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Wavelet Analysis: Applications in Industry

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Abstract

In this paper, we describe how wavelet transforms can be used for analysis of data, signals, and mathematical models and how this analysis has been employed in system prototypes, commercial products and industrial design and manufacturing processes.
key words: multiresolution analysis, time-frequency analysis, wavelet transforms, wavelets

1 Introduction

During the past decade, wavelets have evolved from a trendy field of study to a well established minor branch of mathematical analysis. Particularly in its early stages of development, the importance of the field was overstated by scientific magazines as well as overzealous scientists. However, one of the claims which has stood on solid ground and has led to new technologies is:

> *"... the wavelet transform is a tool that cuts up data or functions or operators into different frequency components, and then studies each component with a resolution matched to its scale"* – Daubechies [21].

A second wavelet-related tool which has proved to be useful is multiresolution analysis (MRA). Although many of the concepts and techniques behind MRA existed long before wavelets became fashionable (see, e.g., [9]), in the past decade, MRA has come to receive attention it richly deserves. A third area in which wavelets have made an impact is in the study of differential equations, more specifically, mathematical analysis, operator theory, and mesh generation. In this paper, we describe how wavelet transforms can be used for analysis of data, signals and mathematical models and how this analysis has been employed – sometimes, gainfully – in system prototypes as well as in commercial products and industrial design and manufacturing processes.

One quantitative means of measuring the usefulness or potential usefulness of wavelet-based technologies (or the perception thereof) to businesses is to count the number of related patents. We ran a search engine on the United States Boolean Search Page [1] using the single keyword "*wavelet*" starting in 1991. Detailed results of our search are given in the Appendix. The annual patent counts are given in Table 1 and plotted Figure 1. After enjoying almost exponential

[1] U.S. Boolean Search Page: http://patents.cnidr.org/access/search-bool.html

growth, it appears that the number of wavelet-related patents issued in 1999 will be about 83, i.e., slightly less than in 1998. Our projections were computed by: counting the number of patents filed in the first quarter of 1999 (i.e., 21 patents as of March 31, 1999); computing the number of patents issued per day; and multiplying the number of patents per day by 365.

Interestingly, the *Intellectual Property Network (IPN)*, the IBM patent search engine [2], which claims to have a database of patents dating back to 1971, retrieved fewer total documents (162 patents) for the wider time span 1971–1999, when we input the same, single word query "*wavelet*". The oldest wavelet-related patent we found using this server dates back to 1971. Titled, *"Wavelet Standardization"*, it was filed by Atlantic Richfield Company [30] with the following abstract:

> *"Processes for improving seismic data whereby a plurality of seismograms is standardized to have the same generating wavelet. A reference seismogram is chosen and its autocorrelation is determined. The crosscorrelation between a first seismogram and the reference seismogram is determined. An operator is computed which transforms the crosscorrelation into the autocorrelation. The first seismogram and the operator are convolved to give the first seismogram the same wavelet as the reference seismogram. Additional seismograms can be standardized by this procedure using the same or a related reference seismogram."*

Table 1: Wavelet-Related Patents Issued

year	number
1991	3
1992	3
1993	4
1994	8
1995	17
1996	34
1997	42
1998	92
1999	(82.4)

Various interpretations of the patent counts are possible. The most straightforward is to note that patent filing and upkeep is costly and conclude that the counts must accurately reflect the impact a technology has on businesses. A more skeptical view is that the numbers alone may mean very little depnding on the context. For example, a skeptic might point out that the patents were filed at the time when zealous, overselling of wavelets was taking place [107]; there is a delay of a few years between the times when a patent is filed and issued. Many of the patents – especially those in image and video signal compression – may have been filed as part of a gambling scheme just in case the technology took off, similar to the current boom of Internet-related stocks. Ultimately, time will come to reveal the usefulness of the patents and their inherent as well as licensing value to the filers.

[2] IBM Intellectual Property Network: http://www.patents.ibm.com/ibm.html

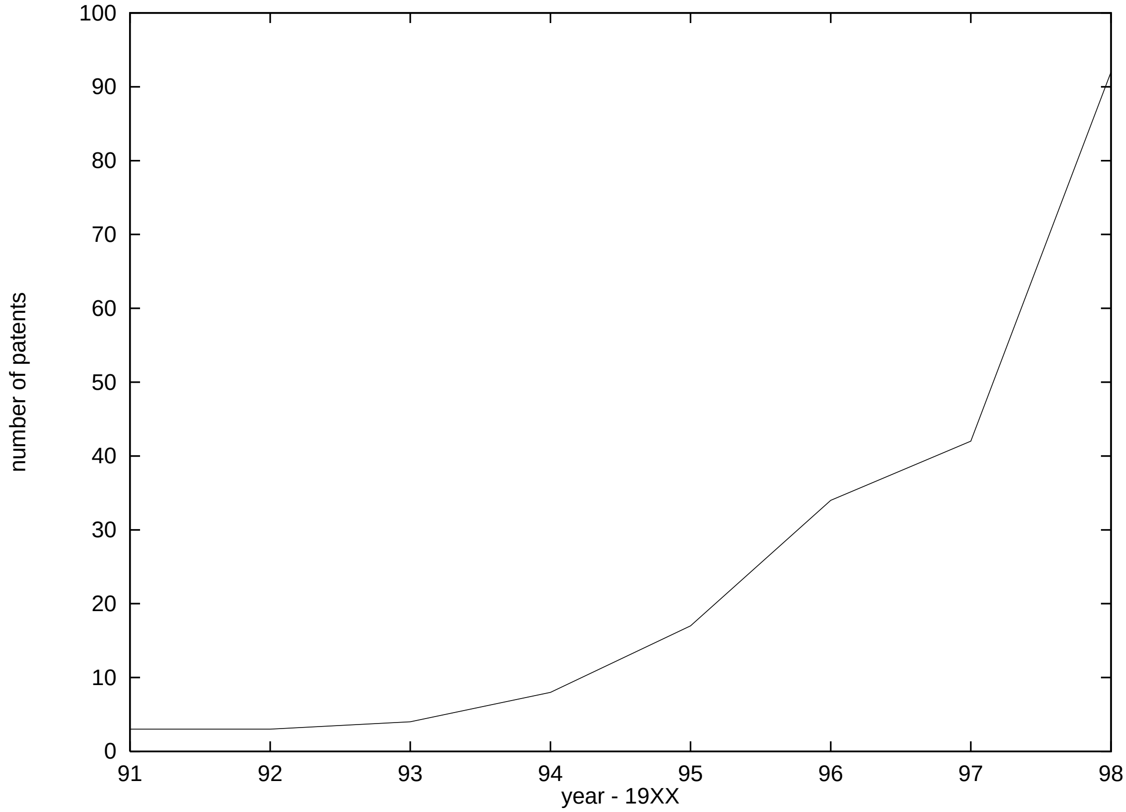

Figure 1: U.S. patents issued with keyword “wavelet”

The remainder of this paper is organized as follows. We will begin with a mathematical definition of wavelets, then examine some concrete examples and their properties. Next, wavelet transforms and time-frequency analysis techniques will be described. The main portion of the paper consists of case studies from research and development labs in which wavelet-based techniques were used to solve a problem. Since research on the subject has become so prevalent, a comprehensive survey on industrial applications would be impossible, and, in any case, is beyond the intended scope of this paper. We present some interesting examples in the hope that they will give prospective wavelet researchers some insight into new approaches to analyzing data and provide some inspiration.

Readers interested in introductory material on wavelets can consult a number of review articles [37], [61], [92], [98], [105]; special journal issues [26], [46], [47], [53], [77]; and texts [5], [13], [15], [72], [95], [106]. More advanced material and reference books include [21], [78], [111], [113]. Some useful applications of wavelet techniques are described in collections of works, e.g., [3], [6], [14], [18], [31], [60], [104] as well as proceedings of conferences and special sessions of the American Mathematical Society (AMS) [3], the Society for Industrial and Applied Mathematics (SIAM) [4], the Institute of Electrical and Electronics Engineers (IEEE) ICIP and ICASSP [5], and the International Society for Optical (SPIE) [6]. A detailed tutorial on time-frequency analysis is [87], and more advanced materials are [40], [46].

2 Wavelets: definition and examples

"Wavelets are families of functions $h_{a,b}$:

$$h_{a,b} = |a|^{-1/2} h\left(\frac{x-b}{a}\right) \quad ; \quad a, b \in \mathcal{R} \ , \ a \neq 0$$

generated from a single function h by dilations and translations" [19]. One of the applications of the theory is to construct a basis set $\{h_{a,b}\}$ for efficient and accurate approximation of functions, operators, and signals [78]. A second class of applications involves the use of wavelet transforms for analysis of non-stationary signals [15], [87], [92]. For wavelets with mother function h, the *continuous wavelet transform* for a function $f \in L^2(\mathcal{R})$ is

$$\langle h_{a,b}, f \rangle = |a|^{-1/2} \int dx \cdot h\left(\frac{x-b}{a}\right) \cdot f(x)$$

for $a, b \in \mathcal{R}$, $a \neq 0$, and the *discrete wavelet transform* is

$$\langle h_{m,n}, f \rangle = |a_0|^{-m/2} \int dx \cdot h(a_0^{-m} x - n b_0) \cdot f(x)$$

for $a_0 > 1$, $b_0 \neq 0$ [20].

Many signal processing applications use convolutions with filter bank coefficients rather than the wavelet function itself [38], [92], [106], [111], [112], [113]. A *filter* is a linear, time-invariant

[3] http://www.ams.org
[4] http://www.siam.org
[5] http://www.ieee.org
[6] http://www.spie.org

operator which acts on an input vector x to produce a vector y by convolution of x with a fixed vector h, i.e.,

$$y(n) = \sum_k h(k) \cdot x(n-k) ,$$

where $x(n)$ and $y(n)$ are the values at times $t = nT$, $h(n)$ are the *filter bank coefficients*, T is the *sampling period*, and n an integer.

There are many types of wavelets and wavelet-based techniques. The optimal choice of wavelet type and method depends on the specifics of the application. In this section we discuss the properties of eight types of wavelets with compact support: Haar, orthonormal wavelets with compact support, three different splines, Gabor, chirp and the Mexican hat.

There are several different ways of describing wavelets. One way is to specify the generating function by a formula and fix the translation and dilation parameters a and b. We use this approach to define the most elementary example, the Haar basis. Set the mother function to be

$$h(x) = \begin{cases} 1 , & 0 \leq x < \frac{1}{2}, \\ -1 , & \frac{1}{2} \leq x < 1, \\ 0 , & \text{otherwise}, \end{cases}$$

and $a_0 = 2$, $b_0 = 1$. The Haar wavelet expansion is a simple and natural choice for representing data sampled at regular intervals, because it amounts to approximation using step functions. The orthonormality of the Haar wavelets in $L^2(R)$ and other properties and their consequences are discussed in [19], [104].

A second method of describing wavelets is by an algorithm for generating the mother function to an arbitrary degree of accuracy. This method is used to define orthonormal wavelets of compact support [19]. The algorithm uses iteration to calculate a scaling function ϕ_N such that

$$\phi_0 = \chi_{[-1/2,1/2[} ,$$

$$\phi_N(x) = \sqrt{2} \sum_n h(n) \; \phi_{N-1}(2x-n) .$$

Here $\chi_{[a,b[}$ denotes the characteristic function on the interval $[a, b[$ and $h(n)$ the filter coefficients of the scaling function. Errors in the table of filter coefficients in [19], were discovered by Hatano, and revised tables (correct to 50 digits) are available in electronically [36]. The wavelet function ψ_N is defined in terms of the scaling functions ϕ_N as

$$\psi_N(x) = \sum_{n=0}^{2N-1} (-1)^n \;\; h_N(-n+1) \;\; \phi_N(2n-x) .$$

Although these wavelets are mathematically elegant, they are ill-suited for time-frequency analysis of speech signals, because their fractal-like nature does not facilitate calculation at half-steps between octaves.

There are many different types of splines, and they are usually generated by iteration or from explicit formulae. Good introductions to the topic are given in [13], [86], [100]. Although iterative methods are computationally faster and more efficient, most algorithms are suited for calculations using 2^n evenly spaced sampling points, where n is an integer. Even spacing of points only allows for octave-to-octave calculations in speech processing so that mathematical formulae or a fast and accurate interpolation scheme are required to determine wavelet transforms for half-steps between octaves. Use of mathematical formulae is the simpler approach.

A simple, second degree polynomial spline with continuous derivatives is

$$S(x) = \begin{cases} 2(x+1)^2 \ , & -1 \le x < -\frac{1}{2}, \\ -6x^2 - 4x \ , & -\frac{1}{2} \le x < 0, \\ 6x^2 - 4x \ , & 0 \le x < \frac{1}{2}, \\ -2(x-1)^2 \ , & \frac{1}{2} \le x < 1, \\ 0 \ , & \text{otherwise} \ . \end{cases}$$

Its appearance is similar to one defined by filter coefficients in [73], [74].

Slightly more sophisticated quadratic and cubic cardinal splines are constructed by first generating the scaling functions from the iterative formulae

$$\begin{aligned} N_1(x) &= \chi_{[0,1[} \\ N_m(x) &= \frac{x}{m-1} \ N_{m-1}(x) \\ &+ \frac{m-x}{m-1} \ N_{m-1}(x-1) \ , \end{aligned}$$

where $\chi_{[0,1[}$ denotes the characteristic function on the unit interval. The splines themselves are given by

$$\begin{aligned} S_m(x) &= \sum_n q_n \ N_m(2x-n) \ ; \\ q_n &= \frac{(-1)^n}{2^{m-1}} \sum_{l=0}^{m} \binom{m}{l} N_{2m}(n+1-l) \ , \\ & \qquad n = 0, 1, \ldots, 3m-2 \ , \end{aligned}$$

where m denotes the order. An overview of polynomial spline approximation techniques and wavelet transforms in a signal processing context is given in [110].

As the order of cardinal B-splines increase, they approach Gabor wavelets as a limit function. For practical applications, Gabor wavelets are not generated by iterative methods from cardinal splines. They are computed from the formula

$$g_k(t) = \exp(-\alpha^2 t^2) \cdot \exp(j 2\pi f_k t) \ .$$

Gabor wavelets (or functions) have been used in signal processing for a long time, even before the birth of the term "wavelet", because

> "*They have the best possible simultaneous concentration in time and in frequency, the set of their finite linear combinations is closed under Fourier transforms, pointwise multiplication and convolution. The scalar product of any two members of this set is given by an explicit formula. And they are among the very few classes of functions where the transition from one to more dimensions is immediate ...*"

[34]. Unfortunately, some bad properties can off-set their use. For example, they do not satisfy the admissibility and progressivity conditions. For wavelets with sufficient decay, the admissibility condition implies

$$\int dx\ h(x) = 0$$

[19]. A wavelet is progressive if it satisfies the admissibility condition and does not have Fourier components on the negative frequency axis. To overcome these problems, the so-called "tails" of the Gaussian are chopped off, and linear combinations of these hybrid-like Gaussians are used in practice. Further details and properties are discussed in [34], [48], [49].

Gabor wavelets belong to a broader class of wavelets known as chirps, which are given by the formula

$$c_k(t) = \exp(-\alpha^2 t^2) \cdot \exp\left(j2\pi f_k t + \frac{1}{2} r t^2\right)$$

[34], [48], [49]. Chirp wavelets contain the frequency shift term $\frac{1}{2}rt^2$, which can be set to zero, by setting $r = 0$, to obtain the Gabor wavelet. A slight shift in the period of the oscillations occur for small $r > 0$. A detailed discussion of chirps is given in [75], [76].

The last wavelet we consider is the Mexican hat

$$M(t) = \frac{2}{\sqrt{3}} \pi^{-1/4} (1 - x^2)\ \exp\left(\frac{-t^2}{2}\right) \quad ,$$

the second derivative of the Gaussian function $\exp(-t^2/2)$ [21].

3 Wavelet-Based Time-Frequency Analysis

Signals can be described in many different ways, however, good descriptions allow facile extraction of meaningful information for a particular purpose. Two fundamental variables in signal analysis are time and frequency. In many situations, these variables are analyzed independently, despite their known interdependency. Joint time-frequency analysis has been studied extensively and has been used as a standard tool for speech signal analysis [90]. Recently, its value is receiving increased recognition by scientists in other fields, such as medical data analysis, manufacturing and machine monitoring, signature analysis, and financial analysis [87].

Traditionally, time-frequency analysis has been carried out using Fourier transforms. Fourier spectral analysis of acoustical signals is, for the most part, serviceable, however, some phenomena, have been elusive, such as small, nonperiodic signals that do not show up after computation of the short-term Fourier transform (STFT). During the 1980's, simple experiments indicated that wavelet analysis – i.e., time-frequency analysis of the wavelet transform of data – could be very useful for the detection of abrupt changes in non-stationary signals [64]. Unlike STFT analysis, wavelet analysis allows arbitrarily good frequency resolution for coarse time resolutions and arbitrarily good time resolution at high frequencies. This means, for example, that when a signal consists of two short bursts, the bursts can be separated during the analysis if sufficiently high frequencies are used [87], [92].

Fourier-based time-frequency data of speech signals have been represented in graphs known as scalograms and phase-shift displays. These displays are three-dimensional representations of speech signal spectra, with time represented on the x-axis and frequency on the y-axis [84], [87], [90]. Color or gray scale maps are used to represent the third dimension, either amplitude (for scalograms) or phase (for phase-shift diagrams), for the transform data.

Recently, analogous time-frequency displays of wavelet transform data are being used by industrial scientists for signal analysis. Success in using wavelet transform methods depends on the specifics of the application, the choice of the family and the associated computational algorithms.

For example, wavelet analysis of acoustical signals has been conducted by many scientists, including [28], [35], [48], [49], [55], [85], [89], [101]. To construct scalogram and phase-shift-like displays, wavelet transforms of these signals must be calculated for several octaves at half-step intervals, where one octave is comprised of twelve half-steps. Straightforward computation of the wavelet transform for applications can often become expensive and perhaps even impossible, depending on the application and data size. The *Algorithme à Trous* by Holschneider et al. [44] and its variations [21], [72], [102] can speed up processing and may be used on many types of signals.

Wavelet transform-based time-frequency analysis of acoustical signals is attractive since it is well localized in time and frequency domains and can be used for constant-Q analysis of a signal in the time-scale plane [87], [92], [98]. For example, experiments which indicate that wavelets show promise in speech event and word boundary detection have been carried out by Dorize and Gram-Hansen [23], Liénard and d'Alessandro [66], and Tan et al. [108]. In particular, the work by Tan et al. [108] shows how to identify four categories of speech: voiced speech, plosives, fricatives and silence. Their goal was to develop technology for improved hearing aid devices. In a separate work, a patent has been filed for hearing aid devices using wavelet technology [103] based, in part, on work by Drake et al. on loudness compensation using wavelets [24].

A variety of pitch marking techniques have been developed over the years, and on-going research contributes to improvements in the field [39]. We modified and enhanced a wavelet-based pitch marking method reported by Kadambe and Boudreaux-Bartels [54] and developed a computer program to detect glottal closure instants, which serve as reference points, for accurate pitch marking. Our approach is a modification of the method outlined in [54] which is based on the abrupt change in a speech waveform at glottal closure instants. We detect the glottal closure instant by searching for a local peak in the wavelet transform of the speech waveform and use it to estimate the pitch period. We use an adaptive threshold to stably and accurately extract the glottal closure using dyadic wavelets and determine the pitch period using the glottal closure instants. Block by block processing delays, which occur during implementation of the algorithm given in [54], are eliminated in our method. And our wavelet transform algorithm can be used for both male and female voices with no parameter changes.

We used our new method to mark pitches in the speech synthesis units for ProTalker 1.0, a Japanese text-to-speech (TTS) system for personal computers [97]. The TTS system relies on the accurate placement of the marks for a overlap-add process to generate a speech waveform, which will lead to natural sounding synthesized speech. The overlap-add technique we developed is an extension of that by Hamon, Moulines and Charpentier [35]. The output from our pitch marking program was double checked when phoneme segmentation was performed manually. In contrast to conventional methods, which use transform extrema for pitch marking, our wavelet method uses the maxima of the transformed signal. Our method has a 97% success rate for identification of glottal closure instants [82], in contrast to the 95% success rate of the method of Moulines et al. [80]. Further details on wavelet transform-based pitchmarking and text-to-speech systems are given in [62], [63]. Other notable wavelet-based pitchmarking techniques have recently been developed by Kawahara and Cheveigine [55], and Yip, Leung and Wong [120].

Wavelet-based time-frequency analysis is being used by scientists in a variety of disciplines other than speech to aid in manufacturing and signal analysis. For example, Sakakibara straightforwardly applied Mallat's decomposition algorithm from multiresolution analysis [71] to clean data from laboratory experiments commonly used by mechanical engineers: dry friction analysis and drop mass tests. For dry friction analysis, he found that when "*data is processed by (his) wavelet-based technique, (his) reults are much better than those computed using finite difference and Butterworth filtering methods*" [96].

A more sophisticated wavelet-based noise reduction method was developed by Sasaki of Kajima Corporation (one of the leading construction companies in Japan) and Yamada (who was at the Disaster Prevention Research Institute (DPRI) of Kyoto Univeristy) to clean noisy seismic acceleration data [119]; Japanese architectural firms commit extensive funds for research on seismic effects on buildings and structures. The scientists constructed new biorthogonal wavelets specially adapted for a class of integral operators. Seismic data correction is carried out in the time-frequency domain using the biorthogonal wavelets. This wavelet-based technique succesfully identified and separated what were inextricably intertwined signal components and enabled correction of noisy data; the signals cannot be cleaned easily using conventional methods. Wavelet-based techniques for cleaning noisy data have been investigated using various approaches by several other scientists, including Coifman and Donoho [16], Donoho and Johnston [22], Hilton et al. [42], Malfait [70] and Saito [94].

Some more unusual scenarios of wavelet-based time-frequency analysis include: identification of irregularities in cement mixes by Aizawa et al. at Chichibu Onoda, a leading Japanese cement company [2]; identification of irregularities in cooling system valves, the development of a wavelet-based time-frequency analyzer with a user-friendly Japanese interface, and the development of a system to compress *keisoku* (control and sensor) data in a manner which preserves features which are essential for automatic, pattern matching-based identification of irregularies in data by Kazato et al. at Yamatake-Honeywell [56], [57]; identification of problems associated with assesmbly line conveyer belts (roller slippage and object drift) by Kitagawa and his colleagues at the Toyohashi University of Technology and a Japanese manufacturer [45], [59]; identification and diagnosis of automobile engine defects by Kikuchi and Nakashizuka of

Niigata University and a leading Japanese automobile manufacturer [58]; and the development of golf balls which emit a more professional and "sporty" sound when they are struck and driven off a tee at angles that have been targeted by pros by Zhang et al. at the Industrial Technology Center of Okayama Prefecture and a Japanese golf equipment manufacturer [121], [122].

4 Wavelet-Based Multiresolution Analysis (MRA)

Wavelet analysis also offers the possibility of robust multiresolution representation and stable, efficient associated numerical computations features not found in Fourier methods. Daubechies and Mallat formalized the concept of a *multiresolution analysis (MRA)* as follows [19], [71]:

Definition: A closed subspace families $\{V_j : j \in \mathrm{Z}\}$ of $L^2(\mathrm{R})$ are said to constitute a *multiresolution analysis (MRA)* when the following five conditions are satisfied:

(i) $\ldots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset \ldots \ ,$

(ii) $\overline{\bigcup_{j\in \mathrm{Z}} V_j} = L^2(\mathrm{R}) \quad and \quad \bigcap_{j\in \mathrm{Z}} V_j = \{0\} \ ,$

(iii) $\varphi(x) \in V_j \iff \varphi(2x) \in V_{j+1} \ ; \quad \forall j \in \mathrm{Z} \ ,$

(iv) $\varphi(x) \in V_j \implies \varphi \quad \varphi(x) \in V_j \implies \varphi(x - 2^{-j}k) \in V_{j+1} \ ; \quad \forall k \in \mathrm{Z} \ ,$

(v) $\{\varphi(x-k) : k \in \mathrm{Z}\}$ form an orthonormal bases for V_0 .

$\varphi(x)$ is called the *scaling function* of the MRA.

At the Earthquake Research Institute (ERI) of the University of Tokyo, Sumiko Hiyama et al. developed a system to accurately display two-dimensional geographical data at user-specified resolutions on a personal computer using wavelet-based MRA [43]. More sophisticated wavelet-based methods for curve and three-dimensional map display using powerful graphics workstations and supercomputers are being developed by a number of scientific teams for a variety of purposes. In particular, we mention Certain et al. [12], De Rose [104], Eck and Hoppe [25], Finkelstein [29], Gross et al. [33], Lindstrom et al. [67], Luebke and Erikson [68], Reissel [91], Salesin and Stollnitz [29],[104], and Zorin, Schröder and Sweldens [123], who developed very sophisticated graphics algorithms for displaying information. A review of wavelet techniques used in computer graphics is [99].

Miller, Wong, Brewster and Foote at the Pacific Northwest National Laboratory in the United States developed a new, wavelet-based approach, called *TOPIC-O-GRAPHY*^{TM}, to "*visualize and explore unstructured text*" [79]. The technique has been implemented in a text visualization system prototype, named *TOPIC ISLANDS*^{TM}. The inventors claim that the system helps users "*determine the thematic content of text, ..., browse a document, generate fuzzy document outlines, summarize text by levels of detail and according to user interests, define meaningful subdocuments, query text content, and provide summaries of topic evolution*". Visualization of the test takes place in several steps:

- a custom digital signal is constructed from the words contained in the document;
- the wavelet tranform is applied to the signal;
- three types of wavelet energy are computed:
  - the *composite energy* is used to detect when a change in topic occurs in the text;
  - the *channel energy* is used to automatically assign keywords to text partitions determined by the composite energy; and
  - the *query energy* is used to determine good matches to an input query.

The system is extremely good at accurately identifying story inserts and major theme changes, however, very complex writing styles tend to be noisier. Local extrema in the wavelet transforms appear to be more important than the global extrema. Different types of wavelets need to be explored to determine which may be more suitable for identifying certain text analysis tasks.

5 Other Examples of Wavelet Analysis

Wavelet analysis has contributed to many disciplines in addition to to those mentioned above; it is being employed in the numerical solution of differential equations, computational fluid dynamics and image processing, just to name a few. In this section, we present references to some interesting applications.

Naoki Saito of developed wavelet techniques for "*simultaneous noise suppression and signal compression, classification, regression, multiscale edge detection and representation, and extraction of geological information from acoustic waveforms*" while at Schlumberger-Doll Research and Yale University [93]. And he developed a library of orthonormal wavelet basis for signal feature extraction and signal compression.

Nobuatsu Tanaka of the Nuclear Engineering Laboratory, Toshiba Corporation, developed a powerful and simple new wavelet-based preconditioning method for solving large systems of linear equations [109]. The preconditioner leads to accurate results while substantially reducing computation time and costs in simulations of fluid flow modeled by Poisson equations. Preconditioning using wavelet bases leads to improved performance since conjugate gradients methods cannot prevent the exponential increase in computing time when the number of gridpoints is increased. The wavelet method developed by Tanaka builds on earlier, more primitive results reported in [7], [32], [50], which can only be applied to problems with restricted conditions, e.g., periodic boundary problems.

Yamada, Higuchi, Mitsuta and scientists at the Disaster Prevention Research Institute (DPRI) of Kyoto University developed and applied a new wavelet to analyze how the topography of the region surrounding a large, man-made structure in Japan influences turbulent and potentially violent characteristics of high winds [41],[118]. They developed a method for classifying wavelet transform coefficients from two distinct components of turbulent wind data. In a second study, the Yamada and Ohkitani used wavelet techniques to study Navier-Stokes equations [119]. The scientists found that the probability density function of the wavelet coefficients of the velocity data has a power-form in regions with very high wavenumbers, more precisely, in the deep dissipation range.

Jameson developed then applied a wavelet-based grid generation technique to computational fluid mechanics at ICASE and Mitsubishi Heavy Industries [52]. Wavelet-based techniques to solve differential equations have also been reported in [1], [4], [117], [7], [8], [17], [18], [27], [51], [65], [69], [88], [114].

One of the more well-publicized examples of a successful wavelet technology is the development of a wavelet-based national standard for digitized fingerprint records by Bradley and Brislawn of the Computer Research and Applications Group at Los Alamos National Laboratory [10], [11].

Another very practical example by Ogunbana, Milliss, De Boer and Fernandes is the real-time, automatic classification of the quality of welds which are made by industrial robots [83]. Wavelet analysis and vector quantization are applied to current and voltage signals from gas metal arc welding robots. Experiments results indicate that the accuracy of the classifications are about 93%.

6 Conclusion

Although wavelets have not lived up to some of the wild speculative claims in the late 1980's, industrial scientists have begun to use them in some specialized development and manufacturing contexts. Wavelets have proved to be a very valuable and inexpensive tool in some of the niche applications mentioned in this paper. Undoubtedly, new contributions by wavelet technolgies will be made in the years to come.

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Appendix 1: Wavelet-related U.S. Patents Issued

1991

1. 5,051,961 Method and apparatus for seismic survey including using vertical gradient estimation to separate downgoing seismic wavefield
2. 5,051,960 Method of removing records of multiple reflection events from seismic data
3. 5,000,183 Device for processing an echographic signal

1992

1. 5,173,879 Surface-consistent minimum-phase deconvolution
2. 5,132,938 Adjusting seismic data to tie to other data
3. 5,101,446 Method and apparatus for coding an image

1993

1. 5,262,958 Spline-wavelet signal analyzers and methods for processing signals
2. 5,253,530 Method and apparatus for reflective ultrasonic imaging
3. 5,214,708 Speech information extractor
4. 5,193,077 Method and apparatus for improved seismic prospecting

1994

1. 5,377,302 System for recognizing speech
2. 5,347,494 Shaped-sweep technology
3. 5,347,479 Small-size wavelet transform apparatus
4. 5,343,994 End of fill detector for a hydraulic clutch
5. 5,325,449 Method for fusing images and apparatus therefor
6. 5,321,776 Data compression system including successive approximation quantizer
7. 5,315,670 Digital data compression system including zerotree coefficient coding
8. 5,287,529 Method for estimating solutions to finite element equations by generating pyramid representations, multiplying to generate weight pyramids, and collapsing the weighted pyramids

1995

1. 5,477,272 Variable-block size multi-resolution motion estimation scheme for pyramid coding
2. 5,471,991 Wavelet analysis of fractal systems
3. 5,461,475 Binary optical spectrum analyzer
4. 5,454,047 Optical method and system for generating expansion coefficients for an image processing function
5. 5,453,945 Method for decomposing signals into efficient time-frequency representations for data compression and recognition
6. 5,439,483 Method of quantifying cardiac fibrillation using wavelet transform
7. 5,436,447 Method and apparatus for determining relative ion abundances in mass spectrometry utilizing wavelet transforms
8. 5,432,870 Method and apparatus for compressing and decompressing images of documents ratus for compressing and decompressing images of documents
9. 5,420,636 Apparatus for and method of transmitting video signal
10. 5,414,780 Method and apparatus for image data transformation
11. 5,412,741 Apparatus and method for compressing information
12. 5,398,067 Picture data processing apparatus
13. 5,396,472 Method for deriving water bottom reflectivity in dual sensor seismic surveys
14. 5,392,255 Wavelet transform method for downward continuation in seismic data migration
15. 5,388,182 Nonlinear method and apparatus for coding and decoding acoustic signals with data compression and noise suppression using cochlear filters, wavelet analysis, and irregular sampling reconstruction
16. 5,384,869 Image processing apparatus
17. 5,384,725 Method and apparatus for encoding and decoding using wavelet-packets

1996

1. 5,590,222 Image signal processing apparatus utilizing a 2D Haar transform and adaptive selection of images based on a parameter such as a ratio of coefficients for reducing block distortion and method thereof
2. 5,588,023 High content information transmission system
3. 5,587,965 Surface multiple attenuation via eigenvalue decomposition

4. 5,587,931 Tool condition monitoring system
5. 5,576,548 Nuclear imaging enhancer
6. 5,565,920 Method and apparatus for video data compression using temporally adaptive motion interpolation
7. 5,563,960 Apparatus and method for emphasizing a selected region in the compressed representation of an image
8. 5,561,724 Method conducting smoothing processing in a linear direction having a minimized magnitude of density change
9. 5,561,431 Wavelet transform implemented classification of sensor data
10. 5,550,788 Method and system of analysis of the behavior of a drill
11. 5,546,477 Data compression and decompression
12. 5,539,842 Method and apparatus for compressing and decompressing images of documents
13. 5,539,658 Electronic presentation system using portable storage media
14. 5,537,493 Apparatus for compressing image data employing entropy encoding of data scanned from a plurality of spatial frequency bands
15. 5,537,365 Apparatus and method for evaluation of picking horizons in 3-D seismic data
16. 5,537,344 High-speed processing apparatus and method, signal analyzing system, and measurement apparatus and method
17. 5,534,927 Method of performing high efficiency coding of image signal and system therefor
18. 5,528,725 Method and apparatus for recognizing speech by using wavelet transform and transient response therefrom
19. 5,526,446 Noise reduction system
20. 5,526,299 Method and apparatus for encoding and decoding using wavelet-packets
21. 5,524,100 Method for deriving water bottom reflectivity in dual sensor seismic surveys
22. 5,517,585 System and method for stable analysis of sampled transients arbitrarily aligned with their sample points
23. 5,513,273 Method for obtaining information about interstitial patterns of the lungs
24. 5,511,155 Method and device for synthesizing all-objects-in-focus images
25. 5,510,678 DC type gas-discharge display panel and gas-discharge display apparatus with employment of the same
26. 5,504,487 System for extracting targets from radar signatures
27. 5,500,902 Hearing aid device incorporating signal processing techniques

28. 5,497,777 Speckle noise filtering in ultrasound imaging

29. 5,497,435 Apparatus and method for encoding and decoding digital signals

30. 5,495,554 Analog wavelet transform circuitry

31. 5,495,292 Inter-frame wavelet transform coder for color video compression

32. 5,490,233 Method and apparatus for reducing correlated errors in subband coding systems with quantizers

33. 5,488,674 Method for fusing images and apparatus therefor

34. 5,481,269 General frame wavelet classifier

1997

1. 5,703,965 Image compression/decompression based on mathematical transform, reduction/expansion, and image sharpening

2. 5,699,286 Wavelet transform processor using a pipeline with a bit unit 3. 5,694,171 Moving image encoding apparatus

3. 5,689,429 Finger wear detection for production line battery tester

4. 5,687,725 Method for motion tracking of interventional instruments with MR imaging

5. 5,684,693 Method for bit-stream data compression

6. 5,682,152 Data compression using adaptive bit allocation and hybrid lossless entropy encoding

7. 5,675,551 Apparatus and method for evaluation of scog

8. 5,675,551 Apparatus and method for evaluation of score failures in picking of 3-D seismic data

9. 5,673,332 Computer-aided method for image feature analysis and diagnosis in mammography

10. 5,673,191 Method and apparatus for identifying geological structures using wavelet analysis of potential fields

11. 5,671,330 Speech synthesis using glottal closure instants determined from adaptively-thresholded wavelet transforms

12. 5,671,294 System and method for incorporating segmentation boundaries into the calculation of fractal dimension features for texture discrimination

13. 5,668,850 Systems and methods of determining x-ray tube life

14. 5,667,244 Method and apparatus for detecting an impact on a vehicle

15. 5,666,475 Method and system for editing multiresolution images at fractional-levels of resolution using a wavelet representation
16. 5,666,434 Computer-aided method for image feature analysis and diagnosis in mammography
17. 5,663,929 Drilling signal transmission method and system
18. 5,661,822 Data compression and decompression
19. 5,657,085 Wavelet transform coding method
20. 5,646,600 Instrument for detecting potential future failures of valves in critical control systems
21. 5,642,166 Bi-directional motion estimation method and apparatus thereof
22. 5,640,490 User independent, real-time speech recognition system and method
23. 5,638,823 System and method for noninvasive detection of arterial stenosis
24. 5,638,338 Seismic processing apparatus and method
25. 5,638,068 Processing images using two-dimensional forward transforms
26. 5,627,907 Computerized detection of masses and microcalcifications in digital mammograms
27. 5,625,745 Noise imaging protection for multi-channel audio signals
28. 5,623,939 Method and apparatus for analyzing uterine electrical activity from surface measurements for obstetrical diagnosis
29. 5,621,699 Apparatus and method of calibrating vertical particle velocity detector and pressure detector in a sea-floor cable with in-situ passive monitoring
30. 5,619,998 Enhanced method for reducing ultrasound speckle noise using wavelet transform
31. 5,615,287 Image compression technique
32. 5,612,700 System for extracting targets from radar signatures
33. 5,610,843 Methods and apparatuses for multi input/multi output control systems
34. 5,604,824 Method and apparatus for compression and decompression of documents and the like using splines and spline-wavelets
35. 5,602,760 Image-based detection and tracking system and processing method employing clutter measurements and signal-to-clutter ratios
36. 5,602,589 Video image compression using weighted wavelet hierarchical vector quantization
37. 5,600,373 Method and apparatus for video image compression and decompression using boundary-spline-wavelets

38. 5,598,481 Computer-aided method for image feature analysis and diagnosis in mammography
39. 5,596,548 Seismic imaging using wave equation extrapolation
40. 5,592,226 Method and apparatus for video data compression using temporally adaptive motion interpolation
41. 5,592,171 Wind profiling radar
42. 5,590,650 Non-invasive medical monitor system

1998

1. 5,852,806 Switched filterbank for use in audio signal coding
2. 5,852,681 Method and apparatus for eliminating artifacts in data processing and compression systems
3. 5,852,243 Method and apparatus for detecting a road pavement surface condition
4. 5,850,482 Error resilient method and apparatus for entropy coding
5. 5,848,193 Wavelet projection transform features applied to real time pattern recognition
6. 5,848,171 Hearing aid device incorporating signal processing techniques
7. 5,845,243 Method and apparatus for wavelet based data compression having adaptive bit rate control for compression of audio information
8. 5,841,890 Multi-dimensional wavelet tomography
9. 5,839,098 Speech coder methods and systems
10. 5,838,815 Method and system to enhance robust identification of abnormal regions in radiographs
11. 5,838,377 Video compressed circuit using recursive wavelet filtering
12. 5,835,129 Multipoint digital video composition and bridging system for video conferencing and other applications
13. 5,831,625 Wavelet texturing
14. 5,828,849 Method to derive edge extensions for wavelet transforms and inverse wavelet transforms
15. 5,828,567 Diagnostics for resistance based transmitter
16. 5,826,232 Method for voice analysis and synthesis using wavelets
17. 5,825,936 Image analyzing device using adaptive criteria

18. 5,825,935 Subband coding method with wavelet transform for high efficiency video signal compression

19. 5,825,909 Automated method and system for image segmentation in digital radiographic images

20. 5,822,459 Method for processing wavelet bands

21. 5,822,436 Photographic products and methods employing embedded information

22. 5,822,370 Compression/decompression for preservation of high fidelity speech quality at low bandwidth

23. 5,821,882 Data conversion method and apparatus

24. 5,821,751 Method for magnetic resonance image acquisition and reconstruction of the basic of wavelet encoding

25. 5,819,215 Method and apparatus for wavelet based data compression having adaptive bit rate control for compression of digital audio or other sensory data

26. 5,818,525 RGB image correction using compressed flat illuminated files and a simple one or two point correction algorithm

27. 5,815,198 Method and apparatus for analyzing an image to detect and identify defects

28. 5,814,897 Vehicle passenger restraint system

29. 5,813,993 Alertness and drowsiness detection and tracking system

30. 5,812,971 Enhanced joint stereo coding method using temporal envelope shaping

31. 5,808,467 RF probe and inspection system using NMR using the same

32. 5,806,038 MBE synthesizer utilizing a nonlinear voicing processor for very low bit rate voice messaging

33. 5,802,481 Adaptive filtering for use with data compression and signal reconstruction

34. 5,802,369 Energy-based wavelet system and method for signal compression and reconstruction

35. 5,801,305 Method and apparatus for detecting a tire inflation pressure

36. 5,799,114 System and method for stable analysis of sampled transients arbitrarily aligned with their sample points

37. 5,799,112 Method and apparatus for wavelet-based universal halftone image unscreening

38. 5,799,100 Computer-assisted method and apparatus for analysis of x-ray images using wavelet transforms

39. 5,798,794 Wavelet transform subband coding with frequency-dependent quantization step size

40. 5,796,921 Mapping determination methods and data discrimination methods using the same
41. 5,796,434 System and method for performing motion estimation in the DCT domain with improved efficiency
42. 5,790,694 Image processing method for inspecting with analysis of binarized high and low pass information in wavelet transformed image data
43. 5,790,185 Video inspection or logging tool
44. 5,790,131 System and method for lossy compression of data with output file size control
45. 5,787,207 Method and apparatus for minimizing blockiness in reconstructed images
46. 5,784,285 Waveform analyzer
47. 5,781,888 Perceptual noise shaping in the time domain via LPC prediction in the frequency domain
48. 5,781,881 Variable-subframe-length speech-coding classes derived from wavelet-transform parameters
49. 5,781,502 Method and device for filtering elliptical waves propagating in a medium
50. 5,781,144 Wide band video signal denoiser and method for denoising
51. 5,778,881 Method and apparatus for discriminating P and R waves
52. 5,777,678 Predictive sub-band video coding and decoding using motion compensation
53. 5,776,073 Method and apparatus for analyzing uterine electrical activity from surface measurements for obstetrical diagnosis
54. 5,774,416 Method and device for attenuating water column reverberations using co-located hydrophones and geophones in ocean bottom seismic processing
55. 5,774,357 Human factored interface incorporating adaptive pattern recognition based controller apparatus
56. 5,772,604 Method, system and apparatus for determining prognosis in atrial fibrillation
57. 5,768,437 Fractal coding of data
58. 5,768,392 Blind adaptive filtering of unknown signals in unknown noise in quasi-closed loop system
59. 5,767,676 MR spectroscopy method for measuring plural voxels at intersections of slices
60. 5,764,805 Low bit rate video encoder using overlapping block motion compensation and zerotree wavelet coding
61. 5,761,341 Image encoding and decoding method and apparatus using edge synthesis and inverse wavelet transform
62. 5,757,974 System and method for data compression

63. 5,757,723 Method of processing data representing energy passing through a medium
64. 5,757,309 Spatial frequency feature extraction for a classification system using wavelets
65. 5,755,666 Method and device for imaging a curved region by means of magnetic resonance
66. 5,754,793 Wavelet image compression/recovery apparatus and method using human visual system modeling
67. 5,754,492 Method of reverberation removal from seismic data and removal of dual sensor coupling errors
68. 5,754,438 Frequency analyzing method and apparatus for time series data
69. 5,751,899 Method and apparatus of analysis of signals from non-stationary processes possessing temporal structure such as music, speech, and other event sequences
70. 5,749,367 Heart monitoring apparatus and method
71. 5,748,786 Apparatus for compression using reversible embedded wavelets
72. 5,748,471 Well collar identification method
73. 5,748,116 System and method for nested split coding of sparse data sets
74. 5,747,749 Acoustic logging to identify oil flow rate
75. 5,746,511 Temperature transmitter with on-line calibration using johnson noise
76. 5,745,604 Identification/authentication system using robust, distributed coding
77. 5,745,392 Method for reducing data storage and transmission requirements for seismic data
78. 5,745,382 Neural network based system for equipment surveillance
79. 5,741,707 Method for quantitative analysis of earth samples
80. 5,740,268 Computer-aided method for image feature analysis and diagnosis in mammography
81. 5,740,036 Method and apparatus for analyzing geological data using wavelet analysis
82. 5,737,448 Method and apparatus for low bit rate image compression
83. 5,734,755 JPEG/MPEG decoder-compatible optimized thresholding for image and video signal compression
84. 5,729,691 Two-stage transform for video signals
85. 5,729,506 3-D multiple attenuation
86. 5,724,138 Wavelet analysis for laser ultrasonic measurement of material properties
87. 5,721,694 Non-linear deterministic stochastic filtering method and system
88. 5,717,791 Image contrast enhancing method

89. 5,710,835 Storage and retrieval of large digital images
90. 5,710,432 Non-contact tension measurement of moving fiber using traveling wave time-of-flight analysis
91. 5,708,759 Speech recognition using phoneme waveform parameters
92. 5,706,220 System and method for implementing the fast wavelet transform

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1. 5,889,559 Method and apparatus for minimally-shifted wavelet decomposition and recomposition
2. 5,889,438 Phase-locked oscillation and control circuit with a wavelet filter
3. 5,886,276 System and method for multiresolution scalable audio signal encoding
4. 5,883,978 Image compressing method and image compressing apparatus
5. 5,881,181 Method and apparatus for compressing the dynamic range of an image
6. 5,881,176 Compression and decompression with wavelet style and binary style including quantization by device-dependent parser
7. 5,880,856 Progressive image transmission using discrete wavelet transforms
8. 5,878,172 Image encoding and decoding method and apparatus using edge synthesis and inverse wavelet transform
9. 5,875,122 Integrated systolic architecture for decomposition and reconstruction of signals using wavelet transforms
10. 5,875,108 Ergonomic man-machine interface incorporating adaptive pattern recognition based control system
11. 5,872,859 Training/optimization of computer aided detection schemes based on measures of overall image quality
12. 5,870,502 System and method for a multiresolution transform of digital image information
13. 5,870,493 Top down preprocessor for a machine vision system
14. 5,867,606 Apparatus and method for determining the appropriate amount of sharpening for an image
15. 5,867,602 Reversible wavelet transform and embedded codestream manipulation
16. 5,867,290 High capacity spread spectrum optical communications system
17. 5,867,118 Apparatus for and method of classifying patterns
18. 5,864,780 Jointly optimized subband coding system and method

19. 5,864,754 System and method for radio signal reconstruction using signal processor
20. 5,862,260 Methods for surveying dissemination of proprietary empirical data
21. 5,859,788 Modulated lapped transform method