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Research Report

(0,G/I) codes are a subclass of MTR codes

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Abstract: Maximum transition run (MTR) codes are defined that avoid quasicatastrophic error propagation in Viterbi detectors matched to partial response channels with spectral nulls both at DC and the Nyquist frequency. It is shown that the new class of MTR codes includes (0,*G*/*I*) codes as a subclass.

Introduction: (0,*G/I*) codes [1] are constrained codes that facilitate gain and timing recovery and avoid quasicatastrophic error propagation [2] in Viterbi detectors matched to partial response channels with spectral nulls both at DC and the Nyquist frequency. The *G*-constraint limits the run of 0's in the encoder output to *G*, whereas the *I*-constraint limits the run of 0's in both the odd and even interleaves of the encoder output to *I*. In general, $G \leq 2I$. Figure 1 shows the (0,*G*/*I*) encoder followed by the $1/(1 \oplus D^2)$ precoder and the partial response channel represented by the polynomial $h(D) = (1 - D^2)f(D)$, where $f(D)$ is a generalized partial-response polynomial that may have spectral nulls, if any, only at DC and/or the Nyquist frequency. At the input of the partial response channel in this configuration, the *G*-constraint limits the runs of identical and alternating symbols to *G*+2, whereas the *I*-constraint limits the run of identical symbols in both interleaves to *I*+1.

MTR(*j*=2,*k*) codes eliminate dominant error sequences to enhance the performance of magnetic recording systems [3]. At the input of the $1/(1 \oplus D)$ precoder, the *j*-constraint limits the run of 1's in the MTR encoder output sequence to *j*, whereas the *k*-constraint limits the run of 0's in the MTR encoder output sequence to *k*. MTR(*j*=2,*k*) codes, however, suffer from rate loss. MTR codes satisfying a *j*=2 or 3 time-varying constraint [4], [5] as well as MTR(*j*=3,*k*) codes [6] both allow a further increase in code rate. Until now, (0,*G/I*) and MTR codes have been studied independently. The purpose of this paper is to introduce a new constraint into MTR codes in order to avoid quasicatastrophic error propagation, and consequently to reveal a connection between (0,*G/I*) and MTR codes.

Pairs constraint: Quasicatastrophic error propagation is inherent in maximum likelihood sequence detection for partial response channels with spectral nulls. In general, MTR(*j*,*k*) codes do not avoid quasicatastrophic error propagation in Viterbi detectors matched to partial response channels with spectral nulls both at DC and the Nyquist frequency. The *k*-constraint avoids channel input error sequences that have spectral energy only at DC, whereas the *j*-constraint avoids channel input error sequences that have spectral energy only at the Nyquist frequency. An additional constraint is needed to limit the maximum length of channel input error sequences of type ... 1 0 1 0 1 0 ... and ... –1 0 –1 0 –1 0 ... that have spectral energy both at DC and the Nyquist frequency. To this end we introduce a constraint at the input of the $1/(1 \oplus D)$ precoder that limits the maximum number of consecutive pairs of 0's or 1's ("twins") in the MTR encoder output to *t* pairs. For example, the sequence ... 0 0

1 1 0 0 1 1 1 1 ... would be allowed if t=5, whereas it would not be allowed if t=4. The t-constraint limits the length of sequences of type ... a_1 1 a_2 1 a_3 1 a_4 1 a_5 ... and ... a_1 0 a_2 0 a_3 0 a_4 0 a_5 ... , $a_i \in \{0,1\}$, at the input of the partial response channel to $2t+3$. It is worth pointing out that in general $j \le 2t+1$ and $k \le 2t+1$. Figure 2 shows the MTR(j,k,t) encoder followed by the $1/(1 \oplus D)$ precoder. A similar constraint has been used in [4] and [6] to eliminate periodic quasicatastrophic sequences of type ... 0 0 1 1 0 0 1 1 0 0 ... at the input of the $1/(1 \oplus D)$ precoder for E^2PR4 and modified E^2PR4 systems. Note though that for $j \ge 4$ these sequences are not necessarily periodic as claimed in [4].

Capacity of MTR sequences: Sequences satisfying a MTR(j,k,t) constraint can be exhaustively characterized by state transition diagrams. We label the states with 3-tuples (x,y,z), where x is the runlength of 1's ending at the current state, y is the runlength of 0's ending at the current state, and z is the runlength of sequences of type ... a_5 a_5 a_4 a_4 a_3 a_3 a_2 a_2 a_1 a_1 or ... a_5 a_5 a_4 a_4 a_3 a_3 a_2 a_2 a_1 ending at the current state. For example, ...1 0 0 0 1 1 1 1 leads to the state (4,0,6), whereas ...1 0 0 0 1 1 1 1 0 leads to the state (0,1,7), indicating that there is a transition from the state (4,0,6) to the state (0,1,7) labeled with 0. The capacity of a constrained system represents the maximum achievable code rate of an encoder generating constrained sequences. It is given by $\log_2 \lambda_{\max}(A)$, where $\lambda_{\max}(A)$ is the greatest real eigenvalue of the nonnegative adjacency matrix A associated with a finite-state transition diagram that presents the constrained system [7]. The capacity of the constrained system of MTR(j,k,t) sequences has been computed from the adjacency matrix associated with state transition diagrams for MTR(j,k,t) constraints that have been constructed. Tables 1-4 list the capacity of MTR(j,k,t) constraints for various values of j, k, and t by truncating the numbers after the sixth digit following the decimal point.

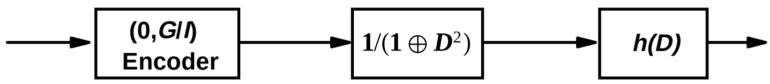

Fig. 1 Block diagram of (0,G/I) constrained partial response channel.

*Connection between (*0,*G/I) and MTR codes:* For our purposes here, the (0,G/I) code is defined as the set of all allowable bi-infinite sequences at the output of the $1/(1 \oplus D^2)$ precoder following the (0,G/I) encoder shown in Fig. 1. Similarly, the MTR(j,k,t) code is defined as the set of all allowable bi-infinite sequences at the output of the $1/(1 \oplus D)$ precoder following the MTR(j,k,t) encoder shown in Fig. 2. In other words, the code is defined in both cases as the constrained system at the partial response channel input.

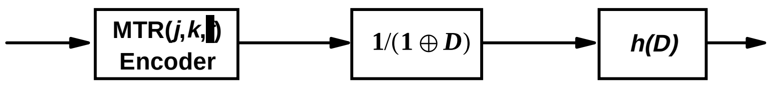

Fig. 2 Block diagram of MTR(j,k,t) constrained partial response channel.

Proposition: (0,G/I) codes are a subclass of MTR(j,k,t) codes.

Proof: The key observation leading to this result is that the $1/(1 \oplus D^2)$ precoder following the (0,G/I) encoder can be represented as the serial concatenation of two $1/(1 \oplus D)$

precoders as shown in Fig. 3. It can be readily shown that the G-constraint at the output of the $(0,G/I)$ encoder translates into the j-constraint, $j = G + 1$, as well as the k-constraint, $k = G + 1$, at the output of the first $1/(1 \oplus D)$ precoder in Fig. 3. Similarly, the I-constraint at the output of the $(0,G/I)$ encoder translates into the t-constraint, $t = I$, at the output of the first $1/(1 \oplus D)$ precoder in Fig. 3. Therefore, the $(0,G/I)$ code is identical to the MTR(G+1,G+1,I) code. Clearly then, the class of $(0,G/I)$ codes is within the larger class of MTR(j,k,t) codes.

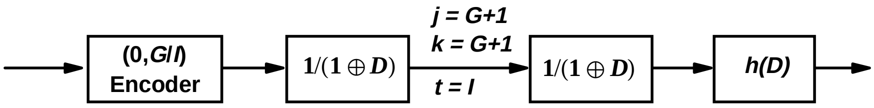

Fig. 3 Connection between $(0,G/I)$ and MTR(j,k,t) constraints.

As the capacity of a constrained system is not affected by the presence of a precoder, a direct consequence of the proposition is that the capacity of the $(0,G/I)$ constrained system is equal to the capacity of the MTR(G+1,G+1,I) constrained system. This can be readily verified by comparing the capacity of the $(0,G/I)$ constraint (see e.g. [8]) with the capacity of the corresponding MTR(G+1,G+1,I) constraint in Tables 1-4. Finally, the connection between $(0,G/I)$ and MTR codes suggests a new approach for constructing a $(0,G/I)$ code that employs the $1/(1 \oplus D)$ precoder instead of the commonly used $1/(1 \oplus D^2)$ precoder.

Conclusion: MTR codes that satisfy an additional constraint to avoid quasicatastrophic error propagation in partial response systems have been introduced. The capacity of this new class of MTR(j,k,t) codes has been computed. It has been demonstrated that $(0,G/I)$ codes and MTR(j,k,t) codes are intimately related.

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TABLE 1

Capacity of MTR(2,*k*,*t*) constraints, *j*=2

	k							
t	1	2	3	4	5	6	7	8
	0.405685	0.551463	0.637088	--	--	--	--	--
	0.405685	0.650899	0.747205	0.772712	0.786634	--	--	--
	0.405685	0.679286	0.781989	0.814167	0.831134	0.836644	0.839656	--
	0.405685	0.688789	0.790071	0.828494	0.847904	0.855144	0.859157	0.860525
	0.405685	0.692203	0.793352	0.833940	0.854013	0.862591	0.867323	0.869204
1	0.405685	0.693470	0.794181	0.836133	0.856337	0.865673	0.870693	0.872960
2	0.405685	0.693948	0.794533	0.837013	0.857283	0.866995	0.872139	0.874618
3	0.405685	0.694130	0.794624	0.837377	0.857653	0.867569	0.872753	0.875360
4	0.405685	0.694199	0.794662	0.837527	0.857803	0.867820	0.873022	0.875694
5	0.405685	0.694225	0.794672	0.837589	0.857863	0.867929	0.873139	0.875846
6	0.405685	0.694235	0.794677	0.837614	0.857887	0.867978	0.873190	0.875915
7								
8								
9								
10								
11								

TABLE 2

Capacity of MTR(3,*k*,*t*) constraints, *j*=3

	k							
t	1	2	3	4	5	6	7	8
1	0.551463	0.637088	0.694241	--	--	--	--	--
2	0.5514630 .551463	0.747205	0.834520	0.850147	0.858590	--	--	--
3	0.551463	0.781989	0.867061	0.893311	0.907863	0.910964	0.912597	--
4	0.551463	0.790071	0.875696	0.907137	0.922734	0.928302	0.931518	0.932231
5	0.551463	0.793352	0.878139	0.911889	0.928056	0.934762	0.938345	0.939691
6	0.551463	0.794181	0.878850	0.913612	0.929879	0.937201	0.941002	0.942654

8	0.551463	0.794624	0.879120	0.914476	0.930778	0.938539	0.942439	0.944341
9	0.551463	0.794662	0.879138	0.914563	0.930865	0.938689	0.942599	0.944543
10	0.551463	0.794672	0.879144	0.914595	0.930896	0.938749	0.942662	0.944626
11	0.551463	0.794677	0.879145	0.914607	0.930908	0.938772	0.942687	0.944661

TABLE 3

Capacity of MTR(4,*k*,*t*) constraints, *j*=4

	k							
t	1	2	3	4	5	6	7	8
	0.617446	0.772712	0.850147	0.864320	0.871955	--	--	--
	0.617446	0.814167	0.893311	0.915723	0.928185	0.930853	0.932243	--
	0.617446	0.828494	0.907137	0.934253	0.948104	0.952674	0.955281	0.955858
	0.617446	0.833940	0.911889	0.941533	0.955975	0.961585	0.964625	0.965677
	0.617446	0.836133	0.913612	0.944539	0.959137	0.965376	0.968639	0.969965
2	0.617446	0.837013	0.914243	0.945812	0.960445	0.967037	0.970393	0.971883
3	0.617446	0.837377	0.914476	0.946359	0.960999	0.967776	0.971168	0.972754
4	0.617446	0.837527	0.914563	0.946595	0.961233	0.968108	0.971514	0.973154
5	0.617446	0.837589	0.914595	0.946698	0.961333	0.968258	0.971670	0.973338
6	0.617446	0.837614	0.914607	0.946742	0.961375	0.968325	0.971739	0.973423
7	0.617446	0.837625	0.914612	0.946762	0.961393	0.968356	0.971771	0.973462
8								
9								
10								
11								
12								

TABLE 4

Capacity of MTR(5,*k*,*t*) constraints, *j*=5

	k							
t	1	2	3	4	5	6	7	8
2	0.650899	0.786634	0.858590	0.871955	0.879146	--	--	--
3	0.650899	0.831134	0.907863	0.928185	0.939505	0.941942	0.943204	--
4	0.650899	0.847904	0.922734	0.948104	0.961366	0.965442	0.967752	0.968263
5	0.650899	0.854013	0.928056	0.955975	0.969628	0.974822	0.977687	0.978607
6	0.650899	0.856337	0.929879	0.959137	0.972929	0.978755	0.981801	0.983009
7	0.650899	0.857283	0.930538	0.960445	0.974273	0.980449	0.983581	0.984952

8	0.650899	0.857653	0.930778	0.960999	0.974828	0.981189	0.984354	0.985820
9	0.650899	0.857803	0.930865	0.961233	0.975059	0.981515	0.984693	0.986210
10	0.650899	0.857863	0.930896	0.961333	0.975156	0.981659	0.984842	0.986387
11	0.650899	0.857887	0.930908	0.961375	0.975196	0.981723	0.984908	0.986467
12	0.650899	0.857897	0.930912	0.961393	0.975213	0.981752	0.984938	0.986504