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A Method for Using Mathematical Optimization to Allocate and Reallocate Geographically Distributed, Reconfigureable Resources with Spatial Considerations and Time Windows

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A METHOD FOR USING MATHEMATICAL OPTIMIZATION TO ALLOCATE AND REALLOCATE GEOGRAPHICALLY DISTRIBUTED, RECONFIGUREABLE RESOURCES WITH SPATIAL CONSIDERATIONS AND TIME WINDOWS

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ABSTRACT: In this report we formulate a model for the scheduling of reconfigurable resources to meet demand over time, with particular consideration for the potential need to reschedule and reconfigure the resources as new demands arise. This is accomplished in the framework of "continual optimization" using dynamic column and constraint generation schemes to accommodate the enormous number of possible variables and constraints.

Keywords: reconfigurable facilities scheduling, integer programming, continual optimization, column generation, constraint generation

1. Introduction

The scheduling of scarce resources over time to meet demand in some optimal sense (e.g. minimum cost or maximum profit) is a problem which has been at the forefront of Operations Research, and Mathematical Programming in particular, since the founding of the field. Examples such as job shop scheduling are ubiquitous in the literature. Another class of examples is the scheduling of facilities, such as classrooms, to accommodate a given number of classes of specified sizes.

In this paper we consider a facilities scheduling problem complicated by another parameter—the ability to reconfigure some of the resources to better meet the demand and objective function criteria. Such problems occur in a number of settings. One of the more obvious is the scheduling of conference rooms and facilities at academic or office buildings, convention centers and conference hotels. Others include athletic fields and facilities, which may be configured into various combinations of (say) soccer, football and rugby pitches, basketball and volleyball courts, and multi-purpose school buildings which may be partitioned into classrooms of various size, assembly halls, etc. Similar allocation problems may occur in microelectric chip design, facility location and urban planning.

To avoid getting lost in the generality of the problem, we will consider aspects specific to the convention facility allocation problem. The extension to other applications is straightforward.

2. Reconfigurable Facility Assignment

In the convention facility allocation problem, the rooms available for booking may be in one or more facilities in a geographical area, and may be reconfigurable, if for example there are partitions which can be added or removed to change the size or shape of the meeting area. When there are several buildings within a geographical area that house conference rooms, this increases the ability to find feasible scheduling solutions when there are multiple, competing requests. Note that not every room assignment might be perceived as attractive or optimal to the meeting group (customer), especially when there are trade-offs in terms of costs and benefits (such as the availability of audio-visual technology built into the room). Nor may it be possible to exactly accomodate all the wishes/requirements of the meeting groups. Thus "soft" constraints may be necessary and should be allowed for.

Given an inventory of space available, with attributes such as size, geographical location, capacity, reconfiguration options, special features (windows, podium, telephone/LAN lines, projection and sound equipment), the problem is to allocate the spaces (possibly re-configured) to demand. We will refer to a particular room with particular fixed characteristics such as seating capacity, podium, projection equipment, etc. as being a room type. A given configuration will result in a specified number of rooms of each type, and customer demand expressed in terms of a number of rooms of these types.

The objective is to maximize profit, given cost recovery (revenue) and cost associated with each assignment. Costs may include re-configuration costs, and apportionment of fixed facility charges.

3. Basic Model Formulation

The basic formulation is a mixed integer programming problem with some elastic (soft) constraints to ensure feasibility and assist in a more efficient branch and bound search. Initially we will assume that each customer wishes to book a single contiguous block of time (a window) in a single facility (if not, they may be considered as several customers), and assume change-over costs are negligible. The entities in this formulation are then as follows:

Indices

$i = 1, ..., I$	The facilities/buildings/properties
$k = 1, ..., K$	The customer set
$c = 1, ..., C_i$	The possible configurations of facility i
$r = 1, ..., R$	Room types
$t = 1, ..., T$	Time periods

Data

D_{krt}	The number of rooms of type r required by customer k begining in period t
t_k^1	The first period customer k wishes to book.
t_k^2	The last period customer k wishes to book.
n_{icr}	The number of rooms of type r when facility i is in configuration c.
R_{ic}	Revenue per time period from a (fully occupied) configuration c at facility i.
P_{irt}^+	Penalty for an unused room type r at i in period t
P_{jt}^-	Penalty for a shortfall of room tpye r at i in period t

Variables

x_{ict}	Equal to 1 if facility i is in configuration c in period t, zero otherwise.
y_{ik}	Equal to 1 if customer k is booked into facility i (starting in period t_k^1).
s_{irt}^+	Surplus of rooms of type r at i in period t
s_{irt}^-	Shortfall in rooms of type r at i in period t

Constraints

Material Balance:

$$\sum_{c=1}^{C_i} n_{icr} x_{ict} - \sum_{k|t_k^1 \le t \le t_k^2} D_{krt} y_{ik} - s_{irt}^+ + s_{irt}^- = 0 \qquad \forall i, r, t \tag{1}$$

Convexity:

$$\begin{array}{ll} \sum_{c=1}^{C_i} x_{ict} = 1 & \forall i, t \\ \sum_{i=1}^{I} y_{ik} = 1 & \forall k \end{array} \tag{2}$$

Bounds:

$$\begin{array}{ll} s_{irt}^+, s_{irt}^- \ge 0 & \forall i, r, t \\ x_{ict} = 0/1 & \forall i, c, t \\ y_{ik} = 0/1 & \forall i, k \end{array} \tag{3}$$

Maximize

$$\sum_{i,c,t} R_{ic} x_{ict} - \sum_{irt} (P_{irt}^+ s_{irt}^+ + P_{irt}^- s_{irt}^-) \tag{4}$$

4. Column generation

Depending on the number of time periods the above model has a reasonable number of constraints, but since the number of configurations might be large, the number of potential (integer) variables is large. We therefore envisage use of column generation, or “Branch and Price” (see Barnhart et al [1] and Johnson et al[4]) as also used in the LimCo model [3]. The columns to be generated correspond to the possible configurations n_{icr}, where reduced cost for the (implicit) x_{ict} variables may be computed from the shadow prices on the material balance constraints (1) and the convexity rows (2), using the known (enumerable) configurations for the facilities. If these configurations are specified in advance this evaluation is rather tedious, but configurations at different facilities can be evaluated in parallel, or by separate threads. If the possible configurations can be generated automatically, as in the 2-dimensional cutting stock problem[2], this may speed up the process.

5. Change-over Costs and Row Generation

Change-over costs can significantly complicate a scheduling model. If they are nontrivial we must model them, using a potentially large number of constraints. If the change-over cost can be considered independent of the configurations (at a given facility i) one method of accounting for all change-over costs is to define new variables:

δ_{it} Equal to 1 if facility i changes configuration at the beginning of period t, zero otherwise.

We may add the following constraints to enforce this property:

$$\delta_{i,t+1} \geq x_{ip,t+1} + \sum_{c \neq p} x_{ict} - 1 \quad \forall i, p, t < T$$

These variables and constraints may be added to the problem, along with the sum of change-over cost terms $M_i \delta_{it}$ in the objective.

It should be clear that if the number of possible configurations is large, the number of such constraints will be even larger. However, it is also clear that only a very small number of these constraints can ever be binding. We therefore use a constraint generation approach, in combination with the column generation to solve the problem.

If the change-over costs are substantially different, we must define a larger set of variables:

δ_{ipqt} Equal to 1 if facility i changes from configuration p to configuration q at the beginning of period t, zero otherwise.

The δ variables are forced to reflect the change-over by imposing the constraints:

$$\delta_{ipq,t+1} \geq x_{ipt} + x_{iq,t+1} - 1 \quad \forall i, p, q, t < T$$

The individual cost terms to be subtracted from the abjective function are now $M_{ipq}\delta_{ipqt}$ summed over t. The number of such constraints is even larger than above, but once again only a tiny fraction can be binding, and the constraint generation approach is needed.

6. Continual Optimization

In reality, demand may change over time—for example, cancellations or a modification of demand to increase or decrease room capacity needs as the scheduled event draws near. When this is the case, an approach based on continual optimization works well. It is envisioned that the optimization will be carried out using a procedure very like that described in [3], but also ideas from [1, 4]. In solving the initial problem to near optimality, a considerable number of configurations will have been generated, and many discarded. When the customer demands change, and the schedule re-optimized, these already computed configurations are available, along with those in the current incumbent solution, to "hot start" the column generation process. The implementation can be carried out using the many facilites of IBM OSL (Optimization Solutions and Library) for customized optimization algorithms.

7. Conclusion

The problem of scheduling reconfigurable facilities can be formulated as a large mixed integer program, suitable for solution by column generation and constraint generation methods. These in turn are amenable to treatment with algorithms developed for continual optimization [3] and provide the basis for an effective, flexible solution to this application.

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