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Bias towards Regular Configuration in 2D Pointing

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ABSTRACT

Extending Fitts' law to more than one dimension has been recognized as having important implications for HCI. In spite of the progress made over the years, however, it is still far from a resolved issue. Our work approaches this problem from the viewpoint of a configuration space, which has served as a useful conceptual framework for understanding human preference in perception. Notably, human are found to be biased towards regular configurations. In this work, we extended the configuration space framework to the domain of motor behavior, analyzed 2D pointing, and developed five models to account for the performance. An extensive experiment was conducted to measure the fit of the derived models and that of three previous models. Consistent with our hypothesis, the model reflecting a bias towards regular configuration was found to have the most satisfactory fit with the data. The paper concludes with discussions on improving understanding of Fitts' law and the implications for HCI.

Author Keywords

Configuration space, Fitts' law, 2D pointing

ACM Classification Keywords

H.5.2 Information Interfaces and Presentation: Evaluation, theory and method

General Terms

Experimentation, Human Factors, Theory.

INTRODUCTION

$$T = a + b\log_2\left(\frac{D}{W}+1\right) \qquad (1)$$

Fitts' law [13] is perhaps the most well-known member of a small repertoire of predictive laws available for the field of human-computer interaction [5]. It is a mathematical formula (Eq. 1 as in its commonly accepted form today) predicting the movement time T of a pointing action, i.e. it predicts the time duration between starting a movement at some distance D away from a target and stopping the movement somewhere within the target. In one dimensional abstraction, where the dimension of the target perpendicular to the movement direction is ignored, the target is defined by the length of the line segment along the movement direction W, see Figure 1. The law's validity as a robust predictor of movement time has been repeatedly confirmed [19, 21, 23]. In HCI, it is generally used as a tool for evaluating user interfaces or devices [28, 42], and has helped launching some important input devices, including the ever-present mouse [4]. Despite its usefulness, a lack of consensus in its fundamental nature limits its applicability and hinders the development of similarly predictive laws. This conceptual deficiency becomes especially apparent when attempts were made to extend the law to more than one dimension.

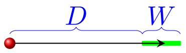

Figure 1. Fitts' Law Task in 1D idealization

Pointing in 2D is the task that subjects in Fitts' law experiments actually do because the targets have two dimensions, and 1D pointing is an idealization. Perhaps more importantly, 2D pointing is a task that every 2D graphic interface user routinely performs, up to thousands of times per day. For such a pervasive task, researchers in HCI have formulated various quantitative models to predict the movement time [1, 2, 16, 27, 31], and have reached some impressive results in term of fitting the models to their experimental data (often obtaining R^2 well above 0.9). In spite of such successes, however, 2D pointing modeling has yet to achieve a conceptually satisfactory resolution. This paper seeks to resolve the issue by bringing in new theoretical point of view.

Background and Motivation

Scope and Approaches

In the literature of 2D pointing, one class of models utilizes the experimentally measured distribution of movement stopping points in space to predict the measured movement time, essentially fitting one type of experimental results to another [16, 31]. Such a modeling approach, though useful, is not the focus of this study, for the following two reasons: We are mainly interested in developing models that in principle could produce predictions without requiring an experiment to be conducted, so that a practitioner could easily use the model to make performance prediction in practice. In addition, we attempt to understand the principles (if any) governing the observed lawful regularities, a desirable model for such purpose should be constructive in nature: predicting experimental outcome from experimental conditions.

Another class of models [1, 2, 27] calculate movement time prediction from some experimental conditions, such as the width of the target, the distance of the starting point to the target, and so on. However, these models often exhibit some conceptual or methodological weaknesses. First, unlike the original Fitts' formulation, many papers did not attempt to propose theoretical justifications for their models. Models could be proposed on the basis of intuitions, or as the outcomes of mathematical exercises of satisfying known trends in data. More principled approach seems to be needed if more theoretical advances are desired. In this paper, we attempt to pursue such a direction by deriving all the models in a theoretical framework.

Methodology

Methodologically, the previous models in general consider only a few factors at a time. Thus, the experiments used to test these models often investigate only limit cases concerning these factors, and hold other factors constant. This type of limited experimentation with a small number of factors is certainly the standard scientific practice of uncovering causal relationships between factors and effects. However, experimentation with a small number of factors alone is insufficient for obtaining a global understanding of the phenomenon. At some point, regression with a large number of testing conditions is necessary. We believe 2D pointing research has arrived at a stage where such a global understanding is desirable.

One side effect of limited experimentation is that the resulting models often contains arbitrary constants that may happen to apply for the subset of investigated cases: the risk of overestimating model fit is high. Although it is undesirable to exhaustively test all the possible conditions, testing models in as many meaningfully varied conditions as possible can reduce such risk. We believe the configuration space viewpoint introduced below offers some methodological guidances on selecting a suitable set of conditions to test.

Overviews

The concept of configuration space refers to a way of treating the state of an entire system as a single point in a higher-dimensional space. It has a long history in physics and mathematics, and has known to be a fruitful method of dealing with complex problems in many engineering fields such as dynamic systems and robotics. Recently it has shown some promises in the psychological research areas such as perceptual organization [10, 11]. We hypothesize that it may also be useful for understanding 2D pointing because the task involves both perception and motor control. This paper documents an effort to explore the idea.

In the following sections, we will briefly review several lines of research that lead to our current position on configuration space. We then analyze 2D pointing task from such a perspective, and derive five models to predict the movement time based on different assumptions on the shape and distribution of 2D pointing configuration space. The five derived models are tested with a 2D pointing experiment. Consistent with our goal, the experiment we use to test the models includes all of the factors as suggested by a configuration space analysis. In total, 384 different conditions are included in the experiment. For comparison purpose, we also test three previous models [1, 2, 27] with our data. Results suggest that the best model is the one assuming that the performance is strongly influenced by regular configurations. That is, the performance data is largely explained by a set of nonexistent but regular experimental conditions, which have dominated the actual experimental configurations presented to the participants. The paper concludes with discussions.

To summarize, the contributions of this paper are:

- Introduced configuration space, a new conceptual framework for understanding user tasks and behaviors to HCI.
- Applied the framework to develop new understandings of 2D pointing, and Fitts' law in general.
- Demonstrated that the new conceptual framework provides some guidance to remedy certain methodological weaknesses in existing research methods.
- Tested the hypothesis that human exhibits a bias towards regular configuration with an extensive 2D pointing experiment, and results are consistent with the hypothesis.
- Discussed theoretical and practical implications of this work to HCI.

THEORETICAL PERSPECTIVES

Our theoretical approach is the result of our effort to integrate several traditionally disparate areas of research. We will start a brief review of these areas by introducing the concept of configuration space.

Concept of Configuration Space

Configuration space is an abstract space of all possible combinations of quantities (parameters) that characterize the state of a system. Every *configuration*, or *state* can be thought of as a point in the space of all possible configurations. The number of dimensions, or degrees of freedom (DOF), of the space equals to the number of parameters specifying the configuration. These parameters can be seen as coordinates of a point in the space. Let us start with a toy example of a "system": a pendulum, the kind seen on some old mechanic clock. The state of the system can be uniquely determined by the angle between the line going through the rod of the pendulum and the line of gravity. The configuration space of the system is an one-dimensional space; in fact, a circle, as the value of an angle goes from 0 to 359 then back to 0. If the pendulum's arm consists of two rods hinged together instead, the configuration space becomes two-dimensional, assuming the hinge is completely flexible: one angle specifies the upper rod's position, another angle the lower rod's position. The shape of the configuration space is a torus: each state of the pendulum system has an one-to-one correspondence with each point of the torus; the two angles, each changing from 0 to 360, are the coordinates of a point on the torus, counting from the "equator" of the torus and its "meridian", respectively.

For complex systems with many parameters, the configuration space can quickly become hard to visualize. However, some seemly complex behaviors can become simple when described in configuration space. Another benefit is that the configuration space viewpoint often forces us to think carefully about all the relevant parameters of a system. Perhaps more importantly, the configuration space viewpoint allows us to move the attention away from the superficial distinction in the ways of parameterization (coordinates systems), and instead focus on the more consequential overall structure of the space. For example, consider a three dimensional space of a ball. We can express a point of the space in term of spherical coordinates of latitude, longitude and altitude, or we can equally express the point in Cartesian coordinates of x, y and z. They are equivalent because one set of parameters can be converted into another without loss of information. For this conversion to be possible, both side of the conversion must have the same number of DOFs. What determine the behavior of the system are the shape of the space and the distribution of points therein, not the coordinates system we choose to describe them. It might be more convenient or simpler to describe a system behavior in one coordinate system than in another, but changing the coordinate system does not alter system behavior.

Adopting a configuration space approach to understand cognitive systems could entail two different levels of commitment. A weak view would regard the concept just as a way of thinking about the problems cognitive systems try to solve. A strong view would assume psychological processes operate in configuration space: psychological space is a configuration space. The published work in psychology that adopts a configuration space approach seem to fall into the later camp. We also take this position in this paper. The underlying assumptions can be laid out in the following: For a given task involving multiple task parameters, we assume that there are mental representation of the parameters, and thus forming a *mental configuration space*; The principles governing the operations in mental configuration space influence task performance, especially the central planning aspect of the performance; Finally, the governing principles can be expressed as some kind of optimizations. Now we will review some possible optimization principles.

Optimization in Mental Configuration Space

One straightforward optimization principle in configuration space is the tendency for the system to seek the shortest distance from one state to another. Such a geometrical idea of optimization has been forcefully advocated by psychologist Shepard in much of his career [38, 39]. His experimental work have shown that mental rotations follow curved paths in the physical Euclidean space that are actually straight paths in the configuration space [6, 40]. Although Shepard's explanation for such a geometrical optimization, as some kind of evolutionary internalization of physical world, may not be convincing [22] or even necessary,[1] the observation that some mental operation is guided by simple optimization principles in configuration space is still compelling.

Regularity in Configuration Space

A central problem of operating in a configuration space is to deal with its high dimensionality. Cognitive systems routinely work with complex tasks that involving a large number of parameters, so the mental configuration space correspondingly must have many dimensions. It is reasonable to hypothesize that cognitive systems employ some kind of strategies to reduce the number of dimensions but still maintain a somewhat degraded yet acceptable performance, in the spirit of bounded rationality[41]. Such strategies, if exist, would certainly exploit the structure of the configuration space.

In perceptual organization research, Feldman proposed a regularity based lattice structure for perceptual configuration space [10, 11]. Object models that are more *regular* (defined as meeting more *constraints*) are placed lower in the lattice, and are the preferred interpretations compared with other models that also apply but are higher on the lattice. For example, equilateral triangles and squares are found to be "better" shapes than the generic ones in both goodness rating and shape production tasks [12].

Such a preference for regular models is consistent with the principle of simplicity [7, 43]: less regular models require more free parameters to specify, whereas more regular models need fewer free parameters to specify due to the increased number of constraints that they must satisfy. Regular models therefore requires less storage space in the cognitive system, thus explaining the preference. In addition, less regular models can always be reconstructed from regular ones through transformations, or be considered as the results of some operations acting on the regular models [26]. Another way to introduce the regularity ideas is to invoke the concept of symmetry: more regular configurations are more symmetric. However, the concept of symmetry has too many connotations to be a precisely defined term, so we will mainly use the number of constraints to define regularity.

The preference for regular configuration seems to be well founded in perception. The question remains as whether such bias exists in other domains of cognition. In particular, would it offer any explanatory power in 2D pointing? This tantalizing question is what we set out to explore in this paper. Of course, one can always argue that since a cognitive optimizing principle should be general, it must be somehow applicable in all domains of cognition. In this case, we should do better by noting the closely coupled relationship between perception and motor behavior.

HYPOTHESIS

The discussions above lead us to the following hypothesis:

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[1] After all, geometry, like other branches of mathematics, "is the study of *mental* objects with reproducible properties"[3]. Mental rotation does not have to be an internalization of physical world to conform to kinematic geometry, since geometry itself is mental.

> Participants exhibit a bias towards regular configurations in 2D pointing.

If this hypothesis is valid, models that reflect such bias should outperform those that do not. Before we proceed to derive a set of models that can be used to test this hypothesis, we briefly review three models that have been previously found to fit data well, as we will use them in comparison.

PREVIOUS MODELS

MacKenzie and Buxton [27] proposed several models for 2D pointing. The best model confirmed by their data is this formula:

$$T = a + b\log_2\left(\frac{D}{min(W,H)} + 1\right) \tag{2}$$

This model reflects the well founded intuition that the smaller of H or W should dominant overall performance. Accot and Zhai [1] built on such understanding, but also noted the importance of W vs H ratio. They formalized these ideas into a set of desirable mathematical properties, and arrived at a set of distance models between D/W and D/H. They finally settled on an Euclidean distance model:

$$T = a + b\log_2\left(\sqrt{\left(\frac{D}{W}\right)^2 + \eta\left(\frac{D}{H}\right)^2} + 1\right) \tag{3}$$

where η depends on the data. For their own data, $\eta = 0.13$; for data in [18], $\eta = 0.32$. Since the authors concluded that the value of η should be in the range of $(1/7, 1/3)$, we use the median value of 0.24 in our model comparison. Recently, Appert et al.[2] proposed a model to more explicitly account for the effect of movement directions:

$$T = a + b\log_2\left(\frac{D}{W} + \frac{D}{H} + 0.6\cos(\alpha)\frac{D}{min(W,H)} + 1\right) \tag{4}$$

where α is the absolute value of the angle between movement direction (assuming straight movement) and the vertical line. We notice that the later two models include constants (η and 0.6, respectively) that are difficult to explain.

DERIVING MODELS OF 2D POINTING

Information Theoretical Framework

To derive a model to account for movement time in 2D pointing, we follow Fitts' original argument in proposing his concept of task difficulty [13]

> The rational basis for this estimate of task difficulty is the maximum relative uncertainty that can be tolerated for a correct movement in a series having a specified average amplitude. Thus the minimum organization required of a particular movement is defined by the specification of one from among k possible categories of amplitude within which the movement is to terminate.

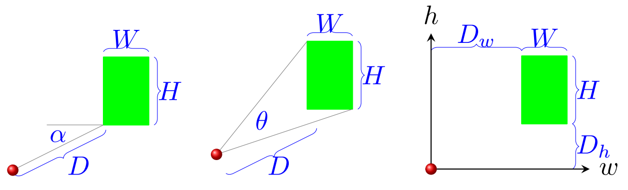
Translating the idea into modern information theoretical term [8], we can say that the index of difficulty (ID) of a pointing task is the logarithm of the inverse of the probability of terminating the movement within the target:

$$ID = \log_2 \frac{1}{P(Terminating\;Within\;Target)} \tag{5}$$

This formulation of ID is general. In principle we can calculate movement time prediction for any pointing task. The key is to estimate the probability. In 1D idealization of the pointing task, all the possible movement terminating positions are on a single line, and thus equation (5) can be expanded into the classic Fitts' law formula:

$$ID = \log_2 \frac{D+W}{W} \tag{6}$$

where D is the distance from the starting position of movement to the beginning of the target line segment, W is the length of the target line segment. $D + W$ determines the total number of possible movement terminating positions, and W determines the number of possible terminating positions *inside* the target. It should be stressed that this entire discussion is within the realm of mental configuration space. The parameters above are mental representation of actual physical parameters, assuming there are direct mappings between them. The ratio $W/(D + W)$ is the probability of terminating within the target.



(a) W, H, D, α (b) W, H, D, θ (c) W, H, D_w, D_h

Figure 2. Three different parameterizations for 2D pointing task. The shaded rectangle is the target of a pointing movement, and the ball represents the starting position of the movement.

Configuration Space of 2D Pointing

Unlike a classic Fitts' experiment, a 2D pointing experiment does not ignore the height H of the target, but treats it as an independent variable. So the number of needed parameters is expanded to three: the distance from starting position to the target D, the width of the target W, and H. However, three is not enough: to fully specify a generic 2D pointing task, an additional angle parameter, α as shown in Fig. 2(a), or alternatively θ as in Fig. 2(b), is necessary. Another way to parametrize the task is to use the parameter set $\{W, H, D_w, D_h\}$, as shown in Fig. 2(c). All three ways of parameterization require four parameters, and they are equivalent since each parameterization can be calculated from another via simple trigonometry. In other words, these different parameterizations characterize the same four dimensional configuration space. However, the shape of the space and the distribution of points therein, which both are unknown, might be easier to discover in one parameterization than in another. We now propose several models by adopting different parameterizations and equipping each with a different set of assumptions on how the parameters are related (probabilistically dependent or independent) and how the points are distributed, resulting in five different models.

Width-Height Independence Model

If we conceptualize 2D pointing as two independent 1D pointing actions combined, one horizontal and one vertical, and further assume that both actions follow 1D Fitts' law, we can write the following formula to model movement time T:

$$T = a + b\log_2\left(\frac{D_w}{W} + 1\right) + c\log_2\left(\frac{D_h}{H} + 1\right) \tag{7}$$

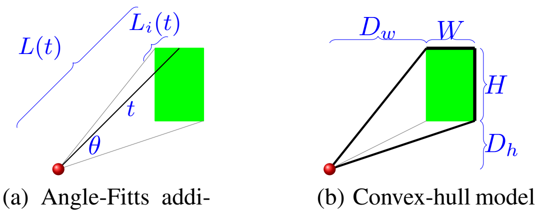
This model assumes total independence between D_w/W and D_H/H, therefore the total movement time is the sum of the two independent movement times, one moves horizontally, another vertically. Obviously, the independence condition is not well justified. We include it as a baseline.

Angle-Fitts Additivity Model

If we switch to another parametrization of the configuration space, with distance D and angle θ as in Fig. 2(b), we have better justification for their independence: we can reasonably assume that users first select an angle, by picking a direction line connecting the start position with the target, then do an 1D Fitts task along the selected line (see Fig. 3(a)). As a result, we have

$$T = a - b\log_2\left(\theta\int_0^\theta \frac{L_i(t)}{L(t)}dt\right) + c\log_2\theta \tag{8}$$

where t is a parameter ranging from 0 to θ, indicating which directional line is chosen, $L(t)$ is the total length of the chosen line segment, $L_i(t)$ is the length of segment inside the target, and $L_i(t)/L(t)$ is the probability of terminating within target when line t is selected. The second term of the formula is then essentially the average Fitts' law performance of all the acceptable direction lines (lines going from the starting position and crossing the target). When the target is rectangular, $L(t)$ and $L_i(t)$ can be calculated via trigonometry and the integral has closed form solution. The third term is the effect of selecting a direction line: $1/\theta$ is proportional to the probability of selecting one direction line.



(a) Angle-Fitts additivity model

(b) Convex-hull model

Figure 3. Explaining parameters of two models

Convex Hull Model

In order to calculate the probability of hitting the target, we want to find a reasonable set of possible terminating positions. Naively, it can be the set of all points within the convex hull enclosing the starting point and the target area, as outlined by thick lines in Fig. 3(b). The probability of terminating within target is then obtained by dividing the area of the target by the area of the convex hull. So we have:

$$T = a + b\log_2\left(\frac{HD_w + WD_h}{2WH} + 1\right) \tag{9}$$

Regular Configuration Model

In order to test our hypothesis that there exists a bias towards regular configuration, we need to first define a regular configuration in 2D pointing. As discussed earlier, we define *regular configurations* as configurations that require fewer free parameters to specify than the generic ones. In other words, a regular configuration satisfies more constraints. To simplify the discussion, in the following discussions, we disregard the configuration's scale, as it has been shown to be largely independent from task performance [17]. So three parameters are needed to specify a scaleless 2D pointing configuration, and they should all be ratios due to the removal of scale. Suppose that we originally choose W, H, D_w and D_h as the set of parameters. To normalize scale, we divide every parameter by D_w, and reduce the set to three: W/D_w, H/D_w, and D_h/D_w. To get a regular configuration, we need to take away one more parameter. Target shape W/H is a hard constraint prescribed by the task; so is the minimal movement distance $D = \sqrt{D_w^2 + D_h^2}$. There is no freedom in these once a participant sees the configuration. The only freedom the participant enjoys is the movement angle. The subject can approach the target in any angle as along as the target of the given shape is hit after the given minimual amount of movement. In other words, movement angle is the only free parameter we can take away if we are to define a regular configuration. Given these constraints, the only reasonable choice is to set $W/H = D_h/D_w$, as illustrated in Fig. 4. This configuration has some interesting features. For example, the diagonal line of the target is perpendicular to the shortest line connecting the starting position to the target; the convex hull is the largest among those configurations with the same D, i.e. when the movement starts from any point on the shaded curve in Fig 4.

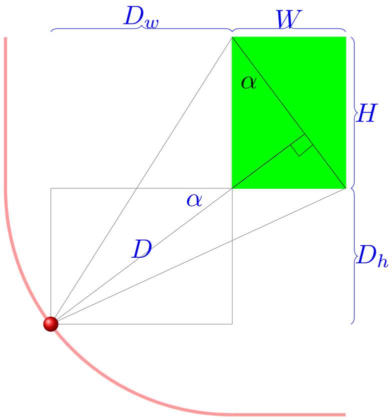


Figure 4. Regular configuration (its movement starting position is indicated by the ball), with a particular relative angle to the target satisfying the constraint $W/H = D_h/D_w$. The shaded curve (a 90° circular arc connecting two line segments) represents all the starting positions with the same shortest distance D to the target.

Applying the convex hull model (Eq. 9) to the regular configuration defined above, we have:

$$\begin{aligned} T &= a + b\log_2\left(\frac{HD_w + WD_h}{2WH} + 1\right) && (10)\\ &= a + b\log_2\left(\frac{HD\cos\alpha + WD\sin\alpha}{2WH} + 1\right) && (11)\\ &= a + b\log_2\left(\frac{D\sqrt{W^2+H^2}}{2WH} + 1\right) && (12)\\ &= a + b\log_2\left(\frac{1}{2}\sqrt{\left(\frac{D}{H}\right)^2 + \left(\frac{D}{W}\right)^2} + 1\right) && (13)\end{aligned}$$

We are surprised to find that this regular configuration model is similar to one of the previous model (Eq. 3), saving one data-dependent constant. However, that proposal did not clearly spell out a cognitive rationale. More importantly, that model was tested only in the limited cases of pointing without varying the movement directions. In the subsequent work of Grossman and Balakrishnan [16], the same model was not tested under the full set of conditions either. The experiment reported below will test the model under variations in all DOFs of 2D pointing.

Augmented Regular Configuration Model

Since previous work have already demonstrated the effect of movement angles [16], the angular aspect of the actual configuration should affect performance. The interesting question is how much the impact would be. In our study, we simply added $c\log_2\theta$, the third term of the angle-Fitts additivity model (8), to the regular configuration model (13) to obtain the augmented regular configuration model. The rationale is that the decision of movement direction is independent from the movement-terminating decision.

EXPERIMENT: 2D POINTING

A 2D pointing experiment was conducted to test the fit of the above models.

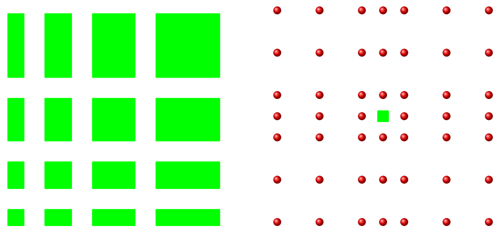
Subjects and Apparatus

Fifteen university students participated in the experiment in exchange for $25 each in compensation. There were six females and nine males. All participants were right-handed except for one female and one male. Preferred hand was used to do the experiment. All participants had normal or corrected to normal visual acuity, and were seated comfortably in front of the computer screen.

The experiment was controlled by a program running on a Thinkpad T61 laptop computer. The screen is 14 inch LCD, set in 1024x768 pixels resolution. A Microsoft USB mouse was used as the main input device in the experiment.

Design

The participants' task was to move the cursor between two identical rectangles drawn on the screen. Participants were asked to click on one rectangle, then the other, and alternate as quickly and accurately as possible. For each experimental condition, the timings for eleven consecutive mouse clicks were recorded. The first two were considered as practice data and excluded from the data set. Clicks outside the correct rectangle were considered as errors and discarded. The average time duration between two correct mouse clicks was taken as the dependent variable.



(a) Target relative sizes and shapes (b) Starting positions relative to the target

Figure 5. Variations of experimental conditions

The experiment was a repeated within-subject design. Each participant did all the conditions, which appeared in dynamically generated random orders different for each participant. There were four primary independent variables: the width W and the hight H of the rectangles, the horizontal distance D_w and the vertical distance D_h between the two rectangles. Variable W and H had four levels of value: 30, 50, 80 and 120 pixels. D_w and D_h also had four levels: 0, 80, 240, 400 pixels. Fig. 5 shows the variations of the task conditions for the experiment.

In addition, the two rectangles were arranged such that four types of movement can be tested: 1) bottom-left to top-right and back, 2) bottom-right to top-left and back, 3) horizontal left to right and back, and 4) vertical bottom to top and back. Combined with four primary independent variables, there were $3 \times 3 \times 4 \times 4 = 144$ conditions for each type of diagonal movements, and there were $3 \times 4 \times 4 = 48$ conditions for horizontal or vertical movements. With horizontal or vertical movements, either $D_h = 0$ or $D_w = 0$, but not both. The total number of conditions for each participant was $144 + 144 + 48 + 48 = 384$, greater than any previous reported experiments. We need to stress here that the great number of conditions is just a side effect of the systematic coverage of all DOFs of the configuration space. The design essentially captured all the possible geometric factor combinations that could be tested in 2D pointing: target size, aspect-ratio, distance to target, approaching angle to target, and any other configurational factors, since they can all be derived from the four independent variables.

Between conditions, the experimental program would pause and wait for the participant to hit the `Space` key to continue, allowing participants to have rest on their own pace. Two mandatory breaks were also programmed in at 1/3 and 2/3 point of the formal experiment. Before the formal experiment, a practice block was included, where four conditions were presented.

Table 1. Summary of model fits

Model	a		b		c		R^2
	Est.	*SE*	*Est.*	*SE*	*Est.*	*SE*	
Width-height independence (Eq. 7)	360.2	8.1	66.1	2.7	68.7	2.7	.753
Angle-Fitts additivity (Eq. 8)	207.8	10.5	107.2	4.9	85.7	9.5	.855
Convex-hull (Eq. 9)	314.2	6.0	143.9	2.7	-	-	.883
Regular configuration (Eq. 13)	191.7	5.8	135.4	1.8	-	-	.937
Augmented regular configuration	157.8	5.6	174.0	3.5	41.8	3.4	.955
MacKenzie and Buxton 1992 (Eq. 2)	227.0	6.0	133.0	2.0	-	-	.924
Accot and Zhai 2003 (Eq. 3)	233.3	5.9	136.5	2.0	-	-	.923
Appert et al. 2008 (Eq. 4)	133.8	7.7	125.6	1.9	-	-	.916

Results

For all participants, we recorded 2487 errors out of 57600 trials. That is a 4.3% error rate, consistent with a normal Fitts experiment. Only correct trials were used in the data analysis. No attempts were made to remove outliers.

Model Fit

Linear least square regression methods were performed to test the fit of the five models proposed above and the three previous models. Results are summarized in Table 1. The first column contains the model names, the last column is the percentage of variance explained by the models, R^2. The columns in between are the estimates of model coefficients a, b, and c (if applicable), and their standard errors. All models and model coefficients are statistically significant with p<.001. We now go over each of the models.

The poor fit of width-height independence model is not surprising, because the model is equivalent to modeling a task where users first move horizontally, then vertically towards the target, or vice versa, but nobody moves in such a city-block way in 2D pointing.

Angle-Fitts additivity model, though conceptually attractive, does not seem to be an accurate model. We speculate that the movement paths are curved, so that a model of following-straight-lines would not work.

It is slightly surprising that the naive convex hull model works better than the aforementioned ones. A model based purely on the ground of the space of all possible terminating positions (a configuration space approach), the assumption of which may be grossly simplistic, could still fare quite well in predicting the performance. This result indicates the value of adopting a configuration space way of thinking.

The augmented regular configuration model fits the data the best. However, its advantage over the regular configuration model is not substantial, especially on account of the one more free coefficient it requires. To measure how much the angle parameter contributed, we used θ alone to fit the data, a significant amount of variance can be explained, R^2=.65, a=387.5, b=111.6, and p<.001. This means that the angle model shares a large amount of contribution with the regular configuration model, but its unique contribution over regular configuration model is minimum.

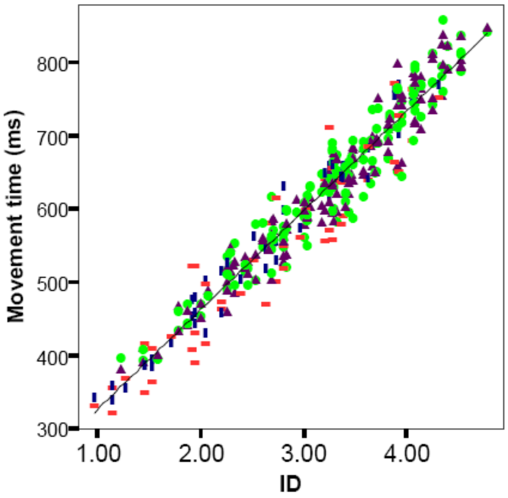


Figure 6. Fit of regular configuration model. Vertical bar marks indicate vertical movements, horizontal bar marks are horizontal movements, triangle and circle marks are the two diagonal movements.

All three previous models tested performed not as well as the regular configuration model. Among them, MacKenzie and Buxton's model (Eq. 2), though the most simple, did surprisingly well. The other two models performed not as good as they were in their original data set. Since the only difference in form between the regular configuration model and the Accot and Zhai's model is that η constant, the extra constant actually hurts the performance of the model. It is likely that the constants introduced are artifacts of overfitting due to the limited set of conditions tested in those experiments.

The superiority of regular configuration model is obvious, in light of its simplicity: it has only two free coefficients instead of three. It does not introduce any constant either. To give an intuitive feel of the model fit, Fig. 6 shows a scatter plot of the data fitting to the regular configuration model. Each data point in the plot represents the average performance of one experimental condition. Data points belonging to different movement directions are represented with different marks. The fit is generally good for all four movement directions. However, Table 2 suggests that there are still slight differences on the level of fit for different move-

Table 2. Regular configuration model fit by movement directions

Movement	a		b		R^2
	Est.	*SE*	*Est.*	*SE*	
Left-bottom ↔ right-top	188.6	9.6	136.7	2.8	.942
Left-top ↔ right-bottom	201.0	10.8	133.1	3.2	.924
Horizontal left ↔ right	198.2	18.7	127.8	6.6	.887
Vertical top ↔ down	187.6	10.4	139.1	3.7	.968

ment directions. Horizontal movements seem to have relatively the worst fit. In any case, considering how different these configurations are, the remarkable fit of the regular configuration model strongly suggests a bias towards regular configuration.

CONCLUSION AND DISCUSSIONS

From the results presented above, we conclude that the hypothesis is supported. The bias towards regular configuration could largely explain 2D pointing performance.

Fitts' law as Reflection of Central Planning

Central Planning and Peripheral Feedback

Since Fitts' formulation of the logarithmic form of the movement time predicting formula, a few attempts have been made to explain or rationalize the form, often as some kind of optimization. One account attributes Fitts' law to a stimulus-response feedback loop. It explains the logarithmic form by an iterative-correction model, where subjects zero-in to the target by making successively smaller corrections [9, 20]. For a time, this explanation has also been an accepted view in HCI [5]. However, the accuracy of the model was seriously challenged over the years due to its incongruence with empirical data [30, 37]. Some examples of the problems include: a) the model's assumption of a constantly proportional decrease of sub-movement distances has not been observed [19, 23]; b) successive motor acts sometimes follow each other too rapidly to be entirely controlled by feedback loops; c) Fitts' law still holds when visual feedback is deprived [33, 44]; d) erroneous acts often reveal that behaviors are planned ahead [24]; and so on. Some contemporary motor behavior models, e.g. posture-based model [34, 36], rely on the ideas that behaviors are planned according to known goal states. Multitude of evidence has been gathered to support such a view [35]. For example, to generate smooth movement [15], a goal posture must be known in advance; uncomfortable hand positions may be adopted initially in anticipation of comfortable goal positions [46]; and so on. The critical role of central planning in motor behavior seems to be undeniable. However, the role of peripheral feedback could not be easily swept away. For example, the behavior modifying role of visual feedback have been clearly demonstrated by experiments that change the target dynamically *during* the movement [14, 29, 45].

Interpreting Fitts' Law

Given the research literature, it seems easy enough to take a hybrid view on this central planning (open-loop) vs peripheral feedback (close-loop) dichotomy: we acknowledge that both play a role. The more interesting question is how much weight we should place on each. The results of this work seem to tilt towards central planning as the more influential factor in determining pointing performance. The primacy of the model of goals has long been recognized in many theories of motor behaviors. For example, the notion of "desired future" in Bernstein's theory [25] and the concept of "reference signal" in perceptual control theory [32] are all models of goals. Our work is consistent with such understanding, and further propose a possible mechanism for the goal setting process. In this sense, we view Fitts' original information theoretical argument as a description of such a process. The decision of planned movement time is made prior to the movement, so as to set the parameters for the first open-loop stage of the movement. This stage determines the main component of the overall movement time, and is exactly what Fitts' law accounts for. The subsequent close-loop feedback stage plays the role of minor adjustment. The above interpretation of Fitts' law is obviously not Fitts' intention. And this discrepancy in understanding may partly explained why Fitts himself abandoned the information theoretical view later on: like his peers at that time, he took information theory literally as a theory of noisy channel communication, therefore the whole movement process is considered as a single communication channel. Obviously such a simplistic view would run into conflict with the more complex reality of possibly two stage process. Our results could only be properly explained when we regard Fitts' law as a computational model of the central planning process.

Principle of Maximum Regularity

Reviewing the literature, a common thread seems to be the quest to find some forms of optimization as the governing principle. Neural noise reduction [30], movement smoothness [15], and minimum effort have all been proposed as the criteria of optimization. We do not aim to dispute these criteria, but to offer another: the preference of maximum regularity in mental configuration space.

Facing the high dimensionality problem of configuration space, cognitive systems are likely to exploit the structure of the space to reduce the number of dimensions. One form of the exploitation could be a tendency to favor more regular configurations as the representatives of more generic ones. In our results, the regular configurations were nonexistent since they were not shown as part of the experimental conditions, but the participants' responded as if they were shown. The source of such response patterns is likely to be the human bias towards regular configuration. Generalizing this tendency, we may call it the *principle of maximum regularity*.

This principle saw support in perceptual organization domain, our results seem to extend its applicability to the realm of motor behavior. Obviously, more studies are needed to verify our results and to support or counter such generalization. Seeing through this lens, we notice that the model of MacKenzie and Buxton [27] and Accot and Zhai[1] can all be seen as ways to regularize the configuration space of 2D pointing, as they both reduce the number of parameters from four to three. However, an explicit consideration in term of configuration space should lead to more systematic exploration of the space.

Practical Implications

Research Methodology

One implication of this work is methodological. Instead of varying only a few factors in model checking, we advocate including all DOFs of the configuration space of a task. Our results indicate that including only a subset of all DOFs in an experiment is likely to produce overfitted models. Another benefit of including all DOFs is the ability to see the relative contribution of individual factors. Properly designed experiments are often capable of finding statistically significant effects of individual factors if they do exist, no matter how small the effects might be. However, to see the factors' contribution to the overall performance requires studies that includes all the relevant factors. For example, our data show that the movement angle parameter does affect performance, but its contribution is relatively small when all parameters are tested.

The analysis of configuration space would tell us the proper set of factors to include for testing purpose. Including less than the number of DOFs of configuration space would likely lead to model overfitting. Including more than the number of DOFs would be wasteful of resources since some of the factors would be redundant, and can be derived from the defining set of DOFs. For HCI, this issue of choosing the proper set of factors to study is more important due to the practical nature of the discipline. It is nice to study causal relationships among a few factors as a scientific endeavor, it would be insufficient for practical purpose, since all the factors are present in the real world.

On the other hand, configuration space viewpoint urges us to move away from the debate of which way of parameterizations is more "correct" than others. If two set of parameters have the same number of DOFs and are characterizing the same configuration space, then they are equivalent. In this view, to understand a behavior is to understand the structure of the configuration space. The coordinate system we choose to impose on the space doe make the job of finding a structure easier or harder, depending on the kind of structure we are looking for. It is desirable to maintain a conceptual flexibility on parameterizations, and to choose the set of parameters that is the easiest to test for the structures we are hypothesizing about.

Interface Evaluation and Task Analysis

The empirical finding of this study, if replicated by future studies, can greatly reduce the number of conditions needed for testing 2D pointing. Since the regular configuration dominants the performance, and is capable of explaining the majority of performance variance, we may only need to test the regular configuration, at least for some engineering purposes of helping design and evaluation of user interfaces. Generalizing the idea, if we can find the dominating regular configurations for other tasks, a standard test suite of regular configurations might be developed and used by practitioners. The potential benefits could be enormous.

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