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Approaches to 3D Free-Form Cutting and Packing Problems and Their Applications: a Survey

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Approaches to 3D Free-Form Cutting and Packing Problems and Their Applications: a Survey

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Abstract

This survey paper describes approaches to and applications of 3D free-form cutting and packing problems. Some initial work on these problems has been done in the last few years. The approaches used, which include simulated annealing, the genetic algorithm, and heuristics with simulated annealing, are described, and applications of the problems are introduced. They include 3D product layout and rapid prototyping, which is a new technology for quickly creating 3D products with computer-aided design systems. Finally, practical aspects of the problems involved in rapid prototyping are discussed.

1. INTRODUCTION

1.1. Term Definition

In this paper, 3D C&P problems are defined as three-dimensional cutting and packing problems. They can be characterized as follows:

(a) There are two groups of basic data whose elements define geometric shapes in a three-dimensional space: a set of containers and a set of items.

(b) The cutting or packing process realizes patterns that are geometric combinations of items assigned to containers. All or some of the items are cut out of or packed into the containers to optimize an objective function.

3D C&P problems include 3D bin packing, container loading, pallet loading (3D palletization), and 3D nesting problems. The 3D free-form C&P problem is a variation of 3D C&P problems in which the containers and items are not restricted to rectangular parallelepipeds and can have any complex shape.

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1.2. Research on the 3D C&P Problem

The earliest work on 3D C&P problems was done around 1980, when George and Robinson [1] first worked on the problem of determining how a set of boxes could be fitted into a container. Early in 1978, Tinarelli and Addonizio [2] had discussed the related problem of determining the minimum number of containers into which a set of boxes could be packed [1]. Before that, in 1970, De Sha [3], in an unpublished master's thesis, had developed a heuristic for loading containers [4].

Since the early 1980s, many researchers have worked on 3D C&P problems. There are too many papers on 3D C&P problems to list here, but some of them are listed in the references of recent papers [5][6][7][8][9]. This paper covers only work done since 1995. Other recent papers can be found in the SICUP(Special Interest Group on Cutting and Packing) Library [10].

This paper focuses on the various approaches to the problem. The most common approach to the 3D C&P problem is to use heuristics [5][6][11]. Another popular approach is to use the genetic algorithm [9]. Only a few researchers have used analytical approaches [12] or integer linear programming [8]. This paper also focuses on applications of 3D C&P problems.

1.3. Framework of This Paper

None of the studies of 3D C&P problems mentioned in this section deal with free-form versions. No work was done on free-form versions of these problems until recently. These recent studies are mentioned in section 2, where they are classified according to their approaches. Section 3 discusses applications of 3D free-form C&P, focusing on rapid prototyping (RP). Practical aspects of the 3D free-form C&P problem in RP are discussed in section 4, and section 5 concludes the paper.

2. THE 3D FREE-FORM C&P PROBLEM

2.1. Approaches Based on Simulated Annealing

In 1995, Szykman and Cagan [13][14] developed the simulated-annealing [15]-based approach to 3D component packing. They applied their method only to blocks and cylinders, and the rotation is limited to multiples of 90 degrees. These limitations, however, are independent of their method. In 1996, Kolli et al. [16] relaxed the limitations on shapes and rotations for problems without spatial constraints between items. In 1998, Cagan et al. [17] presented a general extension to Kolli et al.'s work.

In Cagan et al.'s simulated-annealing-based approach, the annealing moves items within a container. These movements consist of translation, rotation, and swapping the positions of two items. The weighted objective function includes design goals such as maximization of the packing density, minimization of the bracketing of items, and minimization of the number of bends and route lengths of tubes between specific points of items.

In conjunction with the simulated annealing algorithm, a hierarchical decomposition of geometric models is used for efficient and effective interference analysis. Hierarchical decomposition is an approximation of an item at various levels of resolution, based on successive binary division in each coordinate direction. Each division splits a rectangular parallelepiped into eight rectangular parallelepipeds. The first level of a hierarchical decomposition is the minimal rectangular parallelepiped that completely contains the component. Rectangular parallelepipeds are classified into three types — no part of the item lies inside the parallelepiped, the entire parallelepiped lies inside the item, and the item and parallelepiped partially intersect. With this hierarchical decomposition, rough analyses are performed at the beginning of the algorithm and more detailed analyses are performed towards the end.

As a test case, Cagan et al.'s approach was applied to the problem of packing 16 wheels into a cubic container. It took about 820 seconds on the average to solve this problem on a Silicon Graphics R10000 195-MHz Indigo. More complex items were also tested to determine the real power of the approach. One application was the layout of a car engine compartment, which involved packing eleven items into the container or compartment.

2.2. Approaches Based on Pattern-Searching Algorithms

In 1998, Yin and Cagan [18] introduced a pattern search-based algorithm, and compared performance with Cagan et al.'s approach [17] mentioned in section 2.1. Their algorithm generates layouts of the same quality about 14–34 times faster than that of Cagan et al.

The basis of Yin and Cagan's pattern-search [19]-based algorithm is as follows: From an arbitrary initial state of the items, translations are taken as the first set of moves. A series of translations take the items to new states, accepting new states as the current states whenever the objective function is improved, and otherwise keeping the original states. This process is repeated for all the components. If there is no improvement for any of the translations attempted, the length of the translations is scaled down. Rotations are then repeated for all items, with the same rules for the acceptance of new states and updating of the movement size. If none of the translations and rotations in the previous cycle led to a better state, one or more swaps take place between pairs of randomly chosen items, and the same rules are applied for the acceptance of new states. The above cycle of translations, rotations, and swaps is repeated until the movement sizes of both translations and rotations are smaller than predefined thresholds.

In addition, items to be moved are randomly selected, and the direction of the movement is also randomized. Occasional increases of the movement size are allowed, and the hierarchical decomposition of geometric models mentioned in section 2.1 is used.

2.3. Approaches Based on the Genetic Algorithm

Since 1996 Ikonen and Biles [20][21][22][23] have worked on an approach based on the genetic algorithm [24], where the chromosomal representation includes three pieces of information: the order in which items are placed, the orientation of each item, and the "attachment points" of each item, namely, the points at which the next item can be located or "attached." In their method, each item is free to rotate in 45-degree increments around each coordinate axis, and five attachment points are specified. The objective function is to minimize three factors with weight factors. These factors are the volume of the parts of items that are not in the container, the volume of the parts of items that overlap each other, and the sum of the distances of the centers of the boxes bounding the items from the global origin, which is the center of the base of the container.

They showed that good packing is found for seven items, two of which have holes into which other items could be fit. It converged within 1500 generations, where the population size was 300.

2.4. Approaches Based on Heuristics

In 1998 Dickinson and Knopf introduced another approach [25][26] that deals one item at a time, whereas Ikonen's approach deals with all items at the same time. There is no limitation on rotation. Using simulated annealing, items are packed sequentially in ways that are best or that minimize the point moment, which represents the compactness of the free space.

The point moment P is defined by

$$P = \int_R ||\vec{x} - \vec{x_c}||^2 dR$$

where R is the region of 3D space, and $||\vec{x} - \vec{x_c}||^2$is the square of the distance from any point $\vec{x}$ in the region R to the centroid $\vec{x_c}$ of the region R. The point moment provides the smallest value when the region is a sphere, subject to the condition that the volume of the region remain constant.

This approach showed average time improvements over Ikonen et al.'s approach of about a factor of 20. They also tried more complex items, which are typical RP industry packing problems, but it took more than 40 hours to pack 10 items on a computer with a 200-MHz AMD K6 CPU.

2.5. Others

In 1988, Udy et al. [27] worked on a limited 3D free-form C&P problem of packing five convex items into a box. The objective is to pack the items into the bottom of the container, or to minimize the sum of the clearance distances of items from the top wall of the container. Their approach is to iterate a generalized reduced-gradient algorithm, for example, 12 times to obtain the locations of the items, with angles fixed. Their approach was extended by Perry et al. [28] to pack free-form items with rotation [29].

3. APPLICATIONS OF 3D FREE-FORM PACKING

3.1. Rapid Prototyping

3.1.1. What is Rapid Prototyping?

Ikonen et al. [20][21][22][23] and Dickerson et al. [25][26] developed their methods for the SLS (Selective Laser Sintering) machine, which is a rapid prototyping (hereafter abbreviated to RP) technique. Wodziak et al. [30] had also worked on the application of the 3D packing problem to stereo-lithography, which is another RP technique, though their approach is to pack rectangular parallelepipeds as approximations of items.

RP is a new technology for rapidly manufacturing prototypes of products. Traditionally, prototypes have been made by machining, which is a subtractive process. Beginning with a piece of solid stock, a machinist removes material until a desired object is produced. RP is a layer-additive process. It adds material for example resin, paper, plates, or powdered plastic, metal, or ceramic layer by layer until the desired shape is achieved.

Quick product development cycles are essential for companies to compete in today's marketplace. The traditional prototyping method cannot keep pace with current technology, and RP has received more and more attention in recent years [31]. The gross sales of RP machines worldwide began to increase rapidly in 1993 [32]. RP shortens the time to market and makes the cost much lower than that of the traditional method. It was originally used only as a means of visualization, but is now used for many purposes, because of the diversity of forms and materials.

The uses of RP can be classified into four types: concept modeling, communication modeling, function modeling, and production tooling [31]. Concept modeling consists in optimizing a design by creating a model at an early stage. Communication modeling also consists in optimizing a design, since manufactured models improve communication among departments. Both concept and communication modeling are carried out to prevent problems by examining designs at an early stage. Function modeling consists in verifying functions with that models that have exactly the same shapes as the intended real products. It became possible when the materials that can be used to verify functions became available in RP machines. Production tooling consists in manufacturing templates by RP and producing models with the same material as the real products.

3.1.2. 3D Packing in Rapid Prototyping

According to Ikonen and Biles [22][23], in RP machines that use laser beams, the time needed to manufacture one layer with a laser is much less than that needed to lower the platform and to apply material for a new layer. Thus, the more items are fabricated simultaneously in one batch, the less time it takes to manufacture one item. To improve the machine time utilization, as many items as possible should be packed in each batch. Currently, items are manually arranged in the available manufacturing volume by operators.

3D C&P algorithms can be directly applied to RP technologies that use solid materials to build an item, such as SLS, which uses ceramic, plastic, or metal powder and LOM (Laminated Object Manufacturing), which uses paper. Stereo-lithography, however, uses liquid material or resin, and requires supports to be placed under building items to prevent them from sagging or floating away. For an item to be built over another item, supports must be built between them, and this is often unacceptable. Other information on RP can be obtained from a survey paper [33] and the Web pages of Wohlers Associates [34].

3.2. 3D Product Layout

Szykman et al. [13][14] and Udy et al. [27] developed methods for the 3D product layout problem. Examples of 3D product layout include the layout of computers [35], automobile engine compartments [36], mechanical and electromechanical assemblies [14], and aerospace applications [29].

The 3D product layout problem can be divided into two subproblems: the 3D component layout optimization and the 3D routing optimization [37][38]. The objectives of component layout include maximization of the packing density, fitting of components into a container, and satisfaction of spatial constraints on components. The objectives of routing include minimization of the routing length and of the number of bends in the routes. Recently, Szykman and Cagan [39] developed a method for concurrent layout and routing.

4. PRACTICAL 3D FREE-FORM C&P PROBLEM IN RAPID PROTOTYPING

4.1. Type of Problem

3D packing is necessary, especially in RP service bureaus that receive orders from customers and produce prototypes, since they produce many prototypes. Service bureaus have to select items that should be produced by currently unused machines, and select combinations and packing patterns of items. Here, a combination of items means a subset of the items that are produced simultaneously by a certain machine. A packing pattern of items indicates where in the machine each item is manufactured.

Consequently this problem is classified according to Dyckhoff's typology [40] as 3/B/I/M or 3/B/D/M: three-dimensional, a selection of items, many containers of identical or different shapes, and many items of different shapes. On the other hand, all the studies of 3D free-form C&P problems mentioned in section 2 involved packing all of a set of items into a container. Those problems are classified as 3/B/O/F: three-dimensional, a selection of items, one container, and a few items of different shapes.

4.2. Some Constraints

For C&P algorithms to be used in RP service bureaus, they have to satisfy some additional constraints. For example, the delivery period has to be taken into account in selecting which items to pack. In addition, the rotation of items is sometimes constrained. RP is a layer-additive process, which adds material until a desired shape is achieved. A layer-additive process creates small but sharp drops between layers, so faces that must be completely flat have to be placed horizontally or perpendicularly to the layers. These constraints create problems peculiar to RP.

4.3. Similar Problems

To sum up, the practical 3D free-form C&P problem in RP is to select items of different shapes and pack them into containers of identical or different shapes with special constraints. Similar problems can be seen in one- and two-dimensional C&P problems. One example is cutting parts out of steel plates [41]. This is a problem in the ship-building industry, where many parts of complex shapes are produced from rectangle material plates. The idea proposed is to reduce the 2D bin packing problem to a special case of a 1D bin packing problem by approximating plate parts by scan-lines that is, by collections of scalar values. Another example is the problem of cutting steel plates in the steel industry, where small rectangular plates are cut out of large rectangle plates with guillotine cuts. Generally, for optimization approaches to have practical applications in industry, they need to satisfy many constraints, which usually make it difficult to realize practical applications.

5. Conclusions

In the last few years, some approaches to general 3D free-form C&P problems have been proposed by Cagan et al., Ikonen et al, and Dickinson et al. All of them tried packing at most tens of items into a container. Their problems can be classified according to Dyckhoff's typology [40] as 3/B/O/F: three-dimensional, a selection of items, one container, and a few items of different shapes.

3D product layout and RP have been proposed as applications of the 3D free-form C&P problem. In 3D product layout, the routing of wires or pipes, thermal issues, the center of gravity, and so on often need to be taken into account as well as the density of the packing. In RP, the approach to the 3D free-form C&P problem can be directly applied if solid materials are used to build items. However, stereo-lithography, which is the most common RP technology, uses liquid resin and requires supports to be placed under the building items. These are often unacceptable when items are nested three-dimensionally.

The practical C&P problem in RP can be classified as 3/B/I/M or 3/B/D/M: three-dimensional, a selection of items, many containers of identical or different shapes, and many items of different shapes. In addition, some special constraints have to be satisfied. So far, however, no work has been done on ways of dealing with large numbers of items or multiple containers, much less on special constraints.

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