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Valve Placement in Water Networks: Mixed-Integer Non-Linear Optimization with Quadratic Pipe Friction

Bradley J. Eck, Martin Mevissen
IBM Research
Smarter Cities Technology Centre
Mulhuddart
Dublin 15, Ireland

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Bradley J. Eck [1] Martin Mevissen [2]

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ABSTRACT

This paper presents the formulation and solution of an optimization model for placing pressure reducing valves on water networks. A key advance is the development of a quadratic approximation for pipe head loss minimizing the relative approximation error that provides accuracy comparable to the error of the classical equations of Hazen-Williams or Darcy-Weisbach. This approximation enables use of a broader class of optimization engines, provides fast and accurate solutions, and increases the network size solvable by mathematical optimization. The development and utility of the quadratic approximation is demonstrated for the problem of optimal valve placement on benchmark networks having 25 nodes and 1900 nodes, respectively. The quadratic approach compares favorably to EPANET results and is significantly faster than a comparable optimization model that uses a piecewise linearization of the head loss curve.

Keywords: Optimization, Water Networks, Pipe Friction, Modeling

INTRODUCTION

As water systems age and resources to make repairs diminish, optimization techniques are increasingly applied to design and operational problems on water networks. Design problems include placement of pressure reducing valves to minimize pressure or leakage and selection of pipes and pipe sizes for minimum cost rehabilitation. Operational problems include finding set points for pressure reducing valves and scheduling pumps to minimize electricity costs.

These problems have in common the need to simulate a network's hydraulic behavior and the simulation method varies according to the optimization technique. Where meta-heuristics such as genetic algorithms are used, the optimizer is often coupled to a classical hydraulic solver such as EPANET (Rossman 2000).

[1] IBM Research - Ireland, Technology Campus Building 3 F-16, Mulhuddart, Dublin 15 Ireland. Phone: +353 18 269 354, Email: bradley.eck@ie.ibm.com

[2] IBM Research - Ireland, Technology Campus Building 3 F-1, Mulhuddart, Dublin 15 Ireland. Phone: +353 18 269 210, Email: martmevi@ie.ibm.com

While this approach may be the only option for large networks, there is no direct assurance that a solution is optimal. Alternatively, mathematical optimization methods embed hydraulic equations as constraints, and assure that solutions are locally optimal, globally optimal or provide optimality bounds. Design problems with binary decisions are particularly challenging for math optimization methods because the number of possibilities is exponential in the number of binary variables.

The challenge of using hydraulic equations, in particular the energy equation for pipe flow, as a constraint is the strong non-linearity of the curve for friction-induced head loss. Several well known models of this curve are available. The empirical formulas of Hazen and Williams and Manning give explicit equations, but use fractional exponents (Franzini and Finnemore 1997). These models are also often misapplied, resulting solutions with poor accuracy. The Darcy-Weisbach equation for head loss is accurate and theoretically sound, but requires a friction factor that is calculated from an implicit function.

In this paper, a new formulation for pipe head losses in optimization models is developed and applied to the problem of placing pressure reducing valves in an existing network. Valve *placement* is a challenging problem because it combines binary decisions - whether to place a valve on a pipe or not - with continuous decision variables representing nodal pressures, pipe flows and valve set points. Once valve locations are known, their settings are of interest. Finding the settings is a valve *control* problem and is easier because all decision variable are continuous. The control problem is embedded in the placement problem because settings must be chosen for a given placement.

The physical motivation for studying valve placement and control is to reduce leakage on water networks. Leakage may occur on distribution mains, service connections, or at the point of use. Although leakage localization and pipe replacement would be ideal, this is expensive and slow. Lowering system pressure by inserting control valves can reduce, though not eliminate, leakage before the pipes are replaced.

The valve control problem has been studied extensively using mathematical programming starting with Sterling and Bargiela (1984), who used a Taylor-series approximation of the head loss curve to apply sequential linear programming (SLP). An SLP technique was also used by Germanopoulos and Jowitt (1989) and by Jowitt and Xu (1990). Vairavamoorthy and Lumbers (1998) introduced sequential quadratic programming (SQP) for the valve control problem. A parallel computing technique using SQP is given by Alonso et al. (2000), who parallelized by assigning different time steps to each node. The only work to address valve placement using math programming is due to Hindi and Hamam (1991), where a piecewise linearization of the head loss relationship is used. They consider networks having 18 and 72 nodes, respectively.

Meta heuristic approaches have also been applied to valve placement and control. Savic and Walters (1995) use a genetic algorithm to find settings of isolating valves to minimize pressure heads. Reis et al. (1997) find optimal locations and settings for control valves. A multi-objective approach considering the number, location, and setting of valves is proposed by Nicolini and Zovatto (2009). A scatter-search algorithm is used by Liberatore and Sechi (2009).

Recent work on water network design using math optimization is also relevant here because the same hydraulic equations are under consideration. Bragalli et al. (2011) solve a mixed integer nonlinear program (MINLP) using a Branch-and-Bound heuristic to find optimal pipe sizes. They use the Hazen-Williams model for friction loss and fit a quintic polynomial over the small flow range near zero so that the head loss curve is differentiable. Network sizes considered range from 7 to 272 nodes.

A global optimization approach for least cost design of a given water network configuration has been proposed by Sherali and Smith (1997). It is based on a branch-and-bound algorithm using iterations of polynomial lower and upper bounds for the Hazen-Williams formula. This global approach has been demonstrated on water networks with up to 10 nodes and pipes.

A recent approach for globally optimal operation of water networks is due to Gleixner et al. (2012), who compute a pump schedule and flow distribution to minimize energy and water procurement costs. They use a quadratic approximation for Darcy-Weisbach friction loss according to the law of Prandtl-Karman. This approximation assumes hydraulically rough pipes and eliminates the dependency of the Darcy friction factor on the flow rate. The resulting nonconvex MINLP is solved to global optimality for networks having 25 and 88 nodes, respectively.

The remainder of this paper discusses a novel quadratic approximation for pipe head loss and uses the approximation to formulate a MINLP for valve placement on water networks. The optimization problem is solved for two benchmark networks. The MINLP model is compared to Epanet for accuracy and to a MILP formulation for computational effort.

QUADRATIC APPROXIMATION FOR PIPE FRICTION

Friction losses in pipe systems are usually estimated using the formula of Hazen and Williams or Darcy and Weisbach (Franzini and Finnemore 1997). Manning's equation may also be used, but is not discussed here. Using the Hazen-Williams formula and SI units, the frictional head loss in a pipe is computed from (American Water Works Association 2004)

$$h_f = 10.65 C^{-1.852} D^{-4.871} L Q^{1.85} \tag{1}$$

where h_f is the head lost due to friction in meters; C is the Hazen-Williams coefficient; D is the inside diameter of the pipe in meters; L is the length of pipe in meters; and Q is the discharge in cubic meters per second. The Hazen-Williams formula applies only to water flowing in pipes 5cm in diameter or larger and at speeds less than 3 m/s (Franzini and Finnemore 1997).

The Darcy-Weisbach equation applies to any fluid and consistent system of units and is (White 1999)

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

where f is the Darcy friction factor; V is the average fluid velocity; g is the gravitational constant; and other terms are defined previously. The Darcy friction factor f depends on the roughness (k_s) and diameter of the pipe and the Reynolds number (Re). It may be found graphically from the Moody diagram (Moody 1944) or by inverting the implicit relationship of Colebrook (White 1999):

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{k_s}{3.7D} + \frac{2.51}{Re\sqrt{f}} \right) \tag{3}$$

Although Eq. 1 and Eq. 3 are often treated as exact, they are fitted to experimental results and therefore contain some error. Within the flow range where tests were performed, Liou (1998) reports errors from -15% to 10% in the Hazen-Williams formula depending on pipe material. On the Darcy-Weisbach side, the Colebrook equation for f has an average error of 11% with respect to Colebrook's data (Yoo and Singh 2005).

In both Eqs. 1 and 2 the head loss is approximately proportional to the square of the velocity, or equivalently, the discharge. This proportionality suggests that the head loss on a particular pipe can be approximated by a quadratic function. The advantage of making such an approximation is to facilitate the use of broader classes of optimization methods as described below.

In developing an approximation function one must consider several factors:

- the functional form of the approximation;
- a merit function for evaluating candidates;
- the relevant range of the original function; and,
- the method for finding coefficients.

This work uses the same functional form whether Darcy-Weisbach or Hazen-Williams is used as an underlying model:

$$\hat{h}_f(Q) = aQ^2 + bQ \tag{4}$$

with a and b as unknown dimensional coefficients. The intercept is intentionally dropped to enforce the origin as a point on the curve. Coefficients are selected to minimize the *relative* error over the approximation range. Focusing on the relative error is a key element of this approach because large absolute errors at low flows can alter flow distribution in the network. The exact merit function, approximation range and method of finding coefficients differ slightly between head loss models as discussed below.

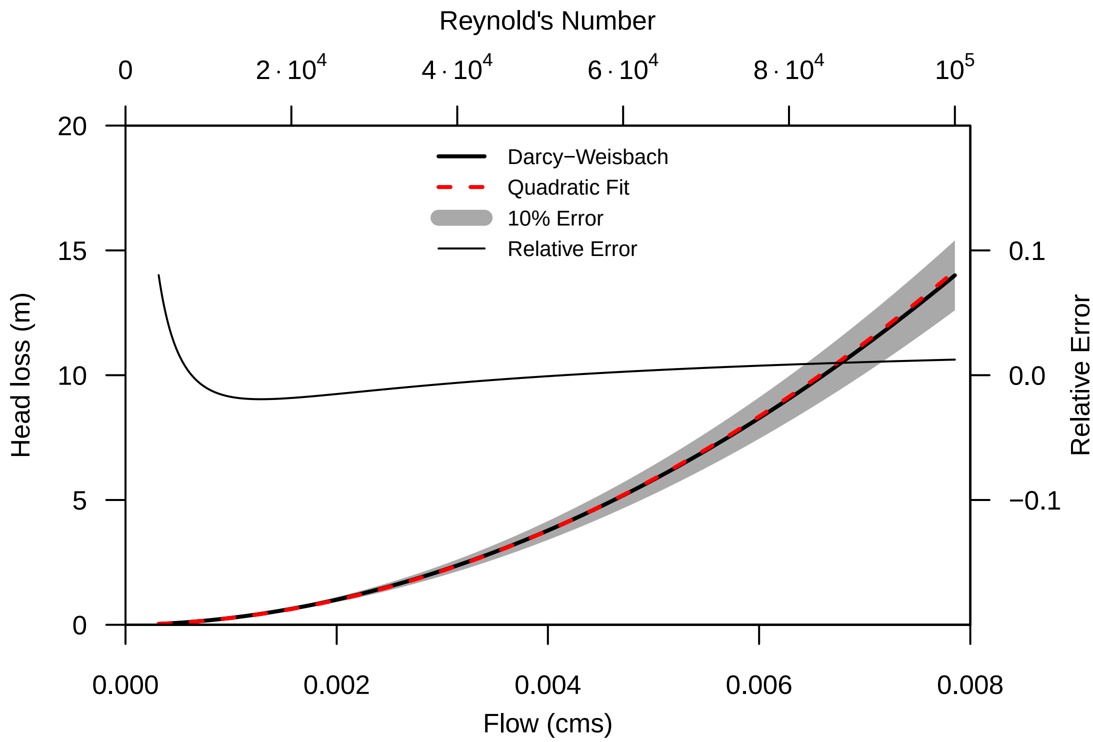

FIG. 1. Quadratic approximation for head loss according to the Darcy-Weisbach formula for an example pipe where L = 1000m, D = 100mm and ks = 0.3mm. The fit used an interval of $4000 \leq Re \leq 10^5$**.**

Darcy-Weisbach

The preferred method for modeling head loss on water networks is the Darcy-Weisbach law of Eq. 2. If the friction factor f were constant the head loss curve would be purely quadratic; this is the primary motivation for developing a quadratic approximation. The fit proposed here minimizes the sum of squared relative errors (SSRE) over the turbulent flow range ($Re > 4000$).

$$SSRE = \sum \left(\frac{\hat{h}_f - h_f}{h_f} \right)^2 \tag{5}$$

Due to the implicit form of Colebrook's equation for f the fit is obtained numerically using weighted multiple linear regression. By using $1/h_f^2$ as the weight for each point, the regression minimizes relative rather than absolute errors.

A quadratic fit to the Darcy-Weisbach head loss curve in a sample pipe is shown in Fig. 1. The quadratic approximation shows good agreement over the approximation range $4 \cdot 10^3 < Re < 10^5$. The relative error ranges from -1.9% to 8% with the largest relative errors occuring at low flow rates.

Because the quadratic fit is developed over the turbulent flow range the accuracy of the approximation in the laminar and transition ranges is of interest.

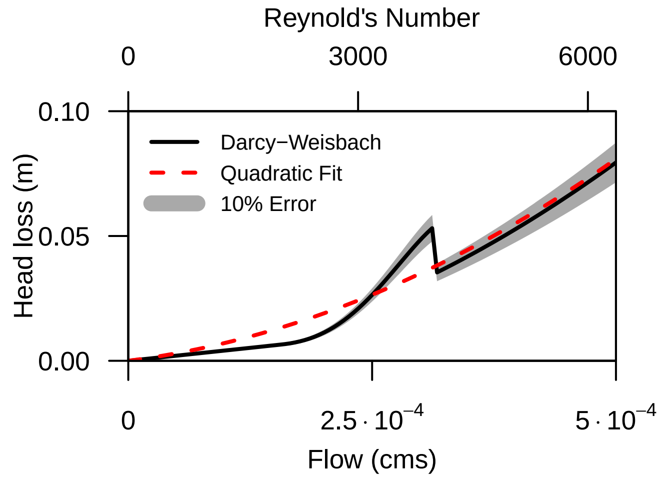

FIG. 2. Head losses at low flows for an example pipe where L = 1000m, D = 100mm and ks = 0.3mm. The quadratic approximation uses an interval of $4000 \leq Re \leq 10^5$**.**

Since head losses in the laminar region are linear, the quadratic approximation over the turbulent range does not provide a very good estimate (Fig. 2). However, the errors noted here do not affect the flow distribution in the networks studied because head losses are low. Fig. 2 also illustrates why the transition flow range is excluded from the approximation: trying to minimize relative errors through the jump in the transition range sacrifices accuracy everywhere else along the curve. Even though the quadratic approximation comes with a considerable relative error at low flows, this does not cause significant constraint violation for the two optimization problems of interest - valve setting and valve placement - since the resulting absolute head loss is low.

Visual inspection of the Moody diagram shows that the friction factor varies over a smaller range as the relative roughness of the pipe increases. This smaller range implies that rougher pipes, such as those in aging distribution systems, are well approximated by a quadratic function. Indeed the fit improves as pipes become rougher (Table 1). The table also illustrates that the approximation's accuracy is not very sensitive to the upper bound of the fitting range. Doubling the size of the flow range had a minimal effect on R^2.

Hazen-Williams

The Hazen-Williams formula for pipe head loss applies only over a limited range; outside this range it does not reflect the underlying physics and has considerable errors compared to experimental results (Franzini and Finnemore 1997); (Liou 1998). These deficiencies notwithstanding, the equation remains in wide use because the explicit formulation is easy to compute. In fact, all previous literature on valve placement in water networks of which the authors are aware use Hazen-Williams for this reason. Since existing models of real networks often use Hazen-Williams, a quadratic approximation is provided here.

TABLE 1. Values of R^2 for Eq. 4 with respect to Darcy-Weisbach head loss for a pipe having length 100m and diameter 10cm. The fit is derived over the intervals $4000 < Re < 10^5$ and $4000 < Re < 2 \cdot 10^5$, which correspond to maximum velocities of 1 and 2m/s.

k_s/D	R^2 for Vmax=1m/s	R^2 for Vmax=2m/s
0.01	1	1
0.001	0.9992	0.9991
0.0001	0.9978	0.9965

A quadratic approximation to the Hazen-Williams formula can be found numerically as described above for Darcy-Weisbach. However, with its explicit functional form a quadratic approximation to the Hazen-Williams formula can also be obtained analytically. Since both the model and approximation functions are continuous, the relevant quantity to minimize is integral of relative errors:

$$F(Q_1, Q_2) = \int_{Q_1}^{Q_2} \left(\frac{aQ^2 + bQ - \alpha Q^{1.85}}{\alpha Q^{1.85}} \right)^2 dQ \tag{6}$$

where Q_1 and Q_2 specify the range of interest and α is the part of Eq. 1 that does not vary with flow rate ($\alpha = 10.65 C^{-1.852} D^{-4.871} L$). The integral of Eq. 6 is found and the resulting form is differentiated by a and b. These differentials are set equal to zero, giving a 2x2 system of linear equations. Solving the system yields expressions for a and b as functions of the approximation interval:

$$\begin{aligned} a &= \frac{C - bA}{B} \\ b &= \frac{\frac{AC}{DB} - \frac{E}{D}}{1 + \frac{A^2}{DB}} \end{aligned} \tag{7}$$

where

$$\begin{aligned} A &= \frac{Q_2^{0.3} - Q_1^{0.3}}{0.3\alpha^2} \\ B &= \frac{Q_2^{1.3} - Q_1^{1.3}}{1.3\alpha^2} \\ C &= \frac{Q_2^{1.15} - Q_1^{1.15}}{1.15\alpha} \\ D &= \frac{Q_2^{-0.7} - Q_1^{-0.7}}{0.7\alpha^2} \\ E &= \frac{Q_2^{0.15} - Q_1^{0.15}}{0.15\alpha} \end{aligned} \tag{8}$$

A quadratic approximation to the Hazen-Williams curve for a typical pipe is shown in Fig. 3 along with the relative error curve. Several points of interest are noted:

1. The quadratic fit provides errors within 10% over the range of flows;
2. since the relative error is minimized, there is a singularity in the relative error at zero flow; and,
3. the shape of the relative error curve suggests a method for choosing the approximation interval.

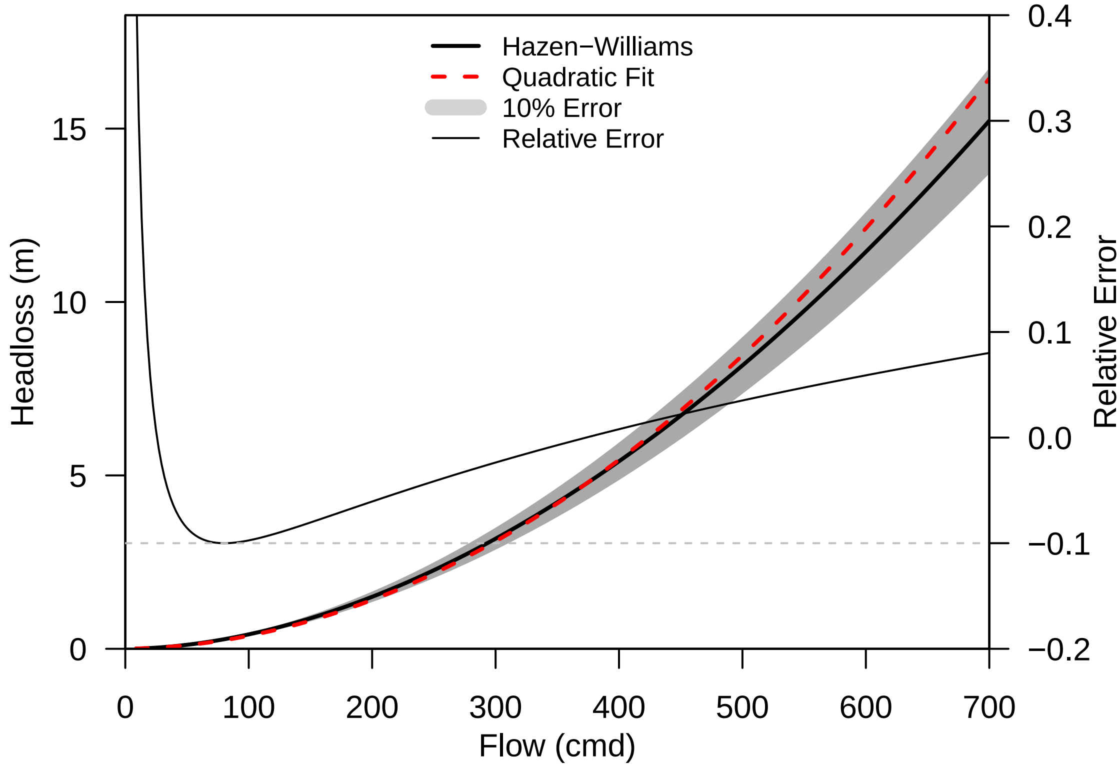

FIG. 3. Quadratic approximation for head loss according to the Hazen-Williams formula for an example pipe where L = 1000m, D = 100mm and C = 120. The fit used an interval of 8.6 to 700 cubic meters per day (cmd) where the lower bound was chosen to give a minimum relative error of 10%.

Choosing the approximation interval $[Q_1, Q_2]$ is a key step because the values of a and b depend on the interval. One method for choosing the interval is to select Q_2 based on the details of the problem and then choose Q_1 so that the minimum of the relative error curve sits at an error tolerance. An upper bound for Q_2 for each pipe is the sum of nodal demands in the network. Other approaches to setting Q_2 include choosing a maximum velocity or power dissipation in each pipe. The location of the minimum relative error is found from calculus:

$$Q_{\epsilon_{min}}(Q_1, Q_2, \alpha) = 0.1275\frac{b}{a} \tag{9}$$

With (9) and an error tolerance, ϵ_{tol}, Q_1 is found at the root of

$$G = \frac{aQ^2_{\epsilon_{min}} + bQ_{\epsilon_{min}} - \alpha Q^{1.85}_{\epsilon_{min}}}{\alpha Q^{1.85}_{\epsilon_{min}}} + \epsilon_{tol} \tag{10}$$

This approach to setting the approximation interval is conservative in that the worst underestimate of head losses is specified. Head losses which are over-estimated occur only at the extreme ends of the interval. The relative error is largest for small flows. However, this does not cause significant absolute constraint violations when solving the resulting optimization problem.

OPTIMIZATION PROBLEM

One application for the quadratic approximation of pipe head loss is the problem of optimal valve placement and setting in water distribution networks. A water network comprised of N_n nodes and N_p pipes is modeled as a directed graph with N_n vertices and $2N_p$ edges called links. Nodes are numbered $i = 1...N_n$ and nodal quantities include demand d_i, elevation e_i, and hydraulic head h_i. Links are identified by source and target node and link quantities include flow rate $Q_{i,j}$, head loss $h_f(Q)_{i,j}$, and a valve indicator $v_{i,j}$. The optimization problem is to place N_v pressure reducing valves on the network links and to determine their set points, such that the pressure distribution is minimized. The setting of each valve is found from the pressure at the downstream node.

This objective can be expressed in several ways, the simplest of which is minimizing the sum of total pressures.

$$\text{minimize } F(p) = \sum p_i \tag{11}$$

where p_i is the nodal pressure at node i. Other objective functions are reasonable to consider including the weighted average pressure on each pipe

$$\text{minimize } F(p) = \sum w_{i,j} \frac{p_i + p_j}{2} \tag{12}$$

The objective is minimized subject to the constraints of mass conservation around each node and energy conservation around each pipe. Mass conservation for the ith node is written

$$\sum_k Q_{k,i} - \sum_l Q_{i,l} = d_i \tag{13}$$

where $Q_{k,i}$ the inbound flows and $Q_{i,l}$ the outbound flows for node i. Energy conservation for the pipe i, j is written as two constraints:

$$Q_{i,j}(p_i + e_i - p_j - e_j - h_f(Q)_{i,j}) \geq 0 \tag{14}$$

$$p_i + e_i - p_j - e_j - h_f(Q)_{i,j} - Mv_{i,j} \leq 0 \tag{15}$$

If $Q_{i,j} > 0$ and $v_{i,j} = 0$, (14)-(15) is simply Bernoulli's equation. With an expression for h_f that is quadratic in Q, (14) is a polynomial inequality constraint of degree three and (15) is a quadratic inequality constraint. One effect of placing a pressure reducing valve on a link from i to j is that the energy constraint (15) is disabled, i.e. friction loss will be greater than predicted by pipe losses alone. Placing a valve on a link is indicated by a binary variable $v_{i,j} \in \{0, 1\}$ and the switch is implemented as a "big M" constraint in Eq. 15. In numerical experiments, M was chosen as small as possible depending on the parameters for each pipe.

There is a trade-off in formulating the energy constraints: by introducing binary variables to indicate the flow direction in each pipe, the energy conservation laws could be modeled by a set of quadratic inequality constraints. However, this results in doubling the number of binary variables in the optimization problem. One advantage of the formulation (14)-(15) is that the optimization problem of setting valves does not involve any binary decision variables, once the location of the valves is known or given. As shown in (Sherali and Smith 1997), under the constraints (14)-(15) a solution where both $Q_{i,j} > 0$ and $Q_{j,i} > 0$ is infeasible.

Pressure reducing valves are modeled as a binary variable associated with each link, but there can only be one such valve on any pipe in the network and no more in total than allowed by the user.

$$\begin{array}{ll} v_{i,j} + v_{j,i} & \leq 1 \\ \sum_{(i,j)\in E} v_{i,j} & \leq N_v \end{array} \tag{16}$$

The user must also specify a minimum pressure as a service requirement.

$$p_{\min} \leq p_i \leq p_{\max} \tag{17}$$

Because the pipe head loss curve is estimated over a range a maximum flow is also needed.

$$0 \leq Q_{i,j} \leq Q_{\max} \tag{18}$$

Advantages of this formulation include the small number of binary variables–there is only one for each link–and the linear objective. The constraints are also linear except for the head loss constraint (14)-(15). We denote optimal valve placement and setting problem to minimize (11) under the constraints (12) - (18) as VP-MINLP (mixed integer nonlinear program). In case the location of the pressure reducing valves is fixed and the sole decision is the optimal setting of these valves, VP-MINLP reduces to a continuous optimization problem where $v_{i,j}$ are input parameters. This reduced problem of valve setting is denoted as VS-NLP. Due to constraints (14)-(15) both, VP-MINLP and VS-NLP, are nonconvex problems. VP-MINLP is a polynomial optimization problem of degree three and dimension $N_n + 4N_p$, VS-NLP (nonlinear optimization problem) is a polynomial optimization problem of degree three and dimension $N_n + 2N_p$.

Due to the scale of water networks of interest in this paper - with $N_n = 1893$ and $N_p = 2469$ for the largest instance - we solve VP-MINLP and VS-NLP by branch-and-bound and interior point methods using the solvers Bonmin (2011) and Ipopt (2011), respectively. Since both optimization problems nonconvex, these solvers are not guaranteed to find globally optimal solutions. However, we follow the recommendations in (Bragalli et al. 2011) for nonconvex MINLP to choose the Bonmin Branch-and-Bound options. Moreover, as a global solver for nonconvex MINLP we considered Couenne (2011).

We compare the performance of this non-linear programming approach for optimal valve placement to simulation results obtained by Epanet and to a MILP (mixed integer linear programming) formulation. The MILP is obtained when choosing a piece-wise linear approximation for the head loss and indicating flow direction by an additional binary variable for each pipe. The MILP is solved using IBM-ILOG-CPLEX v12.4 (2011).

In order to make an accurate comparison between piecewise linear and quadratic approximations of the head loss curve, the quadratic estimate is generated first and then the piecewise linear approximation is developed to have error statistics that are roughly equivalent. The piecewise linear approximation is found using an adaptive bi-section algorithm. The algorithm bisects part of the head loss curve, adding an element to the piecewise approximation, only if the relative error is larger for the linear approximation than the quadratic one. In this way, the level of approximation for each pipe is similar and flatter parts of the curve receive fewer linear segments.

RESULTS AND DISCUSSION

25 Node Network

The first network considered here is the benchmark network of Sterling and Bargiela (1984) that has also been studied by Jowitt and Xu (1990), Reis et al. (1997), Vairavamoorthy and Lumbers (1998) and Nicolini and Zovatto (2009) among others. The network layout is shown in Fig. 4, c.f. Vairavamoorthy and Lumbers (1998) or Sterling and Bargiela (1984) for the pipe and node information. Valve placement on this network is challenging because there are three sources and the network is highly connected. Head loss is modeled using Hazen-Williams. Quadratic approximations for the head loss curve of each pipe were developed using the methods described above. The optimization problem was modeled using AMPL (Fourer et al. 2003) and solved by Bonmin v. 1.5 (2011) and Ipopt v. 3.10 (2011).

The optimal valve placements for minimizing the sum of pressures are on pipes 1, 5 and 11 (Fig. 4). Notably, this result differs from the valve locations in the original version of the problem of Sterling and Bargiela (1984) where optimal settings were sought. A different placement, on pipes 1, 11 and 20, is obtained by Nicolini and Zovatto (2009) using a different objective.

The accuracy of the quadratic approximation on simulation results was assessed by comparing flows and pressures from the optimization model with EPANET 2.0 (Rossman 2000). The flows and pressures from the MINLP solution are very consistent with Epanet (Fig. 5). Differences between pressures in Bonmin and Epanet were small, with the largest relative error of 1% or .2m at node 19. In the flow solution relative errors had a median of 0.5% and relative errors in the middle 90% of the data ranged from -8.8% to 12%. In the remaining 10% of pipes absolute errors ranged from -0.33 to 0.0006 L/s. As expected, high relative errors occurred at low flows and had a negligible impact on the overall mass balance.

The valve placement problem was also solved using a piecewise linear approximation for head loss. The linear method yielded the same valve placement and essentially the same settings. Flow accuracy was slightly better (errors of -7.2% to 4.8% for the middle 90%) since the linearization was generated to be at least

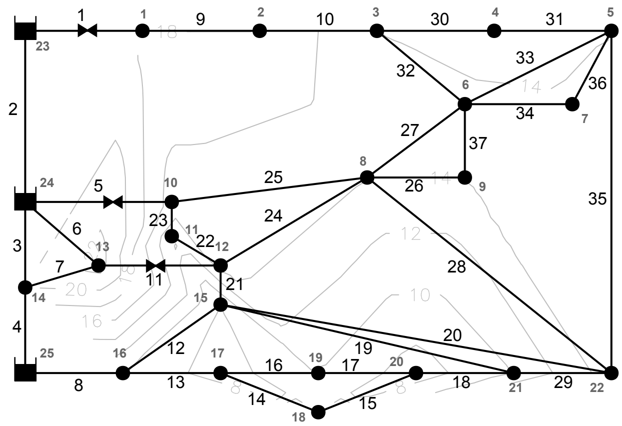

FIG. 4. Twenty five node network due to Sterling and Bargiela (1984) with three pressure reducing valves placed in optimal locations as found in the present paper. Elevation contours shown in the figure are computed from nodal values.

TABLE 2. Computational experience and instance sizes for the benchmark network of Sterling and Barglia (1984) having 25 nodes and 37 pipes (see Fig. 4). The MILP model used a piecewise approximation for the head loss curve having equivalent accuracy to the quadratic fit in the MINLP model.

Problem	Model	Solver	Continuous Variables	Binary Variables	Constraints	Solution Time (s)
Valve Setting	MILP	CPLEX	506	444	753	360
	MINLP	Ipot	99	0	99	8
Valve Placement	MILP	CPLEX	506	518	852	1128
	MINLP	Bonmin	99	74	198	555

as accurate as the quadratic. Computations for this network were performed on a machine with 4GB of RAM and a processor speed of 2.5GHz, running Microsoft Windows 7. The MINLP model run terminated after 685s, while the MILP run required 1128s (Table 2). Results for the the reduced problem of valve setting are also shown in Table 2.

As a global solver for nonconvex MINLP we applied Couenne to the VS-MINLP. However, using the same optimization model, Couenne terminated without finding an optimal solution.

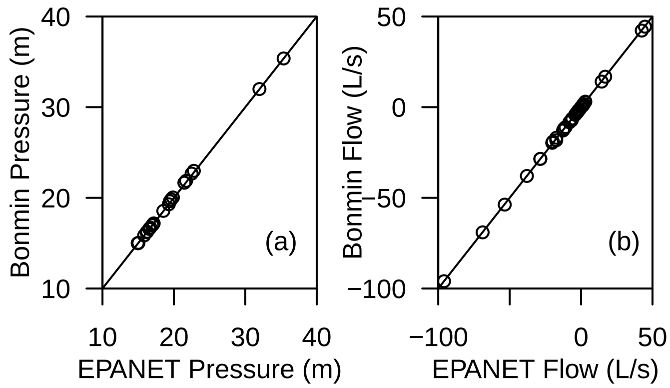

FIG. 5. Comparison of pressures (a) and flows (b) for the 25 node network as calculated by Bonmin with quadratic head loss and EPANET 2.0.

1893 Node Network

To illustrate the scalability of a the quadratic approximation, we also consider the Exnet water system first mentioned by Farmani et al. (2004). The system includes 1893 nodes and is suggested as a realistic benchmark network by University of Exeter Centre for Water Systems (2012). Previous work on the Exnet system studied optimal network design using genetic algorithms. In the present work, optimal locations for three new pressure reducing valves are found.

Since the problem addressed here differs from the problem originally posed for the network, several small modifications were needed to make the system applicable for valve placement:

1. In the downloaded file, node 1610 was disconnected from the network and so was removed;
2. the head of both reservoirs was increased to 80m to create positive pressures everywhere in the network;
3. the valve labeled "prv" was removed and elevations of the valve's nodes were increased from zero to match their adjacent nodes.

The Exnet system studied here includes 1893 nodes and 2469 pipes with 1 existing pressure reducing valve. Head losses are modeled by the Darcy-Weisbach method. Quadratic approximations for pipe head loss were developed for each pipe as described above. An equivalently accurate piecewise linear approximation required a total of 318,000 segments for the entire network. As an illustrative case, optimal placements for three new pressure reducing valves were found in addition to the optimal setting for the existing valve. The simulation used a minimum pressure of 8m.

Optimal locations for three additional pressure reducing valves are shown in Fig. 6. As before, flows and pressures calculated by the optimization model compared favorably to the Epanet solution (Fig. 7). In the pressure solution, errors ranged from -0.18% to 4.4% and averaged 0.29%. In the flow solution

TABLE 3. Computational experience and instance sizes for the Exnet network of Farmani et al. (2004) having 1893 nodes and 2469 pipes (see Fig 6). The MILP model used piecewise approximation with 10% accuracy.

Problem	Model	Solver	Continuous Variables	Binary Variables	Constraints	Solution Time (s)
Valve Setting	MILP	CPLEX	45,823	41,464	63,085	>150,000
	MINLP	Ipot	6,825	0	6,825	121
Valve Placement	MILP	CPLEX	45,823	46,396	69,910	—
	MINLP	Bonmin	6,825	4,932	13,650	461,500

relative errors had a median of zero and relative errors in the middle 95% of the data ranged from -6.8% to 4.9%. In the remaining 5% of pipes absolute errors ranged from -0.75 to .47 L/s.

Computations for the Exnet system were performed running Red Hat Linux on a blade server with with 100GB of RAM and a processor speed of 3.5GHz. The VP-MINLP model run terminated after 461,500s (Table 3). In the case the location of the three additional valves is fixed and one is interested in the optimal setting of all four valves in the network only, the resulting VS-NLP is solved by Ipopt within 121s.

For both cases, optimal valve placement and optimal valve setting, the MILP model of equivalent approximation accuracy for the head loss could not be solved by CPLEX. Even when reducing the approximation accuracy to a relative error of 10% for each pipe - which means ca. 40,000 breakpoints for the piecewise linearization - a solution within a reasonable integer tolerance for CPLEX was not found. Only for very coarse discretizations - basically linearization of the headloss curve in each pipe - integral solutions were obtained for CPLEX with an integer tolerance smaller than 1e-2. However, these solutions are not meaningful since they do not adhere to the hydraulic model of the network. The observed trade-off between accuracy of the piecewise linear approximation and integer tolerance in CPLEX required to solve the resulting MILP is in line with the effect reported by Bragalli et al. (2011).

CONCLUSIONS

This paper has proposed a quadratic approximation for pipe head loss curves and applied the approximation to find optimal valve locations on water networks. The method applies whether a network model has been parameterized by Hazen-Williams or Darcy-Weisbach coefficients. A key feature of the approach is minimal relative approximation error. By focusing on the relative rather than absolute error, the approximation stays more consistent over the range of possible flows. The accuracy of the approximation depends on the range of pipe flow rates and the pipe roughness. Because rougher pipes develop fully turbulent flow at lower Reynolds numbers, the friction factor varies over a smaller range and a quadratic approximation is more accurate. This result means that the approximation is

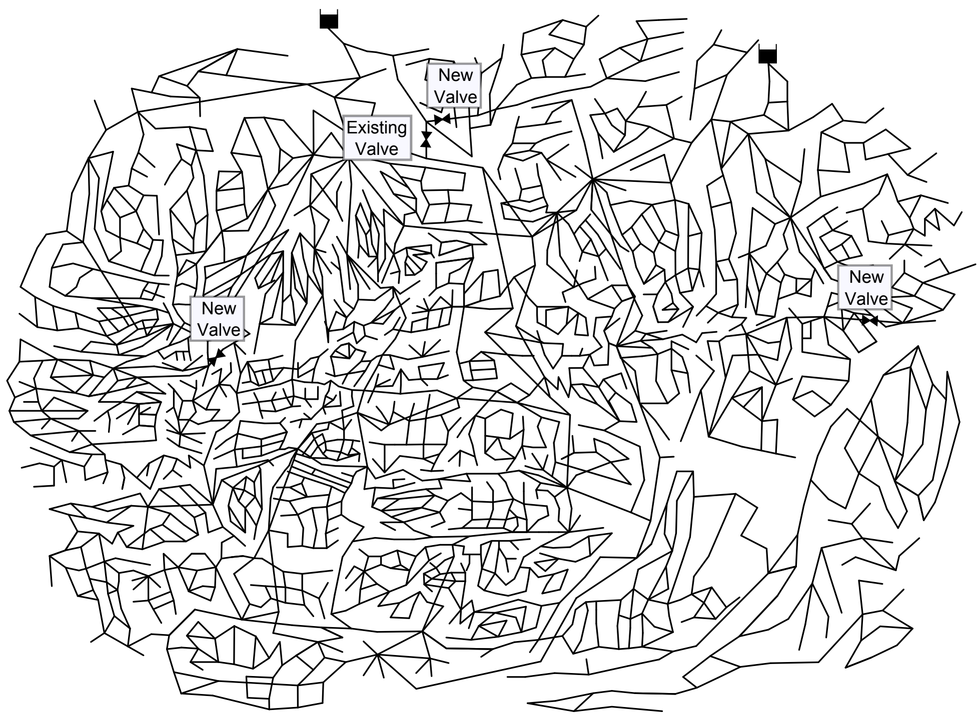

FIG. 6. The Exnet network modified from Farmani et al. (2004) having 1893 nodes and 2469 pipes. Optimal locations for three new valves are shown.

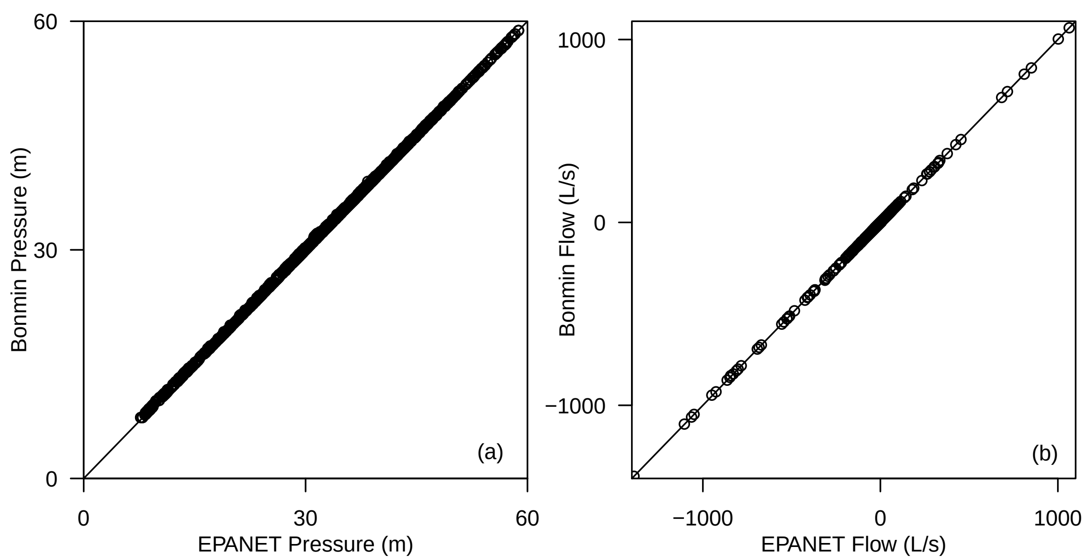

FIG. 7. Comparison of pressures (a) and flows (b) for the Exnet system as calculated by Bonmin with quadratic head loss and EPANET 2.0.

especially well suited to older water systems where pipes tend to be rougher.

Computational results showed that the approximation is fast and accurate. For the two benchmark networks studied here, optimization models achieved accuracy comparable to standard hydraulic solvers. Models based on quadratic head loss were also faster than similar models using piecewise linearizations. The increase in speed is due to the dramatic reduction in the number of binary variables. This speedup is important as optimization models begin to find use in making operational decisions on water networks.

Since the design decision of installing additional valves is a one-time decision, a long run time to find an optimal placement - as for the large-scale Exnet system - seems acceptable. The operational decision to optimally set the valves in a network may need to be addressed on a daily or even hourly basis. Thus, a runtime to find an optimal setting within minutes is crucial. The results presented here underline that the given approach is promising for both optimal design and operation of real world water networks at scale.

Moreover, due to graphs representing a water network often being sparse, both, VP-MINLP and VS-NLP, are sparse polynomial optimization problems. Thus, sparse, convex relaxation techniques proposed by Waki et al. (2006) and Lasserre (2006) may be promising to derive global optimality bounds for the solutions obtained by Bonmin and Ipopt, and to derive starting points for these local methods - in particular for the continuous VS-NLP or the NLP relaxations at each node in the Branch-and-Bound tree.

Although this paper has focused on the valve placement problem, quadratic approximations for pipe head loss are applicable for other problems on water networks where an explicit polynomial form is desirable and an estimate of the relevant flow range is available. As with any approximate method, care is required to obtain proper results. When used carefully, the approach developed here increases the size of network that is tractable for mathematical optimization methods.

Notation

a	dimensional coefficient,
b	dimensional coefficient,
d_i	water demand at node i,
e_i	elevation at node i,
f	Darcy friction factor[–],
g	gravitational acceleration[m/s/s],
h_f	frictional head loss [m],
$\hat{h}_f$	approximation of frictional head loss [m],
k_s	roughness height [m],
$v_{i,j}$	valve indicator,
$A - E$	constants used to compute a and b for Hazen-Williams,
C	Hazen-Williams C-value,
D	Pipe diameter [m],
F	Objective function,
L	Pipe length [m],
N_n	Number of nodes in the network,
N_p	Number of pipes in the network,
N_v	Number of valves in the network,
Q	Flow rate [m^3/s],
Q_1	Flow rate at lower end of approximation interval [m^3/s],
Q_2	Flow rate at upper end of approximation interval [m^3/s],
Re	Reynold's number,
V	Average fluid velocity [m/s],
α	Constant part of Hazen-Williams formula
ϵ	relative error

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