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Research Report

European-Style Forward Derivatives for Telecom Commodities

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European-Style Forward Derivatives for Telecom Commodities

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Abstract

Bandwidth commodity markets are developing, and (dark) fiber swaps are not uncommon. Derivatives, especially derivatives of forward contracts, are likely to be important for risk management and hedging. However there is currently no method available to price contingent claims where the underlying asset is a claim on some part of a network and non-storable. To date, geographical (no-)arbitrage has not been included in the pricing of contingent claims on forwards. We present a method for pricing European-style contingent claims on forwards using both the usual no-arbitrage conditions and geographical no-arbitrage. We make appropriate allowances for the non-storability of the underlying asset (point-to-point) bandwidth and the storability of forward-based contingent claims. We give an example of pricing a call option on a forward contract with a range of underlying network topologies based on realistic forward prices. For this example, a call option on a 10 month forward, we find the option price to be relatively insensitive to the network topology (less than 10%) for a range of strike prices. We speculate that this is due to the long date leading to geographical arbitrage effects being reduced to log-Normality by the Central Limit Theorem. This would imply that increasing differences might be observed for shorter-dated contingent claims. We conclude by discussing the steps required in forward curve modeling to move to any-style contingent claims on network capacity.

1 Introduction

Bandwidth commodity markets are developing as telecommunications moves from proprietary networks to commodity status (see [Gro00] for an introduction to bandwidth markets and [Hul99] for financial terminology). The economic value of these markets could occupy the same fraction of total telecommunications sales as energy derivatives do relative to total energy sales. Derivatives, especially derivatives of forward contracts are likely to be important for risk management and hedging. In fact forward derivatives will be of larger importance for bandwidth than for many conventional commodities because bandwidth cannot be stored for later use; it is a non-storable commodity. However neither forward pricing of bandwidth nor derivative pricing of forward contracts is clearly understood. This paper makes first steps in showing how to price a particular set of forward-derivative contracts. A forward contract is a contract in which capacity is bought today to be used starting at a fixed date in the future and for a fixed duration.

Today the instruments traded on bandwidth markets (mostly over-the-counter or OTC) are typically forward contracts covering long (months to years) periods, see for example Figure 1. This is partly due to inefficient negotiation and contract settlement mechanisms. New switching technologies and public pooling and inter-connection points are expected to hasten automation towards more liquid bandwidth markets and shorter contract periods as well as the development of a spot market. Additionally (dark) fiber swaps are not uncommon[1]. We expect forward derivatives, especially forward call options, to appear given their appropriateness for risk management. Call options on forwards provide the right, but not the obligation, to use capacity in the future starting at some fixed date for a given duration upon payment of some price agreed when the contract is established. Possibly swaptions between different network providers will develop later together with more specialized instruments with unique applicability to a networked commodity. This would mimic the energy markets, which developed specialized quantity-flexible instruments.

For the purposes of this paper we define commodity bandwidth as contracts for point-to-point capacity (e.g. T3 = 45Mb/s, with defined delay, jitter, packet loss, etc). These commodity contracts have standardized, defined start times (e.g. every 15 minutes / hour at :00 / day at 00:00) and lengths (e.g. 15 minutes / 1 hour, day, month, year).

Bandwidth is almost unique as a commodity in that it is not only bought and sold online but may also be used online. The most similar other commodity with an immediate link between trading and use, outside pure financial commodities such as interest rate products, is electricity. Thus, we use methods and insights developed for modeling electricity prices, forward curves and derivatives (for example [CS00], [JRT98], [Joy00], [MT99]) and augment these with the particular features of point-to-point bandwidth. The most important common feature of telecom commodities and electricity (where there is no storage capacity, e.g. in the USA and Australia) is that they are both non-storable commodities. That is, there is no way to store bandwidth and use it later. This has important technical and practical implications that we will explore later. This non-storability is the biggest difference from conventional commodities (e.g. [Sch97], [SS00]) and especially from interest rate modeling [JW00] although that machinery remains very useful. The most important additional feature

[1] Enron with Frontier Communications: *http://www.convergedigest.com/Daily/v6text/v6n014.txt*, World-wide Fiber with KPN Quest: *http://www.convergedigest.com/InternetMediaCenters.htm*, EPIK with Digital Teleport: *http://stlouis.bcentral.com/stlouis/stories/2000/08/28/ daily5.html*

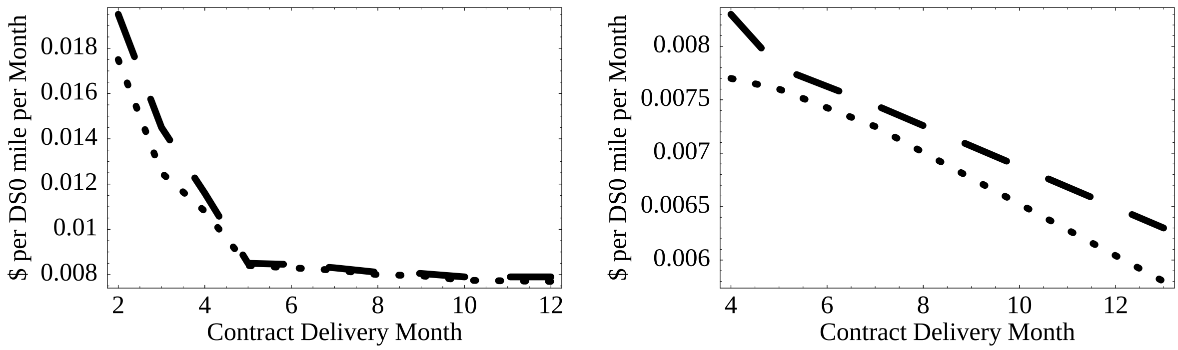

Figure 1: Examples of forward curves for bandwidth for DS3 capacity NY-LA in February 2001 expressed in DS0 units (DS3 = 672 DS0's), delivery months start February 2001 and go to January 2002. Dotted lines are buy quotes, dashed lines sell quotes. *Left panel:* forward contracts of 1-month duration; *right panel:* forward contracts of 1-year duration at monthly rate. Source: *Enron Broadband Services*

for modeling bandwidth, compared to other non-storable commodities, comes from the network aspect in that, in general, several paths are available at a given quality of service between any two points. We will illustrate this further in Section 3.1. This work on forward and derivative modeling for telecom commodities is an extension of our earlier work on stochastic processes for bandwidth spot prices [KC01], where we developed techniques for combining network aspects with more conventional price modeling. The network aspects had not previously been included (e.g. [CCN00]).

In this paper we show how to price European-style contingent claims on telecom commodity forward contracts combining previous work on non-storable commodities, network effects and bandwidth spot-price modeling. We also go through an example calculation for a call option on a forward contract based on realistic information (see Figure 1). European-style claims have a fixed date when they can be exercised. This is in contrast to more general-style claims that may offer a range of exercise possibilities (e.g. American or Bermudan options). Our new contribution is the explicit inclusion of network effects in pricing contingent claims on forwards. This is a first step towards more general contingent claims pricing and hedging for non-storable network commodities.

1.1 Technical Remarks

By convention (e.g. see [Hul99], Chapters 1-3) a forward contract is a transaction between two individuals whereas a futures contract is an exchange-traded instrument. Both contracts are for the future delivery of an asset at a fixed price agreed upon at the start of the contract. Because exchanges usually have margin requirements adjusted daily to reduce default risk, futures and forwards have different sensitivities to daily interest rate fluctuations. As interest-rate modeling is not the objective of this paper we will assume a constant risk-free interest rate r. Thus forward and futures prices are the same, and we will use the two terms interchangeably.

We assume the usual context of a filtered probability space $(\Omega, \mathcal{F}, \mathcal{P})$, with $\mathcal{P}$ the real-world probability measure and events $\mathcal{F}(t)$ revealed over time $0 \leq t \leq T^*$. Forward contracts $F(t,T)$ have maturities $t \leq T \leq T^*$, and trades occur in a fixed time interval $[0, T^*]$ where T^* is a fixed time. We will use $S(t)$, $0 \leq t \leq T^*$ to denote the spot price. This is the usual setting (see e.g. [MT99]) for discussions of forward and derivative pricing.

1.2 Network Model

We represent the bandwidth trading market by a contract graph $G(N, L)$, where N are the nodes of the graph and L are the links between the nodes. A link in the contract graph represents an *indivisible* traded contract for bandwidth between two nodes. For concreteness the reader may imagine that a standard contract is for T3 (45Mb/s) capacity (with defined delay, jitter, packet loss, etc) and several time-scales of contracts are available with standardized starting times (e.g. every 15 minutes / hour at :00 / day at 00:00) and lengths. This degree of liquidity is not yet present (or at least not yet widely observed) for bandwidth trades but, considering how electricity markets developed, may be an eventual situation. Nodes are public pooling or inter-connection points where many carriers are present with the capability to arbitrarily cross connect between networks (several of these have already been deployed worldwide). Thus paths may be assembled in a supplier-neutral manner.

Bandwidth is offered for sale on the market in the form of point-to-point contracts that will generally comprise several underlying links at the network or physical layer. Thus the contract graph is an abstract view of network connectivity at the level of traded contracts, and links/paths in the contract graph do not map one-to-one to physical or network layer links/paths.

The contract graph abstraction is useful for studying the network effects inherent in a bandwidth market. That is, price development on any link is *not* independent of the prices of neighboring links as there is generally more than one way to connect two locations and buyers will choose the cheapest path if the other factors are equal (e.g. QoS).

2 Futures Price Properties

2.1 Martingales

A martingale, say $M(t)$, is a stochastic process whose expected future value is the same as its current value, i.e. for $t_2 > t_1$,

$$E[M(t_2)|\mathcal{F}(t_1)] = M(t_1),$$

where $\mathcal{F}(t_1)$ describes everything that is known up to t_1, including, of course, $M(t_1)$. When we use the risk neutral measure this means (amongst other things) that we are discounting the expected future value back to the present. If this value were not the same as the current value then there would be the opportunity for a certain win at no cost, i.e. an arbitrage opportunity. There are additional technical details to do with the fact that this only makes sense when the risk neutral measure is unique, see [MR97] for details.

2.2 Futures are Martingales

We take the view that the primary traded commodities in the bandwidth telecom commodity market will be futures contracts with fixed maturities and strike prices. The prices of individual futures contracts over time must be martingales under the risk neutral measure (which we will label $\mathcal{Q}$ and which is equivalent to $\mathcal{P}$), otherwise the futures market contains arbitrage opportunities (see [MT99] or [MR97] Chapter 10.1 for details). The risk neutral measure is a construct used in the pricing of derivative contracts, and we will show how to construct it in our case in Section 4 (see also [MR97] or [Nef00] for background). This observation is independent of whether the asset on which the forward is written is storable or non-storable because the forward contract itself is storable.

2.3 Spot Prices are Not Martingales

Because bandwidth is non-storable there is no reason to expect that the spot price will be a martingale under the risk neutral measure. This does not mean that the market now has arbitrage opportunities because the spot is not a tradeable asset. In fact the spot process does not exist in the sense that the underlying asset can be bought at one time and sold at another time — which is usually the definition of tradeable. A spot process can be said to exist when it is possible to substitute the asset bought at one time for the asset bought at another time. For example a share of IBM stock bought today can be substituted for one bought tomorrow, and the person buying cannot distinguish between the two.

In more technical language non-storability means that the spot commodity cannot be part of any self-financing strategy (see [MR97] for details on this). This limitation means that spot process $S(T)$ is really just the set of forward prices $F(T,T)$ at maturity. In this limited sense the spot process does exist and is observable. The forward prices $F(T,T)$ at maturity, for any maturity $0 \leq T \leq T^*$, are equivalent to the spot prices, $S(T)$, at that time — they are both the price at time T for delivery at time T. Note that we are assuming that delivery of forward assets is defined in the same way as delivery of the spot asset. Also note that $F(T,T)$, $0 \leq T \leq T^*$, need not be a martingale because it is non-storable. For non-storable commodities the equivalence between $F(T,T)$ and $S(T)$ is complete. These observations have also been made in the context of electricity forward curve modeling [MT99]. (We make the distinction that in our model our spot process is observable.)

2.4 Futures/Spot (non-)Linkage

For storable investment commodities, the conventional relationship between spot and forward prices is given by comparing forward prices with the strategy of buying the spot and holding it to maturity of the forward to arrive at

$$F(0,T) = S(0)e^{(r+u-y)T},$$

where $S(t)$ is the spot price at time t; r is the (constant) interest rate; u are the storage costs, which are a fixed proportion of the spot price, and y is the convenience yield or fudge factor required to make both sides equal. The fudge factor is generally rationalized as the benefit from actually holding the commodity, e.g. the ability to benefit from temporary increases in the price (shortages), see [Hul99]. A more sophisticated understanding of this term is to do with long and short term price dynamics and has been explored in [SS00] and [Sch97].

As noted in a previous paper on bandwidth [CCN00], this relationship does not hold when the underlying asset (bandwidth) is non-storable. Typically papers on electricity forward modeling start from the forwards as the traded commodities precisely because of this (e.g.

[MT99], [CS00]). For dealing with forward derivatives on bandwidth we also take this position that uses current observed forward prices as an input to the model.

3 European Futures Options and Derivatives

Futures options are perhaps the simplest options to price and we will consider the futures call options. At maturity the payoff from a European futures call option is $\max(F(T,T) - X, 0)$ where X is the strike price, and we have remarked that $F(T,T) \equiv S(T)$ so we also have the payoff equal to $\max(S(T) - X, 0)$. Recall that $S(T)$ is not required to be a martingale although $F(t,T)$ is with respect to (w.r.t. hereafter) t but not w.r.t. T. So in effect we can treat the futures call option as a call option on the spot. The Black-Scholes option pricing formula is not valid here because it assumes storability of the spot market asset. Equivalently the Black futures option formula assumes log-normality of the futures price distribution, which in general, owing to the network effects (see section 3.1.1), will also not be valid.

Now $\mathcal{Q}$ is the risk neutral measure, and we make the assumption that this exists and is unique. From the definition of the risk neutral measure in general we have

$$E_{\mathcal{Q}}[F(T,T)]e^{-rT} = F(0,T)e^{-rT} \tag{1}$$

because all investments must have the same expected return as the riskless rate under the risk neutral measure (by definition — otherwise it would not be the risk neutral measure). In other words $F(t,T)$ is a martingale under $\mathcal{Q}$. In effect Equation (1) is a definition of, or at least a constraint on, $\mathcal{Q}$. Which of the two it is depends on other assumptions, especially the number of parameters in the stochastic process describing forward price development. The e^{-rT} term on both sides is so we compare present values when the present values are taken at time zero.

Note that in Equation (1) we observe $F(0,T)$ today (time zero) on the market (which is the amount to be paid at time T, e^{-rT} converts this to the present value today). We also know r, which we assume is constant, and we know T, the maturity of the futures contract. Now $F(t,t) \equiv S(t)$ so if we possess a model for $S(t)$ then we can calibrate that model to the markets' expectation as expressed by Equation (1). For example when geometric Brownian motion ($dS/S = \mu dt + \sigma dW$) is used to describe stock prices this calibration process results in the stock price drift term μ being replaced by the risk free rate of interest r. After we have calibrated $S(t)$ to the markets' expectations (i.e. we have chosen $\mathcal{Q}$ so equation 1 holds) we have quite directly the price of a call option on a futures contract with strike price X as

$$E_{\mathcal{Q}}[\max(F(T,T) - X, 0)] \tag{2}$$

$$\equiv \quad E_{\mathcal{Q}}[\max(S(T) - X, 0)]. \tag{3}$$

Note that we have not yet proposed a form for $S(t)$, which is vital to these equations and the option pricing. So far all network effects (geographical arbitrage — see Section 3.1.1) have been hidden inside $S(t)$.

We may generalize Equation (2) to obtain the price of any contingent claim $\mathcal{C}(0)$ that depends only on the distribution of $F(T,T)$, some set of deterministic parameters $\mathcal{D}$ and is

European style (i.e. exercise only at T):

$$\begin{aligned} \mathcal{C}(0, \mathcal{D}) &= E_{\mathbb{Q}}[\mathcal{C}(F(T,T), \mathcal{D})] \qquad (4)\\ &\equiv E_{\mathbb{Q}}[\mathcal{C}(S(T), \mathcal{D})]. \qquad (5) \end{aligned}$$

This method of development of a contingent claim price is quite standard so we need to highlight how we used the unique characteristics of bandwidth in the development because there are several subtleties involved. Two important characteristics are non-storability and the potential for geographical arbitrage present in a network.

Non-storability, as we described in Section 2.4 breaks the link between $S(0)$ and $F(0,T)$ thus we cannot go directly from spot prices today to futures prices today and then to futures options. This means that futures must be traded commodities in their own right as this is the only way to assure future bandwidth and leads to Equation (1).

Geographical arbitrage has been used to describe the fact that at a given QoS there may be many equivalent routes between two locations and that the direct route, when this exists, may not always be the cheapest. (A route is a connected set of links.) The difficulty of discovering an alternate and cheaper route leads to possible arbitrage opportunities together with the dynamic nature of link prices. These arbitrage opportunities have been observed in [Che00] together with pseudo-polynomial algorithms for their detection [Che99]. This dynamic is contained within the spot price process and will be described next.

3.1 Spot Price Process $S(t)$

We have already remarked that the spot price process need not be a martingale under any particular measure because bandwidth is non-storable, hence buy-and-hold arguments that lead to martingale requirements are not applicable. However, spot prices for individual links in the contract graph of the telecommunications network are not independent of each other. An equivalent QoS domain may be defined for each link in a contract graph as those paths (sets of connected links) between the link endpoints that offer at least equivalent QoS to the link under consideration. Of course this equivalence will be defined in different ways according to the intentions of the bandwidth consumers and indeed even for the application mix intended for each purchase.

We propose the following spot price process (taken from [KC01]). This takes into account independent spot prices on individual links together with their interaction via load balancing when geographical arbitrage occurs.

$$\begin{aligned} X &= \log(S)\\ dX &= \eta(\overline{X} + GU - X)dt + \sigma dW + GdU + HdV\\ &\quad +\alpha dA \qquad (6)\\ d\overline{X} &= -\nu dt + \rho dZ \qquad (7) \end{aligned}$$

These equations may seem complicated at first glance so we will explain each part in detail. First consider when G, U and V are identically zero and $\overline{X}$ is constant, then we have:

$$\begin{aligned} X &= \log(S) \qquad (8)\\ dX &= \eta(\overline{X} - X)dt + \sigma dW + \alpha dA. \qquad (9) \end{aligned}$$

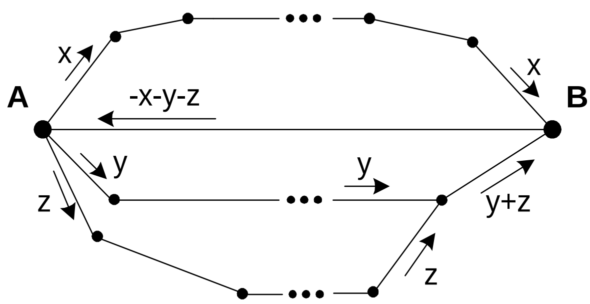

Figure 2: Shifts in spot demand described as network flows

Equation (8) says that we model the logarithm of spot prices and we do this to ensure that observed prices are never negative. Equation (9) is simply an Orstein-Uhlenbeck process with one extra term αdA at the end. An Orstein-Uhlenbeck process is Markov (no dependence on history, only on current values) and mean reverting. Markov-ness is widely assumed for markets because traders are expected to price in everything they could deduce from the past into the current price. Reversion of the price to a mean level is expected (and observed, see [Sch97]) for commodities because if prices go down high-cost suppliers go out of business reducing the supply and supporting the price. Equally when prices are high, more entrants into the market are encouraged and this reduces the upward pressure on prices. In bandwidth markets suppliers can choose how much of their capacity to sell on the market as opposed to keeping to provision their own needs. Equally, dark fiber can be lit.

We will make no further reference to G, U and V except to say that they enable modeling of price spikes and jumps. However the last equation $d\overline{X} = -\nu dt + \rho dZ$ specifies that the long-term mean towards which prices revert is not a constant but decreases over time (ν being taken as non-negative). The actual rate of decline may be regarded as known, in which case $\rho = 0$, or may be uncertain $\rho > 0$. This decline is a reflection of the observation that prices for bandwidth have declined over time because of advances in technology, both in single mode transmission and especially in wavelength division multiplexing. This decline is not expected to stop in the foreseeable future (three to five years) because the technology is still not mature and new technologies are continually being introduced (e.g. most recently all optical cross connects).

3.1.1 Geographical (no-)Arbitrage in $S(t)$

The last term in Equation (6) (or 9), dA, describes a stochastic process that is zero whenever no geographical arbitrage effects are present. When geographical arbitrage effects are present dA gives the appropriate correction to remove arbitrage from the previously observed prices in the contract network. The liquidity of the market is represented by α which gives the extent (i.e. timescale) with which this load balancing actually occurs. The above equations express the fact that market forces are acting at the same time as arbitrageurs acting to remove arbitrage opportunities (by profiting from them). Indeed they are part of the market, just modeled explicitly in this case.

We include a brief sketch of how these appropriate corrections to remove geographical arbitrage are included in dA but for full details and reasoning see [KC01]. We describe how to resolve an arbitrage case by treating demand as a network flow that is conserved point-to-point but allowed to shift from the direct link to alternative paths so as to achieve a load-balancing effect (see Figure 2).

Generally in geographical arbitrage situations there will be $m > 0$ paths connecting points A and B cheaper than p_{AB}. Let x_k be the amount of network flow (demand) shifting from the direct link to all links of path k that has equivalent QoS. Then the no-arbitrage state for the direct link and k can be written as

$$p_d - a_d \sum_{i=1}^{m} x_i = \sum_{i=1}^{n} p_i + \sum_{i=1}^{n} a_i x_k + \sum_{i=1, i \neq k}^{m} a_{ki} x_i,$$

where

$$a_{ki} = a_{ik} = \sum_{j \in k \cap i} a_j$$

is the sum of a's for all links that paths k and i have in common; $a_{(*)}$ is the change in price resulting from a unitary change in quantity demanded on link $(*)$; p_d is the price of the direct link, and p_i are the prices of QoS-equivalent paths. The complexity in the previous expression is necessary because paths will generally not be disjoint. Also,

$$x_i \geq 0,$$

which implies that we allow substitution of alternate paths for the direct link, but that we do not allow the alternate paths to act as substitutes for each other. This is because there may be no end-to-end flow on a substitute path even though each link may have flow. Thus in general end-to-end flow cannot move from a multi-link substitute path to anywhere, which suggests this restriction.

We show in [KC01] that this can be rewritten in the form

$$\mathbf{Ax} - \mathbf{b} = 0, \quad \mathbf{x} \geq 0.$$

This is a linearly constrained linear optimization problem and standard methods are available to solve it e.g. [BT97]. We derived a fast iterative solution method in [KC01] which avoids having to solve a constrained optimization problem and the requirement to know all the QoS-equivalent paths.

3.1.2 Effect of Geographical (no-)Arbitrage

We have introduced a significant complication into the spot price equation in the form of the αdA term, which acts to remove geographical arbitrage from a network of spot prices. Does the inclusion of this term change the actual spot price trajectory significantly for realistic parameter values and network topologies? We showed in [KC01] that geographical arbitrage occurs even for very simple network topologies. We also showed that over a year (252 trading days) there were significant changes to the mean and standard deviations of the observed prices. These changes could be of the same order of magnitude as the changes expected by considering spot price development without taking into account geographical (no-)arbitrage, over quite a range of parameter values. In general the size and direction of these changes depend on the network topology. We will now attempt to measure the importance of geographical arbitrage on option pricing.

4 Example: Forward Call Option Pricing

4.1 Setup

In this section we go through the steps to price a European-style call option on a forward contract for one-month DS3 capacity from New York to Los Angeles (NY-LA) starting 10 months away from February 2001 in December 2001 (or 210 trading days out) based on data from Figure 1 and realistic assumptions which we will detail. We consider three pricing situations[2]:

1. ignore network effects and the possibilities of geographical arbitrage completely;
2. include a possible alternative route via Chicago (CH), so the network is an obtuse triangle NY-CH-LA where we take the side ratio to be 1:1:1.9, and
3. include a possible alternative route via San Francisco (SF), so the network is a highly acute triangle NY-LA-SF where we take the side ratio to be 1:0.1:1.

Clearly these are three different network situations that the market makers would take into account in setting forward prices. In general different network situations will lead to different forward prices with different observed volatilities in the market. These will be strongly affected by the extent of geographical arbitrage[3]. For the purposes of comparing option values in these three situations we will make the rather artificial assumption that the forward prices quoted in all three cases are the same and so are their observed volatilities. While this is artificial it does allow us to ask to what extent option values (but not forward prices) are influenced by network effects.

Common parameter and other values for our simulations are given in Table 1. We used a constant interest rate for discounting forward prices given in Figure 1 back to the present. The *calibrate* entries show the parameters we used in order to calibrate our models (isolated link / NY-CH-LA / NY-LA-SF) to actual market-observed data. The market observed data we used were forward prices for December 2001 ($F(0,T)$) and for February 2001 (which we used as $S(0)$). We have no observations of forward volatility for December 2001 so we just took 50% as a possible value to use. This may seem a little high compared with, say, electricity forwards, but recall from Figure 1 that forward prices are expected to halve on this timescale. Electricity lacks a sensitivity to technological change, which is a significant feature of bandwidth-price development.

4.2 Results

We obtained risk-neutral probabilities (our measure $\mathcal{Q}$) indirectly by calibrating Equations (6) and (7) according to Equation (1) (the no-arbitrage condition) and to the given forward volatility. We basically followed the steps given in [Nef00] (Chapter 14, section 6). Given

[2]Calculation of distances and justification of relative prices given in [KC01] but is not vital to this development

[3]Another major factor will be market liquidity in exploiting geographical arbitrage opportunities, but we ignore that for now.

parameter	symbol	value
number of simulations per parameter combination		1024
simulation length	t_{max}	10 months (=210 trading days)
simulation granularity	Δt	1 day
interest rate (constant)	r	5%
forward volatility (for December)		50%(annualized)
price trend (time to halve)	$-\nu$	*calibrate*
trend uncertainty	ρ	20% per year
short term price volatility	σ	*calibrate*
price reversion to trend (time to halve)	η	3 months
price jumps	G	none
price spikes	H	none
liquidity	α	1

Table 1: Common parameters for forward option pricing in various network topologies. Liquidity refers to how long arbitrage opportunities last before market forces (arbitrageurs, etc) remove them. We consider a highly liquid market. Parameters when referring to equations reference section 3.1 . Note that common random numbers were used across different parameter value combinations for variance reduction.

that we have four equations and two constraints, the risk-neutral measure is not unique. However, we felt that using the short-term volatility and the long-term trend as the calibration parameters was the least ambiguous choice. Note that whilst this fixes the first two moments of our distribution the higher moments are free and so is the shape of the distribution.

Table 2 gives the calibrated parameters as well as some basic shape parameters (skew and kurtosis). The parameters for the networks (NY-CH-LA and NY-LA-SF) are different from those for the isolated link and so are the distribution shape parameters we calculated. Figure 3 shows the actual distributions in the three cases. Although shape parameters may be different, the actual distributions appear very close to each other.

Using our risk neutral probability distribution $\mathbb{Q}$ (which can be seen explicitly in Figure

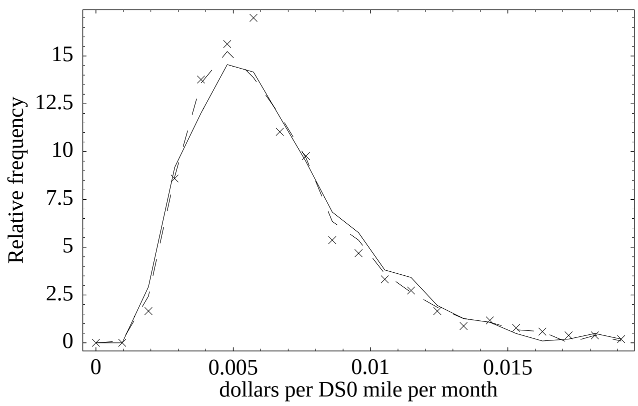

Figure 3: Risk-neutral probability densities. Distribution of $F(T, T) \equiv S(T)$ without a network (isolated link, *crosses*) and with different topologies: *dashes* NY-LA-SF; *line* NY-CH-LA.

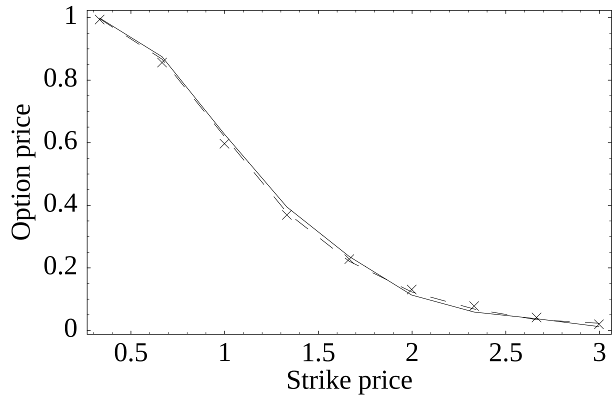

Figure 4: Call option on forward price with and without network, strike and option prices are relative to $F(0, T)$

3, *left panel*) we could then calculate call option values on December forward contracts using Equation (3). Figure 4 shows forward call option values discounted back to the start (0=February) in units of $F(0, T)$ with respect to strike price in the same units. As before, the curves of forward call option values for the three different cases appear very close to each other. Figure 5 shows the percentage difference of the call option prices in the two network cases from the isolated-link case. Up to a strike price of twice the forward price the difference in option values is less than 15%. Above this strike price it appears that there is numerical instability. Looking at Figure 4 confirms this impression as the option values are becoming very low at these high strike prices so we are encountering a division by small-number instability.

4.3 Discussion

The option prices on 10-month forward contracts are very similar to each other for the networks considered (including no network). It appears that for this case inclusion of geographical arbitrage and considering the entire network are not necessary — it suffices to consider an isolated link provided high accuracy (less than, say 5%) is not required and p strike prices are not extreme (less than twice the forward price). This is true even though the presence of geographical arbitrage requires quite different calibration parameters for the spot process. It appears, in this case, that fixing the first two moments of the forward distribution is sufficient for network-independent option pricing. This is very unexpected because geographical no-arbitrage is a highly non-linear condition (it resembles a $a = \min(a, b + c)$ operator). The reason may be that the forward is so far into the future that the accumulation of the no-geographical-arbitrage condition falls under the central limit theorem and results in a log-Normal-like distribution. If this is the case then the small differences seen in forward option prices at ten months may be much greater for shorter-dated options.

We would strongly caution the reader against generalization of the single example presented here to arbitrary networks and forward contingent claims.

network	$-\nu$ (years)	σ (%)	skew	kurt
none	0.345	115	1.5	6.2
NY-CH-LA	0.380	172	1.9	6.1
NY-LA-SF	0.364	172	1.4	13.1

Table 2: Calibrated parameters together with resulting shape parameters for resulting distributions of $F(T,T)$. (kurt stands for Kurtosis)

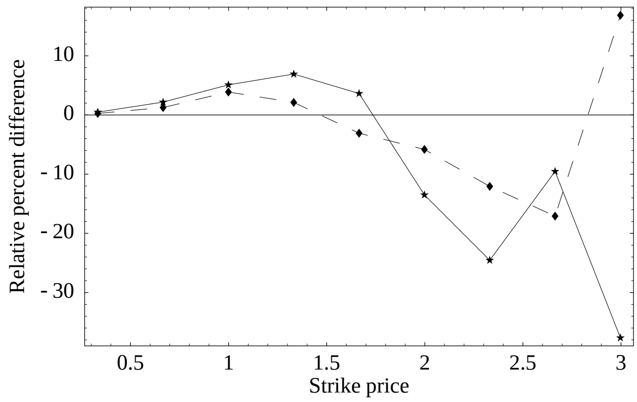

Figure 5: Call option on forward: network case valuation, percentage difference relative to non-network valuation: network topologies *dashes* NY-LA-SF; *line* NY-CH-LA. Strike prices are relative to $F(0,T)$

5 Conclusions and Extensions

We have shown how to price European-style contingent claims on futures contracts of the same maturity, and given a worked example for a call option on a forward. This involved calibrating the spot price process, which represents a non-storable commodity to the market-observed futures price, a martingale under the risk neutral measure. The spot price process explicitly included network effects, notably those from geographical arbitrage, and the markets' response to such conditions through load balancing.

For a European call option on a 10 month forward, we found that the option prices were independent of the networks considered although the presence of geographical no-arbitrage required quite different calibration parameters for different network topologies. Small differences were present (less than 15%), and we speculate that these differences may increase for shorter-dated contingent claims. Equally this result does not necessarily imply anything about non-European-style claims — these generally are affected by differences in price trajectories or by their envelope.

Forward price curve models are generally used to price and hedge derivative products and not to forecast future price curves although forward price curve models must have the flexibility to incorporate observed and expected dynamic features. Testing on historical data is also not a priority, and stability of calibrated parameters is a lesser objective [JW00]. On the other hand accuracy of fit and ease of fit to prices of current liquid market instruments are vital. Generally models are calibrated to a set of instruments and then either used to price similar (interpolation) or different instruments (extrapolation). Given that we are only using forward prices for our calibration and then pricing forward derivatives our work falls into the extrapolation category. If some forward option prices were available that we could include in our calibration before pricing different forward options we would simply be interpolating.

As we chose to focus on European-style contingent claims we did not need to model forward curve development explicitly because we could rely on $F(T,T) \equiv S(T)$ at maturity. To model more general instruments such as swing options, forward curve modeling would be required.

The most direct extension of this work is to pricing arbitrary contingent claims. This requires explicit modeling of forward price curve dynamics (i.e. $dF(t,T)$ w.r.t. t). We leave such for future work and note that this modeling is non-trivial because $F_{ab}(t,T)$ for each link (a,b) must be a martingale and the forward contract graph as a whole must also have no geographical arbitrage. It is not sufficient to simply take a forward curve model without modification from the literature and apply it. These models would have $F_{ab}(t,T)$ as martingales but then corrections for geographical arbitrage would be required which would remove the martingale property. This property would then have to be restored. We leave this for future work.

In conclusion we have shown how to price European-style contingent claims on forward contracts of the same maturity using no-arbitrage considerations, and provided a worked example. Unlike previous work (e.g. [CCN00]) we used forward prices as an input and showed why this was required by the nature of bandwidth: it is not storable. Network effects were included via the spot price process on which no-geographical arbitrage was imposed over the network at any time point and we used $F(T,T) \equiv S(T)$ to make the link between spot and forward prices. Initial results showed that geographical arbitrage was not important in the particular forward call option example considered. Further work is needed to determine whether this result generalizes. The derivative products that can be priced using this method include swaption type contracts as well as European style contingent claims.

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