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Research Report

On the Information-Theoretic Capacity of Magnetic Recording Systems in the Presence of Media Noise

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The compound behavior of the magnetic recording channel is modelled by combining the Lorentzian read-back pulse, the microtrack channel model, and additive white Gaussian noise (AWGN). By noting that at the output of this model the read-back signal is cyclostationary, the average autocorrelation function and corresponding power spectral density over one period are computed. The average power spectral density is then used to characterize the capacity of the magnetic recording channel for various linear density and media noise scenarios by using the conjectured Shamai-Laroia lower bound. It is shown that from a capacity point of view media noise in certain cases is beneficial compared to AWGN.

I. Introduction

THERE are two main difficulties associated with determining the effects of media noise on the information-theoretic capacity of magnetic recording systems. First, the capacity of the magnetic recording channel is unknown even in the absence of media noise. Second, a simple channel model that combines the effects of media noise, electronics noise, and intersymbol interference at high linear densities is difficult to derive.

French and Wolf computed upper and conjectured lower bounds on the capacity for the magnetic recording channel for various noise scenarios (including media noise) by assuming Gaussian inputs and physically motivated channel models [1]. However, the Gaussian assumption fails, in particular at high rates, where our main interest resides. Moreover, the channel models used are not easily describable and are difficult to use for signal processing such as coding.

The purpose of this paper is to present a simple information-theoretic method to study the effects of media noise. It is based on the well-known microtrack channel model [2] and the conjectured Shamai-Laroia lower bound (SLLB) on the capacity of intersymbol interference channels [3].

The pivotal observation behind this approach is that at the output of the magnetic recording channel the read-back signal is cyclostationary. This allows us to compute the average power spectral density at the receiver input. Based on this average power spectrum, achievable information rates are computed by means of the SLLB. For a fixed noise power at the receiver input, it is shown that in certain cases media noise is beneficial from a capacity point of view.

II. Model for the Magnetic Recording Channel

The Lorentzian pulse models the frequency-dispersive nature of the read-back signal and depends on a single parameter, which is called pulse width at 50% amplitude or PW50. The ratio PW50/T, where T is the data rate, is a measure of the normalized linear density in a hard-disk system. A small PW50/T causes less dispersion and therefore less intersymbol interference.

Media noise is data-dependent and due to the random microstructure of the grains in thin-film recording media. The microtrack channel imitates the random zig-zag transition effects. It is specified by three parameters: the number of microtracks N, the transition width parameter a, and the threshold L below which two transitions erase each other [2]. The jitter process is modelled as a process that is independent and identically distributed (i.i.d.) according to the derivative of the average cross-track magnetization profile. If we assume an error-function-shape average cross-track magnetization profile, the jitter exhibits a Gaussian distribution with variance $\sigma_J^2 = \frac{\pi}{2} \cdot a^2$ as in [2]. The output $v(t)$ of our model is given by

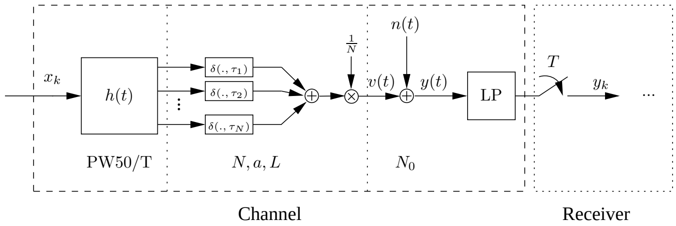

Fig. 1. Five parameter model for the magnetic recording channel.

$$v(t) = \frac{1}{N} \sum_{k=-\infty}^{+\infty} x_k \sum_{i=1}^{N} h(t - kT - j_{i,k}), \tag{1}$$

with $x_k = (u_k - u_{k-1})/\sqrt{2}$. The u_k are generated by a discrete memoryless source and take values from $\{-1, +1\}$ with equal probability (0.5-Bernoulli process). Hence, $x_k \in \{-\sqrt{2}, 0, +\sqrt{2}\}$ and are correlated. Furthermore, $h(t)$ is the Lorentzian pulse, i.e., $h(t) = c \cdot 1/(1 + (2t/\mathrm{PW50})^2)$ with c such that the norm of $h(t)$ equals 1 for PW50 = 1, and $j_{i,k}$ is the jitter of the i-th microtrack at the k-th time step.

The noiseless output $v(t)$ is corrupted with AWGN, $n(t)$, representing electronics noise that is determined by its one-sided power spectral density N_0, and sent through a brick-wall-shaped lowpass filter.[1]

In summary, the behavior of our magnetic recording model is characterized by the five parameters PW50/T, a, N, L, and N_0 (see Fig. 1).

III. Average Power Spectrum

The Lorentzian pulse is a deterministic $L^2(-\infty, +\infty)$ function. Jitter and data process are jointly stationary. Hence $v(t)$ fulfils the conditions for cyclostationary processes [4]. This allows us to compute the average power spectrum of $v(t)$. It can be shown that the average power spectrum of $v(t)$ is given by

$$\begin{aligned}\bar{\Phi}_V(f) \quad &= \quad \underbrace{\left(\frac{N-\varepsilon}{N}\right)^2 \frac{|H(f)|^2}{T} \cdot \Phi_X(f) \cdot |P(f)|^2}_{\mathrm{S}(f)} \\ &\quad + \underbrace{\frac{N-\varepsilon}{N^2} \frac{|H(f)|^2}{T} r_X(0) \cdot \left[1 - |P(f)|^2\right]}_{\mathrm{M}(f)} \end{aligned} \tag{2}$$

where $H(f) = c\pi(\mathrm{PW50}/2)\exp(-|2\pi f|\mathrm{PW50}/2)$ is the spectrum of the Lorentzian, $\Phi_X(f)$ is the spectrum of the input sequence X, $P(f)$ is the Fourier transform of the jitter probability, $r_X(0)$ is the average symbol energy of X, N is the number of microtracks, and ε is the average number of erased microtracks determined by T and L as in [2].

The average power spectrum consists of two terms: a signal term called average signal power spectrum (S(f))

[1] We assume that the suboptimum receive filter (lowpass filter) is part of the channel.

and a signal-dependent noise term called average media noise power spectrum ($\mathrm{M}(f)$). The first models the pulse widening of the input signal, the second reflects the noise caused by the position uncertainty of the transitions in the output signal. To obtain more insight into this formula, we will now consider special cases and relate them to results known from the literature (for ease of interpretation we set $\varepsilon = 0$):

For $N = 1$, Eq. (2) becomes

$$\bar{\Phi}_V(f) = \frac{|H(f)|^2}{T}\left[\Phi_X(f)|P(f)|^2 + r_X(0)\left[1 - |P(f)|^2\right]\right], \quad (3)$$

which is identical to the result in [5]. If we assume a small jitter, the first-order Taylor expansion of the second term yields $r_X(0)(2\pi f\sigma_\mathrm{J})^2$. To obtain an estimate of the shape of the average power spectrum at the output of the lowpass filter, we multiply $\bar{\Phi}_V(f)$ with $1/(2\pi f)^2$, and see that the second term becomes constant. It is therefore present in the entire spectrum, shaped like the first term (signal term), and models the position uncertainty about the transitions. This media noise term can be reduced by increasing the number of microtracks, i.e., decreasing the granularity of the medium.

For $P(f) = 1$, the second term in Eq. (2) vanishes completely. What remains is

$$\bar{\Phi}_V(f) = \frac{|H(f)|^2}{T} \cdot \Phi_X(f). \quad (4)$$

This is the spectrum of an ideal write head causing an infinitely sharp transition.

For $N = \infty$, the granularity of the medium is zero, and we have

$$\bar{\Phi}_V(f) = \frac{|H(f)|^2}{T} \cdot \Phi_X(f) \cdot |P(f)|^2. \quad (5)$$

The medium is ideal and causes no media noise. The pulse widening is due to the non-ideal write head only. The underlying microtrack model allows the influence of write head and media noise to be separated.

To investigate various noise blends of AWGN and media noise for a fixed noise power at the lowpass filter output, we define the media noise power MNP as follows:

$$\mathrm{MNP} \triangleq \int_{-B}^{+B} \mathrm{M}(f)df, \quad (6)$$

where B, the bandwidth of the brick-wall-shaped lowpass filter at the channel output, is chosen sufficiently large, e.g., $B = 10/T$. The media noise factor (MF) indicates the amount of media noise and is given by

$$\mathrm{MF} = \frac{\mathrm{MNP}}{\frac{N_0}{2T} + \mathrm{MNP}}. \quad (7)$$

MF = 0.1 means 10% media noise and 90% AWGN. The amount of media noise is controlled by adjusting N.

IV. Uniform-Input Information Rates (UIIRs)

We are interested in information rates when the input is assumed to be a 0.5-Bernoulli process. These information rates are termed uniform-input information rates (UIIRs) and indicate the achievable rate for random linear codes. In [3], a conjectured lower bound on the capacity of discrete-time binary-input channels with memory was presented. This bound measures for i.i.d. power-limited Gaussian input the loss in SNR due to memory in the channel. This loss is then translated to the same channel with i.i.d. binary input. In [6], it was shown that the SLLB coincides with results for UIIRs obtained by exact computation. This strongly suggests that the SLLB delivers UIIRs, and hence indeed is a lower bound on the capacity. In particular, it has been observed that at high rates the SLLB is tight.

In what follows, we set $T = 1$, $r_X(0) = 1$. As SNR, E_s/N_0 in dB is chosen, where $E_s = 1$ is the energy of the input data signal.

Fig. 2 shows UIIRs for Lorentzian channels with various normalized linear densities as well as for the BPSK AWGN channel without memory. The jitter variance is kept small, i.e. $a = 0.1\ T$. The number of microtracks is infinite, i.e., there is no media noise. This scenario allows us to study the loss in SNR due to increased PW50/T.

In Fig. 3, the UIIRs are computed for PW50/$T = 3.2$ and varying jitter variance σ_J^2. Again there is no media noise, and one can observe the loss in SNR due to wider

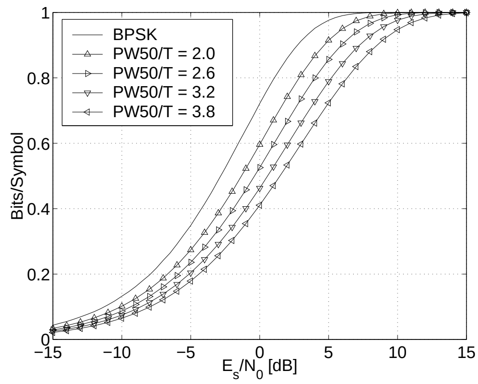

Fig. 2. UIIRs for various PW50/T.

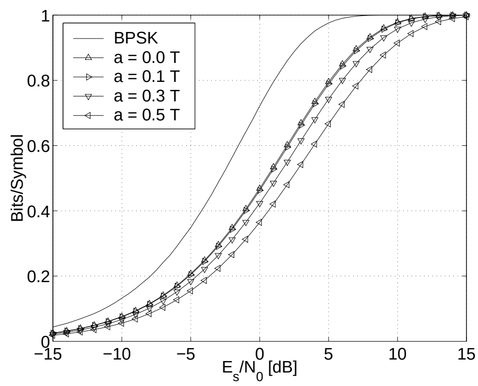

Fig. 3. UIIRs for PW50/$T = 3.2$ and various jitter variances.

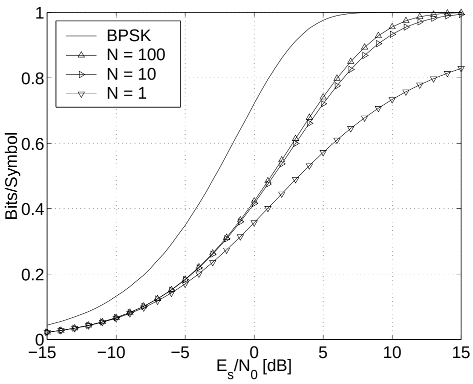

Fig. 4. UIIR for PW50/$T = 3.2$, a=0.3 T, and different N.

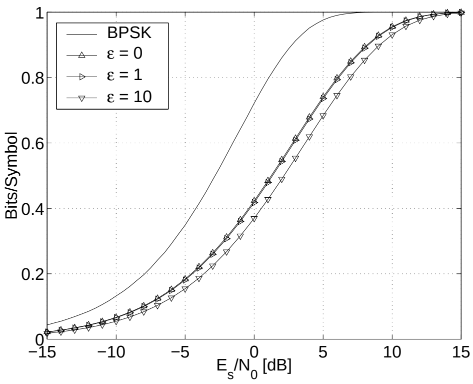

Fig. 5. UIIR for PW50/$T = 3.2$, a=0.3 T, $N = 100$, and $\varepsilon = 0, 1, 10$.

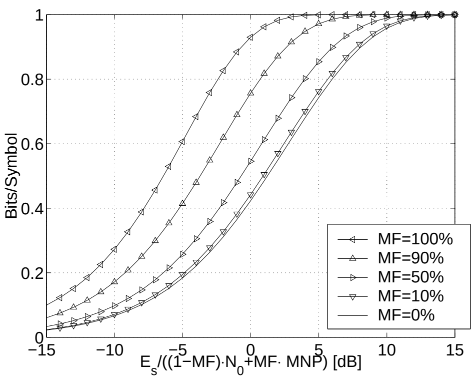

Fig. 6. UIIRs for PW50/$T = 3.2$, a=0.3 T, and varying MF.

transitions. For $a = 0.5\ T$, the transitions are much wider and the alternating output pulses overlap more. This overlapping results in a reduced amplitude and hence in an energy loss in the readback signal.

Fig. 4 shows the limiting effect of media noise for PW50/$T = 3.2$ and a jitter variance determined by $a = 0.3\ T$. E_s/N_0 was chosen as SNR that consists only of electronics noise. The number of microtracks is 100, 10, and 1. The influence of the erasure probability is shown in Fig. 5.

In the final figure (Fig. 6), UIIRs are shown for a fixed noise power at the receiver input and a given noise blend MF. One concludes that media noise from a capacity point of view might be beneficial (at least with a lowpass filter as receive filter). This can be explained by the fact that for a given noise power it is more desirable to have a noise spectral power density that is shaped like the channel than to have white noise (Jensen inequality).

V. Conclusions

The compound magnetic recording channel was modelled as a Lorentzian channel, a microtrack model, and an AWGN channel (including a lowpass filter as receive filter). The behavior of this channel is determined by the five parameters PW50/T, N, a, L, and N_0. By noting that the output of this model is a cyclostationary process, the average power spectrum was computed. With help of this average power spectrum, the influence of the five parameters on the capacity was studied by means of the SLLB. Finally, we found that from a capacity point of view media noise is preferable to AWGN in certain cases.

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