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FAU Discrete Optimization Challenge: Efficient MIP Approach for Energy Efficient Train Timetables

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FAU Discrete Optimization Challenge: Efficient MIP Approach for Energy Efficient Train Timetables

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Abstract

The report presents a mathematical model to solve the Energy Efficient Train Timetable optimisation problem presented in the 2015 FAU Open Research Challenge. The objective of this problem is to minimise the highest level of energy consumption from consecutive 15 minute intervals by making small changes to an existing train timetable. A mixed integer programming (MIP) model is formulated that includes all of the requirements and assumptions listed in given problem description. This model is then solved with commercial MIP solvers to return solutions for ten instances. Reductions in peak energy usage of 12-36% are found across all ten instances.

1 Problem Description

According to [1],

> Railway traffic is the biggest individual electricity consumer in Germany. It amounts to 2 % of the country's total electricity usage, which equals the consumption of the city of Berlin. However, the annual electricity cost (1 billion Euros per year) is not only determined by the total amount of energy used. Electricity contracts of big customers normally also incorporate a price component that is dependent on the highest power value drawn within the billing period, i.e. it depends on the timely distribution of the energy usage. This power component accounts for up to 20 % of the energy bill.

Managing the highest power value drawn, therefore, is an important part of any plan to minimise energy costs. The system-wide power load for railway traffic, as a function of time, can be smoothed by desynchronising acceleration events with each other or synchronising them with braking events. Beginning with an existing train timetable, it is possible to make small changes to train departure times to minimise peaks in energy usage. The problem statement from [1] defines the task.

> The task is to ensure a favourable system-wide power load over the given planning horizon to minimize the power component of the

electricity bill. The system-wide power load is determined by summing up all the individual power profiles of the trains under consideration. Obviously, this total power profile changes when shifting the trains in time. To minimize the power cost of the railway system, you have to minimize the average power drawn by all trains together over any consecutive 15-minute interval within the planning horizon.

1.1 Given Information

Existing train timetable and power profile data have been provided to aid in formulating the problem. Ten instances have been provided containing information on the legs travelled by a set of trains. As an example of problem size, the smallest instance contains 13 trains, making 206 journeys between a network of 30 stations over a period of approximately four hours. For each leg, defined as a unique journey made by a train between two stations along a particular track without stopping on the way, the following information is directly given:

1. The unique leg ID number.
2. The ID number of the train making the journey.
3. The ID number of the station where the leg begins.
4. The ID number of the station where the leg ends.
5. The ID number of the track used on the leg.
6. The earliest departure time from the station at the beginning of the leg.
7. The latest departure time from the station at the beginning of the leg.
8. The current departure time, in the given timetable, from the station at the beginning of the leg.
9. The minimum headway between the departure time for that leg and the departure time of the previous train on the same track.
10. The minimum stopping time that the train must respect at the station at the end of the leg.
11. The time required to travel between the beginning and ending stations for the leg.
12. The power profile, per second of travel, for the leg.

Information on the order of visited stations, the order of trains travelling on the same tracks and connecting trains at the same station can be extracted from the available data.

1.2 Assumptions

A feasible solution must respect rules for train timetabling. As the problem uses an existing schedule and makes changes, some assumptions are drawn from conditions in the original schedule and limit the changes that are allowable. The following assumptions can be found in the problem description in [1].

1. Each train travels the stations and tracks in the order given in data.

2. The order of the trains passing a given track has to be retained.

3. New departure times must lie in this interval defined by the earliest and latest departure times given in the data.

4. The minimum stopping time (in minutes) at each station must be respected

5. Safety distances (in minutes) must be respected. This is the minimal time which the subsequent train passing the same track has to wait before it can depart for the corresponding leg.

6. Passenger connections at the stations are respected. If the arrival time of one train and the departure time of another lie in an interval of 5 to 15 minutes, their new arrival and departure times have to lie in this interval, too.

7. Travel time for each leg is fixed.

8. The time-power curve for each leg is fixed.

2 Mathematical Formulation

A mixed integer linear programming (MIP) approach is proposed for the problem. This section describes the sets,parameters and variables used in the model and then the MIP formulation. The formulation relies on the following definition of a leg: a leg is a unique journey, travelled by any train non-stop between any two stations along a known track.

2.1 Parameters and Set

Here we define the initial sets in the mathematical formulation.

Minutes $= \{0, 1, \cdots, T\}$	The set of all times t in the model, in minutes, where T is the maximum time in minutes
Seconds $= \{0, 1, \cdots, 60T\}$	The set of all times t in the model, in seconds
$\mathbf{I} = \{1, 2, \cdots, \lceil \frac{T}{15} \rceil\}$	The set of all starting times for each 15-minute time interval i given in the model, in minutes)
$\mathbf{L} = \{1, 1, \cdots, L\}$	The set of all unique legs departing over the interval $[0, T]$ in the model, arranged in order of given departure time, where L is the total number of legs

Here we define the parameters in the mathematical formulation.

GD_l	Given departure time, in minutes, for leg l in the orignial timetable
LD_l	The lower bound on the departure time (i.e. earliest time allowed to depart), in minutes, for leg l
UD_l	The upper bound on the departure time (i.e. latest time allowed to depart), in minutes, for leg l
τ_l	The travel time for leg l between the start and end stations
W_l	Minimum waiting time (in minutes) after arrival of the train at the end station for leg l
H_l	Minimum headway (safety distance), in minutes, that must be respected at the beginning of leg l (i.e. minimum difference in departure time between leg l and the previous leg to depart from the same station on the same track)
$Start_l$	The starting station for leg l
$Train_l$	Train assigned to leg l
$Track_l$	Track assigned to leg l
End_l	The ending station for leg l
E_{lt}	Energy consumption, per second, for leg l at time $t \in \{0, 1, \cdots, \tau_l \times 60\}$

2.2 Variables

The decision variable is:

x	The maximum 15 minute average of energy consumption from all consecutive time intervals

A variable for total energy consumption for each second in the model is included.

e_t	Cumulative (total) energy consumption from all legs at any time $t \in \mathbf{Seconds}$

A binary variable,z_{lt}, indicates whether leg $l \in \mathbf{L}$ begins at time $t \in \{LD_l, LD_l + 1, \cdots, UD_l\}$.

$$z_{lt} = \begin{cases} 1 & \text{if } t = \text{departure time of leg } l \in \mathbf{L}, \\ 0 & \text{Otherwise} \end{cases}$$

2.3 Mixed Integer Programming Model

The energy efficient train scheduling problem is now formulated as a MIP in this section. The objective of the problem is to minimise the average energy consumption over consecutive 15 minute (i.e. 900 second) intervals. The decision variable x is therefore divided by a scalar amount to return the average energy consumption in the correct units and minimised.

$$\text{Minimise } \frac{x}{900}$$

s.t.

Constraint (1) extracts the maximum energy consumption from all consecutive 15-minute intervals $i \in \mathbf{I}$. The per interval energy consumption is found by summing over the energy consumption each second in the interval and using a trapezoidal rule to weight the contribution from energy consumed at interval boundaries.

$$x \geqslant \sum_{t=1}^{899} e_{t+(60\times15)(i-1)} + 0.5 \sum_{t\in\{0,(60\times15)\}} e_{t+(60\times15)(i-1)} \quad \forall i \in \mathbf{I} \tag{1}$$

Constraint (2) controls the energy consumption per second, based on positive and negative energy use from all legs. The total number of variables and constraints present in the model is constrained by only considering the possible legs which may contribute to the energy profile at time t.

$$e_t \geq \sum_{l\in L} \sum_{\substack{t'\in[0,\tau_l]: \\ LD_l \leq \lfloor t/60 \rfloor - t' \leq UD_l}} E_{l(t-\lfloor t/60 \rfloor 60+60t')} z_{l(\lfloor t/60 \rfloor - t')} \quad \forall t \in \mathbf{Seconds} \tag{2}$$

Constraint (3) stipulates that each leg begins at exactly one point in time. The constraint sums over the range of feasible departure times, in minutes, for each leg.

$$\sum_{t=LD_l}^{UD_l} z_{lt} = 1 \quad \forall l \in \mathbf{L} \tag{3}$$

Constraint (4) ensures that leg $l' \in \mathbf{L}$ cannot depart any earlier than the time at which the same train departed on any prior leg l, plus the travel time (τ_l) and the minimum required waiting time (W_l) for the prior leg. This also ensures that different legs for an individual train depart in the correct order. This relationship is represented in Figure 1.

$$\sum_{t'=LD_{l'}}^{t+\tau_l+W_l-1} z_{l't'} \leqslant 1 - z_{lt} \quad \forall l \in \mathbf{L}, l' \in \mathbf{L} : Train_{l'} = Train_l \ \& \ GD_{l'} > GD_l,$$
$$t \in \{LD_l, LD_l + 1, \cdots, UD_l\} \tag{4}$$

Constraint (5) forces the departure time of leg l' to obey the minimum required headway (safety distance) from the departure time of any prior leg l using the same track, and maintains the order in which different legs travel on the same track. An illustration of this relationship is provided in Figure 2.

$$\sum_{t'=LD_{l'}}^{t+H_{l'}-1} z_{l't'} \leqslant 1 - z_{lt} \quad \forall l \in \mathbf{L}, l' \in \mathbf{L} : Track_{l'} = Track_l \ \& \ GD_{l'} > GD_l,$$
$$t \in \{LD_l, LD_l + 1, \cdots, UD_l\} \tag{5}$$

Constraint (6) maintains connections between trains at the same station. Whenever leg l arrives prior to the departure of leg l' at the same station, but corresponding to a different train, and the time between the arrival of leg l and the departure of leg l' lies in the interval [5,15] in the original schedule (as determined by arrival time $GD_l + \tau_l$ and departure time $GD_{l'}$) then the new schedule should produce arrival and departure times for these legs within the same interval. The relationship for connections is shown in Figure 3.

$$\sum_{t'=t+\tau_l+5}^{t+\tau_l+15} z_{l't'} \geqslant z_{lt} \quad \forall l \in \mathbf{L}, t \in \mathbf{Minutes},$$
$$l' \in \mathbf{L} : Train_{l'} \neq Train_l, Start_{l'} = End_l \ \& \ 5 \leqslant GD_{l'} - (GD_l + \tau_l) \leqslant 15 \tag{6}$$

The remaining constraints enforce non-negativity for departure times and energy consumption.

$$0 \leqslant x$$
$$z_{lt} \in \{0, 1\} \quad \forall l \in \mathbf{L}, t \in \mathbf{Minutes}$$

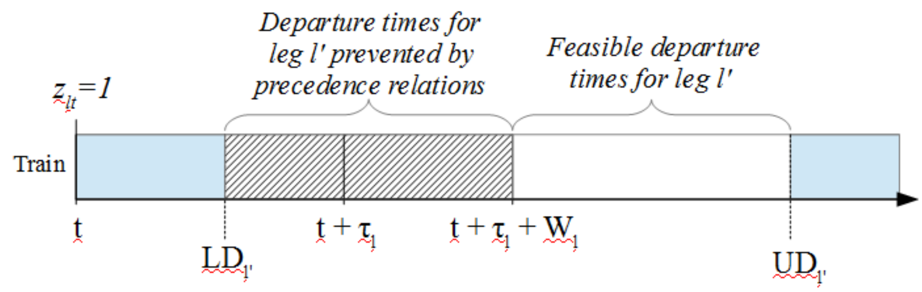

Figure 1: Restrictions on departure times due to precedence relationship between legs for the same train

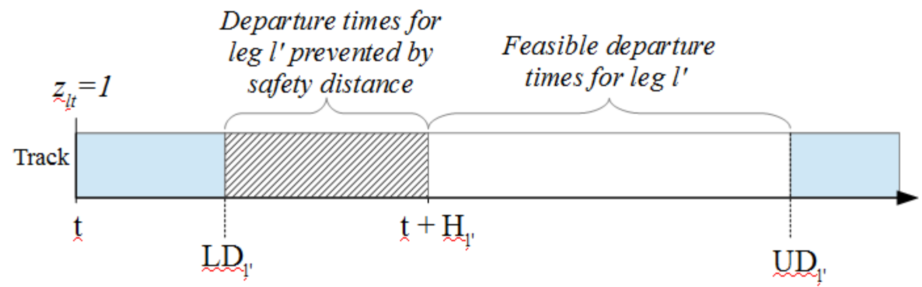

Figure 2: Restrictions on departure times due to headway (safety distance) requirement between legs on the same track

3 Solution Approach

The model has been solved using CPLEX, a commercially available MIP solver. Solutions are also checked for feasibility and correct objective function value using the Python solution checker provided by [1]. The model implemented in CPLEX used a transformed formulation of the model presented above and replaces legs with trains and stations. As each leg has a unique pairing of trains and stations, this does not change the number of variables.

Where possible, pre-processing of the given data has been undertaken to reduce the size of time windows each instance. It was found that the lower bound on departure times in the given data could be tightened for some legs, as these are dependent on the lower bound for the departure time of the previous leg for the same train plus the travel time and minimum waiting time. Wherever the given lower bound for a leg was less than the feasible lower bound calculated from the requirements of previous leg, the lower bound in the model was tightened prior to implementation.

An upper bound on the decision variable is applied to the model. The given train timetable has a feasible solution and the peak energy consumption from this solution (C) can be determined prior to initialising the model. As any new solution should improve the existing solution, the value for energy consumption

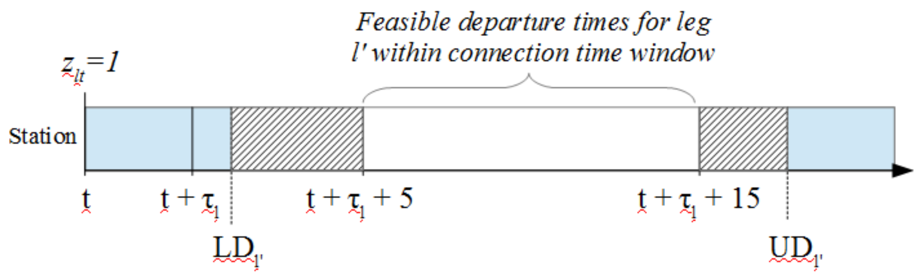

Figure 3: Restrictions on departure times for connecting legs at the same station

from the existing timetable is an upper bound on the decision variable in the model.

$$x \leqslant C \tag{7}$$

Warm starts are also possible and have been applied in multiple stages. Feasible solutions to a restricted versions of the problem can be used as an initial solution to speed up solve time. Specifically, we have used two such restricted versions. The first limits departure times in the solution to within one minute of the given schedule $\left(\sum_{t=max(LD_k,GD_l-1)}^{min(UD_l,GD_l+1)} z_{lt} = 1\right)$; the second limits departure times in the solution to within two minutes of the given schedule $\left(\sum_{t=max(LD_l,GD_l-2)}^{min(UD_l,GD_l+2)} z_{lt} = 1\right)$. For the first subproblem we use the given timetable as a starting solution and allow 1200s as the solution time. The solution to the first restricted problem is used to warm start the second restricted problem, which has a solution time limit of 1200s. Finally, the solution from the second problem is used to warm start the full problem with a solution time of 2 hours and 20 minutes. The full set of three problems solved, therefore, has a combined time limit of 3 hours.

4 Results

The objective function values, for each of the ten instances are listed in Table 4. The size of each instance is $\approx$ 4 hours of real time but the number of trains, stations and legs varies considerably. We are able to improve energy efficiency across all ten instances. The results provided have a small relative optimality gap ($< 1.3\%$) for almost all instances. Improvements may be possible with a longer solve time.

The problem statement listed the objective as minimising the highest system wide power load over consecutive 15 minute intervals. The model we have formulated is able to solve this objective. However, there is the potential for multiple solutions with the same objective function value. While these would be equivalent in terms of satisfying the objective in the problem statement, they may not be equivalent for other criteria. Further work should investigate the solution pool and multiple objectives, for example, the total energy consumed.

Instance	Original timetable	New timetable	Relative Improvement $\left(\frac{old-new}{old} \times 100\%\right)$
1	1.7781	1.1700	34
2	4.6209	2.9682	36
3	20.6357	15.1915	26
4	19.9836	14.97687	25
5	27.5516	19.5421	29
6	26.3494	20.5389	22
7	21.7009	16.6548	23
8	2.7730	2.4326	12
9	113.3918	98.3023	13
10	79.5854	68.2712	14

Table 1: Maximum average power load for a 15 minute interval

References

[1] The Chair of EDOM (Economics Discrete Optimization Mathematics. Discrete optimization: Energy-efficient train timetables, 2015.