seriously the need for visual representations of data to include error and uncertainty information. They note that the geographic information systems community carried out some of the earliest work on two- and three-dimensional representations of error and uncertainty in terrain models. The examples given include representations showing the differences in flow estimation using different integration algorithms, and tubes showing the uncertainty in a particle's path. They point out that blurring can be an excellent choice for conveying uncertainty, as it is intuitively associated with uncertainty by users. However the emphasis in this work is primarily in the scientific visualization arena, where it is often easier to devise a "natural" representation of uncertainty, such as blending away a color in a terrain map.

Perhaps the most thorough body of work on uncertainty visualization is by Zuk ([49],[48]). In [49], Zuk evaluates several previously reported methods for visualizing uncertainty using the perceptual theories of Tufte, Ware, and Bertin[12, 8, 4]. The evaluations presented in this work are primarily related to scientific visualization, where uncertainty is manifested in lack of knowledge about the exact direction, magnitude, size, etc. of entities in a two or three-dimensional visualization. Methods used include arrows with angular extent to signify uncertainty in direction, volume rendering (hence fuzziness) to encode uncertainty in actual location of molecules, and transparency to suggest the speculative nature of archaeological reconstruction. A major contribution of Zuk is to offer the observation that, given the time-consuming nature of full user studies, including the theories of Tufte, Ware, and Bertin early in the design process of a visualization can provide a "light-weight" source of guidance.

In [48], Zuk extends the analysis of [49] to also include some additional application areas, including a medical diagnostic reasoning application. The application involved an evidence-based-medicine (EBM) reasoning system for pulmonary embolism (PE) diagnosis. The situation is complicated by the combination of high mortality if the condition is present, while a significant fraction of patients suspected of PE do not actually have it, and testing and intervention can be invasive and dangerous on its own; that is, false negatives may lead to mortality, and false positives to unnecessary treatment with potentially serious side-effects. EBM attempts to use best-practices, and best-available historical data, to choose the optimal course of action based on the current presented set of patient observables and test results. Naturally it is important to include base rate information in any computation of the likelihood that a particular patient has PE. Given the the existence of conjunction errors even with statistically savvy participants [44], Zuk suggests that it is advantageous to build a reasoning system that explicity exposes the application of Bayes Theorem to the situation at hand. This is similar to the "risk transparency" of Kurz-Milcke, which is discussed in Section 5. An important point which Zuk makes is that the diagnostic decision tree should be shown at any point in which the physician is expected to be guided by the predetermined strategy. Visualizing the tree may reduce unnecessary uncertainty as to why the system makes suggestions, and increase confidence when following or disregarding recommendations. He points out that not providing this information can lead to the physician trying to "game" the system to ensure the recommendation that the physician already believes is the correct one. As much information as is present should be available for the practioner to access to gain confidence in the findings. In this work he also proposes a set of heuristics that would inform a quality visualization (and these are applicable whether we are speaking of uncertainty or data variables in general). These are

- Ensure visual variable has sufficient length
- Preserve data to graphic dimensionality
- Put the most data in the least space
- Provide multiple levels of detail
- Remove the extraneous
- Consider Gestalt Laws
- Integrate text wherever relevant
- Don't expect a reading order from color
- Color perception varies with size of colored item
- Local contrast affects color and gray perception
- Consider people with color blindness
- Preattentive benefits increase with field of view
- Quantitative assessment requires position or size variation

He also presents seven "directives" to support uncertainty visualization (which are of course described in detail in his thesis):

- Provide support for cognitive task simplification
- Support emphasis and de-emphasis of uncertainty information
- Support viewing of uncertainty as metadata and separately as data

[1] Specifically, people tend to prefer 1 Million (monetary units) for sure to a lottery offering a 10% chance of winning 5 Million, a 89% chance of 1 Million and a 1% chance of winning nothing. At the same time they prefer a lottery offering a 10% chance of winning 5 Million and 90% chance of winning nothing to a lottery offering an 11% chance of winning 1 Million and a 89% chance of winning nothing. Such preferences are inconsistent with expected utility decision theory, when one considers that in both cases, 89% of the time exactly the same result will occur within the choices offered (either a win of 1 Million in the first example or a win of nothing in the second example), and the remaining probabilities remain the same for either scenario (the person must now choose between a 1% chance of winning nothing and a 10% chance of winning 5 Million on one hand, *vs.* a sure win of 1 Million on the other). Even though the two scenarios appear equivalent, most people make opposite choices.

- Allow the user to select realizations of interest
- Mitigate cognitive heuristics and biases with reasoning support
- Provide interaction to assist knowlege creating
- Assess the implications of incorrectly interpreting the uncertainty

In [32], Pang *et al.* focus primarily on the scientific visualization arena in suggesting a variety of ways to encode uncertainty. The authors describe a classification of data and its associated uncertainty. This classification includes value of datum and its associated value uncertianty, location of datum and its associated positional uncertainty, extent of datum location and value, visualization extent, and axes mapping. The visualization methods they discuss include adding glyphs, adding geometry, modifying geometry, modifying attributes, and animation. Of these, the last two are perhaps the most relevant to an information visualization application. An example of modifying attributes would be to use texture, transparency, or color to indicate more or less uncertainty, and an example of animation would be to show two or more versions of a graphical display with animation transitioning between them.

Skeels *et al.* [37] deal specifically with the issue of uncertainty in an information visualization context. They began with an initial taxonomy of uncertainty: approximation, predictions, inconsistency, incompleteness, and credibility. They then recruited a set of 18 participants who self-identified as dealing with uncertainty in their work. They conducted extensive interviews with these people and then attempted to classify their responses with respect to the sources of uncertainty in their work. From this analysis they derived a new classification scheme wherein uncertainty can be thought of as residing in three "levels": level 1 is uncertainty due to limited measurement precision, level 2 is uncertainty due to measurement incompleteness, and level 3 is uncertainty due to modeling, predictions, or extrapolation. The concept of disagreement spans all these levels; for example at the measurement level (level 1), measurements may disagree when taken multiple times or by different measurement devices, At the completeness level (level 2), disagreement may come from overlapping but not identical datasets, and at the inference level (level 3), disagreement may come from different models used to describe the process. Another aspect of uncertainty which spans all the levels is credibility, which is also the most difficult to characterize precisely; credibility often comes from past experience and built relationships. When asked about how they visualized uncertainty, the most common response by participants was error bars or some generalization of error bars. Some participants described using color to indicate regions of greater or lesser uncertainty.

In [5], Bisantz *et al.* evaluate the relative effectiveness of several methods of encoding the uncertainty in the true value of a stock. Participants in the study are given the goal of maximizing their profit based on the (imperfect) information they receive. Uncertainty is shown as either a linguistic expression, a colored icon, or an arrow icon. The speed of decision-making increases the "profit" of the participant, so it is important that the information be presented in a way that is quickly grasped. The authors found that degraded graphical icons are a viable method for communicating uncertainty.

Sanyal *et al.* [34] present a user study that evaluates the perception of uncertainty amongst four methods for displaying it: error bars, scaled size of glyphs, colormapping on glyphs, and colormapping of uncertainty on a data surface. These techniques are again primarily appropriate for scientific visualization application areas. In their study, they applied these techniques to both one-dimensional (defined as samples from a curve) and two-dimensional (defined as samples from a surface) synthetic data sets, with well-defined criteria for successful interpretation of the graphics. The application area which motivated this work was geoscience, so for example remotely sensed data, observed data a buoys, or simulated weather data. They considered both measurement uncertainty (assumed to be normally distributed about the true value) and systematic uncertainty in particular regions of the data. The researchers found that some graphical methods were more effective in searching for regions of highest uncertainty, while others were more effective in counting the number of actual features in the data. Overall, however, error bars performed poorly compared with the other evaluated techniques.

Thomson *et al.* [41] present a typology of the various kinds of uncertainty that must be considered in a visual presentation, with an emphasis on the application area of intelligence analysis. They discuss a typology of different sorts of uncertainty in a variety of application areas and present their own list of categories, including accuracy, precision, completelness, consistency, lineage, currency, credibility, subjectivity, and interrelatedness. However, they do not extend their work to presenting visual metaphors for the different sorts of uncertainty.

In [31], Olston and Mackinlay carefully distinguish between statistical uncertainty, which can be represented with error bars, and bounded uncertainty, which has very different properties. Unlike statistical uncertainty, which has a potentially infinite distribution of possible values, with a peak representing a most likely value, with bounded uncertainty the exact value is known to lie within an interval, though no most likely value can be defined. Error bars, a staple of data display when there is statistical uncertainty, are typically used in conjunction with an estimated exact value, and thus can be misleading in the case of bounded uncertainty. The authors suggest that a method they call *ambiguation* be used to represent bounded uncertainty. In ambiguation, graphical elements are elongated in one or more directions to represent the range of possible value. For example, a scatterplot in two dimensions would display rectangles rather than dots, where the extent of the rectangle in the x and y dimensions would represent the possible range of the value. The technique can be applied to line plots, pie charts, barcharts, etc. as described by the authors, and they give explicit direction as to how to best approximate the appropriate display when an exact manifestation of the bounded uncertainty is impossible.

5 Health Risk Communication

When considering the topic of risk communication, it is natural to consider the health domain. The practice of medicine is full of uncertainties which need to be communicated to patients. Diagnoses are rarely certain as symptoms can be linked to a variety of causes and are not uniformly expressed among the population. Tests are imperfect, yielding both false positives and false negatives. Treatments do not necessarily have a certain outcome as they depend on a variety of factors such as existing conditions. Drugs rarely come free from potential side effects. In other words, heath and medicine provide a wealth of opportunities to communicate risk and uncertainty.

The website http://www.yourdiseaserisk.wustl.edu/english/ provides an example of health risk communication. There, one can take a variety of questionnaires to assess one's own medical risk for a diverse set of conditions (several cancers, diabetes and heart disease among others). In that website, they have chosen to communicate the risk as shown in Figure 1.

One of the most salient requirements of risk communication in the health setting is the need to have an easily comprehensible and non-ambiguous representation of a difficult concept: probability. The fact that in this application area probabilities are generally small compounds the difficulty because of known biases related to small probabilities [24]. Indeed, people have a tendency to distort their importance, either overestimating them or simply neglecting them.

One example of ill-managed communication of risk is about the increased risk of venous thromboembolism associated with third generation birth control pills. In Britain in 1995, a public announcement by the Committee on Safety of Medicines reported that third generation contraceptives were associated with roughly a doubling of the adjusted odd-ratios of having thromboembolism. By this they meant that the risk increased from 15 per 100,000 person-years for women taking the second generation pill to 25 per 100,000 for women taking the third generation pill. However, if simplified as "third generation pills increase the risk of thrombolembolism by 66%" it makes it unclear whether the total risk is high or not and led to a widespread discontinuation of the pill and to thousands of unwanted pregnancies and

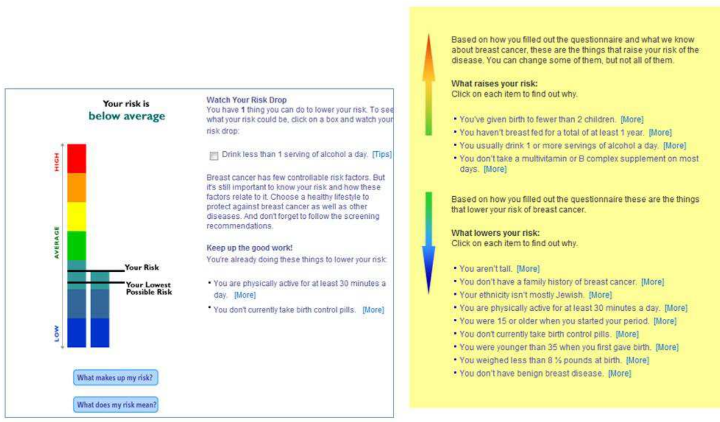


Fig. 1. Screen capture from http://www.yourdiseaserisk.wustl.edu/english. Figure A displays the risk of breast cancer for the hypothetical person relative to the general population and Figure B provides some explanation as to the contributors to risk.

abortions.

5.1 Survey Papers

Lipkus and Hollands [29] provide an early survey of the work to date in risk visualization, with a concentration on medical risk. They point out that at a minimum, a graph illustrating risk must communicate different risk characteristics such as risk magnitude, relative risk, cumulative risk, uncertainty and interactions or synergy among risk factors. They summarize the literature around several different sorts of visual displays. The risk ladder, which displays a range of risk magnitudes such that increasing risk is portrayed higher up the ladder, was found to help people anchor a risk between upper and lower reference points. Perceived risk was often influenced more by the location of the risk on the ladder than by the actual numerical value of the risk. Risk ladders are especially useful to convey the risk associated with unfamiliar events against more familiar ones. Figure 2 provides an example of such risk scale in the case of risk of death from various causes.

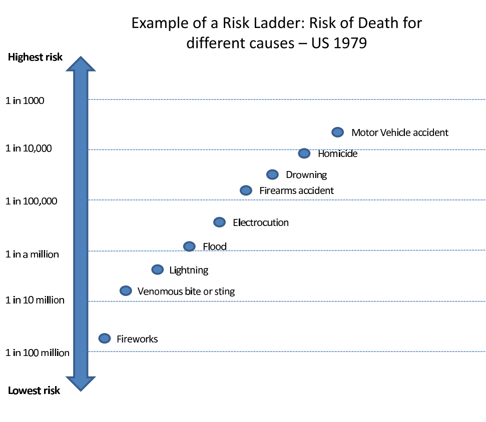


Fig. 2. Example of a risk ladder.

Stone *et al.* [40] point out that when possible, graphs should contain a reference point indicating when a hazard has reached a level that requires action, along with advice about the action to take. They also provide some guidelines for maximizing the effectivness of graphs: to avoid area or volume to depict quantities due to the optical illusion issues that are involved, to consider the aim of the visual representation (is the viewer being asked to compare two risks, to assess a trend, etc.) They also apply the work of Cleveland and McGill [7] to determine which visual methods are most applicable to particular tasks. For example, when you are interested in perceiving a precise risk magnitude or to compare two risks, then line charts, bar charts, and histograms are likely to lead to the best accuracy. They recommend minimizing the number of mental operations, thus to directly present that which the viewer is expected to compare.

The 2006 review by Ancker *et al.* [3] provides a recent survey of the main issues related to the communication of risk in a health setting, along with the current experimental research in this domain. They focus more specifically on quantitative information. It is in fact an update of Lipkus and Hollands [29]. In this paper they discuss (i) the relationship between graphical features and the ability of people to understand the risk (ii) the consequences of graphical features in terms of induced risk behavior (iii) the factors (numeracy, other) that influence the user-friendliness of graphical representations over others.

Indeed, depending on the purpose of the communication, one may follow different objectives. A public policy pamphlet may well seek to influence people's behavior while in a one-to-one discussion between a patient and his/her doctor, the focus is more likely to be on making sure the understanding of the risk is accurate. Finally designers of a decision support tool may also seek to identify factors that make communication about risk more pleasant and thereby increase the chances of adoption of their tool. In fact, one of the findings of the review is that

> "graphical features that improve the accuracy of quantitative reasoning appear to differ from the features most likely to alter behavior or intentions. For example, graphs that make part-to-whole relationships available visually may help people attend to the relationship between the numerator and the denominator, whereas graphs that show only the numerator appear to inflate the perceived risk and may induce risk-averse behavior."

Two other reviews are Fagerlin *et al.* [13], and Kurz-Milcke *et al.* [26]. Kurz-Milcke *et al.* review best practices for communicating risks such as the use of graphical representation over text format, the specification of part-to-whole relationship to help scale the risk, the use of icon arrays, and the use of simple decision trees for doctors making emergency decisions. Several examples in the paper focus more specifically on the communication of tests results such as HIV tests or Down syndrome tests during pregnancy.

An extensive overview of the status of the research on risk communication to date, including a list of best practices in health risk communication can be found in Fagerlin *et al.* [13]. In this paper, the authors discuss several specific examples of communication of risk, identifying their advantages and limitations and suggesting improvement for increasing the clarity of the information.

In addition to risk ladders, other common representations of risk are icon arrays and mortality curves. An example of icon array is presented in Figure 3.

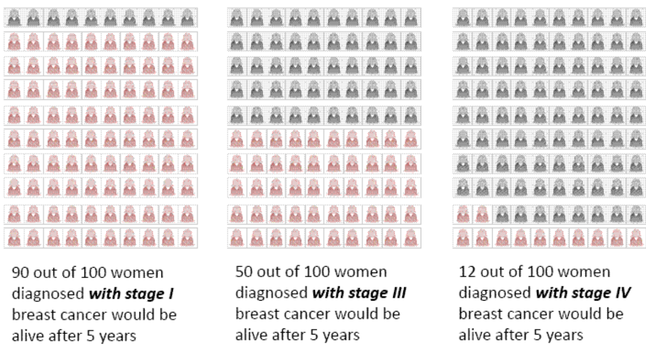


Fig. 3. Example of an icon array.

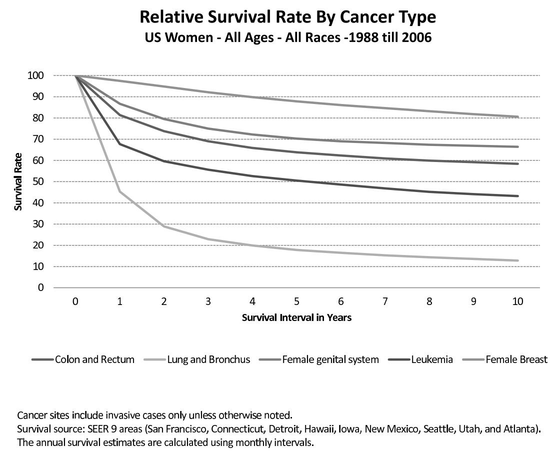


Fig. 4. Example of a survival curve.

The risk is represented by coloring a subset of the icons in a chosen color so as to represent the incidence of the risk within a reference population. As discrete representations of risk, they are more intuitive to comprehend. Many studies have compared icon arrays with the more traditional bar charts. The Shapira *et al.* ([35]) study is based on focus groups. The frequency representation through icon arrays was felt easier to understand and more accessible than the bar chart which was perceived as analytical. However, bar charts were felt to be useful for comparison purposes. The denominator appears to have some influence on the risk perception: larger denominators are associated with lower risk. This finding applied in fact both to graphical representations (icon arrays) as to text (1 in 10 versus 10 in 100). Also, they report that random arrangements of the icons do not provide many benefits and significantly hamper the ease of interpretation. Similarly, results from Zikmund-Fisher *et al.* ([46]) shows that displaying fewer options and displaying them in an icon array format has significant influence on the understanding of the patients (measured by increased knowledge accuracy). Another of the advantages of icon arrays is that they can display the part-to-whole relationship, thus ensuring a less biased perception of the risks.

Finally, survival and mortality curves are used to indicate the evolution of risk over time. The addition of a dimension (i.e. time) makes them cognitively challenging. Figure 4 provides an example of a survival curve in the case of survival rate for various types of cancer.

Survival and mortality curves often provide part-to-whole information but in a less straightforward way as other representations (stacked bars or icon arrays for instance). Studies have shown that instruction is very efficient at improving the understanding of such curves. In addition, it appears that survival curves are easier to grasp than mortality curves [3]. The choice of timeframe to represent such risks is also important. Schapira *et al.* [35] report that the younger focus group had a strong preference for 10 years while older focus group leaned toward a lifetime risk representation.

5.2 Specific Topics in Health Risk Communication

We discuss in this section a few of the important research topics related to health risk communication.

While many researchers focus on visual support, there also has been research into the verbal and numerical text presentation of risk information. In Ancker *et al.* [3], the authors state "Future research should also integrate the literature on comprehension of different number format (percentages, rates) to avoid confounding from the use of hard-to-understand numbers in graphs". In fact, Shapira *et al.* [35] study whether to present information as frequency (1 in 10) or as a probability (10%). Investigation has shown that frequency was perceived as simpler, easier to interpret and conveyed a "human" dimension whereas probability was felt as mathematical but more strongly related to personal risk. For the frequency format, some felt that a low denominator conveyed an idea of lack of reliability of the data (because it was associated with a small sample size). Stone *et al.* [40] investigate the possible reasons for increased risk avoidance when graphical rather than numerical displays are used. They found that in fact it was due to the fact that the graphical display highlighted the number of people harmed, introducing a "foreground salience" effect. This points to the importance of transparency in presenting both the numerator and denominator of the risk equation if one wants to communicate the true risk of a situation. Of course if "influence" is the goal of the communication (for example, decreasing risky behavior) then the foreground salience effect may be used to advantage. Regarding verbal representations, there is usually consensus against relying solely upon a verbal description because of the lack of reliability of mapping terms to a numeric scale [13].

Another specific topic in risk communication, yet less researched than the communication of single risk estimate, is the ability to provide a fair comparison of the risk evaluations. In Zikmund-Fisher *et al.* ([45]) and in Zikmund-Fischer *et al.* ([47] the authors consider the problem of communicating the risk of side effects from medication. Currently, in the United States, it would be presented as: "9% of people using Drug X experience heartburn, 5% of people in control group experience heartburn," thus describing for each case the total risk. Such a framing overemphasizes the risk from the medication. The authors postulate that presenting the baseline risk and the incremental risk instead would be better. For instance "5% of people experience heartburn without medication, but an additional 4% experience heartburn when taking Drug X". They test their hypothesis through two large internet-administered surveys and one with an actual patient sample where they present side effect risk under a variety of forms (incremental or total risk, text only or text + graphics, with 100 or 1000 denominator). The results confirm that presenting risk incrementally does lower the risk perception and reduces the worry level of the patients. However, in the study on actual patients [47] they found that it has a negative effect on the qualitative understanding of the risk (such as being able to correctly identify what group -those taking medicine or not- is at a higher risk of developing specific side effects). This negative consequence can be alleviated by presenting the risk in a pictograph / icon array format rather than in a numerical text format.

Finally, communicating uncertainty in risk estimates is essential in providing patients with a fair picture of the information although it adds even more complexity to the communication hurdle. Few studies seem to have looked at this specific issue [3]. One exception would be Shapira *et al.* [35], where the authors tested whether or not to provide an idea of the uncertainty along with the risk estimates. Specifically, they presented a risk either as a single point estimate or as a range. Educational level had a strong influence there on the preferences. Less-educated groups felt that the range representation conveyed vagueness while complicating the understanding. On the contrary, more educated groups felt that it did convey a notion of the scientific uncertainty related to the data and should be communicated to the patients.

In the decision analysis community, there has also been some research into defining a unit for small risks related to life-and death. Howard [19] advocates the use of the micromort, representing one chance in a million of dying as a more natural measure than chance of dying. While a simple rescaling, it enables a more meaningful comparisons among risks from those associated with commuting by car to skydiving to medical risks. In that sense it is akin to risk ladders.

In parallel with core health risk communication research, there has been several investigations into the link between numeracy, defined as the quantitative skills required for understanding numerical information, and people's ability to correctly interpret health risk information. Fagerlin *et al.* [13] report that poor numeracy skills are associated with a higher chance of poor health outcome, because of the difficulty of following complicated treatment instructions. This is an important challenge for the healthcare community as it is reported [1] that in 2003 20% of Americans have poor (below basic) numeracy skills It has been shown in particular that poor numeracy is linked with an over-estimation of risk and a difficulty in estimating the benefits of a procedure. In turn, this implies that people with low numeracy are more likely to choose a treatment option that is not aligned with their preferences.

Given the importance of numeracy, Fagerlin *et al.* [14] developed an alternative to the objective numeracy test that asks patients about their preferences regarding the communication of numerical information. Such test presents the advantage of being faster and more practical to administer as it can be done over the phone or the internet (no cheating needed as there is no wrong answer) , and perceived as much less judgmental than the objective test.

There are also tests specifically designed to evaluate functional health literacy in patients which combine both reading and comprehension exercises with numeracy exercise such as (i) determining the next time a pill should be taken if one needs to take a medicine every six hours and has last taken it at 1pm or (ii) determining if ones vital reading (blood pressure for instance) is within the normal range, with the normal range being clearly specified. One such test is described in Baker et al (1999) which according to the authors can be administered in almost half the time (from 22 minutes to 12 minutes), as the TOFLHA [33] test with similar reliability (self-consistency) and validity (as compared to a third health literacy test REALM [9]).

Some research has focused also on the appraisal of the communication of risks in existing decision support tools. Glasspool et al (2010 Interactive Decision Support for Risk Management). This paper presents the REACT tool that has been developed researchers in England with the objective of communicating the risks associated with breast cancer. In particular the REACT tools enables to gather information- facts(demographics) and past decisions (such as taking the birth control pill or breastfeeding)- and to evaluate their effect on the lifetime risk, thereby providing a personalized risk profile to the patients. The tool also provides the ability to explore the effect of specific interventions (oophorectomy for instance) on the risk profile. In the paper, the authors report the reaction of eight genetic counselors that used the tool as support during test interviews with actors. Overall, the reaction of counselors is positive in particular they appreciate the dynamic aspect of the risk profile. However, they were also concerned about whether and under what format to communicate the risk profile, fearing that it over-estimates both the baseline risk and the effect of interventions.

Zikmund-Fisher *et al.* [46] describe an experiment where they compared the current risk displayed provided by Adjuvant Online with alternate designs. Adjuvant! Is an online calculator target at clinicians but oftentimes used during patient-doctor discussion about adjuvant therapy decisions (chemotherapy and/or hormone therapy) in order to reduce the likelihood of cancer recurrence and thus mortality risk. The current display of risk for adjuvant was a bar chart with four bars, one for each option (no therapy, chemotherapy only, hormone therapy only or combined therapy). A study on more than 2000 randomly selected women was undertaken to compare the performance of the current graphic with three other designs:

- Four options but displayed as icon arrays side-by-side
- Only two options (removing the no therapy and chemotherapy only options) as bar chart
- Only two options as icon array.

Beyond improved accuracy when displaying information through icon arrays, the results suggest that simplifying the graphics in that manner appeared to significantly reduce the answer time. Nonetheless, the authors conclude that the accuracy is still far from perfect (around 70%) and therefore that further improvements are required.

Before concluding this section on health risk communication, one nuance to keep in mind is that there can be a discrepancy between people's preferences (in terms of how they would like to be communicated risk) and the support that they interpret best. Elting *et al.* [10] found that doctors performed worst with the format they liked best, and best with the one they strongly disliked.

6 Risk Visualization in General

The risk visualization literature in the medical risk area focuses on risk primarily in the situation of a discrete probability distribution (*eg.* the chance of having a particular side effect from a medicine or treatment, vs. not experiencing such an effect, or the chance of having a disease based on a positive result on a test, or not having such a disease). We are interested as well in continuous probability distributions which complicates the communication task.

Epper and Aeschimann [11] summarize the state of the art of visual risk communication with a number of examples. These examples are primarily in the area of flow charts and cause-and-effect diagrams which illustrate which factors may lead to particular risks, or "metaphorical" diagrams which aim to translate a set of risks into an image of icebergs with possibly hidden dangers below the surface.

In Li *et al.* [27], the authors present a model for predicting the risk and severity of electric power outages. A statistical model is necessary because weather predictions are imprecise. In addition, because weather data is only available at discrete points, interpolation is necessary, and the accuracy of the prediction will depend on the distribution of the weather station locations. They use a texture to indicate the uncertainty of the forecast of the number of outages, with white and color "striping." A higher proportion of white indicates a more imprecise forecast of a particular severity of outages.

Feather *et al.* [15] discuss a variety of risk visualizations in the application area of a large software and hardware development project. There are multiple sources of risk to the project, with a variety of mitigation strategies with varied cost and effectiveness. They explore different ways of presenting this information to the user, including overviews of the level of risk for different parts of the project, comparisons of different approaches to mitigating risk, and explorations of the optimal boundary when considering the cost vs. the benefit extracted within the huge range of strategies.

Finally we consider work which explicitly considers probability *distributions* rather than simply binary events. Shortle and Mendel [36] present a method for drawing probability densities. While it has several useful properties, such as allowing recovery of event probabilities, and reducing the dimensionality required to draw the plot than other methods, it also violates one of the basic principles which we feel are important in providing an intuitive interface: that the graphical space occupied by a visual element should correspond linearly to the data value represented. In fact their form plot has exactly the opposite behavior, with the possible result that the typical user may find it difficult and potentially misleading to interpret.

Haisley *et al.* study the communication of financial risk [18] and thus also explicitly address a continuous, as opposed to a binary, probability distribution. Interestingly, they found that involving users in a simulation where they "experienced" the risk scenario resulted in greater risk-taking along with a more accurate understanding of the true probability of loss.

7 Alternative Representations of Risk

Here we make some proposals for some alternative ways to visualize a probability distribution. In particular, we focus specifically on the communication of continuous probability distributions, first in the general case and then for a specific application area, i.e., transportation.

7.1 General Case

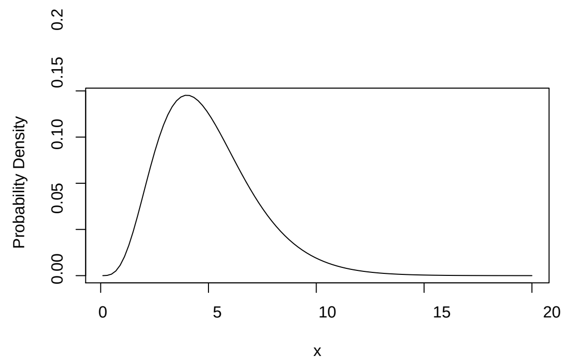
The most common methods of displaying continuous probability distributions are

- **Probability density functions (abbreviated as pdf)**, as displayed in Figure 5(a) for a gamma distribution.
- **Cumulative distribution functions (abbreviated as cdf)**, as displayed in Figure 5(b) for a gamma distribution.
- **Histograms**, which are a discretized version of the probability density function representation, as displayed in Figure 6(a). This is one of the most common representations of probability functions for non-scientific audiences.
- **Boxplot (also called "box and whisker" plots)** as shown in Figure 6(b). They represent a summary of some of the main statistics of the distribution: mean and quartiles. For empirical distributions, they also highlight outliers. They are beloved of statisticians, as they can show multiple attributes of a probability distribution simultaneously, but they are rarely used for a more general audience, as they require a significant amount of training to understand.

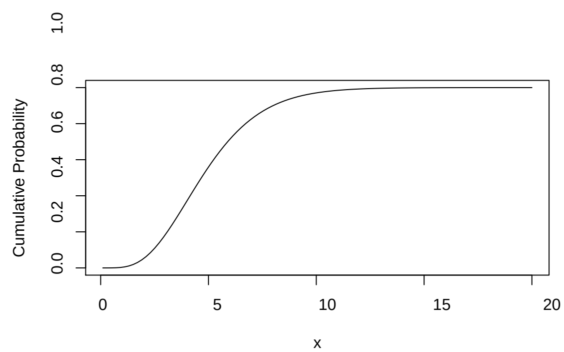
Note that when representing sample data (rather than a theoretical distribution), it is customary to rely on histograms and boxplots rather than cdfs and pdfs.

Each of these representations has pros and cons with respect to deriving useful information in an easy and intuitive way. Typically, the information that people seek to extract from a probability distribution for decision making purposes is as follows:

- **Central value** (also called location), a single number statistical parameter representative of the distribution. The most common candidates for capturing central tendency are the mean, the median and the mode (most likely value).
- **Notion of variability** (also referred to as spread), which is often conveyed through the standard deviation, but can also be obtained from the inter-quartiles range or simply from the shape of the distribution.
- **Percentiles and cumulative probabilities**, where percentile is defined as the value of the random variable below which a certain percent of observations fall, so in mathematical terms, for a random variable X, the n^{th} percentile is the value v_n such that $P(X \leq v_n) = \frac{n}{100}$. Oftentimes however, people are interested in the likelihood of reaching at least (or at most) a certain value v, which is simply $P(X \leq v) = F(v)$, i.e., the cumulative distribution function evaluated at value v.



(a) Probability density for a gamma function.



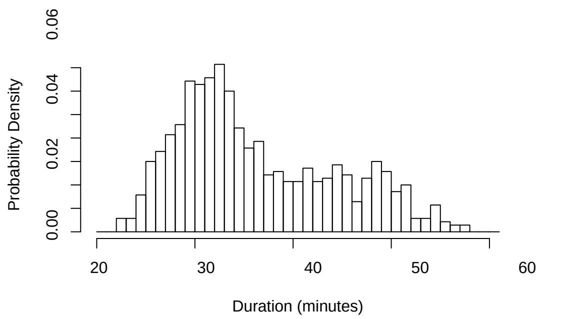
(b) Cumulative distribution function for a gamma function with the same parameters as Figure 5(a).

Fig. 5. Some standard ways to a probability distribution, in this case for a gamma distribution with $\alpha = 5$ and $\beta = 1$.

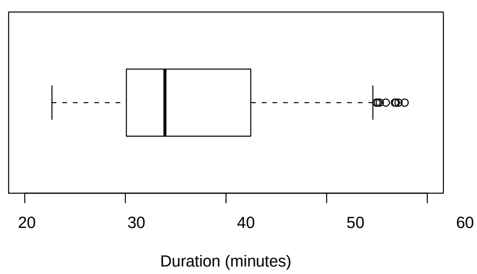
Both the probability density function and the cumulative distribution function representations can provide all the information listed above but often this is through mental calculations. For instance, while the central value and variability can be somewhat assessed (and compared) from the shape of pdfs, percentiles require estimation of the area under the curve (integration). The cdfs, by contrast, provides direct information of the percentiles but are much less intuitive in terms of understanding central value or variability. Histograms, because they are a discretised version of the pdfs, imply a loss of information. This feature however makes the estimation of percentile easier, as area estimation is replaced by the addition of the relevant categories. Finally, boxplots, which represent a summary of the pdfs through a set of statistics, also enable, for the trained user, a fast evaluation of central value and variability but provide limited ability, even with mental calculations, to obtain percentiles or their cumulative probabilities.

We are interested in alternatives which may be simultaneously easier to understand and information-rich, particularly for the typical consumer of information, who is probably not scientifically-trained. For example, we seek to find representations that enable at the same time to estimate central value, variability and cumulative probabilities. We feel that providing sufficient granularity is essential in a number of real-case scenarios such as, for instance, some persons may need to determine the probability of being *more than* 10 minutes late, while another may be interested in the probability of being more than *one* minute late. A graphical presentation that allows both of these to be easily derived is advantageous.

As discussed in Section 5, icon arrays have been found to be well-understood by people. One characteristic of icon arrays which may contribute to their success is that the icons naturally represent a proportion of the whole. When arranged in a rectangular form, they also naturally convert the probability, or percentage, to a length. As discussed in Cleveland and McGill [7], using length within an image is



(a) A histogram showing probability density for duration of travel time (simulated data).



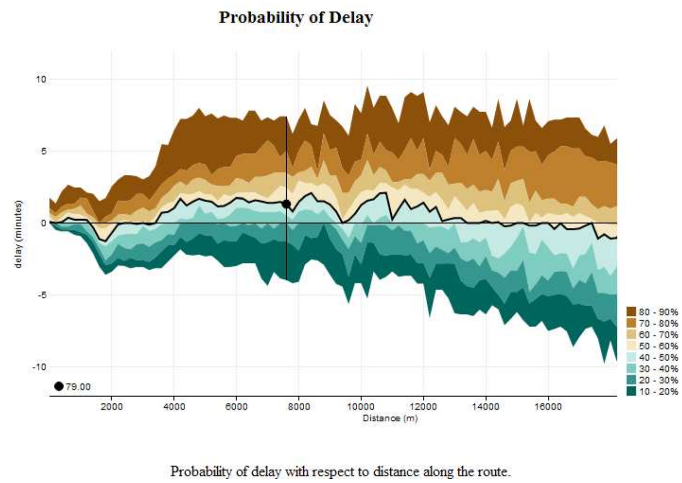
(b) Boxplot showing the statistics of the duration of travel time for the same data as shown in Figure 6(a). The box itself outlines the region between the 25th and 75th percentiles, with the median represented by the vertical line within. The plot "whiskers" extend to the most extreme data point which is no more than 1.0 times the interquartile range (between the 25th and 75th percentiles) from the box. The many overlapping circles to the right represent the "outlier" points which in this case, are all at the upper end of the data range.

Fig. 6. Some standard ways to display the statistics of sample data. Note how both the histogram and the boxplot capture essential details of the distribution such as central tendancy, skew, and outliers.

appropriate for comparing quantities. We keep this fact in mind when designing possible graphical representations of probability.

7.2 Application Area: Travel Times

As risk perception and communication greatly varies with the application domain, we decided to focus on a particular area to provide some practical examples. With the collaboration of a metropolitan bus authority we researched novel ways to represent travel time within the city bus network. Using recorded GPS traces from buses, we were able to compute the distribution of deviation times from expected at points along a bus's route. Figure 7(a) shows a representation of this data. Here we show the (cumulative) probability of different amounts of "delay" (negative delays correspond to the bus being ahead of schedule). As can be seen, the delay starts at zero at the time the bus leaves the starting depot. This is a snapshot from an interactive visualization in which the user can select different spots along the route; here the mouse is at a point approximately 6km along the route, and the label at the bottom left indicates that the median delay at this point is negative 25 seconds. The probability distribution itself is shown using a diverging map using ColorBrewer [6], with a breakpoint at the median of the probability distribution. We plan to integrate this visual presentation with a map of the city bus system, so that the location along the route can be interactively associated with the two-dimensional location in the city. Figure 7(b) shows such a map which is currently in use to highlight in real time not only the location of a particular bus, but portions of the route for which the bus was ahead of (green) or behind (red) schedule.



(a) Cumulative probability of delay relative to scheduled time for a particular bus route. Color map is a diverging color map based on ColorBrewer [6]

(b) One bus route in the system. A current bus location is indicated with an icon, and regions of advance or delay relative to scheduled time are shown in green or red respecitvely.

Fig. 7. Screen shots from visual representations of the probability of delay on an urban bus route.

8 Conclusion

We provide in this paper an overview of the literature related to the visual communication of risk, which we understand as continuous or discrete distributions. We focus in a large part on the healthcare domain which has received significant attention in terms of communicating risks associated with conditions, treatments and side effects. We observe however that most of the risk information represented is about discrete probability distributions, while many real life situations, whether within the medical domain or beyond, call for the communication of continuous probability distributions. We discuss standard representations of the latter and propose a modified representation which we feel provides relevant information more easily. Finally, we discuss the communication of continuous distributions in the specific case of transportation.

This paper specifically investigates the visual communication of risk through the representation of its *mathematical* estimation. However, as we discussed in section 3, public perception of risk is not limited to quantities but also includes qualitative factors about the characteristics of the risk. Therefore, a natural direction for future research would be to explore how to best communicate the qualitative aspect of the risk along with its quantitative characterization. In the case of travel time for instance, people may want to obtain a description of the source of the risk, whether it is traffic jams, mechanical failures, transfers or even strikes, specially as they probably would associate different *dread* and *unknown* grades to each of the sources.

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