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The Role of Communication in Emergency Evacuations: An Analysis of a Ring Network with a Static Disruption

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Abstract— **Whilst technology has advanced rapidly over the last few decades, emergency evacuation procedures have not kept up-to-date with this technological progress. This paper explores the benefits of information and communication for agents in a simple ring network. The majority of the existing literature focuses on using agent-based simulation models to predict outcomes. One of the major benefits of this technique is that it allows the incorporation of many different factors that drive evacuation outcomes; however, simulation-only techniques lack power when attempting to explain the relationship between these factors. This paper provides an analytical framework to derive closed-form solutions for the relationship between the time it takes an agent to evacuate, the size of the network, the number of agents in the network, the level of information available and the communication broadcast range. Performance is measured by expected evacuation time and worst-case analysis.**

I. Introduction

Increasing populations and lifestyle choices have led to an increase in population density in natural disaster prone regions over the last few decades. This, coupled with an increase in extreme weather events, has led to more lives being affected by such disasters [1], [2]. In light of this trend, there now exists an increasing body of work related to modelling large-scale evacuation dynamics. This body of work can be broadly split into three categories: a) network design, b) optimal path-finding solutions and b) evacuation behaviour.

The design of a network or the architecture of a building plays an important role in assisting individuals to evacuate effectively [3]. For example, contraflow lanes were introduced to augment network capacity when directing traffic away from a disaster [4]. Helbing et al. [5] discovered that having a small obstruction in front of doorways, such as a pillar, could in fact improve evacuation efficiency by slowing the flow of traffic to avoid congestion.

Many path-finding algorithms have been offered in the literature, including distributed algorithms [6] and multi-objective, evolutionary algorithms [7]. Other real-time traffic models have been developed to guide road-users to help avoid congestion [8].

In terms of evacuation behaviour, many researchers have proposed various models for crowd dynamics. Although most of these models deal with small-scale evacuations, such as escape from a burning building, many of their findings can be extended to large-scale scenarios where way-finding and congestion play a significant role in the probability of survival. Henderson [9] was the first to model human crowd motion using gas fluid dynamics. Helbing has also done extensive work on modelling pedestrian behaviour and developed a "social force" model in which force equations from physics are used to model the interactions between individuals [10], [11], [12]. He and others later extended this model to include panic as it became clear that it played an important role in decision-making during emergencies [5], [13], [14].

Other studies have made use of game-theoretic approaches which account for the interactions between agents [15], [16]. This improves the power of behavioural models giving us more accurate predictions. In particular, these game-theoretic models have been used to model congestion along a bottleneck [17], [18], [19], [20], [21], [22], [23], [24] and exit or route selection [25], [26], [27], [28].

These studies and the various methodological approaches to evacuation modelling are summarised succinctly in Zheng et al. [29].

This paper focuses on the role of communication and information during evacuations. These aspects are discussed by Corlett et al. [30] in the context of signage for direction-finding. Others have also looked at what happens when signage is incorrect or misleading [31], [32]. Whether or not evacuees heed evacuation instructions and the factors that affect their level of obedience have also been studied [33], [34]. Richter et al. [35] ran a simulation study on what would happen if centralised communications broke down leaving only localised communication available.

This paper builds on the work of Richter et al. We study how passing on new knowledge can help others escape. Most of the existing literature discussed above uses simulations to test their models. This research provides an analytical model with analytical results that can be compared and used to explain the driving factors behind the outcomes observed from simulated studies.

The remainder of this paper is organised as follows: Section II describes the model and introduces a ring network with differing levels of information and communication. Section III provides the analytical results of the model, whilst Section IV compares these results and discusses the implications. Lastly, Section V concludes the paper and suggests directions for further research.

II. Model

Consider a ring network with n nodes, a single entry/exit point and a single static disruption. The nodes are numbered 1 to n in a clockwise direction, with the entry/exit node, E, at the n^{th} node. The nodes are connected by edges of unit length which are labelled using the midpoint between nodes; e.g. the edge between nodes 1 and 2 is situated at location 1.5. The disruption, X, occurs on an edge located at k. The agents are located on nodes with agent i located at a_i, for $i = 1, 2, \ldots, m$. Figure 1 shows an example scenario with one agent in the network when the disruption occurs.

Performance is measured by the distance travelled to exit the network. We assume that agents travel at the same speed, hence distance is a sufficient measure and is comparable to time.

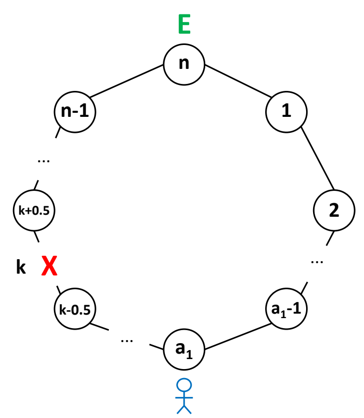

Fig. 1. A single agent with one disruption in a ring network.

A. Levels of Information

The level of information available to the agent will determine the heuristic used to find the exit. Three information levels and their associated heuristics are explored. There are obviously infinitely many other heuristics that can be examined, however we believe the ones discussed cover a suitable range of knowledge from complete information to having only minimal information. The information levels and their associated heuristics are described below. We assume that agents enter the network and initially travel in a clockwise direction.

1. Complete information (C): The agent knows the size and structure of the network and the location of the disruption. The agent will choose the direction that avoids the disruption.

2. Network information (N): The agent knows the size and structure of the network, but not the location of the disruption. The agent will choose the direction closest to the exit, however they might encounter the disruption and be required to turn back. If the agent is equal distance from the exit, then the agent chooses their direction randomly.

3. Historical information (H): The agent does not know the location of the disruption and only knows the path they have already travelled. The agent will opt to backtrack in the event of an emergency rather than to explore new nodes unless stopped by the disruption.

B. Communication

In this model we also allow agents to communicate. Let us call the agent who discovers the disruption the "discoverer" and all other agents are "receivers". The discoverer will immediately broadcast the location of the disruption upon discovery. The broadcast range is a distance α to either side of the agent. This range of communication is defined as "α-hop communication". The discoverer continues to broadcast until they exit the network. We assume that this broadcast range does not wrap around the network at the exit node.

We also introduce the concept of a "communication zone". This is the section of the network where communication is potentially beneficial. Similarly, we define the "non-communication zone" to be the portion of the network where communication will not benefit the agents. The separation of these zones depends on the information level of the agents.

As we increase the number of agents in the model, the combinations to consider under a discrete system becomes increasingly unwieldy. Therefore, when analysing communication outcomes with m-agents for $m \geq 3$, we alter our model to consider an interval of unit length on $[0, 1]$ with agents arriving via a Poisson process with arrival rate λ. The exit is now simultaneously located at 0 and 1 and agents are assumed to enter the interval at position 0 and initially travel from left to right at a constant pace. This process continues until the disruption occurs, after which arrivals cease. In addition to this process, we also assume that communication is propagated instantaneously; that is, any agent within the range of a broadcasting agent will automatically begin to broadcast the information themselves without delay.

We begin our analysis from the time the disruption occurs. At this point, agent i is located on the interval at $a_i \in [0, 1]$ for all $i \in \{1, 2, \ldots, m\}$. The disruption also sits on this interval, but is assumed not to be at the exit. It is located at $k \in (0, 1)$.

Figure 2 shows a graphical representation of the model in its interval form. Moving from left to right is the equivalent of travelling clockwise around the previous network. In this example, the agents have historical information, hence they will attempt to exit by initially travelling to the left. For any agent i located at $a_i \in (k, 1)$, they would ordinarily encounter the disruption and be forced to backtrack if communication was not available. Since communication can benefit these agents, the communication zone is given by the interval $(k, 1)$ and the non-communication zone is the remaining interval $[0, k]$.

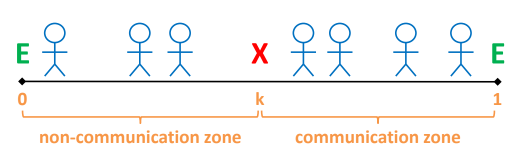

Fig. 2. An interval representation with historical information.

Section III provides our derived expressions for the distance travelled to exit and Section IV evaluates the importance of information and communication by performing some comparative statistics.

III. Results

In this section, we derive the expressions for the expected distance travelled to exit by an agent as a function of the size of the network, n, the location of the disruption, k, the initial location of each agent i when the disruption occurs, a_i for all $i \in \{1, 2, \ldots, m\}$ and their information level, I. In addition, when communication is available, these expressions for distance will depend on the broadcast range, α, and the arrival rate of agents to the network, λ.

A. Single Agent without Communication

We begin by analysing the distance travelled by a single agent under the discrete formulation as a function of n, k, a_1 and I. Since no communication is available to a lone agent, we make the assumption that the agent will only turn back if they reach the disruption and no sooner. Let $d^I_{a_i}$ denote the distance it takes agent i to reach the exit, given information level I.

Under complete information, there are two possible cases:

$$d^C_{a_1} = \begin{cases} a_1, & a_1 < k \quad \text{(1a)} \\ n - a_1, & a_1 > k \quad \text{(1b)} \end{cases}$$

Equation (1a) arises when the agent travels anticlockwise to exit, whilst (1b) is when the agent travels in a clockwise direction to exit. There is no need to account for when $a_1 = k$ since the agent is assumed never to be at the location of the disruption.

Randomising over the possible starting locations of the agent and the disruption, we can calculate the expected distance to exit for a single agent with no communication. We introduce $\widehat{k} = k + \frac{1}{2}$ to facilitate summation over whole indices.

$$\mathbb{E}\left[d^C_{a_1}\right] = \frac{1}{n^2}\left[\sum_{\widehat{k}=1}^{n}\left(\sum_{a_1=1}^{\widehat{k}-1} a_1 + \sum_{a_1=\widehat{k}}^{n}(n - a_1)\right)\right] = \frac{n^2-1}{3n} \quad (2)$$

With network information, there are four cases to consider:

$$d^N_{a_1} = \begin{cases} a_1, & a_1 \le \frac{1}{2}n \text{ and } a_1 < k \quad \text{(3a)} \\ a_1 + n - 2k, & a_1 \le \frac{1}{2}n \text{ and } a_1 > k \quad \text{(3b)} \\ 2k - a_1, & a_1 > \frac{1}{2}n \text{ and } a_1 > k \quad \text{(3c)} \\ n - a_1, & a_1 > \frac{1}{2}n \text{ and } a_1 < k \quad \text{(3d)} \end{cases}$$

Equation (3a) represents the situation where the agent attempts to exit by travelling in an anticlockwise direction and is not impeded by the disruption. In (3b), the agent also initially travels anticlockwise, but encounters the disruption and is forced to turn around. Equations (3c) and (3d) begin with the agent travelling clockwise. The agent avoids the disruption in (3c), but encounters it in (3d).

Due to symmetry, we can construct the expected distance travelled simply by looking at half the network. We consider the case where $k \le \frac{n}{2}$. Recall that if the agent is located at the midpoint of the network, they will choose their initial direction randomly with equal probability.

$$\begin{aligned}\mathbb{E}\left[d^N_{a_1} \mid \widehat{k}\right] &= \frac{1}{n}\left[\sum_{a_1=1}^{\widehat{k}-1} a_1 + \sum_{a_1=\widehat{k}}^{\frac{n}{2}-1}\left(a_1 + n - 2\left(\widehat{k} - \frac{1}{2}\right)\right)\right. \\ &\quad + \frac{1}{2}\left(\frac{3n}{2} - 2\left(\widehat{k} - \frac{1}{2}\right)\right) + \frac{1}{2}\left(n - \frac{n}{2}\right) \\ &\quad \left. + \sum_{a_1=\frac{n}{2}+1}^{n}(n - a_1)\right] \\ &= \frac{1}{n}\left[n - 2n\widehat{k} + \frac{1}{2} - 2\widehat{k} + 2\widehat{k}^2 + \frac{3n^2}{4}\right] \quad (4)\end{aligned}$$

$$\mathbb{E}\left[d^N_{a_1}\right] = \frac{1}{n^2}\left[\sum_{\widehat{k}=1}^{n}\left(n - \widehat{k} + \frac{1}{2} - 2\widehat{k} + 2\widehat{k}^2 + \frac{3n^2}{4}\right)\right] = \frac{5n^2-2}{12n} \quad (5)$$

By symmetry, the expression found in (4) is the same for the case when $k > \frac{n}{2}$. Therefore, in (5), when randomising over the location of the disruption, only the expression in (4) needs to be summed over.

For historical information, since the agent will always choose to backtrack first, there are only two possible scenarios for the distance travelled to exit.

$$d^H_{a_1} = \begin{cases} a_1, & a_1 < k \quad \text{(6a)} \\ a_1 + n - 2k, & a_1 > k \quad \text{(6b)} \end{cases}$$

In (6a), the disruption is not encountered, but in (6b), the disruption is found and the agent must turn back.

We assume that if the agent is at the exit, they will simply leave the network rather than go back the way they came. This gives us the following expression for the expected distance travelled under historical information.

$$\begin{aligned}\mathbb{E}\left[d^H_{a_1}\right] &= \frac{1}{n^2}\left[\sum_{\widehat{k}=1}^{n}\left(\sum_{a_1=1}^{\widehat{k}-1} a_1 + \sum_{a_1=\widehat{k}}^{n-1}\left(a_1 + n - 2\left(\widehat{k} - \frac{1}{2}\right)\right)\right.\right. \\ &\quad \left.\left. + \sum_{a_1=n}^{n} 0\right)\right] \\ &= \frac{4n^2 - 3n - 1}{6n} \quad (7)\end{aligned}$$

Section IV-A uses these results to compare the effect of information on an agent's ability to exit a ring network efficiently.

B. Two Agents with Communication

Here we introduce a second agent into the network and allow for localised communication between the agents. For simplicity, we will only look at the case where the agents have historical information. Note that with complete information, communication will not have any effect on the agents.

We will derive our expressions from the point of view of agent 1. Since the agents are symmetric, all expressions derived for agent 1 will be the same for agent 2. We distinguish the distance travelled with communication by denoting it $d^{I,\alpha}_{a_i}$. There are four possible cases to consider here:

$$d_{a_1}^{H,\alpha} = \begin{cases} a_1, & a_1 < k \quad \text{(8a)} \\ a_1 + n - 2k, & k < a_1 < a_2 \le n \quad \text{(8b)} \\ a_2 + n - 2k & \\ \quad - \min\{\alpha, (a_1, a_2)\}, & k < a_2 < a_1 < n \quad \text{(8c)} \\ 0, & a_1 = n \quad \text{(8d)} \end{cases}$$

Equation (8a) represents the case where agent 1 exits without encountering the disruption. The distance travelled in this case does not depend on the location of agent 2. In (8b), agent 1 discovers the disruption before agent 2, hence agent 1 will need to backtrack and cannot benefit from communication (although agent 2 will). Equation (8c) represents the case where agent 1 can benefit from communication since agent 2 discovers the disruption first and can communicate this to agent 1 before agent 1 reaches the disruption. Finally, (8d) represents the case where agent 1 is already at the exit when the disruption occurs and hence will simply leave the network immediately.

Due to the minimum function in (8c), the expected distance equation becomes a piecewise function.

$$\begin{aligned} &\mathbb{E}\left[d_{a_1}^{H,\alpha} \mid \widehat{k}\right] \\ &= \frac{1}{n^2}\left[\sum_{a_2=1}^{n}\sum_{a_1=1}^{\widehat{k}-1} a_1 + \sum_{a_2=1}^{n}\sum_{a_1=\widehat{k}}^{n-1}\left(a_1 + n - 2\left(\widehat{k} - \frac{1}{2}\right)\right)\right. \\ &\left. - \begin{cases} \sum_{i=0}^{\alpha} i\left(n - \widehat{k} - i\right) + \sum_{j=\alpha+1}^{n-\widehat{k}} \alpha\left(n - \widehat{k} - j\right), & \alpha \le n - \widehat{k} \\ \sum_{i=0}^{n-\widehat{k}} i\left(n - \widehat{k} - i\right), & \alpha > n - \widehat{k} \end{cases}\right] \end{aligned} \quad (9)$$

For brevity, we have excluded the terms where agent 1 is at the exit and the distance travelled is zero. The first line of the piecewise component in equation (9) refers to when the broadcast range is not large enough to immediately cover the entire communication zone. The indices i and j represent the possible distances between the agents, namely $(a_1 - a_2)$, with $\left(n - \widehat{k} - i\right)$ and $\left(n - \widehat{k} - j\right)$ representing the number of occurrences for each distance i and j. When the broadcast range is large enough such that the receiver becomes aware of the disruption immediately as it is discovered, i.e. $\alpha \ge (a_1 - a_2)$, the receiver will save a distance of $(a_1 - a_2)$, which is given by and dependent on i. This is seen through the first term in the first line of the piecewise component. When the broadcast range is not large enough for the receiver to be immediately aware of the disruption as it is discovered, then the distance the receiver saves is only equal to α. This is represented by the second term in the first line of the piecewise function. The second line of the piecewise component represents the case where the broadcast range is larger than the communication zone. In this situation, the receiver will always be immediately aware of the disruption as it is discovered and hence will save a distance of $(a_1 - a_2)$, which again is given by i.

We can now randomise over the location of the disruption to get an expression for the expected distance to exit as a function of n and α. Rearranging the conditions in the piecewise function to give $\widehat{k} \le (n - \alpha)$ and $\widehat{k} > (n - \alpha)$, respectively, we use these as the bounds on our summation indices for our randomization over $\widehat{k}$.

$$\begin{aligned} &\mathbb{E}\left[d_{a_1}^{H,\alpha}\right] \\ &= \sum_{\widehat{k}=1}^{n-\alpha} \mathbb{E}\left[d_{a_1}^{H,\alpha} \mid \widehat{k} \le (n-\alpha)\right] + \sum_{\widehat{k}=n-\alpha-1}^{n} \mathbb{E}\left[d_{a_1}^{H,\alpha} \mid \widehat{k} > (n-\alpha)\right] \\ &= \frac{\alpha^4 - 4\alpha^3 n + 2\alpha^3 + 6\alpha^2 n^2 - 6\alpha^2 n - \alpha^2 - 4\alpha n^3}{24n^3} \\ &\quad + \frac{6\alpha n^2 + 2\alpha n - 2\alpha + 15n^4 - 10n^3 - 3n^2 - 2n}{24n^3} \end{aligned} \quad (10)$$

Section IV-B discusses how the expected distance travelled is affected by the broadcast range.

C. m-Agents with Communication

In this section, we look at how information propagates as the number of agents in the system increases. We now consider an interval to facilitate our calculations, as discussed in Section II-B. Again for simplicity, we will only consider historical information in this section.

Let there be m agents in the system when the disruption occurs, where $m \ge 3$. To calculate the expected distance to exit per agent, we decompose our interval into its communication and non-communication zones. We are able to treat these zones separately due to the memorylessness of the Poisson process. Due to the independence of these two zones, when randomising over the number of agents in the system, we can simply multiply the probabilities of there being m_{C} agents in the communication zone and m_{NC} agents in the non-communication zone, where $m_{\text{C}} + m_{\text{NC}} = m$.

Firstly consider the non-communication zone and let $T_{m_{\text{NC}}}^{H,\alpha}$ be the total distance travelled by all the m_{NC} agents in the zone given historical information and a broadcast range of α. Since communication does not play a part in this zone and the agents are identical and independent, the total expected distance travelled by the agents is equivalent to the expected distance travelled for a single agent multiplied by the number of agents in the zone.

$$\mathbb{E}\left[T_{m_{\text{NC}}}^{H,\alpha} \mid k, m_{\text{NC}}\right] = m_{\text{NC}} \int_0^k \frac{1}{k} a \; \mathrm{d}a \quad (11)$$

$$= \frac{m_{\text{NC}} \times k}{2} \quad (12)$$

In equation (11), a is the location of a representative agent when the disruption occurs such that $0 < a < k$. It also represents the distance travelled by this agent in order to reach the exit. Since our agents enter the interval via a Poisson process, each agent's location is uniformly distributed in our system and by extension, on the interval of the non-communication zone, $[0, k)$.

The probability of having m_{NC} agents in this zone is

$$\mathrm{P}\left(M_{\text{NC}} = m_{\text{NC}}\right) = \frac{(k\lambda)^{m_{\text{NC}}} e^{-k\lambda}}{m_{\text{NC}}!} \quad (13)$$

where λ is the arrival rate of the Poisson process. Equation (13) is derived from coverage processes found in [36].

Randomising over the number of agents in the non-communication zone, we get

$$\mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{NC}}} \mid k\right] = \sum_{m_{\mathrm{NC}}=0}^{\infty} \mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{NC}}} \mid k, m_{\mathrm{NC}}\right] \mathrm{P}\left(M_{\mathrm{NC}} = m_{\mathrm{NC}}\right)$$
$$= \sum_{m_{\mathrm{NC}}=0}^{\infty} \frac{m_{\mathrm{NC}} \times k}{2} \times \frac{(k\lambda)^{m_{\mathrm{NC}}} e^{-k\lambda}}{m_{\mathrm{NC}}!}$$
$$= \frac{\lambda k^2}{2} \tag{14}$$

To calculate the total expected distance travelled by agents in the communication zone, we need to consider the behaviour of the agent closest to the disruption separately, since they will become the discoverer of the disruption and communication will not benefit them. We separate our expression into the expected distance travelled when there is one agent in this zone versus when there are multiple.

The expected distance travelled for one agent is

$$\mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k, m_{\mathrm{C}} = 1\right] = \int_k^1 \frac{1}{1-k}\left[(a-k) + (1-k)\right] \,\mathrm{d}a$$
$$= \frac{3(1-k)}{2} \tag{15}$$

We can similarly deconstruct the expression for the expected distance travelled when there is more than one agent in the communication zone.

$$\mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k, m_{\mathrm{C}} > 1\right]$$
$$= \int_k^1 \mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k, m_{\mathrm{C}} > 1, y\right] f_Y(y) \,\mathrm{d}y$$
$$= \int_k^1 \left[(y-k) + (1-k) + \frac{(m_{\mathrm{C}}-1)}{2}\left[3y - 4k + e^{-\alpha\lambda}(1-y) + 1\right]\right]$$
$$\times \frac{m_{\mathrm{C}}\,(1-y)^{m_{\mathrm{C}}-1}}{(1-k)^{m_{\mathrm{C}}}} \,\mathrm{d}y$$
$$= \frac{m_{\mathrm{C}}(1-k)\left[e^{-\alpha\lambda}\,(m_{\mathrm{C}}-1) + m_{\mathrm{C}} + 5\right]}{2\,(m_{\mathrm{C}}+1)} \tag{16}$$

where y represents the location of the discoverer and $f_Y(y)$ is the probability density function for the minimum order statistic with m_{C} observations and uniformly distributed, independent random variables. In order to calculate the expectation in (16), the coverage of the broadcast needs to be taken into account. Due to page limitations, we will not go into the calculation here.

Below, we use equations (15), (16) and (13) to calculate the total expected distance travelled by agents in the communication zone, given k.

$$\mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k\right] = \mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k, m_{\mathrm{C}} = 1\right] \mathrm{P}\left(M_{\mathrm{C}} = 1\right) +$$
$$\sum_{m_{\mathrm{C}}=2}^{\infty} \mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k, m_{\mathrm{C}} > 1\right] \mathrm{P}\left(M_{\mathrm{C}} = m_{\mathrm{C}}\right)$$
$$= \frac{4e^{-(1-k)\lambda} - 2e^{-(1-k+\alpha)\lambda}}{2\lambda}$$
$$+ \frac{e^{-\alpha\lambda}\left[\lambda^2(1-k)^2 - 2\lambda(1-k) + 2\right]}{2\lambda}$$
$$+ \frac{\lambda^2(1-k)^2 + 4\lambda(1-k) - 4}{2\lambda} \tag{17}$$

Putting both zones together, the total expected distance travelled by all agents in the network is

$$\mathbb{E}\left[T^{H,\alpha}_{m} \mid k\right] = \sum_{m_{\mathrm{C}}=0}^{\infty} \sum_{m_{\mathrm{NC}}=0}^{\infty} \mathrm{P}(M_{\mathrm{C}} = m_{\mathrm{C}})\mathrm{P}(M_{\mathrm{NC}} = m_{\mathrm{NC}})$$
$$\times \left(\mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{C}}} \mid k\right] + \mathbb{E}\left[T^{H,\alpha}_{m_{\mathrm{NC}}} \mid k\right]\right)$$
$$= \frac{4e^{-(1-k)\lambda} - 2e^{-(1-k+\alpha)\lambda}}{2\lambda}$$
$$+ \frac{+e^{-\alpha\lambda}\left[\lambda^2(1-k)^2 - 2\lambda(1-k) + 2\right]}{2\lambda}$$
$$+ \frac{\lambda^2(2k^2 - 2k + 1) + 4\lambda(1-k) - 4}{2\lambda} \tag{18}$$

Again, for brevity, the specifics of this calculation will be omitted and can be sought from the authors, if desired.

It turns out that the expression for the expected distance travelled to exit per agent can be derived analytically, however the exact form is ungainly and difficult to interpret. Hence, in this paper, we use an approximation for this average by taking the total expected distance travelled by all agents (18) and dividing it by the expected number of agents in the network, λ. Due to symmetry, all agents have the same expected distance to travel.

$$\mathbb{E}\left[d^{H,\alpha}_{a_1} \mid k\right] = \mathbb{E}\left[d^{H,\alpha}_{a_i} \mid k\right], \qquad \forall\, i \in \{1, 2, \ldots, m\}$$
$$\simeq \frac{1}{\lambda}\,\mathbb{E}\left[T^{H,\alpha}_{m} \mid k\right]$$
$$= \frac{4e^{-(1-k)\lambda} - 2e^{-(1-k+\alpha)\lambda}}{2\lambda^2}$$
$$+ \frac{e^{-\alpha\lambda}\left[\lambda^2(1-k)^2 - 2\lambda(1-k) + 2\right]}{2\lambda^2}$$
$$+ \frac{\lambda^2(2k^2 - 2k + 1) + 4\lambda(1-k) - 4}{2\lambda^2} \tag{19}$$

Sensitivity analysis showed that the error from approximation was negligible and tends towards zero as λ increases.

Finally, we randomise (19) over k.

$$\mathbb{E}\left[d^{H,\alpha}_{a_i}\right] = \int_0^1 \mathbb{E}\left[d^{H,\alpha}_{a_i} \mid k\right] f_K(k) \,\mathrm{d}k$$
$$= \frac{6e^{-(1+\alpha)\lambda} - 12e^{-\lambda} + e^{-\alpha\lambda}\left(\lambda^3 - 3\lambda^2 + 6\lambda - 6\right)}{6\lambda^3}$$
$$+ \frac{2\left(\lambda^3 + 3\lambda^2 - 6\lambda + 6\right)}{6\lambda^3} \tag{20}$$

Again, we assume that the disruption is uniformly distributed across the interval which is of unit length, hence $f_K(k) = 1$.

Section IV-C discusses how the expected distance travelled will vary depending on the density of the agents in the system.

IV. Discussion

In this section, we discuss comparative statistics and analyse the effect of information, the broadcast range and the density of agents on the expected travel distance.

A. Comparison of Information Levels

For a more meaningful interpretation of the expressions from Section III-A, we take the ratio of these expressions with respect to one another. We also look at the value of these ratios as the size of the network gets very large.

Using complete information as a baseline, we can compare how much worse network information is by dividing (5) by (2) and taking the limit as $n \to \infty$.

$$\lim_{n\to\infty} \frac{\mathbb{E}\left[d_{a_1}^N\right]}{\mathbb{E}\left[d_{a_1}^C\right]} = \lim_{n\to\infty}\left(\frac{5n^2-2}{4n^2-4}\right) = 1.25 \tag{21}$$

On average, as the network size gets large, network information performs 25% worse than complete information. Figure 3 shows the convergence of this ratio as the network size increases.

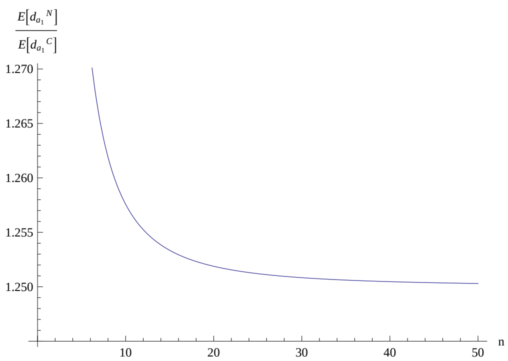

Fig. 3. Network information versus complete information.

Doing the same for historical information by dividing (7) by (2) and taking the limit as $n \to \infty$, we get

$$\lim_{n\to\infty} \frac{\mathbb{E}\left[d_{a_1}^H\right]}{\mathbb{E}\left[d_{a_1}^C\right]} = \lim_{n\to\infty}\left(\frac{4n+1}{2n+2}\right) = 2 \tag{22}$$

As the network size gets large, historical information will, on average, cause the agent to travel twice the distance as with complete information. Figure 4 shows this ratio as it converges from below. This implies that historical information performs worse as the network size increases.

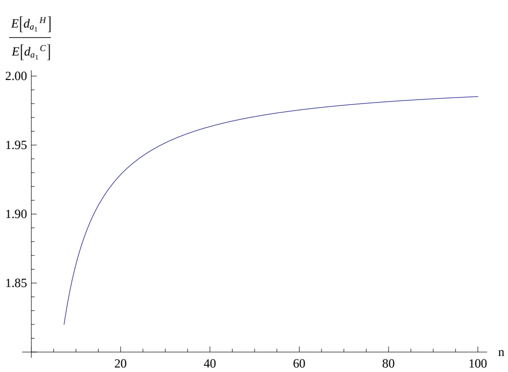

Fig. 4. Historical information versus complete information.

For completeness, one can do the same comparison for historical information versus network information. On average, historical information performs 60% worse than network information as the network size tends to infinity. This convergence occurs from below, again showing that historical information performs worse as the network size increases.

We also analyse how detrimental having incomplete information is to the distance travelled in the worst case. These results are calculated by choosing a starting location for the agent that would result in the shortest distance to exit under complete information and the largest distance to exit for the comparison information level using (1), (3) and (6). For network information, an agent will travel up to three times the distance than with complete information as the network size gets large. If an agent only has historical information, the agent will travel $2(n-1)$ times the distance than with complete information. This factor clearly worsens linearly with the size of the network, therefore the larger the network, the more detrimental it is for an agent with only historical information in the worst case. We should note that these worst case scenarios only occur with probability $\frac{1}{n^2}$, so despite their undesirable implications, their occurrences are fortunately rare.

B. Broadcast Range

We compare the distances travelled with and without communication and observe what happens as the network size gets large. To do this, we need to consider the broadcast range as a proportion of the network size. Let $\beta = \frac{\alpha}{n}$ for $0 \le \alpha \le n$ and divide (7) by (10), then we have

$$\lim_{n\to\infty} \frac{\mathbb{E}\left[d_{a_1}^H\right]}{\mathbb{E}\left[d_{a_1}^{H,\beta}\right]} = \frac{16}{15-4\beta+6\beta^2-4\beta^3+\beta^4} \tag{23}$$

For notational convenience, let $g(\beta) = \lim_{n\to\infty} \frac{\mathbb{E}\left[d_{a_1}^H\right]}{\mathbb{E}\left[d_{a_1}^{H,\beta}\right]}$. Partially differentiating (23) with respect to β, we get

$$\frac{\partial g(\beta)}{\partial \beta} = -\frac{64\,(\beta-1)^3}{\left(15-4\beta+6\beta^2-4\beta^3+\beta^4\right)^2} \ge 0, \text{ for } \beta \in [0,1] \tag{24}$$

Unsurprisingly, (24) shows that as the broadcast range increases as a proportion of the network size, the ratio of expected distance travelled without communication on the expected distance travelled with α-hop communication increases. This implies that as the broadcast range increases as a proportion of the network size, communication becomes more valuable. This relationship can be clearly seen in Figure 5.

Plugging $\beta = 0$ into (23), we find that when the agents must meet in order to communicate, then having no communication increases the expected distance travelled by 6.7% compared to when they are able to communicate. When $\beta = 1$ and communication is guaranteed immediately, then on average, having no communication increases the agents' expected travel distance by 14.3%.

C. Number of Agents

This section explores what happens to the expected travel distance per agent as the density of agents along the interval increases.

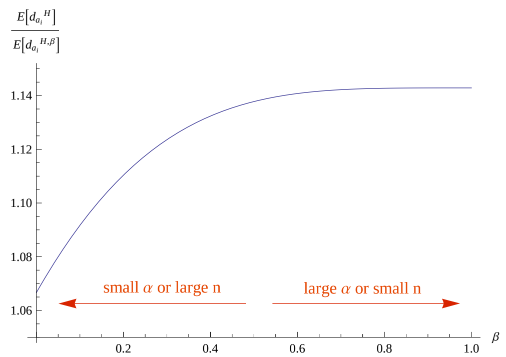

Fig. 5. No communication versus α-hop communication as β increases.

Figures 6 and 7 show the behaviour of the approximated expected distance to exit per agent, randomised over k, as the intensity rate and the broadcast range increase, respectively. In Figure 6, we fix $\alpha = 0.2$ and in Figure 7, we fix $\lambda = 10$.

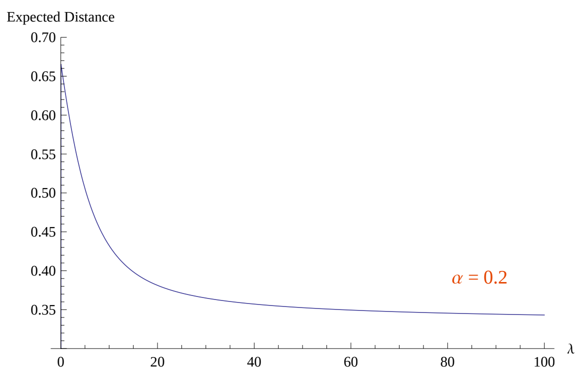

Fig. 6. Approximate expected distance to exit per agent as λ increases.

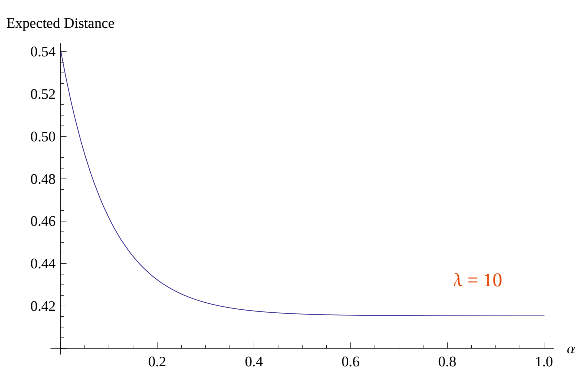

Fig. 7. Approximate expected distance to exit per agent as α increases.

From our approximated values, when $\alpha = 0.2$ and $\lambda = 1$, an agent will, on average, travel a distance of 0.63 units to exit an interval of unit length. If we increase the intensity rate to $\lambda = 100$, an agent will now, on average, travel a distance of 0.34 units to exit. This is a reduction of 45.2% from when $\lambda = 1$. This reduction slows dramatically with increasing λ as the effectiveness of communication diminishes once everyone knows about the disruption.

Similarly, when we fix $\lambda = 10$ and set $\alpha = 0$, an agent will, on average, travel a distance of 0.54 units to exit an interval of unit length. If we increase the broadcast range to cover the whole interval, i.e. $\alpha = 1$, then an agent will, on average, travel 0.42 units to exit. This is a reduction in distance travelled of 23.2%.

V. Conclusions and Future Work

In this paper, we introduced a simple model of information and communication in the event of an emergency. We have provided various measures of performance and investigated the importance of information and some factors that impact the effectiveness of communication using analytical results.

This research has shown that different levels of information can impact an agent's ability to escape an emergency situation. Unsurprisingly, the more information an agent has regarding their surroundings and the disruption, the easier it is for the agent to find the optimal path to exit the system.

Given that information matters, it is natural to ask: what if we can pass this information on? We found that communication between agents could further reduce the distance travelled to exit. Agents were able to broadcast the location of the disruption once they discovered it so other agents did not need to discover the disruption themselves to determine the best direction to exit.

We also found that the average distance travelled decreased as the available broadcast range increased. This is intuitive, since the larger the communication range, the sooner other agents will hear about new information, saving them from potentially wasting time by travelling in the wrong direction. In reality, increasing or maintaining the size of the broadcast range can be costly and further work could be done on modelling the trade-off between the size of the broadcast range and its cost, e.g. limited battery life on the communication device. This could lead to an "optimal" broadcast range that could be employed.

We also explored the effect of having many agents in the system on the effectiveness of communication. We found that the denser the agents were in the system, the easier it was for information to propagate and hence allowed the agents to exit more efficiently. Again, this continued improvement as we pack more agents into the system seems unrealistic. As further work, congestion can be introduced into the model to counteract the benefits of communication propagation, which could suggest there might be an "optimal" number of agents to be allowed in the system at any one time. This has implications for setting capacity restrictions in buildings and population limits in natural disaster prone communities.

The ring model developed in this paper is obviously simplistic and could be extended to a larger, more connected network with more exits and dynamic disruptions. Despite its simplicity, many high-rise buildings and store layouts have been known to mimic this ring design with a single exit, albeit with a layered approach for each storey.

This paper has shown that information can be invaluable in helping agents escape in an emergency situation. Increasing the ability to communicate and facilitating the propagation of information can also increase the chances of survival. Mobile apps that improve localised communication in the advent of centralised communication failure is one way of achieving this. This work highlights the importance of assessing the technology available today and ensuring we are making the most of the capabilities on offer in order to maximise evacuation success and minimise the risk to individuals due to a lack of information.

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