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Identifying leakage likelihood using state estimation and bad data identification methods

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1 Introduction

As the number and type of sensor deployments on water distribution networks (WDNs) increases, there is an opportunity to use the sensor information to improve the management and operation of the network.

Several techniques have been proposed in the literature to exploit data coming from pressure/flow sensors in order to provide an initial guess for the location of the leaks within a water network, thus reducing the time required by physical exploration. Most of the proposed methods are based on the analysis of residuals between the sensor data and an estimate calculated from prior knowledge of the nodal demands (Vento, 2009; Perez et al., 2010; Gertler et al., 2010). An integration of the residual analysis with state estimation, where the demands are estimated from the sensor data, was also proposed in (Andersen et al., 2000; Fusco et al., 2012). In (Wu, 2008; Wu et al., 2010), an optimization-based technique was proposed, where the leakage at the nodes is estimated by minimising the residuals between sensor data and model prediction, using genetic algorithms. Existing techniques, however, are not reliable in practical setups characterized by sparse sensors and may produce misleading diagnosis, with high rates of false positives or false negatives. Even though more sensors are going in all the time, in fact, the number of sensors is still usually small compared to the number of nodes in a skeletonised system model.

In this paper, tools based on state estimation and bad data analysis, popular in the power systems industry, are combined with factor analysis into a new method for detecting flow anomalies in water systems. A key feature of the new technique is an aggregation scheme whereby detected anomalies are mapped to a sub-graph of the network consistent with the density of sensors. This mapping contrasts with reporting anomalies for a single node when such resolution is not justified by the measurement density. The size of the flow anomaly is estimated along with its uncertainty to support decisions on possible corrective actions.

The proposed methodology, along with some background on state estimation and bad data analysis, is detailed in section 2. Both real and semi-synthetic results, on a real municipal DMA, are presented in section 3. Final comments are then given in section 4.

2 Methodology

The proposed method for the localisation and estimation of leaks is based on state estimation and bad data analysis, which are the typical tools in power systems for the identification and estimation of anomalous errors in measurement data. In a power network, as well as in a WDN, a set of m measurements, $y \in \mathbb{R}^m$, can be expressed, in steady state, as functions of a minimum independent set of n variables, $x \in \mathbb{R}^n$, which fully specifies the *state* of the system:

$$y = h(x) + \varepsilon. \tag{1}$$

In (1), the noise term ε is included to model the uncertainty in the observations, due for example to sensor noise, and it is typically considered to be zero-mean, white and Gaussian with diagonal covariance matrix Σ.

The model $h(\cdot)$, in (1), is a mathematical description of the system which expresses any physical variable of interest as a function of x, which is referred to as the state of the system. The choice of the minimum set of n independent quantities composing x is not unique. In a WDN, for example, the nodal demands could be chosen as state variable, since they allow the calculation of all the flows and pressures, based on mass conservation and and head-loss equations (as in a hydraulic simulation software). Note that, if loops are present, an explicit algebraic function relating pressures to demands cannot be found. An extension of the state variable and of the measurement set, to include some pipe flows, is required for the representation in (1) to be possible, as detailed in (Andersen et al., 2000) and (Fusco et al., 2012). In the following, without lack of generality, x represents the set of nodal demands and n is the number of nodes in the network.

Based on an estimate of the state, calculated from the available measurements, the objective of bad data analysis is to identify and possibly estimate anomalous errors ε, which do not belong to the expected distribution. Some background on state estimation and bad data analysis is given in sections 2.1 and 2.2. The reader is referred to (Abur et al., 2004) and (Monticelli, 2000), among others, for extensive reviews on the topic.

2.1 State estimation

State estimation is the process of using (1) and the measurements set to reconstruct an estimate, $\hat{x}$. State estimation is typically solved by minimisation of the weighted least squares (WLS) error:

$$\min_x J(x) = [y - h(x)]^T W [y - h(x)], \tag{2}$$

through the following iterative scheme:

$$\hat{x}_{k+1} = \hat{x}_k + (H_k^T W H_k)^{-1} H_k^T W [y - h(x_k)]. \tag{3}$$

In (2) and (3), $W \in \mathbb{R}^{m \times m}$ is a diagonal weighting matrix and $H_k \in \mathbb{R}^{m \times n}$ is the Jacobian matrix of the model $h(x)$, evaluated at $\hat{x}_k$. In a statistical setting, the weighting matrix is typically chosen as $W = \Sigma^{-1}$, where more weight is given to less uncertain measurements.

Note that it is necessary, albeit not sufficient, that enough measurements are available, $m \geq n$, to make $(H_k^T W H_k)^{-1}$ invertible and the state estimation solvable.

2.2 Traditional bad data analysis

The detection and identification of bad data is based on the analysis of residuals, $r \in \mathbb{R}^m$, defined as the difference between the measurements, y, and their estimation, $h(\hat{x})$, based on a state estimate :

$$r \triangleq y - h(\hat{x}). \tag{4}$$

For small deviations, (4) can be linearised to:

$$r = S\varepsilon, \tag{5}$$

where $S \in \mathbb{R}^{m \times m}$ is termed the *residual sensitivity matrix* and it is given by:

$$S = I - H(H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1}. \tag{6}$$

In (6), $I \in \mathbb{R}^{m \times m}$ is the identity matrix and H is the Jacobian of the model $h(x)$, evaluated around the state estimate, $\hat{x}$.

Based on (5) and on the assumptions about the distribution of the measurement error ε, the residual is distributed as a zero-mean Gaussian variable with covariance matrix given by:

$$\Omega = S\Sigma. \tag{7}$$

Therefore, if no anomalies are present, the normalised residuals, defined as:

$$r_i^N \triangleq \frac{|r_i|}{\sqrt{\Omega_{ii}}} \qquad i = 1, \ldots m, \tag{8}$$

have a zero mean and unitary standard deviation.

A statistical test can therefore be designed on the normalised residuals:

$$|r_i^N| \geq C_\alpha, \tag{9}$$

to decide whether the measurement y_i is affected by anomalous error. The choice of the threshold C_α depends on the desired level of confidence, α (rate of expected true positives), and it is evaluated from the properties of the Gaussian distribution. As an example, a value of $C_\alpha = 3$ corresponds to $\alpha \approx 99.7\%$.

A naive method for bad data identification, which we will refer to as Normalised Residual (NR), flags as bad data all measurements for which the test in (9) is true. An estimate of the anomalous error is given by the residual itself.

A single error, however, can affect many residuals, based on (5). A slightly more advanced method, named largest normalised residual (LNR), has been proposed. In LNR, only the the maximum r_i^N is tested. If the test is positive, the corresponding measurement is flagged as anomalous, a new state estimate is calculated after removing such measurement and the procedure is repeated. The method stops when the statistical test in (9) is negative on the largest normalised residual, or when $m - n$ bad measurements have been found, after which there are not enough measurements left to solve the state estimation. Again the residuals give an estimate of the anomalous errors.

In the presence of multiple bad data, LNR may fail to correctly identify the erroneous measurements. An alternative, more effective method, called hypothesis test identification (HTI) was therefore proposed by (Mili et al., 1988), where the residuals are decorrelated before the diagnosis. In HTI, the observations are partitioned in suspect, y_s, and true, y_t, measurements, based on the value of the corresponding normalised residuals. The residuals are also partitioned accordingly:

$$\begin{bmatrix} r_s \\ r_t \end{bmatrix} = \begin{bmatrix} S_{ss} & S_{st} \\ S_{ts} & S_{tt} \end{bmatrix} \begin{bmatrix} \varepsilon_s \\ \varepsilon_t \end{bmatrix}. \tag{10}$$

By assuming that $\varepsilon_t = 0$, the error of the suspect measurements is then estimated as:

$$\hat{\varepsilon}_s = S_{ss}^{-1} r_s, \tag{11}$$

with covariance matrix $\Delta = S_{ss}^{-1}\Sigma_s$, where Σ_s is the covariance matrix of ε_s. A statistical test similar to (9) is then designed to decide whether the $\hat{\varepsilon}_s$ are anomalous. The estimate of the error, in this case is more accurate than using the residual. Again, a maximum of $m - n$ measurements can be chosen as suspect, otherwise the matrix S_{ss} is not invertible, because the rank of S is, at best, $m - n$.

2.3 Application to leak identification

In principle, the method of bad data analysis can immediately be applied to the problem of leak localisation and estimation in WDNs. Given sensor data measuring flow and pressure at some pipes and nodes of the network a model of the type in (1) can be written. Additional observations come from some knowledge of the nodal demands, available from metering infrastructure or,

more often, from expected consumption patterns (not real measurements but mathematically similar, possibly with a larger error). By assuming that the pressure and flow measurements (sensors) are correct, the effect of leakage in the WDN is the same of an anomalous error on the measured (or expected) demands. Identification and estimation of errors corresponding to demand measurements, with NR, LNR or HTI, will therefore indicate the size and location of possible leaks, approximated to the nearby node of the skeletonised model.

In practice, however, most WDNs only avail of very sparse measurements, for example consisting of the water flow at the inlet and the pressure at a few nodes. It is recognised that, in cases of low measurement redundancy, many residuals could be strongly correlated, the corresponding columns in the sensitivity matrix S, given in (5), being linear dependent. As a consequence, a single error in ε has a *similar* effect on many residuals, so that it cannot be uniquely identified. Groups of measurements with highly correlated residuals are termed *critical sets* in the literature of bad data analysis and they were studied in (Ayres et al., 1986; Korres et al., 1991; Fusco et al., 2014) among others. It is clear that traditional methods based on LNR or HTI, which try to identify the exact erroneous measurement, could give quite misleading results in this case. In the particular case of leak detection, the presence of critical sets also means that most traditional methods, by trying to identify the exact location of a leak, could produce incorrect results. Such methods include optimisation-based techniques (Wu, 2008; Wu et al., 2010), as well as methods based on residual analysis (Andersen et al., 2000; Perez et al., 2010; Gertler et al., 2010; Fusco et al., 2012).

In this paper, we propose a solution where strongly-correlated residuals are grouped together first, and bad data analysis is then performed for each group as a whole, based on an extension of HTI. Each group can be conveniently interpreted as a sub-graph of the WDN model, so that bad data analysis produces, for each group, a likelihood of the presence of leakage, as well as an estimate of the total anomalous water demand. Nothing definite can be said about individual nodes within the groups. The method is summarised in the following sections, and the reader is referred to Fusco et al. (2014) for a complete treatment, in the context of power systems.

2.3.1 Clustering residuals in strongly-correlated groups

The clustering of the residuals is based on factor analysis (FA), which is a statistical tool used to express the covariance relationships among many variables in terms of a few underlying quantities called factors (Johnson et al., 2007). Base on FA, the covariance matrix of the n residuals corresponding to the demand measurements, given in (7), is expressed as:

$$\Omega = LL^T + \Psi, \tag{12}$$

where $L \in \mathbb{R}^{n \times p}$ is the matrix of the factor loadings and $p < n$ is the desired number of factors. Based on (12), FA approximates Ω with LL^T, which spans a space of dimension p. The matrix Ψ measures the error of the approximation error. Implicitly, the n residuals are being expressed as a linear combination of p independent factors. The matrix L can be obtained by maximum likelihood estimation, as implemented in the `factoran` function of Matlab Statistics Toolbox (Mathworks, 2013), which is used in this paper.

The maximum possible number of factors, p, is given by the rank of Ω, which equals the rank of S and is exactly given by the sensor redundancy over the size of the state variable, that is $m - n$, as explained in section 2.2. We therefore set $p = m - n$. Note that, by assuming the pressure/flow measurements are correct (error within the assumed sensor noise) and setting the focus on leak detection, FA is applied to a subset of the m residuals corresponding to the n demand measurements, so that it may happen that $m - n > n$ (if we have many sensors!). In this case we set $p = n$, which is the dimension of the Ω considered in (12), so that we have as many factors as residuals and the procedure converges to traditional methods discussed in section 2.2.

The factor loadings l_{ij}, elements of L, express how much the residual i loads on factor j. Residuals loading a lot on the same factor are strongly correlated. Therefore, for each factor, we

build a cluster of measurements, namely $\mathcal{M}_k$, $k = 1, \ldots p$, such that $i \in \mathcal{M}_{k_i}$ if the corresponding factor loading is maximum in absolute value, that is if $|l_{ik_i}| \geq |l_{ij}| \, \forall j$.

2.3.2 Bad data identification in clusters

After clustering the measurement set, we employ a similar procedure to HTI, described in section 2.2. One measurement per each of the p clusters is flagged as suspect. We propose to choose the one with largest normalised residual, but the result is, in theory, independent on this choice.

Based on the partition in (10), and similarly to (11), an estimate of the error in the suspect measurements is calculated as:

$$\hat{\varepsilon}_s = S_{ss}^{-1} r_s. \tag{13}$$

The matrix S_{ss} is invertible by construction, since each of its columns corresponds to a residual that is mostly approximated by one of the p independent factors. By similar argument one can verify that errors in the same cluster are not distinguishable, since S_{ss} would be close to singular.

Very conveniently, and under conditions typically verified in practice, it can be shown that the error estimate, calculated from (11), is such that:

$$\hat{\varepsilon}_i \simeq \frac{1}{S_{ii}} \sum_{j \in \mathcal{M}_{k_i}} S_{ij} \varepsilon_j \qquad \forall i \in \mathcal{S}. \tag{14}$$

The result in (14) gives the convenient interpretation of $\hat{\varepsilon}_s$ as a measure of the total error within each cluster. In the specific case of leak detection, $\hat{\varepsilon}_s$ is the total anomalous demand (possibly leakage) in each group of nodes. Statistical testing, similar to (9), can be designed to decide whether a leak is present or not, given a certain significance and threshold.

3 Results

To method has been applied to a real-world test DMA in Chapelizod, Dublin city. The DMA consists of a total of nearly 3.6 Km of pipelines, serving approximately 400 households, modelled with $n = 122$ demand nodes and 127 connecting links, as shown in Fig. 3(a). A flow meter at the inlet, along with five pressure loggers, were available and provided measurement data for 18 days between February and March 2013, sampled at 15 minutes.

While the presence of leakage in the DMA is acknowledged by the water utility, the position and size of the leaks is not known, and a field investigation could not be carried out in order to validate the results of our methodology. Before presenting the results on the real test-case, in section 3.2, the effectiveness of the method is demonstrated on a semi-synthetic example applied to the same network model, in section 3.1.

3.1 Semi-synthetic example

In order to build a *realistic* example, a hydraulic simulation of the DMA was run with the nodal demands given by real smart meter data, available from Dublin City council (collected in a different DMA). It was assumed that the actual demands were not known, but average daily patterns were available for each demand node. The demand patterns come from 4 generic shapes derived from the smart meter data by use of Gaussian mixture models and clustering techniques, as described in detail in Mckenna et al. (2013), where information about the smart meter data-set utilised here can also be found. Figure 1 compares the available profile against the actual demands at two of the nodes, to exemplify the significant amount of uncertainty in the system.

The total demand in the network peaks about 2.5 L/s. Two leaks were simulated as constant demands of 0.3 and 0.5 L/s at the two circled nodes in Fig. 3(b), where the position of the
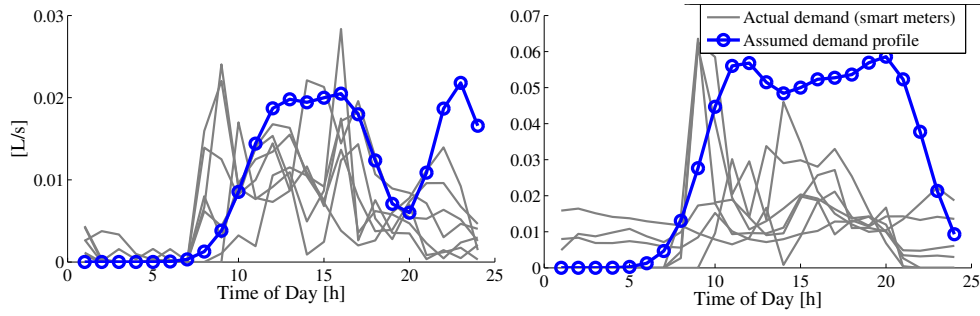
Figure 1: Example of assumed demands patterns (used as measurements in the state estimation) against actual demands coming from smart meters, at two nodes of the network (Mckenna et al., 2013).
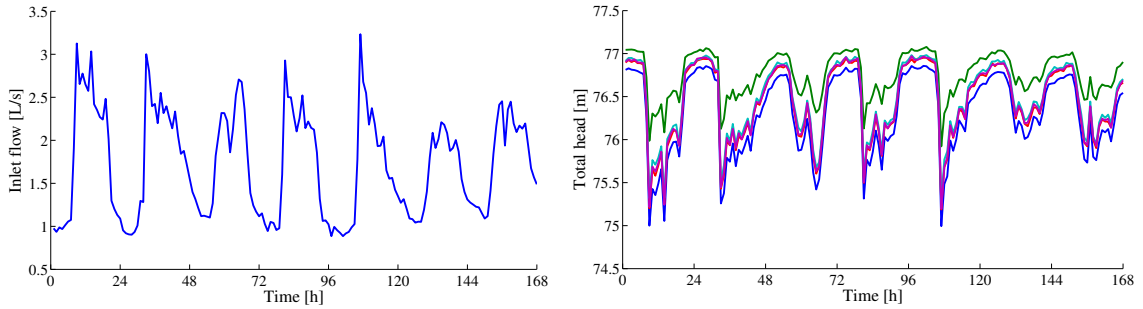

Figure 2: Simulated measurements of the: (a) Flow at the inlet; (b) Total head at 5 nodes. A Gaussian random noise with 0.01 L/s and 0.01 m was applied to model sensor noise.

5 pressure loggers is also indicated by squared boxes. The pressure and flow measurements were generated from the results of the simulation, by applying a random noise with a standard deviation of 0.01 L/s and m, respectively, to model sensor noise, and are shown in Fig. 2. The simulation was run over a 7-days time window and data were sampled at hourly intervals, for a total of 168 samples.

The state estimation, as described in section 2.1, was run at each of the 168 time steps to produce an estimate of the nodal demands. Factor analysis of the residual covariance matrix, obtained from (7) and based on an average of the sensitivity matrix across the whole simulation, was utilised to cluster highly-correlated demand nodes, according to the methodology outlined in section 2.3.1. The order of the factor analysis was set to $p = m - n = 6$, which is the redundancy of measurements, $m = 129$ (6 sensors, 123 demand profiles), over the dimension of the state, $n = 123$ (123 demand nodes), and represents the degrees of freedoms (rank) of the residual correlation matrix. Figure 3(a) shows how the 6 resulting node groups are distributed on the network. It is interesting to note how the clusters are composed of connected nodes, which is not a constraint of the method, but only an implicit result of the mathematical description of the system and of how the pressure/flow data are related to the nodal demands.

The residual analysis and error estimation, as outlined in section 2.3.2, is then run for each of the 168 time steps. In particular, based on (14), the estimated errors represent the total amount of anomalous demand (possibly leakage) in each group of nodes previously identified. Based on the evaluation of the covariance of the estimated error, given in (11), the maximum α, such that a statistical test of the type in (9) gives positive outcome, is evaluated. Such a value of α will give the confidence that the estimated error represents an actual anomaly (leakage).

Figure 3(b) shows the overall results, averaged over the 168 time steps. An average error $\hat{\varepsilon} \approx 0.887$ L/s, corresponding to a confidence of $\alpha > 99\%$, is evaluated at the red-colored nodes, corresponding to $\mathcal{M}_3$, which actually contains the two leaks of 0.3 and 0.5 L/s. The rest of the network is flagged as anomalous with low confidence of $\alpha < 50\%$ (green-colored nodes) or $\alpha < 30\%$ (black-colored nodes). Given the strong correlation between the residuals
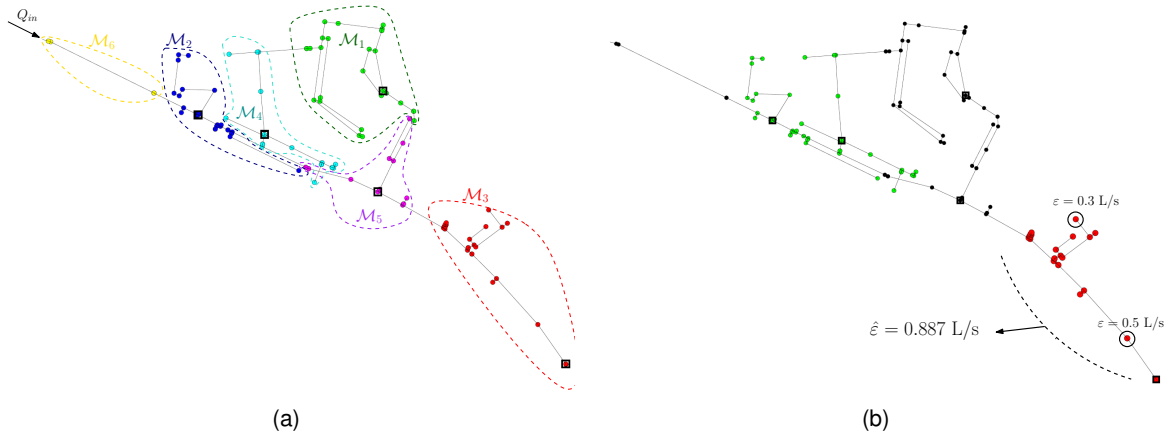
Figure 3: Scheletonised model of Chapelizod (Dublin, Ireland) DMA: (a) Identified clusters; (b) Output of proposed leak detection method ($\hat{\varepsilon}$ is the average estimated leakage in group $\mathcal{M}_3$, red color indicates $\alpha > 99\%$). The location of the 5 pressure loggers is indicated by boxed nodes, while the actual leaks are indicated by circled nodes.
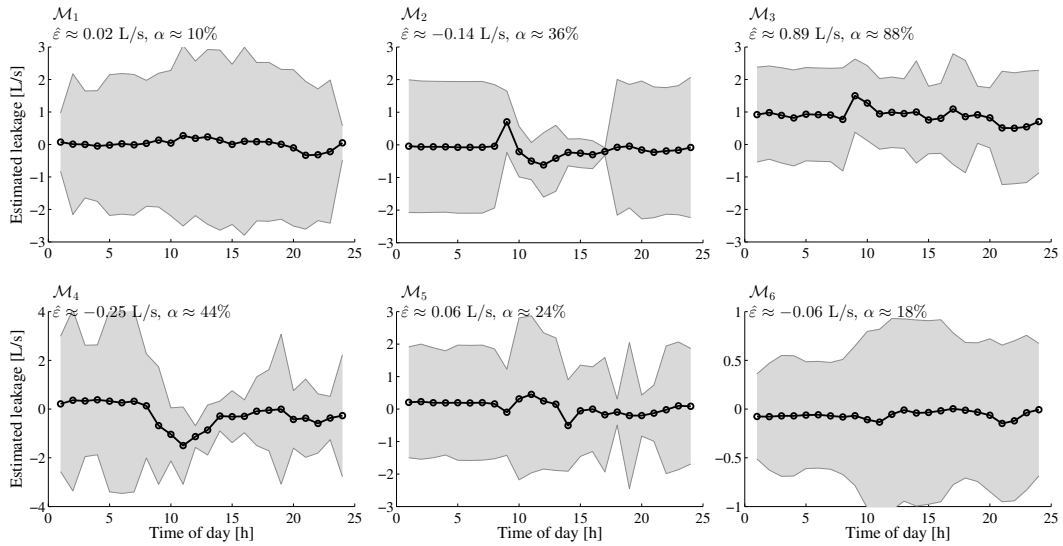

Figure 4: Average daily pattern of estimated error in each cluster. The error represents the total anomalous water consumption in each area of the network.

corresponding to the two leaking nodes (they belong to the same cluster), the method declares them indistinguishable, with the given data (and no method would be able to distinguish them!).

It is also interesting to look at the daily profiles of the estimated errors, where the values at each time of the day are averaged over the 7 days of the analysis. Figure 4 shows the estimated daily profiles of anomalous water consumption in each of the 6 clusters. They can be intuitively interpreted as difference between expected (from consumption profiles) and actual demand (estimated from the available real-time sensors). At cluster $\mathcal{M}_3$ there is a consistent under-estimation (positive error) of the expected demand, whose cause may likely be a leak (and it is, in fact). Given the large uncertainty in the demands, however, we can see different anomalous patterns appearing also in other groups. Their average is usually close enough to zero, but the daily shape may give indications about significant errors in the expected demand patterns. Consider, for example, the significant negative error (over-estimation) in $\mathcal{M}_4$ at around 11.00 am. If we look at the demand profile in Fig. 1, right-most graph, the smart meter data clearly indicate that the actual consumption is consistently much lower, just at around 11am, which may be the cause of the estimated anomaly.

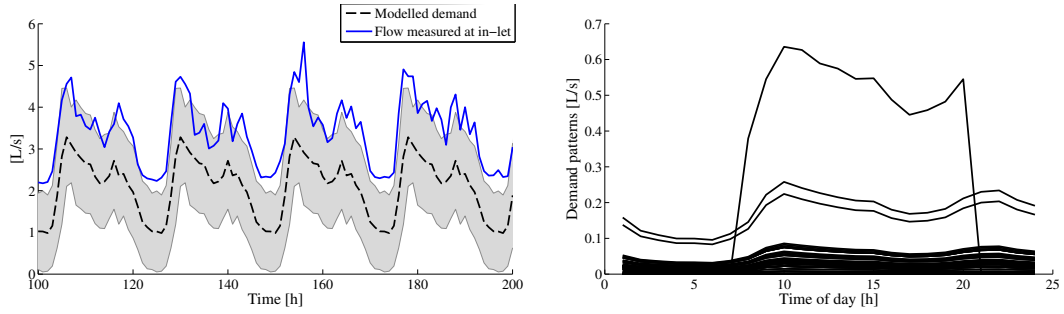The proposed method, therefore, can be used as a generic tool to investigate mismatches

Figure 5: Measured in-let flow against aggregated demand (graph on left), based on profiles of the nodal demands (graph on the right). The shaded area around the aggregated demand represents their ± 3 standard-deviations confidence interval.

between expected and actual water consumption, including but not limited to leakage. The results are very intuitive and offer themselves to immediate interpretation of the potential source of the anomaly, since the daily patterns of the estimated error indicate whether they may be caused by a leak, unexpected peak in demand, or something else.

3.2 Real test-case

The real data available from the DMA under study were also used to run the proposed method, although, as previously mentioned, a validation of the results in the field was not possible to date. However, the results are still of some value, in providing insight in other interesting aspects of the proposed solution.

The data consist of 18 days of measurements of the inlet flow and of the pressure at 5 locations, indicated by squared boxes in Fig. 6(b). Daily profiles of the nodal demands were also provided by the utility. The given profiles, and their uncertainty (standard deviation) were calibrated with the available flow measurements, also accounting for an estimated amount of leakage of 1.2 L/s, independently calculated by the utility. The resulting profiles are shown in Fig. 5 (graph on the right), together with a comparison of the total aggregated demand against the measured flow at the inlet (graph on the left).

The first interesting result is the fact that, as shown in Fig. 6(b), the methodology identified only 2 clusters of nodes, given the same sensors as in the example of section 3.1. The reason behind this behavior is mostly due to the fact that the available model didn't seem to be well calibrated, so that there was a significant mismatch between the predicted and measured pressures. All of the residuals were, therefore, mostly approximated by only two factors, in (12), due to the high level of uncertainty in the pressures.

Although losing in resolution, the method is still able to come up with a meaningful result, as shown by the daily profiles of the estimated anomalous water consumption, in Fig. 6(a). The highest confidence, $\alpha \approx 100\%$, in the presence of leakage is flagged in $\mathcal{M}_1$, and the profile clearly indicates significant under-estimation of the night-flow. The anomalous flow in $\mathcal{M}_2$, on the other hand, seems to be composed of a pattern similar to a demand, which may indicate the fact that the given demand profiles may under-estimate the actual water usage in that area.

4 Conclusion

This paper presented a new methodology for the localisation of flow anomalies in WDNs, where the anomalies may represent unexpected demands or leakage. Compared to existing solutions, the method implicitly adapts to level of information available about the network, including hydraulic sensors, physical model, and profiles. The WDN is first divided in areas (group of nodes) where the leak is not identifiable, given the available information. An estimate of the total
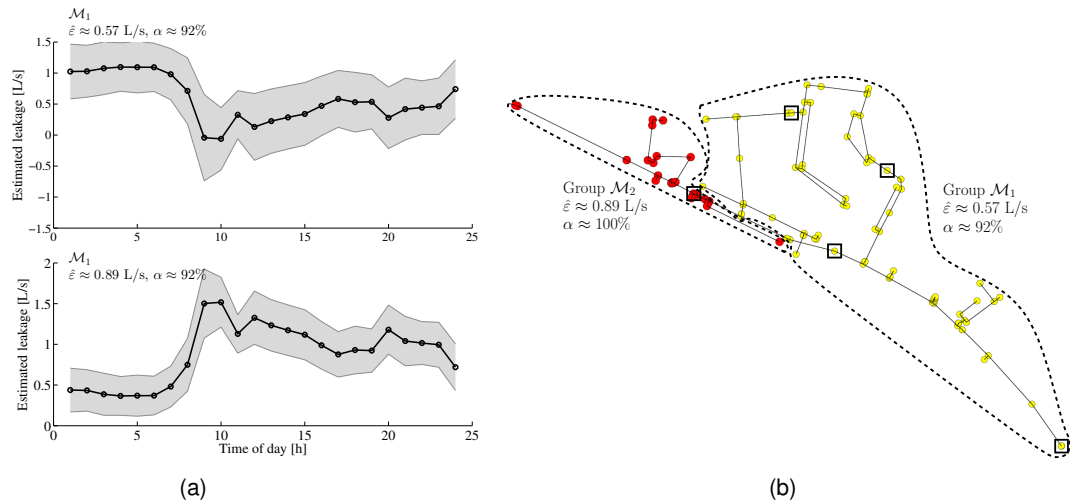
Figure 6: Estimated anomalous water consumption (graph on the left) in the two clusters identified (graph on the right).

anomalous water demand in each area of the network is then produced, as well as a statistical bound (standard deviation) indicating the confidence of the estimate. The spatial resolution of the method (size of the areas) is strictly relates to the amount of sensors and to the quality of the model and of the available demand models.

Overall, the proposed method provides the means for a hotspot analysis, where areas of the network are assigned an amount of anomalous consumption and confidence in the estimate. The daily profiles of the estimated patterns of anomalous flow can be intuitively processed for root-cause analysis of the problem (leakage, unexpected customer demands, . . .). The solution would indeed provide an invaluable tool for the WDN operator, who, based on the output, can then decide to investigate further the issue by increasing the number of sensors (or move the available ones) in a flagged area for a finer diagnosis, until the source of anomaly is finally pinned down.

Acknowledgments

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