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Research Report

A Quasi-solid 3D Shape Description - An Improvement of S.M.F. -

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A Quasi-solid 3D Shape Description

- Improvement of S.M.F. -

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At last document, we described new method to describe the shape of polyhedral surface by using Spherical Moment Functions (S.M.F.) for the purpose of similarity retrieval. In this document, we describe two techniques to improve accuracy of S.M.F in similarity retrieval.

There were two major problems when we describe polyhedral data by using S.M.F. Firstly, the accuracy of comparisons are irregular, since sampling points on the polyhedral surfaces were not uniform. Secondary, S.M.F. blends all the sampling points, which are in the same direction seen from the center of polyhedral surface.

We introduced two techniques namely a normalization of sampling points and a quasi-solid description method to a S.M.F. procedure and improved the accuracy of similarity retrieval. At the experiments, an efficiency of our improvement is shown comparing original S.M.F and improved S.M.F.

1. Introduction

There were two major problems, when we describe polyhedral surface by using S.M.F, namely an irregular accuracy of comparisons and a blending of sampling points, which are in the same direction seen from the center of polyhedral surface.

In the case of similarity retrieval, many comparisons will be performed between key model and various models. The accuracy of each comparison will be affected by the density of input meshes and this causes an irregular accuracy of comparisons. Further more, since each input mesh is not uniform on the surface of polyhedral data, the similarity computed form S.M.F. will be affected. We will solve this problem by normalizing sampling points of polyhedral surface as to dense of sampling points become even and the direction of each sampling points seen form the center of polyhedral surface become uniform.

When we are to compare complex models, in many cases, there are several pieces of surface overlapped, when seen from the center of polyhedral surface. If we place normalized sampling points on the surface, in many cases, several sampling points are in the same direction when seen from the center for polyhedral data. Though S.M.F. distincts the directions of sampling points seen from the center of polyhedral surface, it does not distincts overlaps of sampling points in the same direction. Let us call this problem a “vertical blending problem.” To solve a vertical blending problem, we introduced the idea of quasi-solid that distincts front face, and back face according to the direction of normal of surface at the each sampling point and the direction of each sampling point seen from the center of plolyhedral surface.

In this document, we firstly describe terminology used in this document, then describe the method to normalize sampling points of polyhedral surface, then describe entire process of improved S.M.F using quasi-solid description technique, then show efficiency of improved S.M.F. by comparing older S.M.F, and finally describe conclusion of this document.

2. Terminology

In this section, we describe several terminology of used in this paper. Suppose we are going to compare following polyhedral data named P_a and P_b.

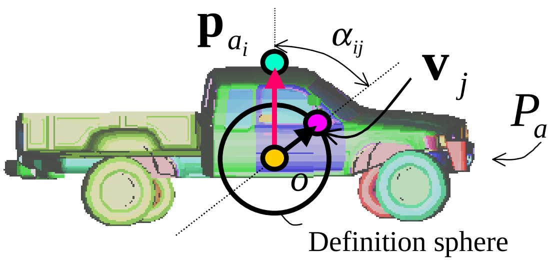

Fig. 1 Terminology

In this paper, we use following terms.

P_a **:** Polyhedral data a

o **:** The center of mass of each polyhedral data

$\mathbf{p}_{a_i}$ **:** Normalized Radial vectors from the center of mass to each sampling point on the surface of polyhedral data a. The identifier i denotes the direction of each $\mathbf{p}_{a_i}$.

r_{a_i} **:** The length of each $\mathbf{p}_{a_i}$

Definition sphere: A sphere, whose center coincides the center of mass of P_a.

$\mathbf{v}_{a_j}$ **:** Radial vectors form the center to each sampling point on the surface of a definition sphere. The identifier j denotes the direction of each $\mathbf{v}_{a_j}$.

3. Normalization

Before we begin, all the models should be normalized their positions, and should be normalized their scales if necessary. Normalization of orientation may also needed before we begin normalize sampling points. It is depends on dataset that we perform queries.

Suppose polyhedral surface consist of triangle mesh. The normalization of position is performed using the center of mass of polyhedral surface. The center of mass of polyhedral surface is given as follows:

$$C = \sum_i \frac{s_i}{S} c_i , \qquad (1)$$

where C denotes the center of mass of polyhedral surface, c_i denotes the center of mass of each triangle, s_i denotes the area of each triangle, and S denotes the total area of polyhedral surface.

By translating each model as to the center of mass coincides the origin of local coordinates, the normalization of position is performed

The normalization of scale is performed by using following scale factor.

$$f_s = \sum_i \frac{s_i}{S} r_i , \qquad (2)$$

where f_s denotes the scale factor, and r_i denotes the distance form the center of mass of polyhedral surface to the center of mass of each triangle. By scaling each model by $1/f_s$, the normalization of scale is performed.

The normalization of orientation is performed by using primary moment functions, which is described in section five. By computing primary moment function using the coordinates of nodes of triangle mesh insetad of normalized sampling point. We can compute the rotation matrix to normalize the orientation of each model.

4. Normalization of sample points

In this section, we describe the method to solve irregular accuracy of each comparison described in introduction. Objective of this section is to calculate sample points $\mathbf{p}_{a_i}$ by resampling polyhedral surface and to calculate weights w_{a_i} correspond to sample points $\mathbf{p}_{a_i}$.

Normalized sample points of polyhedral surface P_a are calculated as intersection points between a polyhedral surface and a set of vectors which share starting points, and whose directions are uniform.

The concrete procedure to calculate normalized sapling points is as follows.

1. Place a sphere, whose radius is 1.0, as to the center of a sphere coincides center of mass of polyhedral surface P_a.
2. Generate points on a sphere in even dense, and in uniform directions seem from the center.

3. Generate rays form the center of a sphere to points on a sphere that is generated in step 2.
4. Calculate intersection points between each ray and polyhedral surface. The calculated intersection points are the normalized sampling points $\mathbf{p}_{a_i}$.

A step 2 can be implemented by following steps

1. Place a cube as to inscribe to a sphere.
2. Insert a vertex to the center of each face of a cube.
3. Subdivide each face into four.(fig)
4. Make vectors from the center of a sphere to points generated by subdivision of a cube surface.
5. Normalize the length of vectors to 1.0.

By processing procedures above, we can get uniform sampling points on each polyhedral surface.

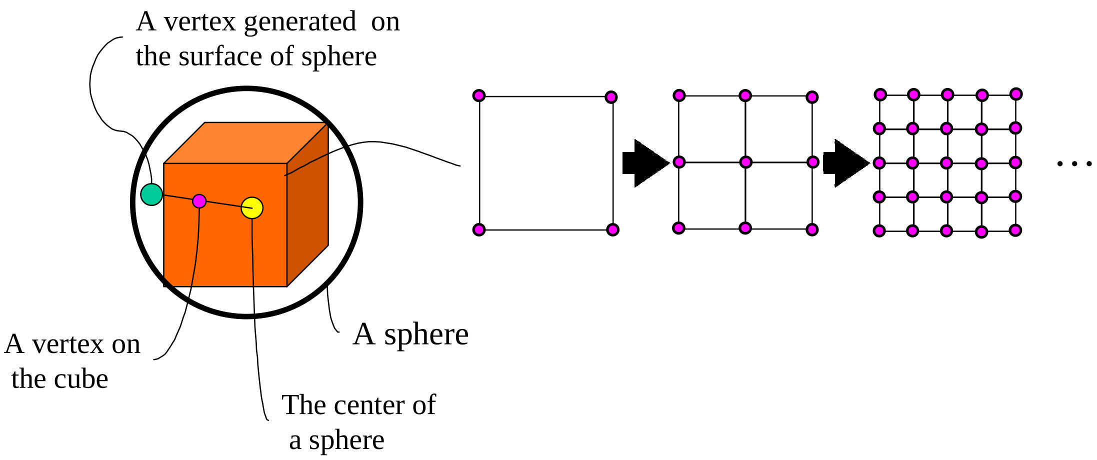

Fig. 2 generating points on sphere in even dense

The weight for each normalized sampling point is calculated as a ratio of the area of sphere surface, which is occupied by each sampling point, to the area of sphere surface. It can be calculated very simple as follows.

$$w_{a_i} = \frac{1}{n}, \qquad (3)$$

where n denotes the number of points generated on the surface of a sphere.

5. Quasi-solid description

In this section we firstly describe a basic idea of quasi-solid description, then describe the method of making quasi-solid description of S.M.F, and finally the method to compare polyhedral surfaces using quasi-solid description of S.M.F.

Let us focus on one of radius vectors that starts from the center of polyhedral surface to one of normalized sampling points on the polyhedral surface. In many cases, there are several piece of surface that intersects this radius vector. Let us focus on a couple of pieces of a surface namely back face and front face that intersects a radius vector $\mathbf{p}_{a_i}$. A back face and a front face are determined by the direction of normal vectors of piece of a surface and the direction of a radius vector as follows. If $\mathbf{p}_{a_i} \cdot \mathbf{n}_{a_i} \geq 0$, $\mathbf{p}_{a_i}$ is on a front face other wise, $\mathbf{p}_{a_i}$ is on a back face, where $\mathbf{n}_{a_i}$ denotes normal vector of a surface at the point of $\mathbf{p}_{a_i}$.

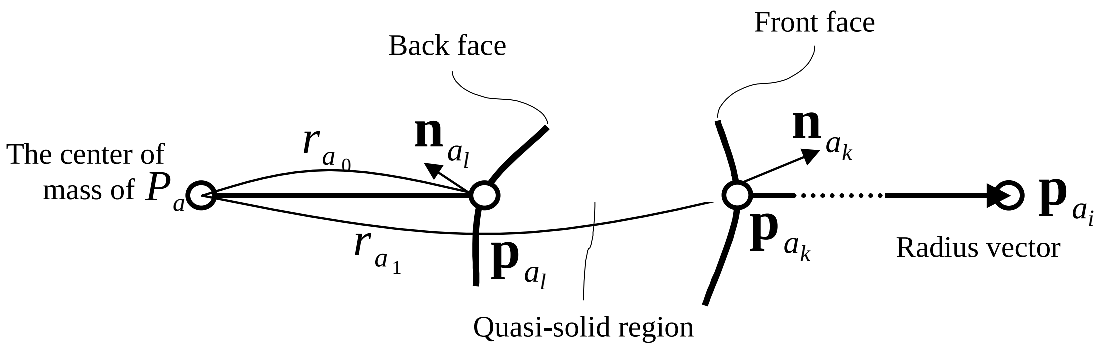

Fig. 3 Back face and front face on a radius vector

Let us denote identifier k as identifier for front faces, and identifier l as an identifier for back faces. In the figure, $\mathbf{p}_{a_k}$ denotes normalized sample point on a front face, $\mathbf{p}_{a_l}$ denotes a normalized sample point on a back face, $\mathbf{n}_{a_k}, \mathbf{n}_{a_l}$ as normal vectors of surfaces at the point of $\mathbf{p}_{a_k}, \mathbf{p}_{a_l}$, and r_{a_k}, r_{a_l} as the length of $\mathbf{p}_{a_k}, \mathbf{p}_{a_l}$. The region that in between neighboring $\mathbf{p}_{a_k}$ and $\mathbf{p}_{a_l}$ is regarded as inside of quasi-solid.

5.1. Description Generation

In this subsection, we describe the method to describe polyhedral surface P_a by using quasi-solid description of S.M.F.

In the S.M.F. a n-th degree of primary moment function M_{a1} that corresponds to polyhedral data P_a is defined as follows:

$$\mathrm{M}_{a1}(\mathbf{v}_j) = \sum_i w_{a_i} \cos^2 \alpha_{ij} r_{a_i}{}^n . \qquad (4)$$

A primary moment function can be explained by using radius vectors of a polyhedral data $\mathbf{p}_{a_i}$ and points on a definition sphere $\mathbf{v}_j$ as follows:

$$\mathrm{M}_{\mathrm{a1}}(\mathbf{v}_j) = \sum_i w_{a_i} \mathbf{v}_j^T \left(\left(\left|\mathbf{p}_{a_i}\right|^{\frac{n}{2}-1} \mathbf{p}_{a_i} \right) \left(\left|\mathbf{p}_{a_i}\right|^{\frac{n}{2}-1} \mathbf{p}_{a_i} \right)^T \right) \mathbf{v}_j \; . \tag{5}$$

This equation shows that we can easily compute a primary moment function form radius vectors of a polyhedral data.

Further more, since $\sum_i w_{a_i} \left(\left|\mathbf{p}_{a_i}\right|^{\frac{n}{2}-1} \mathbf{p}_{a_i} \right) \left(\left|\mathbf{p}_{a_i}\right|^{\frac{n}{2}-1} \mathbf{p}_{a_i} \right)^T$ is a symmetry matrix, it can be diagonized. A primary moment function can be explained as follows

$$\mathrm{M}_{\mathrm{a1}}(\mathbf{v}_j) = \mathbf{v}_j^T \mathbf{R}^T \begin{pmatrix} \lambda_0 & & \\ & \lambda_1 & \\ & & \lambda_2 \end{pmatrix} \mathbf{R} \mathbf{v}_j \, , \tag{6}$$

where $\mathbf{R}$ denotes rotation matrix, and $\lambda_o, \lambda_1, \lambda_2$ denotes eighgen values ordered as to $\lambda_o \ge \lambda_1 \ge \lambda_2$ holds. This equation shows that the orientation of each polyhedral data normalized by aligning the eigenvectors with the x, y, and z-axis with a rotation matrix $\mathbf{R}$. After a normalization of orientation, we just need to keep three real value λ_o, λ_1, and λ_2 to explain each polyhedral data as follows.

$$\mathrm{M}_{\mathrm{a1}}(\mathbf{v}_j) = \mathbf{v}_j^T \begin{pmatrix} \lambda_0 & & \\ & \lambda_1 & \\ & & \lambda_2 \end{pmatrix} \mathbf{v}_j \tag{7}$$

In the case of quasi-solid description, we use two types of primary moment functions, defined as follows:

$$\mathrm{M}_{\mathrm{a10}}(\mathbf{v}_j) = \sum_k w_{a_k} \cos^2 \alpha_{kj} r_{a_k}{}^n + \sum_l w_{a_l} \cos^2 \alpha_{lj} r_{a_l}{}^n \, , \tag{8}$$

$$\mathrm{M}_{\mathrm{a11}}(\mathbf{v}_j) = \sum_k w_{a_k} \cos^2 \alpha_{kj} r_{a_k}{}^n - \sum_l w_{a_l} \cos^2 \alpha_{lj} r_{a_l}{}^n \, . \tag{9}$$

These moment functions are easily calculated by changing the sign of w_{a_i} in equation (5). Two set of $\lambda_o, \lambda_1, \lambda_2$ will be kept for each n-th degree of primary moment functions.

A n-th degree of secondary moment function is defined as follows.

$$M_{a2}(\mathbf{v}_j) = \sum_i w_{a_i} \cos\alpha_{ij} r_{a_i}{}^n \, , \tag{10}$$

By using radius vectors $\mathbf{p}_{a_i}$ of a polyhedral data, a secondary moment function can be explained as follows after normalization of orientation:

$$M_{a2}(\mathbf{v}_j) = \mathbf{v}_j \mathbf{R}\left(\sum_i w_{a_i} \mathbf{p}_{a_i} \left| \mathbf{p}_{a_i} \right|^{n-1} \right), \qquad (11)$$

Since $\mathbf{R}\left(\sum_i w_{a_i} \mathbf{p}_{a_i} \left| \mathbf{p}_{a_i} \right|^{n-1} \right)$ denotes three-dimensional vector, we need to keep three real value to describe a secondary moment value. We need to keep a total of only six real value to explain a primary and a secondary moment function: the three eigenvalues of a primary moment function and the three components of three-dimensional vector of a secondary moment function.

In the case of quasi-solid description, we use following two types of secondary moment functions.

$$M_{a20}(\mathbf{v}_j) = \sum_k w_{a_k} \cos\alpha_{kj} r_{a_k}{}^n + \sum_l w_{a_l} \cos\alpha_{lj} r_{a_l}{}^n, \qquad (12)$$

$$M_{a21}(\mathbf{v}_j) = \sum_k w_{a_k} \cos\alpha_{kj} r_{a_k}{}^n - \sum_l w_{a_l} \cos\alpha_{lj} r_{a_l}{}^n, \qquad (13)$$

Again these moment functions can be calculated by changing the sign of w_{a_i} in equation (11). Two sets of three-dimensional vectors will be kept for each n-th degree of secondary moment functions.

The description of polyhedral surface is done by using m(m>0) to n-th(n>=m) degree of moment functions, and we need to keep 12 real values for each n-th degree of moment functions, as described in this subsection. The resolution of the descriptor can be controlled by m and n.

5.2. Comparison of polyhedral surfaces

In this subsection we describe the method to compute similarity of two given polyhedral surface P_a and P_b by using quasi-solid description of S.M.F.

Let us consider the similarity computed by using n-th degree of moment functions. The difference of primary moment functions, is calculated as follows:

$$d_1 = \sum_j \left(M_{a10}(\mathbf{v}_j) - M_{b10}(\mathbf{v}_j)\right)^2 + \left(M_{a11}(\mathbf{v}_j) - M_{b11}(\mathbf{v}_j)\right)^2, \qquad (14)$$

where M_{a10} M_{a11} denotes primary moment functions, which describe polyhedral surface P_a, and M_{b10} M_{b11} denotes primary moment functions, which describe polyhedral surface P_b.

The points on the definition sphere should be generated as to be distributed in the even dense on the surface of definition sphere. The technique that described in section three (3.Normalization of sample points) can be used.

In the same manner, the difference of secondary moment functions is calculated as follows:

$$d_2 = \sum_j \left(M_{a20}(\mathbf{v}_j) - M_{b20}(\mathbf{v}_j)\right)^2 + \left(M_{a21}(\mathbf{v}_j) - M_{b21}(\mathbf{v}_j)\right)^2 . \qquad (15)$$

The similarity S is computed as follows:

$$s = \frac{d}{v - d}, \qquad (16)$$

where

$$d = \left(\sqrt{d_1} + \sqrt{d_2}\right)^{\frac{1}{n}},$$

$$v = \left(\sqrt{v_1} + \sqrt{v_2}\right)^{\frac{1}{n}},$$

$$v_1 = \sum_j \left(M_{a10}(\mathbf{v}_j)\right)^2 + \left(M_{a11}(\mathbf{v}_j)\right)^2 + \left(M_{b10}(\mathbf{v}_j)\right)^2 + \left(M_{b11}(\mathbf{v}_j)\right)^2 ,$$

$$v_2 = \sum_j \left(M_{a20}(\mathbf{v}_j)\right)^2 + \left(M_{a21}(\mathbf{v}_j)\right)^2 + \left(M_{b20}(\mathbf{v}_j)\right)^2 + \left(M_{b21}(\mathbf{v}_j)\right)^2 .$$

Let us consider multiple degree of moment functions. If we describe polyhedral surface by using m-th to n-th degree of S.M.F., the similarity is computed as the mean of similarity:

$$s = \sum_{i=m}^{n} \frac{s_i}{n - m + 1}, \qquad (17)$$

where, s_i denotes the similarity computed from i-th degree of S.M.F.

6. Experiments

We compared improved S.M.F. and S.M.F to show efficiency of our improvement by using a dataset that consist of about 250 car models. The models are categorized in several categories, for example coupes, pickup trucks, and vans,

The comparison was performed for models categorized to pickup trucks. We first separated the pickup truces from rest of models, then selected 10 models from pickup trucks, then put selected 10 pickup trucks back to rest of models. Ten test queries were performed by specifying 10 pickup trucks as keys. We scored each test query by counting the number of pickup trucks from top ten of retrieved models. The following shows the results:

Table 1 Results of test queries

Key models	vp2588	vp1986	vp2232	vp1981	vp1991	vp1980
Quasi-solid description + S.M.F.	10	10	10	10	10	10
S.M.F. only	10	10	9	7	9	9

Table 1 cont.

	vp2210	vp1894	vp1990	vp1993	Total
Quasi-solid description + S.M.F.	10	10	10	10	100
S.M.F. only	9	6	1	8	78

Following figure shows snapshot of query results for vp1990, which showed significant difference in between both of methods.

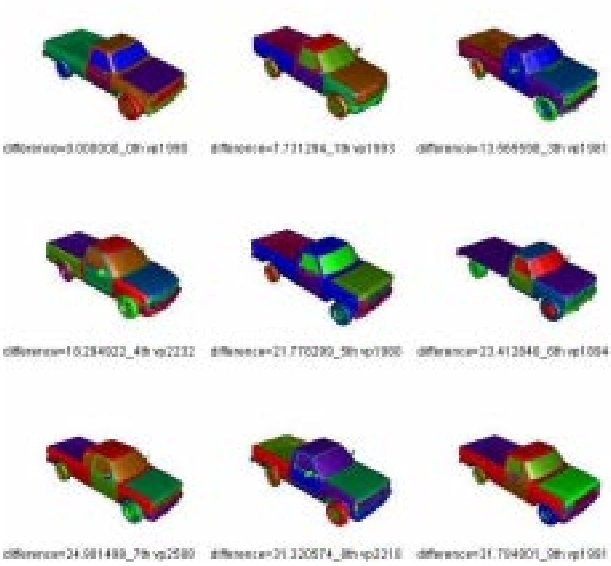

Fig. 4 Query result for vp1990 (quasi-solid description + S.M.F.)

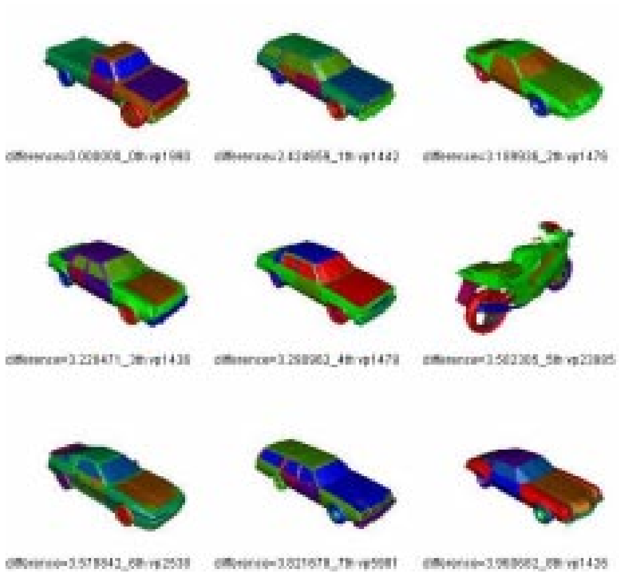

Fig. 5 Query result for vp1990 (S.M.F. only)

As seen in table 1, our improvement showed visible efficiency. The efficiency of our improvement also showed in figure 4 and figure 5. In the worst case of S.M.F., we could not retrieve other pickup trucks within top ten, on the other hand in the case of improved S.M.F we could retrieve all the pickup trucks from the same key model. All the retrieval was performed same condition and we used 2-nd degree of S.M.F. only for this test query.

7. Conclusion

We have described two major problem of S.M.F. namely an irregular accuracy of comparisons and a vertical blending problem. To solve an irregular accuracy of comparisons, we introduced the technique to normalize sampling point of polyhedral surface. To solve a vertical-blending problem, we introduced quasi-solid description technique.

The efficiency of both of the techniques was shown by performing test queries using improved S.M.F. and older S.M.F.