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## **Research Report**

# Optimizing Watermarking to Improve the Robustness without affecting the Fidelity

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### Optimizing Watermarking to Improve the Robustness without affecting the Fidelity

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### Abstract

One of the expected application areas for the practical use of the watermarking technology is for protecting contents from illegal distribution and/or copying, such as for DVD copy protection. In such application areas, the watermarked contents should be robust with regards to various kinds of post-processing, such as digital-to-analog or analog-to-digital (D/A,A/D) conversion, MPEG compression, and VHS recording. This paper introduces a watermark embedding method that maximizes the strength of the detected watermark signal, an embedding method robust against MPEG compression, and an embedding method robust against VHS recording, without affecting the fidelity of the watermarked contents.

### **1. Introduction**

In watermarking application areas such as protecting content from illegal distribution and/or copying, the technology must satisfy the requirements for fidelity, robustness, reliability, and security. And, of course, the original contents are not referred to during detection. Among them, fidelity and robustness are very important requirements, but they involve trade-offs. The more the watermark is embedded, the more the robustness is improved, but the less the fidelity is. Therefore, it's an interesting and practical research area to investigate how we can improve the robustness when the fidelity of the watermarked contents is predefined.

This paper introduces a watermark, especially for video, embedding method to maximize the detection value under the condition that the fidelity of the watermarked contents is predefined.

The structure of this paper is as follows: in Section 2, we introduce the statistical method [1, 2] we use for embedding and detection. In Section 3, we discuss the embedding method that provides the maximum detection value under

the condition that the fidelity of the watermarked contents is predefined. In that section, we first discuss the case of no post-processing, then take the most common kinds of video post-processing, MPEG compression and VHS recording, into consideration. As for evaluating the fidelity, we use the square sum of the amount of modification. For an embedding robust against MPEG compression, we predict the quantization value, and then discuss an embedding method that provides a large detection value based on that value. For an embedding robust against VHS recording, we first discuss the features of the VHS recorder that attenuates the watermark, then discuss the embedding method that provides the maximum detection value. In Section 4, we provide experimental results and in Section 5, we offer our conclusions.

### 2 Statistical method

In the detection process of the statistical method we propose, we first calculate a sequence of values,  $x_0, ..., x_{N-1}$ , from the contents. In video watermarking, each  $x_i$  is an inner product of the 2-dimensional luminance block  $Y_i$  and a detection pattern  $M_i$  calculated as:

$$x_i = \boldsymbol{Y}_i \cdot \boldsymbol{M}_i \tag{1}$$

where stands for the inner product.  $M_i$ 's are pseudo random patterns, and sum of the components of each  $M_i$  is zero. For the original images,  $x_i$ 's are assumed to have zero-mean independent identically distributed (IID) values.

The embedding is performed by slightly changing the luminance values as follows:

$$\boldsymbol{Y}_{i}^{\prime} = \boldsymbol{Y}_{i} + \boldsymbol{W}_{i} \tag{2}$$

where  $Y'_i$  and  $W_i$  are for the luminance block of the watermarked content and the embedding pattern, respectively.  $M_i$  and  $W_i$  are highly correlated.

In the detection process, we calculate a detection value D as follows:

$$D = \sqrt{N} \frac{\mu}{\sigma} \tag{3}$$

where  $\mu$  and  $\sigma$  are the average and standard deviation of  $\{x_i\}$ , respectively.

In this paper, we discuss how to yield the largest D from the watermarked and post-processed contents without affecting the fidelity of the watermarked contents.

### **3** Embedding method

The embedding discussion consists of three subsections: an embedding method to maximize the strength of the detected watermark signal without post-processing, an embedding method robust against MPEG compression, and an embedding method robust against VHS recording.

Let w(x) denote the amount of the modification for  $x_i$  at the value x. Following are popular methods to embed the watermark into x [3]:

$$w(x) = a \tag{4}$$

$$w(x) = a|x|$$
(5)  
  $0 < a$ 

To generalize above methods, we define w(x) as follows:

$$w(x) = a|x|^b$$

$$0 < a, 0 < b$$
(6)

b = 0 is based on Eq. (4) (hereafter, Method A) and b = 1 is based on Eq. (5) (hereafter, Method B).

Next, let f(x) and  $f_e(u)$  denote the probability density functions (PDF) of  $x_i$  and  $x_i + w(x_i)$ , respectively.

f(x) is approximated by zero-mean generalized Gaussian Distribution (GGD) as follows [4, 5, 6, 7, 8]:

$$f(x) = \frac{\nu \alpha(\nu)}{2\sigma \Gamma(1/\nu)} \exp\left[-\left(\frac{\alpha(\nu)}{\sigma}|x|\right)^{\nu}\right]$$
(7)

where

$$\alpha(\nu) = \sqrt{\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}}$$
(8)

and  $\Gamma(\cdot)$  denotes the usual gamma function.

Now, let  $w_a(x) = x + w(x)$ . Then, when  $w_a(x)$  is strictly increasing and the inverse function of  $w_a(x)$  exists,  $f_e(u)$  is written as follows [9]:

$$f_e(u) = \left. \frac{f(x)}{w'_a(x)} \right|_{x = w_a^{-1}(u)} \tag{9}$$

Using f(x) and w(x), we define the embedding volume V as follows:

$$V = \int_{-\infty}^{\infty} w^2(x) f(x) dx \tag{10}$$

We use V to evaluate the fidelity of the watermarked contents. In this paper, we discuss how to choose the optimizing b in Eq. (6) to maximize D in Eq. (3) while holding V constant for each post-processing.

#### **3.1** Embedding to maximize the detection value

First, we discuss an embedding method to maximize the strength of the detected watermark signal when V is given.

The expected detection value from the watermarked contents,  $D_n$ , is calculated as follows:

$$D_n = \sqrt{N} \frac{\mu_n}{\sigma_n} \tag{11}$$

where

$$\mu_n = \int_{-\infty}^{\infty} w_a(x) f(x) dx$$
$$= \int_{-\infty}^{\infty} w(x) f(x) dx \qquad (12)$$

$$\begin{aligned} x_n^2 &= \int_{-\infty}^{\infty} w_a^2(x) f(x) dx - \mu_n^2 \\ &= V + \sigma^2 - \mu_n^2 \end{aligned}$$
(13)

where

σ

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) dx \tag{14}$$

Once V and b are given, a is calculated by using Eq. (6) and Eq. (10). So,  $D_n$  is written as a function of b. Figure 1 shows the theoretical expected detection value  $D_n$  from without post-processing contents with given V at  $\nu = 1.0$ and  $\nu = 0.5$ . The horizontal axis shows b and the vertical axis shows  $D_n$ . The graph shows the attenuation from the detection value from the content embedded using Method A. The embedding strength is  $a = 0.2\sigma$  when b = 0, From this graph, we see that Method A provides the larger expected detection value for both of the  $\nu$ s.

### 3.2 Embedding robust against MPEG compression

The attenuation of the watermark signal by MPEG compression is caused by quantization. Using quantization and dequantization, a value x is converted to  $x_q$  as follows:

$$x_q = \lceil \frac{x}{q} \rfloor q \tag{15}$$



Figure 1. Theoretical expected detection value from the without post-processing contents

where q is the quantization value (hereafter, Q value) and  $[\cdot]$  is the rounding function.

In MPEG compression, each DCT (Discrete Cosine Transform) coefficient is quantized by the Q value of each index, but here, to simplify the problem, we discuss the case where  $M_i$  consists of only one DCT index.

Let  $D_q$  denote the expected detection value from the contents compressed by Q value q. Then,  $D_q$  is calculated as follows:

$$D_q = \sqrt{N} \frac{\mu_q}{\sigma_q} \tag{16}$$

where

$$\mu_{q} = q \sum_{n=-\infty}^{\infty} nA(n)$$

$$\sigma_{q} = q^{2} \sum_{n=-\infty}^{\infty} n^{2}A(n) - \mu_{q}^{2}$$

$$A(n) = \int_{\left\lceil \frac{x}{q} \right\rfloor = n} f_{e}(u) du \qquad (17)$$

Figure 2 and 3 show the values of  $D_q$  according to theory for quantized contents with given Vs, and with PDFs of  $\nu = 1.0$  and  $\nu = 0.5$ . The horizontal axis shows b and the vertical axis shows  $D_q$ . We plotted the cases of  $q = 0.5\sigma, \sigma, 2\sigma$ , and  $4\sigma$ . The graph shows the attenuation from the detection value from the content embedded using Method A without post-processing. The embedding strength is  $a = 0.2\sigma$  when b = 0. The graph shows that as the Q value becomes larger, the value b which provides the largest expected detection value becomes larger.



Figure 2. Theoretical expected detection value from the quantized contents,  $\nu = 1.0$ 



Figure 3. Theoretical expected detection value from the quantized contents,  $\nu = 0.5$ 

### 3.2.1 An embedding method not to embed in truncated ranges

In the MPEG domain, if the quantization value for each DCT index is known prior to embedding, we can improve the detection result by embedding only for DCT coefficients which are greater than q/2 after embedding. However, it's hard to predict the Q value because, unlike the classical JPEG case, the Q values change for each macroblock, and furthermore, DVD titles are usually created at professional studios in which the quantization values are manually adjusted. But, if the compression ratio (BPS) is known prior to embedding, we can predict the approximate quantization value. By predicting this value, we can improve the detection result from the compressed contents by not embedding for DCT coefficients which are less than q/2 before and after embedding, and instead, embedding more strongly for other coefficients.

In the MPEG domain,  $q_k$ , Q value for DCT coefficients



### Figure 4. Average of the *quantizer\_scale*, predicted vs observed

of index k, is calculated as follows [10]:

$$q_k = qs \times Qtable[k]/16 \tag{18}$$

where qs is the *quantizer\_scale* which is the scaling factor defined for each macroblock and common to all the indexes, and *Qtable* is the quantizer matrix, which is a default (fixed) quantizer table or a table loaded in the sequence header or quantizer matrix extension header. By using the method recommended in TM5 [11] and tuning up the value for each encoder, we can predict the approximate value of qs for each macroblock.

Figure 4 shows the predicted (calculated) and observed (actual) averages of qs using ten 15-second video clips which include several kinds of material such as movies, animation, landscapes, etc. This shows that our prediction works well.

Using qs predicted above, we introduce a new embedding function  $w_{new}(x)$  as follows:

$$w_{new}(x) = \begin{cases} 0, & |\beta w(x) + x| < q/2 - \Delta \\ & \text{and } |x| < q/2 - \Delta \\ & \beta w(x), & \text{otherwise} \end{cases}$$
(19)

where  $\Delta$  is the margin in case the prediction of the quantization value doesn't match, and  $\beta \geq 1$ .

To hold V of the watermarked contents constant,  $\beta$  must satisfy the following equation.

$$V = \int_{-\infty}^{\infty} w^2(x) f(x) dx$$
  
=  $\beta^2 (\int_{-\infty}^{-(q/2 - \Delta)} w^2(x) f(x) dx$   
+  $\int_{\gamma}^{\infty} w^2(x) f(x) dx)$  (20)

where

$$\gamma + \beta w(\gamma) = q/2 - \Delta \tag{21}$$



Figure 5. Theoretical expected detection value from the quantized contents,  $\nu = 0.5$ ,  $\Delta = q/4$ , Method  $M_0$  vs  $M_1$ 

Figure 5 shows the theoretically expected detection value  $D_q$  from the quantized contents under the given V with PDF of  $\nu = 0.5$ ,  $\Delta = q/4$ . The horizontal axis shows b and the vertical axis shows  $D_q$ . The graph shows the attenuation from the detection value from the content embedded using Method A without post-processing. The embedding strength is set to  $a = 0.2\sigma$  when b = 0. Hereafter, we call this method, i.e. the method not embedding in the range where is truncated to 0 by quantization, Method  $M_1$ , and the method that embeds in the range which is truncated to 0 by quantization method  $M_0$ . From this graph, we can see that Method  $M_1$  improves the detection result when b is small.

### 3.3 Embedding robust against VHS recording

VHS recording includes several features that attenuate the watermark, such as Band Pass Filtering (hereafter, BPF) including D/A and A/D conversion and jitter. Normally, BPF is a big issue for the watermark signal attenuation but it can be analyzed by using a frequency response model. Figure 6 is an example of the horizontal frequency response for a series of conversions: D/A conversion, VHS recording/play back, and A/D conversion.

Besides these features, VHS has characteristics that cannot be analyzed by a simple frequency response model. For example, the horizontal frequency response at 1.5 MHz in Figure 6 is approximately -1dB, but the actual attenuation is often worse than that. These are often caused by Line Noise Cancellation (LNC) and High Pass Noise Cancellation (HPNC).

LNC is a function to interpolate the two adjacent lines when there is little motion in the video to improve the video image quality when playing it back. Let p(x, y) and p'(x, y)denote the luminance at pixel (x, y) during recording and



Figure 6. Horizontal frequency response of a VHS (NTSC), including D/A and A/D conversions

play back, respectively. Then, when LNC is enabled, the following equation holds:

$$p'(x,y) = (1-t)p(x,y) + tp(x,y+1)$$
(22)  
$$0 \le t \le 1/2$$

To analyze the attnuation of the watermark signal by LNC, let  $M_i = W_i = (m_{i_{xy}})$ , and

$$m_{i_{xy}} = s\cos(r_x x + r_y y + \phi_i) \tag{23}$$

where  $\phi_i$ 's are pseudo random phases, and let  $D_l(t)$  denote the expected detection value after LNC is applied with t in Eq. (22) to the content embedded using Method A. Then  $D_l(t)$  is calculated as follows:

$$D_l(t) = \sqrt{N} \frac{\mu_l(t)}{\sigma_l(t)}$$
(24)

where

$$\mu_l(t) = (1 - t + t\cos(r_y))\mu_l(0)$$
  

$$\sigma_l^2(t) = ((1 - t)^2 + t^2 + 2t(1 - t)\cos(r_y))\sigma_l^2(0)$$
(25)

Let  $\eta(t)$  denote the attenuation by LNC. Then  $\eta(t)$  is written as

$$\eta(t) = \frac{\mu_l(t)\sigma_l(0)}{\mu_l(0)\sigma_l(t)}$$
(26)

Figure 7 shows the values of  $\eta(t)$  when  $t = 1/2, 0 \le r_y \le \pi$ .

BPF (Figure 6) is a problem caused by the attenuation of the horizontal frequency, and LNC (Figure 7) is a corresponding problem for the vertical frequency, and these lead to the problem of choosing the embedding and the detection



Figure 7. Attenuation due to LNC, t = 1/2

patterns from a frequency range that doesn't cause much attenuating. But, we also need to investigate the characteristics of the video contents. When the patterns include significant amounts of low vertical frequency components, attenuation by LNC will be reduced, but  $\sigma$  tends to increase, because video clips often include vertical lines and, as a result, the detection value will be reduced. Also, when the patterns include significant amounts of low horizontal frequency components, attenuation caused by BPF will be reduced, but horizontal lines will similarly cause  $\sigma$  to become larger. The horizontal frequency range which isn't attenuated by BPS and whose  $\sigma$  is not too large is between 1 MHz and 2 MHz, but this frequency range is attenuated by another feature, HPNC.

HPNC is a feature to remove some small high horizontal frequency components (mainly 1 to 2 MHz) from the video to improve the image quality when playing it back. This means that the watermarks in this frequency range that are embedded in solid portions of the image tend to be removed by this feature, and the watermark signal is attenuated. In figure 8, the solid line is the observed attenuation and the dotted line is the simulated attenuation with  $g(x) = c_0(1 - c_1 \exp(-c_2|x/\sigma|^{c_3}))$  as a result of fitting. This is just an attenuation of a horizontal line by HPNC, and we need to know the attenuation for each  $M_i$  we use for detection. But it is still expected that the attenuation has the similar characteristics as g(x) above.

Let h(x),  $f_{off}$ , and  $D_h$  denote the attenuation function of the HPNC that converts a value x to h(x), the PDF from the contents embedded and VHS converted with HPNC disabled (OFF), and the expected detection value after VHS converted with HPNC enabled (ON), respectively. Then  $D_h$ is calculated as follows:

 $D_h = \sqrt{N} \frac{\mu_h}{\sigma_h}$ 

(27)

where

$$\mu_h = \int_{-\infty}^{\infty} h(u) f_{off}(u) du$$



Figure 8. Attenuation by HPNC at 1.5MHz range



Figure 9. Theoretical expected detection value when HPNC is OFF and ON

$$\sigma_h^2 = \int_{-\infty}^{\infty} h^2(u) f_{off}(u) du - \mu_h^2 \qquad (28)$$

Figure 9 shows the theoretical expected detection value  $D_h$  when HPNC is OFF and ON with  $h(x) = 0.9(1 - 0.95 \exp(-0.02(|x|/\sigma)^{0.7}))x$ , which is an observed function using a set of  $M_i$ 's, at  $\nu = 0.5$ , assuming that  $f_{off} = f_e$ . The embedding strength is  $a = 0.2\sigma$  when b = 0. The graph shows the attenuation from the detection value from the content embedded using Method A, with HPNC OFF. From this graph, we can see that b = 0.2 provides the largest detection value for the h(x) described above.

### 4 Experimental results

In order to evaluate the approaches discussed above, we embedded the watermark in video content and observed the detection results after performing following postprocessing: 1) No post-processing, 2) MPEG compression, 3) VHS recording.



Figure 10. Detection values from the contents without post-processing, Method A vs Method B

For all the experiments, we used the same video clips as those used for the experiment for Figure 4.

#### 4.1 No post-processing

We embedded the contents with Method A and Method B with various Vs of the watermarked contents, and detected without any post-processing. The PDF for the original contents had the value  $\nu = 0.45$ . Figure 10 shows the theoretical and experimental detection results. The horizontal axis shows the average of the PSNR of the watermarked contents, and the vertical axis shows the attenuation of the theoretical detection value from the contents embedded using Method A, PSNR=30dB.

This shows that Method A provides better results than Method B as is discussed in section 3.1 for all the PSNRs, and the theoretical results are close to the experimental results.

### 4.2 MPEG compression

We embedded the contents using Method  $M_0$  and Method  $M_1$  with various bs. PSNR of the watermarked contents were fixed at 45dB. The margin  $\Delta$  for Method  $M_1$  was set to q/4. The PDF of the original contents had the value of  $\nu = 0.45$ . The contents were embedded, MPEG compressed by CBR, and detected after decoded to YUV domain. We compressed with the compression ratios whose average of the Q values is  $q/\sigma = 1.14$ . Figure 11 shows the theoretical and experimental detection results. The horizontal axis shows b and the vertical axis shows the attenuation from the detection value from the content embedded using Method A without post-processing. The theoretical curve simulates the case detected only from I pictures, but the experimental result is the detection average from all of the of



Figure 11. Detection value from the MPEG compressed contents, Method  $M_0$  vs Method  $M_1$ 

decoded pictures including I, P, and B pictures. Therefore, the experimental results are 1 to 3dB worse than the theoretical curve, but, by shifting the graphs of the experimental results 1 to 3dB up, they become close to the theoretical curve. This figure shows that as is theoretically predicted, Method  $M_1$  improves the detection result at smaller b, and the b that provides the maximum detection values is around 025 to 0.5 for Method  $M_0$  and 0 for Method  $M_1$ .

### 4.3 VHS recording

We embedded the contents with various bs, VHS recorded and played back with HPNC ON and OFF, and observed the detection results. We used a VHS recorder introduced in section 3.3. PSNR of the watermarked contents were fixed to be 45dB. The PDF for the original contents had the value  $\nu = 0.45$ . Figure 12 shows the theoretical and experimental detection result with HPNC ON. The graph shows the attenuation from the detection value from the content embedded using Method A, VHS recorded and played back with HPNC OFF. From this graph, we can see that Method A provides the best detection result in this case, and that the experimental curve is close to theoretical curve.

### 5 Conclusion

In this paper, we discussed watermarking schemes to improve the robustness under the condition that the fidelity of the watermarked contents is predefined. First, we discussed an embedding scheme that provides the largest expected detection value from the contents without post-processing, and both theoretical and experimental results proved that Method A provides larger detection value. As for robustness with regards to post-processing, we focused on the



Figure 12. Detection after VHS conversion, HPNC ON

most common post-processing, MPEG compression and VHS recording. We first analyzed the attentions caused by these forms of post-processing, and recommended embedding schemes that provide large detection values after these specific conversions.

Specifically as regards MPEG compression, we confirmed that Method  $M_1$ , which does not embed in the range which is truncated by quantization, improves the detection value much, and the *b* which provides the largest detection value exists near 0 when embedded using Method  $M_1$  and between 0 and 1 when embedded using Method  $M_0$ , and this *b* becomes larger as the Q value becomes larger. So, for applications whose Q values are large, such as low bit coding, we need to carefully choose the *b* to maximize the detection value.

Specifically as regards VHS recording, we analyzed the effects of LNC and HPNC, and offered a method to find an optimizing embedding method. LNC is the problem of how to choose the detection pattern, and the attenuation of HPNC can be simulated with the attenuation function h(x). In the case of the VHS recorder we used, Method A was the best case, and it was proved from both theoretical and experimental points of view.

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